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"DESIGN ANALYSIS METHODOLOGY FOR COMPOSITE ROTOR BLADES"

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DESIGN ANALYSIS METHODOLOGY FOR COMPOSITE ROTOR BLADES

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INTRODUCTION

Composite material systems are now the primary materials for helicopter rotor system applications. Bearingless rotor designs proposed for the LHX helicopter are an example. In addition to reduced weight and increased fatigue life, these materials provide designs with fewer parts which means increased service life and improved maintainability. Also, in terms of manufacturing, it is possible to achieve more general aerodynamic shapes including flapwise variation in planform, section and thickness.

The aeroelastic environment in which rotor blades operate consists of inertial, aerodynamic and elastic loadings. Because of the directional nature of the composite materials, it is possible to construct rotor blades with different ply orientations and hybrid combinations of materials exhibiting coupling between various elastic modes of deformation. For example, plies with fiber orientations placed appropriately in the upper and lower portions of the blade can produce elastic coupling between twist and flapwise bending or between twist and extension. This provides a potential for improving the performance of a blade through elastic tailoring of the primary load-bearing structure.

A working definition of elastic tailoring is the use of structural concept, fiber orientation, ply stacking sequence and a blend of materials to achieve specific performance goals. In the design process, materials and dimensions are selected to yield specific elastic response characteristics which permit the goals to be achieved. Common choices for goals for the application of elastic tailoring are the creation of favorable deformations, often for the purpose of preventing or controlling aeroelastic phenomena or vibration, and damage tolerance.

Current design practice for composite rotor blades is to treat them similar to metal designs. The only distinguishing feature is that the effective extensional modulus is not related to the shear modulus. This approach, therefore, does not permit description of general composite layups and cannot be applied if unusual ply layups are introduced in order to create favorable elastic coupling for enhanced performance.

A composite rotor blade structural model and corresponding theory are presented herein which are created to accurately, but simply, characterize response. Simplicity is achieved by considering a primary structural box or single closed-cell spar, the primary load bearing element, as a thin walled beam made of an arbitrary composite layup. The full potential is included to account for the influences of elastic tailoring. In addition, two nonclassical influences - - - transverse shear deformation and torsion-related warping - -are included in the theory as these effects are far more pronounced for laminated composite materials than for monolithic metallic materials.

EARLIER WORK

Any acceptable theoretical model must account for the anisotropic character of composite materials. In addition, in order to be of use in design, it should be simple and reliable so that a clear physical picture of the cause-effect relationship between configuration and response is obtained. An early model of this type has been used by Weisshaar[1] for the study of aeroelastic tailoring of lifting surfaces. He uses an engineering beam theory that incorporates a plate-like behavioral model to represent the structure. The torsional stiffness, therefore, is underestimated as the enhanced stiffness of closed cell construction is not represented.

In recent work on vibration tailoring, Weisshaar and Foist[2] describe and compare three different stiffness models that have been used in tailoring studies. Two of them are plate-like and the third is a closed cell model. The latter is of the type developed by Mansfield and Sobey[3].

The pioneering work of Mansfield and Sobey[3] is of particular interest because it is intended for rotor blade applications. The model represents the load bearing structure or spar as a closed cell cylindrical tube with its thin wall constructed of composite plies. This is a Batho-Bredt type of thin wall structural theory that is commonly used for aeronautical structures. The authors have a very clear idea of the potential of elastic tailoring for rotor blades and discuss pitch-flap (torsion-bending) and pitch-stretch (torsion-extension) elastic coupling in some detail. The influence of actual aerodynamic forces is not considered so the discussion is conceptual.

The type of structural model utilized in Reference 3 is appropriate for preliminary design studies. However, the theoretical development is unusual, sometimes hard to follow as ad hoc assumptions are strategically introduced and inconsistent regarding some details. A clear, straightforward and consistent theory does not emerge. Two special cases are analyzed - - - (1) bending and twisting by constant moments and (2) bending and twisting by transverse shear forces on cantilever beams. The latter case is used to determine the shear center for the section. In the first case, the shear flow is taken to be constant around the section of the tube. This assumption, of course, does not apply to the second case and, although not specifically stated, it must be modified in that instance. In spite of these troublesome points and some apparent sign errors and omissions in the equations of the text, this is an extremely important work and serves as a foundation for the present study. It has been extended by Mansfield[4] to two-cell construction.

An important conclusion emerging from Reference 3 is that the effect of initial pretwist on longitudinal tension is small and can be discounted in preliminary design.

Other European researchers in References 5 and 6 present numerical models based upon finite element idealizations for composite rotor blades. Large scale simulation is utilized in place of insight. These methods are more appropriate for the analysis of configurations that have been designed by other, simpler methods. Such approaches will not be considered here.

Hong and Chopra[7] have conducted a pioneering study of the influence of ply layups on the aeroelastic stability of a composite rotor blade in hover. The structural heart of the blade is taken as a closed cell rectangular box. Stability is studied as the ply layups of the sides of the box are varied. The analysis is based upon the nonlinear kinematics of Hodges and Dowell[8]. Unfortunately, the configurations studied do not resemble practical configurations that can be easily manufactured by the usual means. The analytical results do illustrate, however, that ply orientation effects are extremely important and offer great promise for enhancing performance.

Very little detail on the actual analysis is provided by the authors in Reference 7 or its predecessor[9]. It is not clear what structural theory or approximations were actually made. It apears that a thin walled theory of the Mansfield and Sobey[3] type was not used from the Appendices in Reference 9. Consequently, a realistic appraisal of the results is difficult.

Recent theoretical research[10] has contributed a new appreciation for nonclassical effects in structural behavior. The nonclassical influences relevant to rotor blades are those due to transverse shear, bending-related warping, stretching-related warping and torsion-related warping. Laminated composites are in general strong and stiff in the plane of lamination and weak and flexible in the transverse direction. Consequently, transverse shear deformation becomes much more pronounced. Bending-related section warping also affects response in a similar way, but it is due to the fact that bending strain does not strictly correspond to planar deformation. Torsion-related warping arises whenever a section is restrained against out of plane deformation. The key to improving the stress predictive capability of a theory is to account for these effects correctly. A theory of the thin walled closed cell type has been developed by Valisetty and Rehfield[11] which is based upon a proven methodology for accounting for nonclassical effects in bending theories.

Results presented in Reference 11 indicate the great promise of tailoring to alter behavior and that the nonclassical effects are probably small for main rotor blades. A modern glass-epoxy material system is assumed along with typical blade dimensions based upon those of the CH-47 main rotor blade.

STRUCTURAL MODEL AND KINEMATICS

Model, Coordinates and Overview

The variety of the types of construction, materials and structural concepts that have been employed for composite rotor blades prevent the development of a single, all-encompassing theory. Instead, a general approach will be adopted which can be tailored to the unique features of any particular concept, manufacturing methodology and choice of materials. Attention is restricted to the most commonly used type of blade, one which utilizes a single closed cell thin walled spar as the primary load bearing and stiffness producing element. An example appears in Figure 1, the main rotor blade of the CH-47.

The spar is modeled as a closed cell thin walled beam of the classical aeronautical type. The wall construction is of general laminated composite construction which allows the flexibility for elastic tailoring. The model and coordinate system are shown in Figure 2. The coordinate directions x, y and z have displacement components u, v and w associated with them. The circumferential coordinate is imbedded in the middle surface of the wall.

The objective is to create a theoretical model suitable for representing composite rotor blade designs. The level of detail or definition is envisaged as appropriate for overall stress analysis and sizing in preliminary design, dynamic stability analysis and elastic tailoring. The model is beam-like with response determined as a function of the axis coordinate x.

The essential structural features of the model can be established on the basis of static considerations and small displacement response. The scope of this paper is confined to these limitations. The fundamentals consist of the definition of generalized displacements, corresponding generalized internal forces and the force-deformation equations which relate them.

Geometrical Matters and Transverse Displacement Components

Let $\bar{i}_x$, $\bar{i}_y$ and $\bar{i}_z$ be unit vectors in the coordinate directions. From any point on the beam reference axis, the x-axis, the centerline of closed curve defining the beam cross section is determined by the position vector $\bar{r}$ from the axis. It is written as

$$\bar{r} = \bar{i}_y y(s) + \bar{i}_z z(s) \tag{1}$$

The unit tangent vector, $\bar{t}$, to the cross section closed curve (CSCC) is defined as

$$\bar{t} = \frac{d\bar{r}}{ds} = \bar{i}_y \frac{dy}{ds} + \bar{i}_z \frac{dz}{ds} \tag{2}$$

A unit normal vector, $\bar{n}$, radially directed toward the center of the cross section is constructed from $\bar{i}_x$ and $\bar{t}$.

$$\bar{n} = \bar{i}_x \times \bar{t} = -\bar{i}_y \frac{dz}{ds} + \bar{i}_z \frac{dy}{ds} \tag{3}$$

With this definition, the normal projection of the radius vector, r_n, is easily determined.

$$r_n = -\bar{n}\cdot\bar{r} = -(z\frac{dy}{ds} - y\frac{dz}{ds}) \tag{4}$$

It is useful in geometrically describing motion due to twisting of the cross section about the beam axis.

The beam undergoes stretching, bending, twisting and transverse shearing. The displacement vector, $\bar{u}$, is

$$\bar{u} = \bar{i}_x u + \bar{i}_y v + \bar{i}_z w \tag{5}$$

Bending and twisting of any cross section are properly represented by transverse displacement components of the form

$$v = V(x) - z\phi(x) \tag{6}$$

$$w = W(x) + y\phi(x) \tag{7}$$

where V and W are transverse displacement components of the beam axis and ϕ (x) is the twist angle, which is assumed to be a small angle, positive for counter-clockwise rotation consistent with s in Figure 2.

<u>The Axial Displacement Component</u>

The tangential component of displacement, v_t, is

$$v_t = \bar{t}\cdot\bar{u} = V\frac{dy}{ds} + W\frac{dz}{ds} + r_n\,\phi \tag{8}$$

Let $\gamma^o_{xy}(x)$ and $\gamma^o_{xz}(x)$ be the transverse shear strains of any cross section, which are assumed to be uniform for each cross section; that is, due to transverse shear, the cross section remains planar. Further, let $\gamma(x)$ be the shear strain due to twisting. In the usual theory of torsion of thin walled beams of isotropic materials, the shear flow and, hence, the shear strain are independent of s. Consequently, the twisting contribution $\gamma(x)$ is consistent with this observation. Therefore, from the stress transformation law and elementary physical considerations, the membrane shear strain in the beam wall, γ^o_{xs}, is

$$\gamma^o_{xs} = \gamma^o_{xy}\frac{dy}{ds} + \gamma^o_{xz}\frac{dz}{ds} + \gamma \tag{9}$$

From strain-displacement considerations

$$\gamma^o_{xs} = u_{,s} + v_{t,x} \tag{10}$$

If any effects of taper of the cross section along the length of the beam are ignored, Equation (8) yields

$$v_{t,x} \cong V_{,x}\frac{dy}{ds} + W_{,x}\frac{dz}{ds} + r_n\phi_{,x} \tag{11}$$

Equations (9) - (11) result in

$$u_{,s} = (\gamma^{o}_{xy} - V_{,x})\frac{dy}{ds} + (\gamma^{o}_{xz} - W_{,x})\frac{dz}{ds} + \gamma - r_n \phi_{,x} \tag{12}$$

Define the section rotations, β_y and β_z, as

$$\beta_y = \gamma^{o}_{xz} - W_{,x} \tag{13}$$

$$\beta_z = \gamma^{o}_{xy} - V_{,x} \tag{14}$$

The axial displacement component must be continuous around the circumference of the cross section. Consequently,

$$\oint u_{,s}\, ds = 0 \tag{15}$$

consistent with theory of elasticity, namely, rigorous compatibility conditions.

which results in

$$\gamma(x) = \frac{2A_e}{c}\phi_{,x} \tag{16}$$

where

$$2A_e = \oint r_n\, ds \tag{17}$$

and

$$c = \oint ds \tag{18}$$

The enclosed area of the cross section is A_e and c is the circumference of the CSCC.

Integration of Equation (12) and use of Equations (13), (14) and (16) produce the following result:

$$u = U(x) + y\,\beta_z + z\,\beta_y + \psi\,\phi_{,x} \tag{19}$$

where $U(x)$ is the extension of the axis and ψ is the torsion-related warping function. The latter is defined as

$$\psi = \frac{2A_e}{c}s - 2\omega \tag{20}$$

where

$$\omega = \frac{1}{2}\int_0^s r_n\, ds \tag{21}$$

is the sectorial area swept out as s increases.

The warping function satisfies the equation

$$\oint \psi \, ds = 0 \tag{22}$$

which is consistent with Equation (15).

In the classical St. Venant theory of torsion of beams under end torques, $\phi_{,x}$, the rate of twist, is constant. Consequently, in that case, no axial strain is produced by twisting. In the present theory, is allowed to vary arbitrarily so that nonuniform warping effects are accounted for on a rational basis.

EQUATIONS OF EQUILIBRIUM

Strains

The displacement field is defined by Equations (6), (7) and (19) and the membrane shear strain in Equation (9). If Equations (13), (14) and (16) are utilized, the membrane shear strain may be written as

$$\gamma^{o}_{xs} = (\beta_z + V_{,x})\frac{dy}{ds} + (\beta_y + W_{,x})\frac{dz}{ds} + \frac{2A_e}{c}\phi_{,x} \tag{23}$$

The membrane axial strain is

$$\epsilon^{o}_{xx} = u_{,x} = U_{,x} + y\beta_{z,x} + z\beta_{y,x} + \psi\phi_{,xx} \tag{24}$$

The strain and displacement fields are completely specified by the six kinematic variables

$$U, V, W, \phi, \beta_y, \beta_z$$

These variables will be varied in order to derive the governing equilibrium equations and natural boundary conditions in terms of naturally defined generalized internal forces.

Stress Resultants and Generalized Internal Forces

The beam reacts external forces by membrane action in the wall. For thin walled beams, local shell bending and twisting moment resultants can be ignored. Consequently, only the membrane stress resultants N_{xx}, N_{xs} and N_{ss} need be considered. Furthermore, by virtue of the beam-like geometry, the hoop stress resultant N_{ss} is quite small and will be ignored. (This assumption would be abandoned if the cell was pressurized.) Thus

$$N_{ss} \approx 0 \tag{25}$$

The variation of the internal strain energy, $\bar{U}$, is

$$\delta \bar{U} = \int_0^L \oint (N_{xx} \delta \varepsilon_{xx}^o + N_{xs} \delta \gamma_{xs}^o) ds\, dx \tag{26}$$

where L is the beam length. The following generalized internal forces arise naturally from Equations (23), (24) and (26):

$$N = \oint N_{xx}\, ds \qquad \text{(Axial Force)} \tag{27}$$

$$Q_y = \oint N_{xs} \frac{dy}{ds} ds \qquad \text{(Chordwise Shear Force)} \tag{28}$$

$$Q_z = \oint N_{xs} \frac{dz}{ds} ds \qquad \text{(Flapwise Shear Force)} \tag{29}$$

$$M_x = \frac{2A_e}{c} \oint N_{xs}\, ds \qquad \text{(Direct Torque)} \tag{30}$$

$$M_y = \oint N_{xx}\, z\, ds \qquad \text{(Flapwise Bending Moment)} \tag{31}$$

$$M_z = \oint N_{xx}\, y\, ds \qquad \text{(Chordwise Bending Moment)} \tag{32}$$

$$Q_w = \oint N_{xx}\, \psi\, ds \qquad \text{(Generalized Warping Force)} \tag{33}$$

(9-24) into (67) true 27-33

The direct torque expression (30) can be put in a more familiar form by defining the mean shear flow $\bar{N}_{xs}$ as

$$\bar{N}_{xs} = \frac{1}{c} \oint N_{xs}\, ds \tag{34}$$

Consequently

$$M_x = 2A_e \bar{N}_{xs} \tag{30A}$$

which is the obvious counterpart of Bredt's formula for constant shear flow. The systematic approach utilized herein avoids the confusion regarding shear flow that emerges from Reference 3.

The generalized warping force Q_w is defined naturally as a consequence of the form of the axial displacement component. It is the continuous counterpart of the warping group of forces utilized in classical aeronautical structural analysis to study shear lag effects due to restrained warping[12]. Its units are force-(length)^2.

Virtual Work of External Forces

In order to fix ideas, let the end of the beam corresponding to x = L be subjected to net force and moment resultants $\bar{N}$, $\bar{Q}_y$, $\bar{Q}_z$, $\bar{M}_x$, $\bar{M}_y$ and $\bar{M}_z$. The end x = 0 will be supported. In addition, an effective applied traction to the outer surface of the beam with components $\bar{\sigma}_{nx}$, $\bar{\sigma}_{ny}$ and $\bar{\sigma}_{nz}$ is considered. The virtual work of the external forces, δW_e, is

$$\delta W_e = \bar{N}\,\delta U(L) + \bar{Q}_y\,\delta V(L) + \bar{Q}_z\,\delta W(L)$$

$$\bar{M}_x\,\delta\phi(L) + \bar{M}_y\,\delta\beta_y(L) + \bar{M}_z\,\delta\beta_z(L)$$

$$+ \int_0^L \oint (\bar{\sigma}_{nx}\,\delta u + \bar{\sigma}_{ny}\,\delta v + \bar{\sigma}_{nz}\,\delta w)\,ds\,dx \tag{35}$$

From the applied surface tractions and the form of the displacement field, the following definitions for generalized external loadings follow directly:

$$q_x = \oint \bar{\sigma}_{nx}\,ds \qquad \text{(Distributed Axial Force)} \tag{36}$$

$$q_y = \oint \bar{\sigma}_{ny}\,ds \qquad \text{(Distributed Chordwise Axial Force)} \tag{37}$$

$$q_z = \oint \bar{\sigma}_{nz}\,ds \qquad \text{(Distributed Flapwise Axial Force)} \tag{38}$$

$$\bar{m}_x = \oint (\bar{\sigma}_{nz}y - \bar{\sigma}_{ny}z)\,ds \qquad \text{(Distributed Torque)} \tag{39}$$

$$m_y = \oint \bar{\sigma}_{ny}\,z\,ds \qquad \text{(Distributed Flapwise Bending Moment)} \tag{40}$$

$$m_z = \oint \bar{\sigma}_{nx}\,y\,ds \qquad \text{(Distributed Chordwise Bending Moment)} \tag{41}$$

$$q_w = \oint \bar{\sigma}_{nx}\,\bar{\psi}\,ds \qquad \text{(Generalized Distributed Warping Force)} \tag{42}$$

All of the above are familiar with the exception of the generalized distributed warping force q_w. It is the external counterpart of Q_w.

Governing Equations and Boundary Conditions

The governing equations of equilibrium and natural boundary conditions are derived from the Principle of Virtual Work, which is

$$\delta\bar{U} = \delta W_e \tag{43}$$

As a result of the definitions (27)-(33), the variation of the internal strain energy (26) is written as

$$\delta\bar{U} = \int_0^L \Big[N\,\delta U_{,x} + Q_y(\delta\beta_z + \delta V_{,x})$$

$$+ Q_z(\delta\beta_y + \delta W_{,x}) + \bar{M}_x\,\delta\phi_{,x} + M_y\,\delta\beta_{y,x}$$

$$+ M_z\,\delta\beta_{z,x} + \bar{Q}_w\,\delta\phi_{,xx}\Big]\,dx \tag{44}$$

With the aid of the definitions (36) - (42), the virtual work of the external forces (35) is

$$\delta W_e = \bar{N}\,\delta U(L) + \bar{Q}_y\,\delta V(L) + \bar{Q}_z\,\delta W(L)$$

$$+ \bar{M}_x\,\delta\phi(L) + \bar{M}_y\,\delta\beta_y(L) + \bar{M}_z\,\delta\beta_z(L)$$

$$+ \int_o^L \Big[q_x\,\delta U + q_y\,\delta V + q_z\,\delta W + \bar{q}_w\,\delta\phi_{,x}$$

$$+ m_x\,\delta\phi + m_y\,\delta\beta_y + m_z\,\delta\beta_z \Big] dx \qquad (45)$$

Application of the calculus of variations and the usual assumptions regarding continuity result in the following equations of equilibrium:

$$N_{,x} + q_x = 0 \qquad \text{(x - Force)} \qquad (46)$$

$$Q_{y,x} + q_y = 0 \qquad \text{(y - Force)} \qquad (47)$$

$$Q_{z,x} + q_z = 0 \qquad \text{(z-Force)} \qquad (48)$$

$$M_{x,x} - Q_{w,xx} + m_x - q_{w,x} = 0 \qquad \text{(x - Torque)} \qquad (49)$$

$$M_{y,x} - Q_z + m_y = 0 \qquad \text{(y - Moment)} \qquad (50)$$

$$M_{z,x} - Q_y + m_z = 0 \qquad \text{(z - Moment)} \qquad (51)$$

The corresponding boundary conditions are obtained as well. The natural boundary conditions emerge at $x = L$ by virtue of choice for applied and resultants at that section. The results are

$$N = \bar{N} \qquad (52)$$

$$Q_y = \bar{Q}_y \qquad (53)$$

$$Q_z = \bar{Q}_z \qquad (54)$$

$$M_x - Q_{w,x} = \bar{M}_x + q_w \qquad (55)$$

$$M_y = \bar{M}_y \qquad (56)$$

$$M_z = \bar{M}_z \qquad (57)$$

$$Q_w = 0 \tag{58}$$

As an alternative to the above, geometric boundary conditions may be prescribed. The counterparts to Equations (52) - (58), respectively, are to prescribe U, V, W, ϕ, β_y, β_z and $\phi_{,x}$. These alternatives in the present development correspond to the end $x = 0$.

Most of these results are familiar and require no explanation. Equations (49), (55) and (58) involving torque are unusual and require comment. From Equations (49) and (55), there is an equivalent internal torque, $(M_x)_{eq}$, that reacts the external loads. It is

$$(M_x)_{eq} = M_x - Q_{w,x} \tag{59}$$

The first contribution is the direct, St. Venant contribution which is familiar. The second contribution is the secondary torque due to restrained or nonuniform warping. It is less well known and seldom is developed in the manner used here. The presence of q_w in equation (55) is variationally consistent; it is likely to be zero in most applications.

Equation (58) is equivalent to permitting the cross section at the end to be free to warp. This is the usual "free end" condition. If warping is restrained, ϕ_x must be set to zero, as is clear from the form of u, Equation (19). In principle a prescribed value, $\bar{Q}_w$, could be imposed at an end. It is difficult to think of such a case in practical applications.

GENERALIZED FORCE-DEFORMATION RELATIONS

Constitutive Relations

Up until this point, the theory created is general - - - no specification to composite materials has been introduced. Composite thin walled construction is characterized by the membrane stiffness matrix $\underline{A}$ which relates the stress resultants to the membrane strains. The constitutive relations are

$$\begin{Bmatrix} N_{xx} \\ N_{ss} \\ N_{xs} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^o_{xx} \\ \varepsilon^o_{ss} \\ \varepsilon^o_{xs} \end{Bmatrix} \tag{60}$$

The convention for labeling the $\underline{A}$-stiffness coefficients is the standard one given in Reference 13.

For a laminate of N plies, the stiffnesses are determined by simply adding the plane stress stiffnesses, $\bar{Q}_{ij}$, for each ply. Thus

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}^{(k)}_{ij} h_k \qquad (i,j = 1, 2, 6) \tag{61}$$

where h_k is the thickness of the k-th ply. The ply stiffnesses depend upon the material system, material form (fabric or tape, for example) and fiber orientation.

The equations can be reduced with the aid of Equation (25). The hoop strain ε^o_{ss} can be eliminated since N_{ss} is zero. It is found to be

$$\varepsilon^{o}_{ss} = -(A_{12}\varepsilon^{o}_{xx} + A_{26}\gamma^{o}_{xs})/A_{22} \tag{62}$$

Consequently, the remaining equations may be written

$$\begin{Bmatrix} N_{xx} \\ N_{xs} \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{12} & K_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon^{o}_{xx} \\ \gamma^{o}_{xs} \end{Bmatrix} \begin{matrix} \rightarrow (24) \\ \rightarrow (9) \end{matrix} \tag{63}$$

The K-stiffnesses correspond to uniaxial extension and shear. They are

$$K_{11} = A_{11} - (A_{12})^2/A_{22} \tag{64}$$

$$K_{12} = A_{16} - A_{12}A_{26}/A_{22} \tag{65}$$

$$K_{22} = A_{66} - (A_{26})^2/A_{22} \tag{66}$$

For the familiar case of isotropic materials, the above reduce to

$$K_{11} = E\,h,\ K_{12} = 0,\ K_{22} = Gh \tag{67}$$

where E is Young's modulus, G is the shear modulus and h is the wall thickness.

The shear-extension coupling stiffness K_{12} is responsible for elastic tailoring. It vanishes for locally balanced laminates for which A_{16} and A_{26} are zero. The use of unbalanced angle ply layups, therefore, is the fundamental mechanism employed in tailoring at this level of modeling. For this reason, care must be exercised in manufacture to properly account for the tendency of warping in the design of tooling.

Generalized Strains

The deformation-related variables or generalized strains that are natural to consider are easily identified from the strain expressions (23) and (24). Arrayed in a vector $\underline{u}$ they are

$$\underline{u} = (U_{,x}\ \gamma^{o}_{xy}\ \gamma^{o}_{xz}\ \phi_{,x}\ \beta_{y,x}\ \beta_{z,x}\ \phi_{,xx})^{T} \tag{68}$$

Choice of Axes

The x-axis or beam axis has not been concretely specified other than to require that it be parallel to the span. It is convenient to choose it in such a way that

$$\oint K_{11}\, y\, ds = 0 \tag{69A}$$

and

$$\oint K_{11}\, z\, ds = 0 \tag{69B}$$

This choice defines the <u>tension axis</u>[3]. This is the axis for which the application of a resultant tensile force will not produce any <u>bending</u>. For general elastic coupling, a twist may be produced, however.

The tension axis is the counter part of the centroidal axis for homogeneous, isotropic beams.

It is also possible to define the y-axis and z-axis as principal flexural axes which uncouple bending about these orthogonal axes in the cross section. The necessary condition is that

$$\oint yz\, K_{11}\, ds = 0 \tag{70}$$

As for the tension axis, twist may accompany bending about these axis for unbalanced angle ply layups.

The above choices for the axis system are adopted.

<u>Generalized Force-Deformation Relationships</u>

A generalized internal force vector, $\underline{F}$, that corresponds to $\underline{u}$ is obtained from Equations (27) - (33). It is

$$\underline{F} = (N \quad Q_y \quad Q_z \quad M_x \quad M_y \quad M_z \quad Q_w)^T \tag{71}$$

The <u>beam stiffness matrix</u>, $\underline{C}$, is defined such that

$$\underline{F} = \underline{C}\ \underline{u} \tag{72}$$

It is a 7 x 7 symmetric matrix which is constructed in a straightforward, consistent manner.

Determination of the C_{ij} elements proceeds as follows: (1) the strains from Equations (9) and (24) are substituted in Equations (63); (2) these results are inserted into Equations (27) - (33); and (3) the stiffness elements are identified directly. Because of the choice of axes defined by Equations (69)-(70)

$$C_{15} = C_{16} = C_{56} = 0 \tag{73}$$

There are , therefore, in general, 25 independent stiffnesses to be determined.

For convenience, the equations for the stiffnesses are given in the Appendix. The classical St. Venant theory of bending and torsion is recovered if γ^o_{xy}, γ^o_{xz} and ϕ_{xx} are set to zero in $\underline{u}$, Equation (63), and the second, third and seventh equations in the system (72) are ignored.

SUMMARY AND CONCLUDING REMARKS

A complete, variationally consistent static theory that is valid for small displacements of single closed cell composite beams of arbitrary ply layup has been developed. Such a beam model serves as a first approximation to many commonly used rotor blade configurations. The fundamental mechanism for elastic tailoring appears in the wall coupling stiffness K_{12}, which is commonly associated with unbalanced ply layups. The nonclassical influences of transverse shear strain and nonuniform torsion related warping are accounted for in a simple, rational manner.

Initial pretwist has been ignored, partially based upon the desire for brevity and partially based upon the findings of Reference 3. Dynamics and nonlinear geometric effects due to large displacements have likewise not been considered for brevity. All of these issues will be considered in future work.

The foundation provided by the present work provides consistency and clarity, as well as a straightforward development that facilitates understanding. The "mystery" of elastic tailoring, hopefully, has been diminished as well.

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APPENDIX: STIFFNESS COEFFICIENTS

Coefficient	Description		Eq.
✓ $C_{11} = \oint K_{11}\, ds$	(Extensional Stiffness)		(A-1)
✓ $C_{12} = \oint K_{12} \frac{dy}{ds}\, ds$	(Coupling Stiffness)	extension-shear	(A-2)
✓ $C_{13} = \oint K_{12} \frac{dz}{ds}\, ds$	(Coupling Stiffness)		(A-3)
(–) $C_{14} = \frac{2A_e}{c} \oint K_{12}\, ds$	(Coupling Stiffness)	ext-torsion	(A-4)
$C_{15} = C_{16} = 0$	(Choice of Axes)		(A-5)
(–) $C_{17} = \oint K_{11} \psi\, ds$	(Coupling Stiffness)	ext-warp.	(A-6)
✓ $C_{22} = \oint K_{22} (\frac{dy}{ds})^2\, ds$	(Transverse Shear Stiffness)		(A-7)
✓ $C_{23} = \oint K_{22} (\frac{dy}{ds})(\frac{dz}{ds})\, ds$	(Transvere Shear Stiffness)		(A-8)
(–) $C_{24} = \frac{2A_e}{c} \oint K_{22} \frac{dy}{ds}\, ds$	(Coupling Stiffness)	torsion-shear	(A-9)
✓ $C_{25} = \oint K_{12} \frac{dy}{ds} z\, ds$	(Coupling Stiffness)	bending-shear	(A-10)
✓ $C_{26} = \oint K_{12} \frac{dy}{ds} y\, ds$	(Coupling Stiffness)		(A-11)
(–) $C_{27} = \oint K_{12} \frac{dy}{ds} \psi\, ds$	(Coupling Stiffness)		(A-12)
✓ $C_{33} = \oint K_{22} (\frac{dz}{ds})^2\, ds$	(Transverse Shear Stiffness)		(A-13)
(–) $C_{34} = \frac{2A_e}{c} \oint K_{22} \frac{dz}{ds}\, ds$	(Coupling Stiffness)		(A-14)
✓ $C_{35} = \oint K_{12} (\frac{dz}{ds}) z\, ds$	(Coupling Stiffness)	bending-shear	(A-15)
✓ $C_{36} = \oint K_{12} (\frac{dz}{ds}) y\, ds$	(Coupling Stiffness)		(A-16)
$C_{37} = \oint K_{12} \frac{dz}{ds} \psi\, ds$	(Coupling Stiffness)		(A-17)
(–) $C_{44} = \frac{4A_e^2}{c^2} \oint K_{22}\, ds$	(Torsional Stiffness)		(A-18)

$$C_{45} = \frac{2A_e}{C} \oint K_{12}\, z \, ds \qquad \text{(Coupling Stiffness)} \qquad (A-19)$$

$$C_{46} = \frac{2A_e}{C} \oint K_{12}\, y \, ds \qquad \text{(Coupling Stiffness)} \qquad (A-20)$$

$$C_{47} = \frac{2A_e}{C} \oint K_{12}\, \psi \, ds \qquad \text{(Coupling Stiffness)} \qquad (A-21)$$

$$C_{55} = \oint K_{11}\, z^2 \, ds \qquad \text{(Bending Stiffness)} \qquad (A-22)$$

$$C_{56} = 0 \qquad \text{(Choice of Axes)} \qquad (A-23)$$

$$C_{57} = \oint K_{11}\, z \psi \, ds \qquad \text{(Coupling Stiffness)} \qquad (A-24)$$

$$C_{66} = \oint K_{11}\, y^2 \, ds \qquad \text{(Bending Stiffness)} \qquad (A-25)$$

$$C_{67} = \oint K_{11}\, y \psi \, ds \qquad \text{(Coupling Stiffness)} \qquad (A-26)$$

$$C_{77} = \oint K_{11}\, \psi^2 \, ds \qquad \text{(Warping Stiffness)} \qquad (A-27)$$

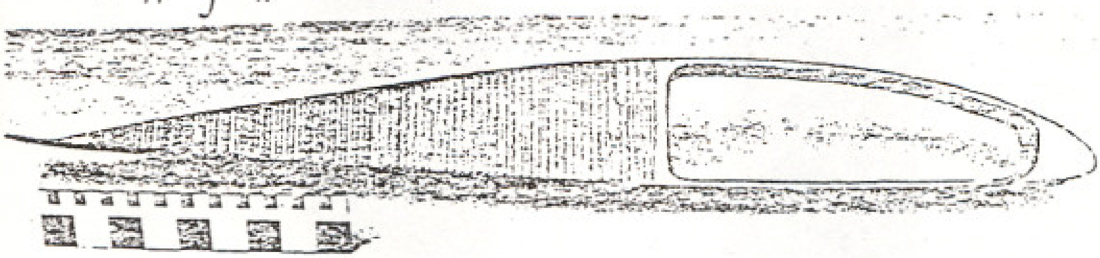

Figure 1. CH-47 Composite Main Rotor Blade Section

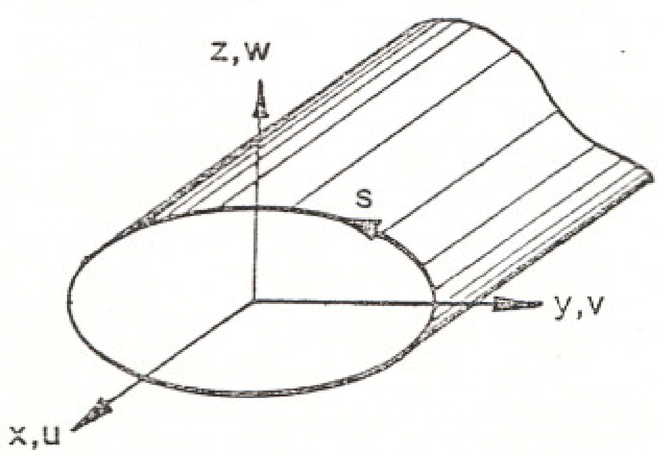

Figure 2. Closed Cell Thin Wall Beam Model