p.2
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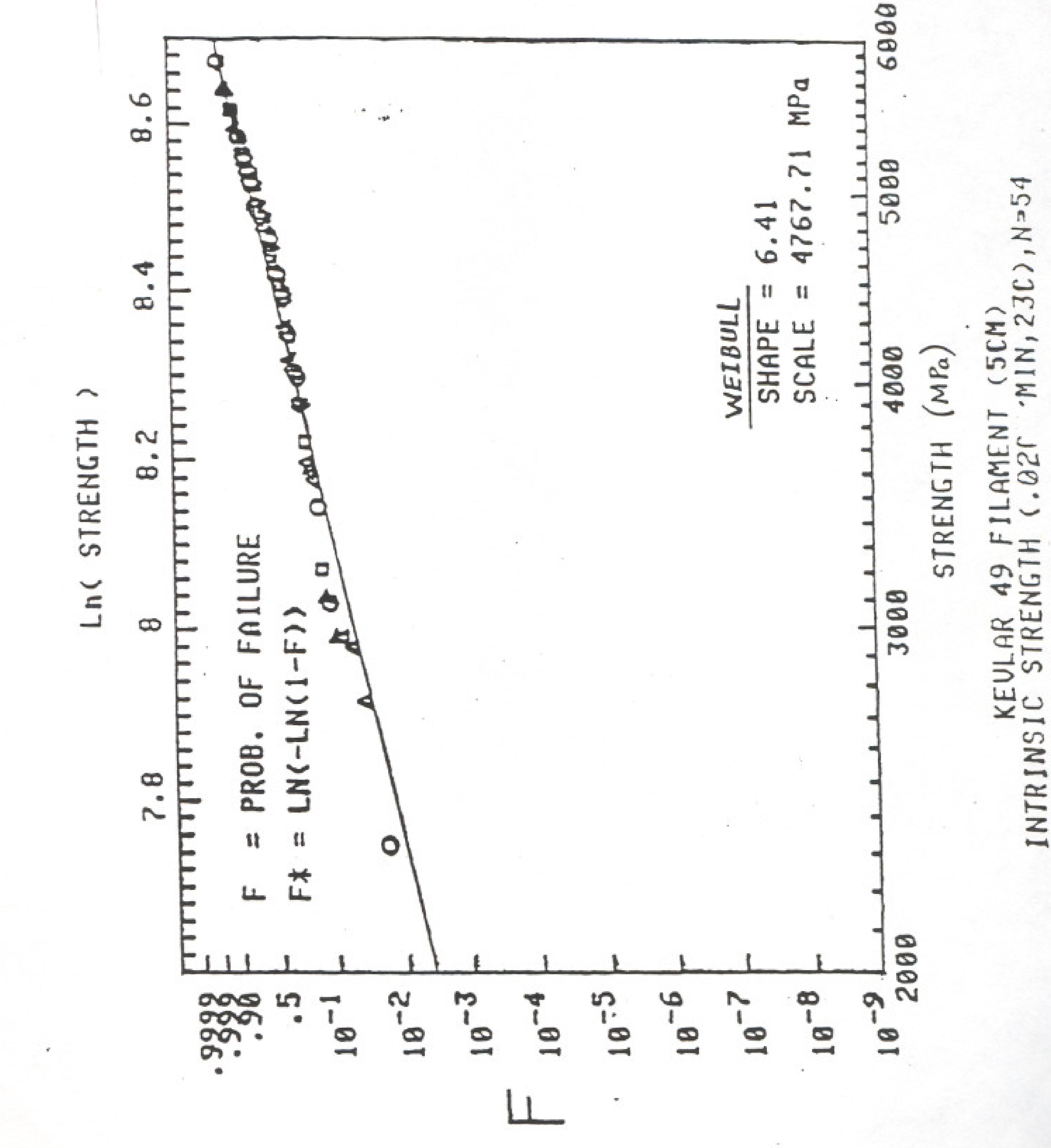
DRY FILAMENT STRENGTH



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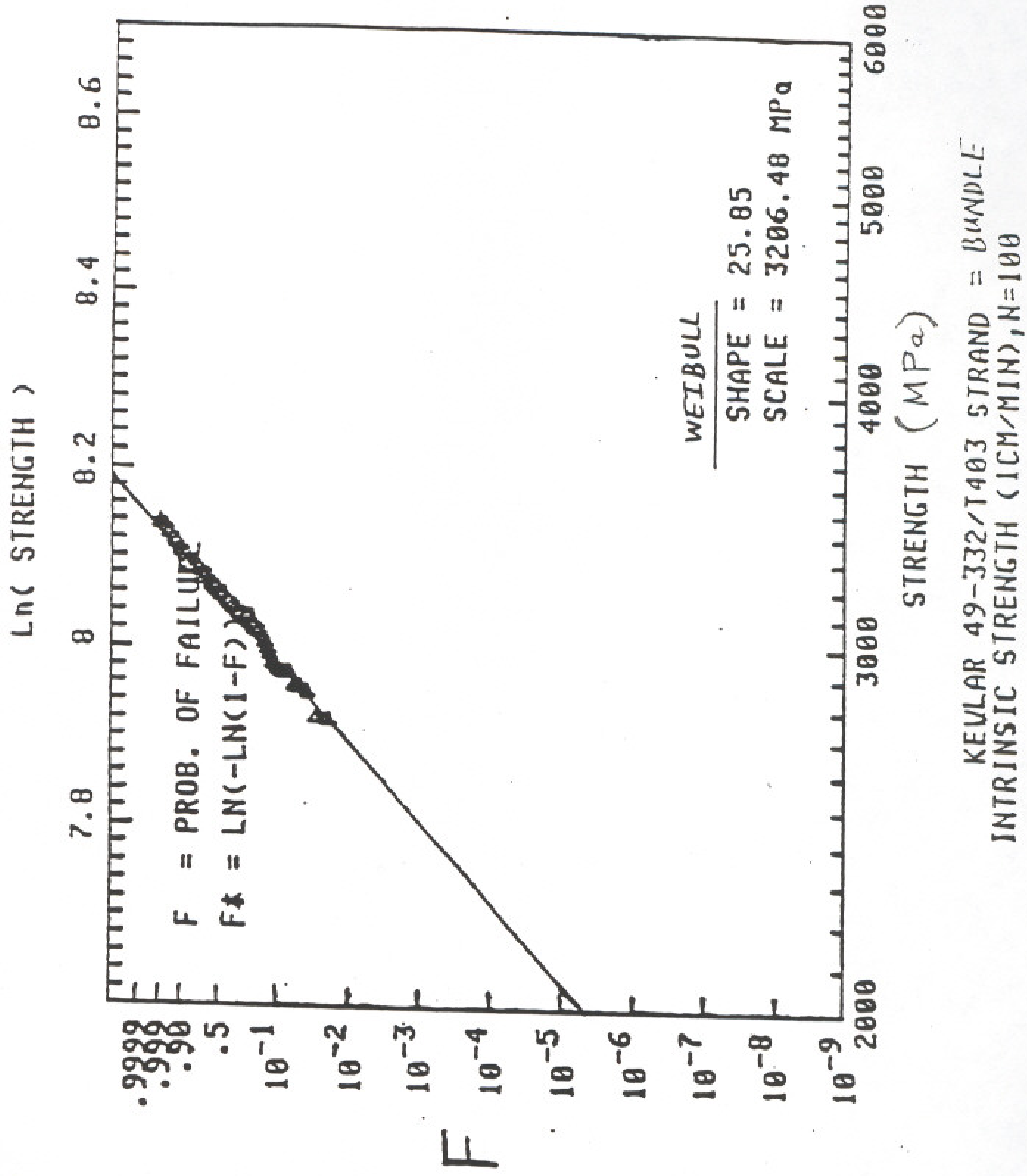
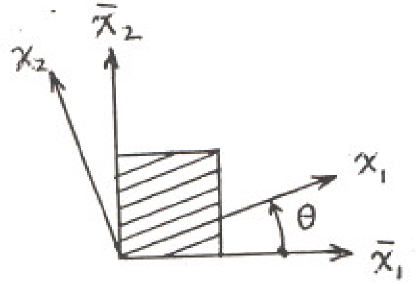
IMPREGNATED BUNDLE STRENGTH



General Laminate Transformations



$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \underbrace{\begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & (c^2 - s^2) \end{bmatrix}}_{T_\sigma} \begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \underbrace{\begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & (c^2 - s^2) \end{bmatrix}}_{T_\varepsilon} \begin{Bmatrix} \bar{\varepsilon}_1 \\ \bar{\varepsilon}_2 \\ \bar{\varepsilon}_6 \end{Bmatrix}$$

$T_\sigma^{-1} = T_\varepsilon^T \qquad T_\varepsilon^{-1} = T_\sigma^T$

$\sigma = Q \ \varepsilon \qquad \bar{\sigma} = \bar{Q} \ \bar{\varepsilon} \qquad \bar{Q} = T_\varepsilon^T \, Q \ T_\varepsilon$

$$\bar{Q}_{11} = c^4 Q_{11} + s^4 Q_{22} + 2 c^2 s^2 (Q_{12} + 2 Q_{66}) = I_1 + I_2 + R_1 \cos 2\theta + R_2 \cos 4\theta$$

$$\bar{Q}_{12} = c^2 s^2 (Q_{11} + Q_{22} - 4 Q_{66}) + (c^4 + s^4) Q_{12} = I_1 - I_2 - R_2 \cos 4\theta$$

$$\bar{Q}_{16} = c^3 s \, Q_{11} - c s^3 Q_{22} - cs(c^2 - s^2)(Q_{12} + 2 Q_{66}) = \tfrac{1}{2} R_1 \sin 2\theta + R_2 \sin 4\theta$$

$$\bar{Q}_{22} = s^4 Q_{11} + c^4 Q_{22} + 2 c^2 s^2 (Q_{12} + 2 Q_{66}) = I_1 + I_2 - R_1 \cos 2\theta + R_2 \cos 4\theta$$

$$\bar{Q}_{26} = c s^3 Q_{11} - c^3 s \, Q_{22} + cs(c^2 - s^2)(Q_{12} + 2 Q_{66}) = \tfrac{1}{2} R_1 \sin 2\theta - R_2 \sin 4\theta$$

$$\bar{Q}_{66} = c^2 s^2 (Q_{11} + Q_{22} - 2 Q_{12}) + (c^2 - s^2)^2 Q_{66} = I_2 - R_2 \cos 4\theta$$

$$I_1 = \tfrac{1}{4} (Q_{11} + Q_{22} + 2 Q_{12})$$

$$I_2 = \tfrac{1}{8} (Q_{11} + Q_{22} - 2 Q_{12} + 4 Q_{66})$$

$$R_1 = \tfrac{1}{2} (Q_{11} - Q_{22})$$

$$R_2 = \tfrac{1}{8} (Q_{11} + Q_{22} - 2 Q_{12} - 4 Q_{66})$$

Anisotropic Plate Equations

$$A_{11}u_{,xx} + 2A_{16}u_{,xy} + A_{66}u_{,yy} + A_{16}v_{,xx} + (A_{12} + A_{66})v_{,xy} + A_{26}v_{,yy}$$
$$-B_{11}w_{,xxx} - 3B_{16}w_{,xxy} - (B_{12} + 2B_{66})w_{,xyy} - B_{26}w_{,yyy} = -p_x$$

$$A_{16}u_{,xx} + (A_{12} + A_{66})u_{,xy} + A_{26}u_{,yy} + A_{66}v_{,xx} + 2A_{26}v_{,xy} + A_{22}v_{,yy}$$
$$-B_{16}w_{,xxx} - (B_{12} + 2B_{66})w_{,xxy} - 3B_{26}w_{,xyy} - B_{22}w_{,yyy} = -p_y$$

$$D_{11}w_{,xxxx} + 4D_{16}w_{,xxxy} + 2(D_{12} + 2D_{66})w_{,xxyy} + 4D_{26}w_{,xyyy}$$
$$+ D_{22}w_{,yyyy} - B_{11}u_{,xxx} - 3B_{16}u_{,xxy} - (B_{12} + 2B_{66})u_{,xyy} - B_{26}u_{,yyy}$$
$$-B_{16}v_{,xxx} - (B_{12} + 2B_{66})v_{,xxy} - 3B_{26}v_{,xyy} - B_{22}v_{,yyy} = +p_z$$

Also can express as,

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} -p_x \\ -p_y \\ +p_z \end{Bmatrix}$$

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + 2A_{16}\frac{\partial^2}{\partial x \partial y} + A_{66}\frac{\partial^2}{\partial y^2}$$

$$L_{12} = A_{16}\frac{\partial^2}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2}{\partial x \partial y} + A_{26}\frac{\partial^2}{\partial y^2}$$

$$L_{13} = -B_{11}\frac{\partial^3}{\partial x^3} - 3B_{16}\frac{\partial^3}{\partial x^2 \partial y} - (B_{12} + 2B_{66})\frac{\partial^3}{\partial x \partial y^2} - B_{26}\frac{\partial^3}{\partial y^3}$$

$$L_{22} = A_{66}\frac{\partial^2}{\partial x^2} + 2A_{26}\frac{\partial^2}{\partial x \partial y} + A_{22}\frac{\partial^2}{\partial y^2}$$

$$L_{23} = -B_{16}\frac{\partial^3}{\partial x^3} - (B_{12} + 2B_{66})\frac{\partial^3}{\partial x^2 \partial y} - 3B_{26}\frac{\partial^3}{\partial x \partial y^2} - B_{22}\frac{\partial^3}{\partial y^3}$$

$$L_{33} = D_{11}\frac{\partial^4}{\partial x^4} + 4D_{16}\frac{\partial^4}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial x^2 \partial y^2}$$
$$+ 4D_{26}\frac{\partial^4}{\partial x \partial y^3} + D_{22}\frac{\partial^4}{\partial y^4}$$

ation for moist[illegible] diffusion can be derived similarly from of mass as

$$\frac{\partial}{\partial z}\left(K_z^H \frac{\partial H}{\partial z}\right) = \frac{\partial H}{\partial t} \tag{8.24}$$

K_z^H is constant, Equation 8.24 can be reduced to Fick's

$$K_z^H \frac{\partial^2 H}{\partial z^2} = \frac{\partial H}{\partial t} \tag{8.25}$$

rison of Equations 8.23 and 8.25 reveals that these equa- ntical in form to each other, indicating the similarity of the processes. The thermal diffusivity $K_z^T/(\rho C)$ and the moisture efficient K_z^H are a measure of the rate at which the temper- he moisture concentration respectively change within the general, these parameters depend on the temperature and ncentration. However, over the range of temperature and ncentration that prevails in typical applications of com- thermal diffusivity is about 10^6 times greater than the fusion coefficient. Thus, the thermal diffusion takes place ster than the moisture diffusion. As a result, the tempera- ach equilibrium long before the moisture concentration bservation allows one to solve Equation 8.25 separately n 8.23.

dy of the hygrothermal behavior of composites the specific centration defined by

$$c = H/\rho \tag{8.26}$$

used in lieu of H. Physically, c represents the amount of fraction of the dry mass of composite, i.e.

$$= \lim_{\Delta V \to 0} \frac{\text{mass of moisture in } \Delta V}{\text{mass of dry composite of volume } \Delta V} \tag{8.27}$$

In [illegible]ns of c Equation 8.25 becomes

$$K^H \frac{\partial^2 c}{\partial z^2} = \frac{\partial c}{\partial t} \tag{8.28}$$

where the subscript has been dropped off K_z^H for convenience. The appropriate boundary conditions are

$$\begin{aligned} c &= c_o \text{ for } 0 < z < h \text{ at } t \leqslant 0 \\ c &= c_\infty \text{ for } z = 0 \text{ and } h \text{ at } t > 0 \end{aligned} \tag{8.29}$$

Here h is the thickness of the laminate in Figure 8.2. The solution to Equations 8.28 and 8.29 is given by [1]

$$\frac{c - c_o}{c_\infty - c_o} = 1 - \frac{4}{\pi} \sum_{j=0}^{\infty} \frac{1}{2j+1} \sin \frac{(2j+1)\pi z}{h} \exp\left[-\frac{(2j+1)^2 \pi^2 K^H t}{h^2}\right] \tag{8.30}$$

Equation 8.30 is shown graphically in Figure 8.3 where the non-dimensional time $K^H t/h^2$ has been used. Note that c eventually reaches c_∞ throughout the laminate. Therefore, c_∞ is also called the equilibrium (specific) moisture concentration.

In a moisture absorption test the final moisture concentration c_∞ is always greater than the initial one c_o. The converse is true in a moisture desorption test. However, Equation 8.30 is equally valid in either case.

In actual experiments the sample frequently is weighed to determine the moisture content which is the total mass of the absorbed moisture divided by the dry weight of the sample. The moisture content is in fact the same as the average specific moisture concentration $\bar{c}$ defined by

$$\bar{c} = \frac{1}{h} \int_o^h c\, dz \tag{8.31}$$

Substituting Equation 8.30 into Equation 8.31 and noting that

$$\begin{aligned} \bar{c} &= c_o \text{ at } t = 0 \\ \bar{c} &= c_\infty \text{ at } t = \infty \end{aligned} \tag{8.32}$$

obtain

$$\frac{\bar{c}-c_o}{c_\infty-c_o} = 1 - \frac{8}{\pi^2}\sum_{j=0}^{\infty}\frac{1}{(2j+1)^2}\exp\left[-\frac{(2j+1)^2\pi^2 K^H t}{h^2}\right] \tag{8.33}$$

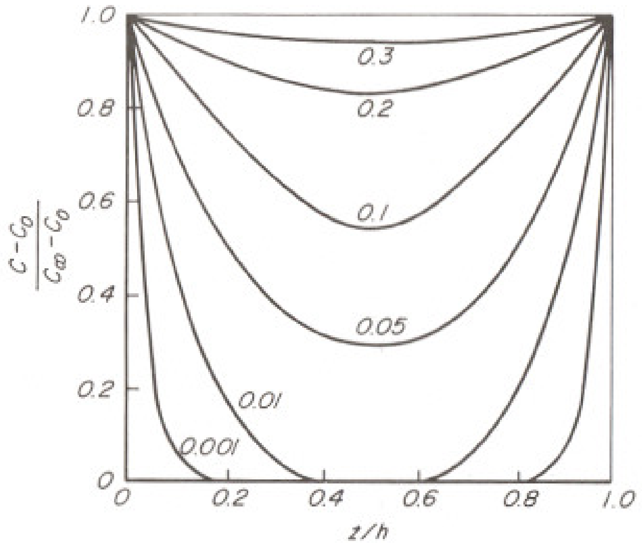


Figure 8.3 Moisture profile as a function of time. The numbers are the values of the nondimensional time $K^H t/h^2$.

quation 8.33 is compared with experimental data in Figure 8.4. [2] e the moisture diffusion is through the thickness, it does not end on the type of laminate. Also, Equation 8.33 is applicable rdless of the type of diffusion.

or t sufficiently large Equation 8.33 can be approximated by the term in the series,

$$\frac{\bar{c}-c_o}{c_\infty-c_o} = 1 - \frac{8}{\pi^2}\exp\left(-\frac{\pi^2 K^H t}{h^2}\right) \tag{8.34}$$

the other hand, for short times an approximation can be obtained an alternate solution [1] as

$$\frac{\bar{c}-c_o}{c_\infty-c_o} = 4\left(\frac{K^H t}{\pi h^2}\right)^{1/2} \tag{8.35}$$

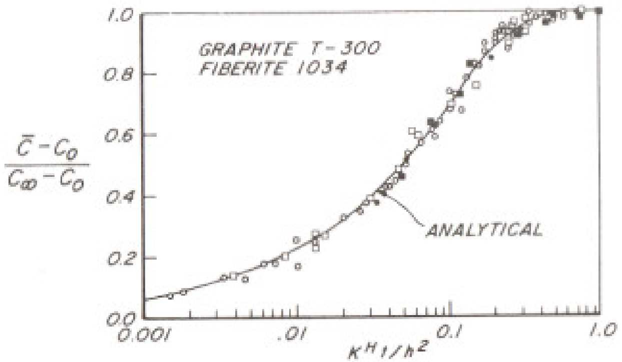


Figure 8.4 Experimental correlation of Equation 8.33 for graphite/epoxy laminates: unidirectional (○,●) and quasi-isotropic (□,■). [2] Open and filled symbols represent absorption and desorption, respectively.

Thus, the initial increase in moisture content is proportional to $(t/h^2)^{1/2}$.

Equations 8.34 and 8.35 are frequently used to determine K^H from the measurements of moisture concentration. From Equation 8.34, the time $t_{1/2}$ for which $(\bar{c}-c_o)/(c_\infty-c_o) = 1/2$, is given by

$$t_{1/2} = \frac{h^2}{\pi^2 K^H}\ln\frac{16}{\pi^2} \tag{8.36}$$

Therefore, the diffusion coefficient is determined from the half-time of sorption process as

$$K^H = \frac{0.04895\, h^2}{t_{1/2}} \tag{8.37}$$

The applicability of Equation 8.35 becomes apparent if the moisture content is plotted as a function of $\sqrt{t}$. A relationship between $\bar{c}$ and $\sqrt{t}$ is schematically shown in Figure 8.5. From the figure we choose, in the linear region, two moisture contents $\bar{c}_1$ and $\bar{c}_2$ corresponding to t_1 and t_2, respectively. Substituting these values into Equation 8.35 and solving the resulting equations for K^H, we can determine K^H as

$$K^H = \frac{\pi}{16}\left(\frac{\bar{c}_2-\bar{c}_1}{c_\infty-c_o}\right)^2\left(\frac{h}{\sqrt{t_2}-\sqrt{t_1}}\right)^2 \tag{8.38}$$

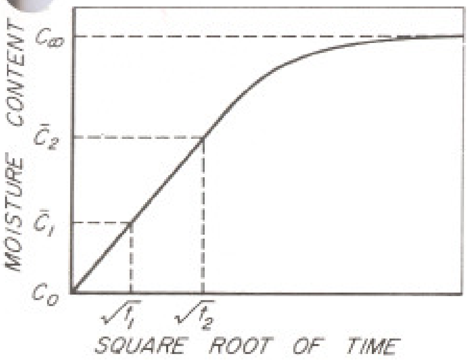


Figure 8.5 Determination of diffusion coefficient.

The equilibrium moisture concentration depends on the environ-ient. In humid air it is related to the relative humidity ϕ in percent by power law

$$c_\infty = a\left(\frac{\phi}{100}\right)^b \tag{8.39}$$

here a and b are material constants. A set of data bearing such rela-onship is shown in Figure 8.6.

The moisture diffusion coefficient strongly depends on temperature. he relationship can be described by an equation of the form

$$K^H = K_o^H \exp\left(-\frac{E_d}{RT}\right) \tag{8.40}$$

here K_o^H and E_d are the pre-exponential factor and activation energy, spectively, and R is the gas constant (=1.987 cal/(mole · K)). For the aphite/epoxy composite of Figure 8.6 a relationship between K^H and is shown in Figure 8.7. [3]

Typical hygrothermal properties of a graphite/epoxy composite are ımmarized in Table 8.2. From the table we can find, for example, that e equilibrium moisture content at 100% relative humidity is 1.8%. At om temperature (=23°C) the moisture diffusion coefficient is only 62×10^{-8} mm^2/s whereas the thermal diffusivity in the transverse rection is 0.45 mm^2/s. Thermal expansion coefficients of other com-sites are listed in Table 8.3.

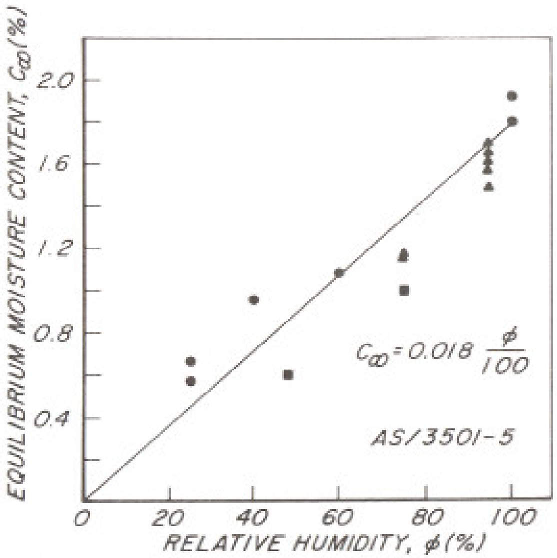


Figure 8.6 Equilibrium moisture content as a function of relative humidity for AS/3501-5. (● [3], ▲ [4], ■ [5]).

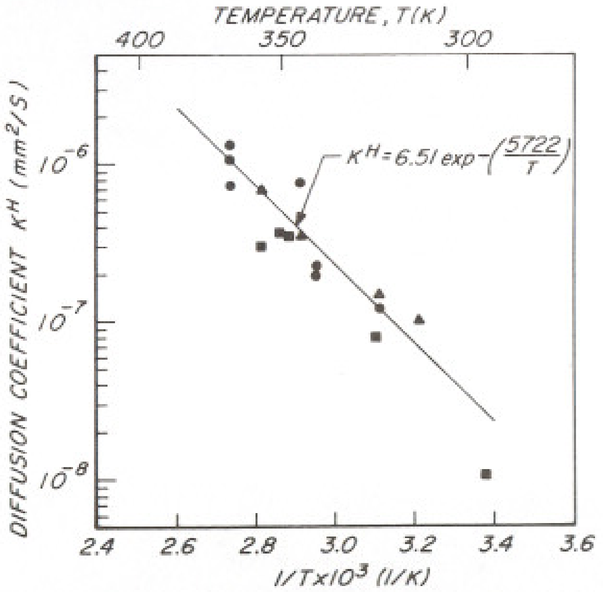


Figure 8.7 Transverse diffusion coefficient as a function of temperature for AS/3501-5. (● [3], ▲ [4], ■ [5]).

$$M_c = (M_{100})(M_c^*) \qquad (16.24)$$

where M_{100} is the maximum moisture content corresponding to 100 percent relative humidity (Equation 16.14)

$$M_{100} = a(100)^b \qquad (16.25)$$

M_c^* is the moisture content in the material at the change over time $t = t_c^*$. The change over time t_c^* is given by Figure 16.6. At time t_c^* the moisture concentration c_c^* as well as the moisture content M^* can be calculated [16–5]. The calculated values of M_c^* are summarized in Figure 16.8. The value of M_c^* at which the change over from $\phi = 100_s$ to $\phi = \phi_d$ is performed can readily be determined from Figure 16.8 and Equations 16.24 and16.25. The weight of the witness coupon also indicates the end of the conditioning. The conditioning is complete when the weight of the coupon becomes constant.

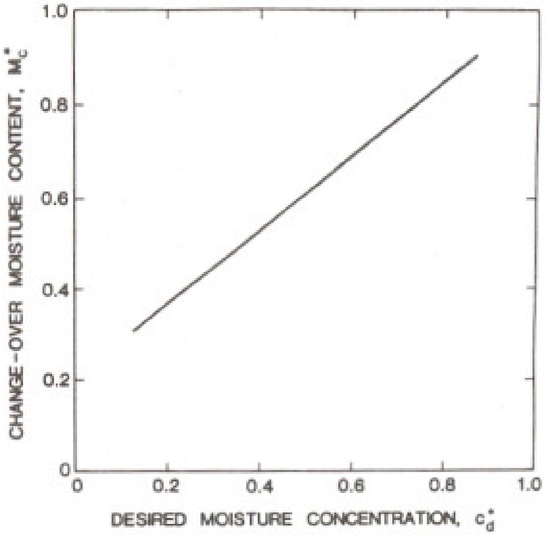


FIGURE 16.8 THE DIMENSIONLESS CHANGE–OVER MOISTURE CONTENT Mc* VERSUS THE DIMENSIONLESS MOISTURE CONCENTRATION Cd*.

Sample Problem

The following example illustrates the use of the method described above in choosing the conditions for accelerated moisture conditioning.

We consider a 0.25 inch thick plate made of Fiberite T300/1034 graphite epoxy composite. The material properties are specified in Tables 16.2 and 16.5. The plate is exposed to humid air in which both the temperature and relative humidity vary with time. Let us suppose that after a long period of time the weight gain of the plate becomes nearly constant, having the value of $M_d = 0.68$ percent. It is desired to establish the test conditions which result in the same moisture content (0.68 percent) as well as the same moisture distribution as the actual ambient. Solution to the problem proceeds as follows:

1) The dimensionless desired moisture concentration is calculated (see Equations 16.21 and 16.25)

$$M_d = c_d/c_{100} = M_d/M_{100} = 0.68/0.017/100 = 0.4 \qquad (16.26)$$

2) The relative humidity corresponding to M_d is calculated

$$\phi_d = (M)^{1/b}/a = 0.68/0.017 = 40 \text{ percent} \qquad (16.27)$$

3) The dimensionless change–over time is selected. From Figure 16.6 the value of t_c^* corresponding to c_d^* is

$$t_c^* = 0.05 \qquad (16.28)$$

4) The change–over time is calculated. The accelerated test is assumed to take place at 170°F. Then Equation 16.23 and the data in Table 16.5 give

$$t_c = t_c^* h^2/D = 0.05x(0.25)^2/1.56x10^{-6}$$

$$= 2000 \text{ hrs} = 84 \text{ days} \qquad (16.29)$$

5) The dimensionless conditioning time is determined. From Figure 16.7 (corresponding to the 99 percent level), we obtain

$$t_t^* = 0.104 \qquad (16.30)$$

6) The actual conditioning time is calculated

$$t_t = t_t^* h^2/D = 0.104x(0.25)^2/1.56x10^{-6}$$

$$= 4200 \text{ hrs} = 170 \text{ days} \qquad (16.31)$$

If a "witness coupon" were to be placed in the chamber, steps 3–6 would be as follows

3a) The dimensionless change over weight M_c^* is determined from Figure 16.8

$$M_c^* = 0.43 \qquad (16.32)$$

4a) The actual change over weight M_c calculated. Equations 16.24 and 16.25 give

$$M_c = (M_{100})(M_c^*) = 1.7x0.43 = 0.73 \text{ percent} \qquad (16.33)$$

5a)—6a) The final conditioning time is determined from the observed weight change of the witness coupon.

According to this example the plate is to be kept in humid air at 100 percent relative humidity for 84 days. The relative humidity is to be changed then to 40 percent. After 170 days the moisture concentration will be within 99