FORMULATION OF A LAMINATED SHELL THEORY INCORPORATING EMBEDDED DISTRIBUTED ACTUATORS

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ABSTRACT

Smart materials have created new paradigms for structural design by introducing new concepts for vibration, damage, and structural control. Shape memory alloy reinforced composites are some of the newest and most versatile of this category of novel materials. They have shown tremendous versatility to adaptively and actively tailor mechanical and physical properties of structures and to perform shape and damage control. Moreover, they have generated new concepts for acoustic and vibration control. However, the unique behavior of the shape memory alloy fibers used as active elements within the composite also poses some difficult and interesting problems for describing the mechanical behavior of SMA reinforced structures. This paper will describe the formulation of a generalized laminate shell theory that incorporates embedded distributed actuators, i.e., shape memory alloy fibers or piezoelectric films. The theories consider the nonlinear strain-temperature-stress coupling for shape memory alloy actuators and the simplifications for analyzing piezoelectric actuators. Some of the computational difficulties of predicting the behavior of SMA reinforced shells will be discussed.

NOTATION

A_1^2, A_2^2	first fundamental magnitudes.
E^*	Young's modulus.
F_1, F_2, F_3	external forces.
h	thickness of the shell.
M_{11}, M_{22}, M_{12}	resultant moments.
N_{11}, N_{22}, N_{12}	resultant forces.
R_1, R_2	radii of curvatures.
$\bar{R}$	position vector of an arbitrary point in the shell.
$\bar{r}$	position vector of a point in the reference surface.
U_1, U_2, W	displacements of a point in the shell.
u_1^0, u_2^0, w	displacements of a point in the reference surface.
V_a	volume fraction of the SMA fiber.
n, z	normal coordinate direction, and value.
α_1, α_2	curvilinear coordinates in the shell coordinate system.
δ_i	thickness of ith layer.
ε_ϕ, ε_T	electric field or temperature induced strains.
$\varepsilon^0_{\alpha_{11}}$, $\varepsilon^0_{\alpha_{22}}$	strains of the reference surface in the curvilinear coordinate
$\varepsilon_{\alpha_{11}}$, $\varepsilon_{\alpha_{22}}$	strains of shell in the curvilinear coordinates.
$\varepsilon_{\alpha_1 n}$, $\varepsilon_{\alpha_2 n}$	
$\varepsilon_{\alpha_{12}}$	shear strains in shell coordinate system.
σ_r^*	recovery stress of the SMA fibers.
ρ	density.
$[C]^*$	stiffness matrix of the laminate.
$[T]^*$	transfer matrix from principal to shell global coordinate system.
superscript(*)	electric field or temperature related quantities.
subscript (a)	actuator related quantities.

Other notation will be explained in the text or will be self-explanatory.

INTRODUCTION

Shell structures are frequently encountered in engineering practice. Their complex, curved geometry makes them more difficult to deal with than plates. Even more difficult is analyzing shell structures with various embedded actuators. In this paper, the characteristics of embedded distributed actuators such as shape memory alloy (SMA) fibers and piezoelectric films are incorporated into general thin laminated shell theory. Following the description of the model formulation is a discussion of the solution to this complex nonlinear problem and the new problems that result. This model establishes the first step toward reliable design and behavior prediction of SMA reinforced shells.

Shape Memory Alloys

In 1965[1] Buehler and Wiley received a United States Patent on a series of engineering alloys that possess a unique mechanical (shape) "memory." The shape-memory effect (SME) can be described basically as follows: an object in the low-temperature martensitic condition, when plastically deformed and the external stresses removed, will regain its original (memory) shape when heated. The process, or phenomenon, is the result of a martensitic transformation during heating. Although the exact mechanism by which the shape recovery takes place is a subject of controversy, a great deal has been learned in the past twenty years [2-4] about the unique properties of this class of materials. Clearly, however, the process of regaining the original shape is associated with a reverse transformation of the deformed martensitic phase to the higher temperature austenite phase.

Nickel-titanium alloys (Nitinol, NiTi) of proper composition exhibit unique mechanical "memory" or restoration force characteristics. The name is derived from Ni (Nickel) - Ti (Titanium) - NOL (Naval Ordinance Laboratory). The shape recovery performance of Nitinol is phenomenal. The material can be plastically deformed in its low-temperature martensite phase and then restored to the original configuration or shape by heating it above the characteristic transition temperature. This unusual behavior is limited to NiTi alloys having near-equiatomic composition. Plastic strains of typically six to eight percent may be completely recovered by heating the material so as to transform it to its austenite phase. Restraining the material from regaining its memory shape can yield stresses of 100,000 psi (the yield strength of martensitic Nitinol is approximately 12,000 psi).

For some applications, creating large internal forces within the material or structure is not needed or desired. Shape memory alloys have the unique ability to change their material properties reversibly, and this characteristic can be exploited without embedding plastically deformed SMA "fibers" or creating large forces and deformations of the structure. This capability is exploited in the concept termed "Active Modal Modification" and will be further explained below.

Substantial progress has been made in understanding the nature of the "shape memory effect" (SME). A great deal of literature has been published over the past twenty years presenting detailed thermal, electrical, magnetic, and mechanical characterizations of this unusual alloy [2-4]. However, there is still much to be learned about the influence of residual stress and high temperatures that may be used in composite fabrication and processing; about the extent, duration and repeatability of SME; and about the dynamic actuation and sensing characteristics of Nitinol.

Shape Memory Alloy Reinforced Composites

The class of the material referred to as SMA composites in this paper is simply a composite material that contains shape memory alloy fibers (or films) such that the structure's stiffness can be controlled by the addition of heat (i.e., a current applied through the fibers). SME recovery forces can be applied in the same way[5,6].

Shape Memory Alloy (SMA) reinforced composites have tremendous potential for creating new paradigms for material-structures interaction [6]. The list of scientific areas that can be influenced by novel approaches possible with SMA reinforced composites is quite large. For example, vibration control can be accomplished by using the distributed force actuator capabilities similar to the common piezoelectric systems. However, two unique approaches to active control are possible with a material that can change its stiffness and physical properties and, in the second case, apply large distributed loads throughout the structure: i) Active Strain Energy Tuning and ii) Active Modal Modification. Simulation results showing the potential for SMA reinforced composites to vary the modal response of a composite plate will be presented below.

Applications for SMA reinforced composites extend far beyond vibration control tasks. Active buckling control or, more generically, active structural modification schemes can be imagined in which SMA fibers are stiffened within a composite to alter the critical buckling load of the structure. SMA composites that are used for various vibration control tasks could also be used for motion or shape control, allowing a structure to maintain a given shape or orientation for an extended period of time. Motion and shape control will, in all likelihood, involve the simultaneous use of force actuators (SMA) and stiffness actuators (the technique in which the SMA is heated to change its modulus of elasticity) to create a structure that behaves much like a mechanical muscle.

Transient and steady-state vibration control can be accomplished with SMA reinforced composites using several techniques. Transient vibration control is defined here as the ability to suppress or damp structural vibration by applying forces (distributed and/or point) to the structure in such a way as to dissipate the energy within the structure. This is accomplished generally by applying point transverse loads to the structure or applying an "actuator film" to the surface of the structure. The approach with SMA reinforced composites is simply to embed the actuators (shape memory alloys) in the structure so that when actuated correctly, they exert agonist-antagonist forces off the neutral axis, thereby reducing vibrations [5].

Motion and shape control can be accomplished using the same technique as described above for transient vibration control except that the structure is expected to maintain a given shape or orientation for an extended period of time. The physical, thermal, and controller design will be much more critical than in the transient vibration control scenario. Another possible design approach is to actuate single fibers with pulse-type signals, much like the all-or-nothing actuation of the individual muscle fibers in the human muscle.

Active Control Concepts

Steady-state vibration control, which may also be used for structural acoustic control, can be accomplished with SMA reinforced composites using a novel technique termed "Active Modal Modification" [7]. The modal response of a structure or mechanical component (i.e., plate or beam) can be tuned or modified by simply heating the SMA fibers or laminae to change the stiffness of all or portions of the structure. When Nitinol is heated to cause the material transformation from the martensitic phase to the austenite phase, the Young's modulus changes by a factor of approximately four. Not only is the stiffness increased by a factor of four, but also the yield strength is increased by a factor of ten. This change in the material properties occurs because of a phase transformation and does not result in any appreciable force. Nor does it need to be initiated by any plastic deformation.

In "Active Strain Energy Tuning" [7] the shape memory alloy fibers are placed in or on the structure in such a way that when activated there is no resulting deflections, but instead, the structure is placed in a state of "residual" strain. The resulting stored strain energy (tension or compression) changes the energy balance of the structure and modifies the modal response in the same way tuning a guitar string does.

BEHAVIOR OF SHAPE MEMORY ALLOY FIBERS[8]

Plastically elongated shape memory alloy fibers will tend to

return to their original shape when activated by increasing their temperature to a certain degree T, but the amount of shape change or restoring strain depends on the activation temperature and the initial plastic deformation, ε_0. The significant dimensional change is only in the fiber direction because the shape memory phenomenon is due to the crystal structure change (twinning), which will not result in a large volume change representative of diffusion phase changes and therefore will not affect the dimension in directions other than the fiber direction beyond Poisson contractions. Figure 1 shows the typical nonlinear coupling that exists between the recovering stress, initial plastic strain, and the activation temperature.

In the shape memory alloy reinforced composites explained above, shape memory alloy fibers or actuators with an initial plastic strain, ε_0, are embedded in a "matrix." Recall that the "matrix" referred to in this discussion may be a composite material itself, such as graphite-epoxy. The entire material system may then be strained, ε, which is "added" to the initial plastic strain, ε_0. The induced global strain, ε , may be the result of external mechanical loading or of contraction of the material system resulting from activation of the shape memory alloy fibers or actuators. Therefore, the total strain of the actuator, ε_a, will be

$$\varepsilon_a = \varepsilon_0 + \varepsilon \tag{1}$$

The resulting stress in the SMA fibers upon activation is therefore also a nonlinear function of initial plastic strain and of the activation temperature of the fibers. This relationship is expressed simply as

$$\sigma_a = f(T, \varepsilon_a). \tag{2}$$

When the SMA actuators are employed in our concepts for active dynamic tuning, shape control, structural (i.e., buckling) control, etc., the incremental strain, ε, will be small. In this

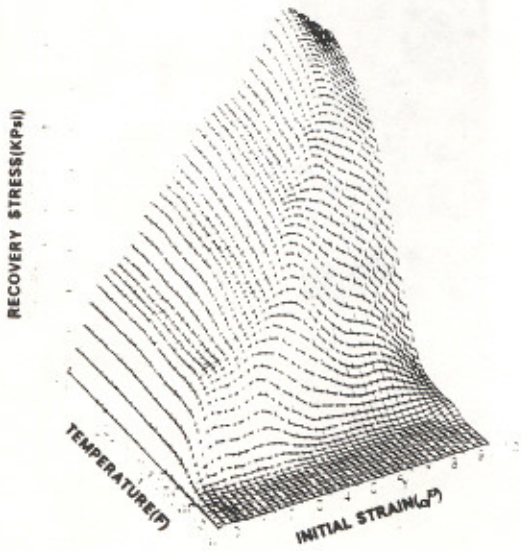

Fig. 1 Stress-Strain-Activation Temperature Coupling of Nitinol

paper, we assume this strain is within elastic limit of the shape memory alloy fibers, even though the fibers have been plastically elongated first. Figure 2 illustrates the operational range of the actuators being considered and also the individual strain components as defined above. Confining the strain to this linear-elastic range simplifies the development and results

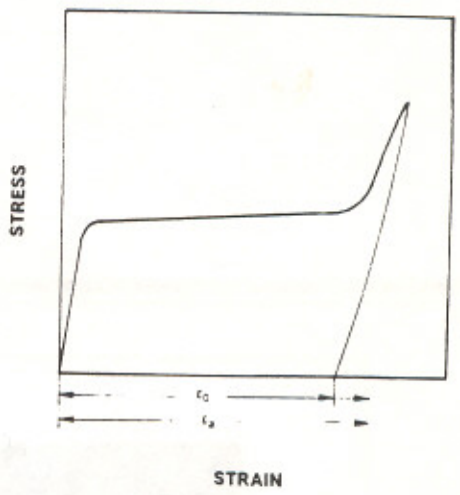

Fig. 2 Plastic and Elastic Strains of SMA Actuator.

in the following quasi-linear expression relating the actuator stress and strain:

$$\sigma_a = E_a^* \varepsilon + \sigma_r^* \tag{3}$$

where

$$E_a^* = E(T), \tag{4}$$

and

$$\sigma_r^* = \sigma_r(T, \varepsilon_0 + \varepsilon). \tag{5}$$

The superscript (*) denotes variables that are activation temperature dependent and typically highly nonlinear.

In equation (3), E_a^* is the elastic modulus of the SMA fibers at a given activation temperature, T. The recovery stress of the SMA fibers at a specified strain and activation temperature is denoted as σ_r^*. Figure 3 shows the relationship between E_a^* and σ_r^*.

Rewriting Eq. (3) to solve for the resulting elastic strain in the actuator as a function of initial strain and activation temperature yields

$$\varepsilon = \frac{\sigma_a}{E_a^*} - \frac{\sigma_r^*}{E_a^*} \tag{6}$$

To solve for the strain, ε, in Eq. (6), an iterative method must be used as σ_r^* is a function of strain, $(\varepsilon_0 + \varepsilon)$. The iterative form of Eq. (6) can be written as

$$\varepsilon_i = \frac{\sigma_a}{E_a^*} - \frac{\sigma_r^*(T, \varepsilon_{i-\Delta i})}{E_a^*} \tag{7}$$

TWO DIMENSIONAL MODEL

The two-dimensional model is a natural extension of the one-dimensional model. The general two-dimensional model essentially involves incorporating the tensor transformations appropriate for describing the planar stress-strain behavior of transversely isotropic laminae. However, the most fundamental two-dimensional relationship simply describes the stress-strain behavior in the principal coordinate system. Unlike conventional materials, be they linear-elastic or elastic-plastic or possessing other nonlinear behavior, SMA reinforced composites have a characteristic or tenser quantity that must be incorporated into the basic stress-strain relationship – the shape recovery stress tenser. In the section above, a one-dimensional stress-strain relationship was derived that contained a shape recovery stress term that added vectorially to

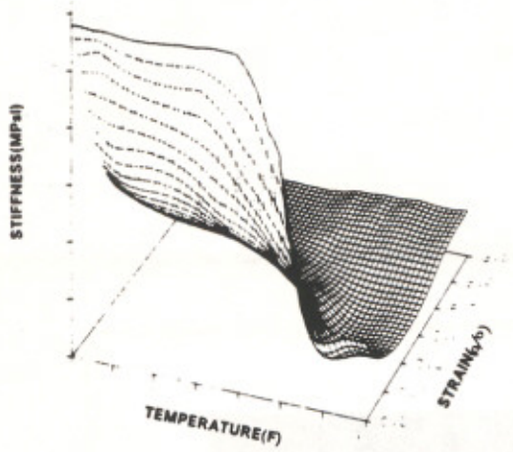

Fig. 3 Stiffness-Strain-Activation Temperature Coupling of Nitinol

the classical rule-of-mixtures "linear-elastic" constitutive relationship. Using the resulting temperature-strain laminae properties, the following two-dimensional relationship for a transversely isotropic lamina can be expressed in the principal coordinates as

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{21} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66}\end{bmatrix}^{*} \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_{12}\end{Bmatrix} - \begin{Bmatrix} V_f\sigma_r^{*} \\ 0 \\ 0\end{Bmatrix} \tag{8}$$

Extending Eq. (8) to describe the mechanical behavior of laminae in the general coordinate system involves performing the well known tenser transformations and yields,

$$\begin{Bmatrix}\sigma_x^{*}\\ \sigma_y^{*}\\ \sigma_{xy}^{*}\end{Bmatrix} = [T]^{-1}\begin{Bmatrix} V_f\sigma_r^{*} \\ 0 \\ 0\end{Bmatrix} = \begin{Bmatrix} c^2V_f\sigma_r^{*} \\ s^2V_f\sigma_r^{*} \\ 0\end{Bmatrix} \tag{9}$$

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix} = [C]^{*}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix} + \begin{Bmatrix}\sigma_x^{*}\\ \sigma_y^{*}\\ \sigma_{xy}^{*}\end{Bmatrix} \tag{10}$$

Equation (10) is the stress-strain relationship for a single layer, or lamina, of a SMA reinforced composite. Each lamina of a laminate will therefore have a different effective stiffness and resulting stress-strain relationship depending on the fiber/actuator orientation, the actuator volume fraction, and the activation temperature of each lamina. For laminate analysis, each lamina must be first evaluated as described above and then denoted by using the superscript (k) to identify individual plys.

Piezoelectric Layers

Another actuation element often used in smart structures is the piezoelectric ceramic and polymer film actuator[9].

Piezoelectric materials can generate an electrical charge in response to mechanical deformation or, conversely, provide a mechanical strain when an electric field is applied across them. The charge generated by the mechanical deformation can be used for distributed sensing. However, only the actuator effect of mechanical strain induced by the electric field will be considered in this formulation.

The electric-strain coupling of the piezoelectric material is similar to the coupling between temperature and the thermal strain of most engineering materials. Therefore, the constitutive relation for a piezoelectric layer has the same form as a SMA reinforced layer:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix} = [C]^{*}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix} + \begin{Bmatrix}\sigma_x^{*}\\ \sigma_y^{*}\\ 0\end{Bmatrix} \tag{11}$$

where ε_e the electric field induced strain, and

$$\sigma_x^{*} = E_{11}\varepsilon_e$$

$$\sigma_y^{*} = E_{22}\varepsilon_e$$

For the piezoelectric layers, the superscript * is used to denote either electric field induced quantities, ε_e, or the properties after the electric field is applied such as [C]. Without the nonlinear recovery stress and the highly temperature dependent mechanical properties, the piezoelectric structures are more easily dealt with than the SMA reinforced layers. The following discussion will therefore concentrate on SMA reinforced structures as they represent the most general case; however, the discussion uses the theory presented below to account for embedded piezoelectric layers.

THEORY OF MULTI-LAYERED THIN ORTHOTROPIC SMA SHELLS

Assumptions

- The shells covered in this paper will be confined to the class of thin shell. A thin shell has a thickness considerably less than either of the other two dimensions.
- A line originally normal to the shell reference will remain normal to the deformed reference surface.
- Shell deflections are assumed to be small.
- The SMA composite shell is composed of a number of thin, orthotropic laminae, each of which may have a different thickness δ_i. However, the radii of curvature of the shell, R_1 and R_2, are much greater than the thickness of each layer, i.e., $R_j >> \delta_j$. The interfaces of the layers are parallel to each other.
- The strains in the shell are smaller than the initial plastic strains of the SMA.

Shell Coordination

The middle surface of the shell is chosen as the reference surface. In this surface we establish an orthogonal curvilinear coordinate system which coincides with the orthogonal lines of the principal curvature of the surface. Then, in the thickness direction, the normal of the reference is chosen as the third coordinate direction. This constitutes a shell coordinate system. Thus, to an arbitrary point within the shell, a position vector can be defined in this coordinate system as (see Figs. 4,5)

$$\vec{R}(\alpha_1, \alpha_2, z) = \vec{r}(\alpha_1, \alpha_2) + z\vec{n}(\alpha_1, \alpha_2) \tag{12}$$

where $\vec{r}$ is the position vector to a point on the reference surface, $\vec{n}$ is the unit vector normal to the reference surface, and z a coordinate measured from the reference surface, through the thickness, along $\vec{n}(\alpha_1, \alpha_2)$. The α_1 and α_2 are bounded by some values that define the shell boundaries.

Define Lame's Constants:

$$A_1^2 = \frac{\partial\vec{r}}{\partial\alpha_1} \cdot \frac{\partial\vec{r}}{\partial\alpha_1} \tag{13}$$

and

$$A_2^2 = \frac{\partial\vec{r}}{\partial\alpha_2} \cdot \frac{\partial\vec{r}}{\partial\alpha_2} \tag{14}$$

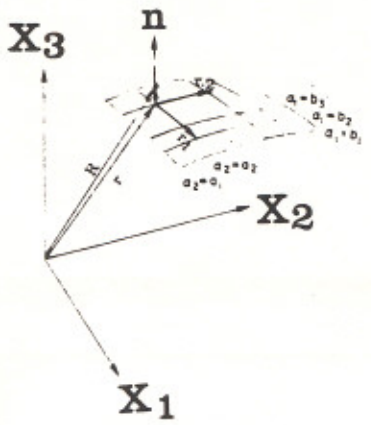

Fig. 4 Reference Surface and the Shell Coordinate System.

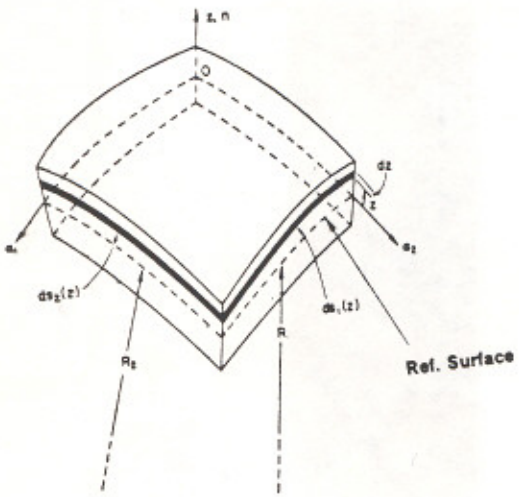

Fig. 5 A Shell Differential Element.
(Note that ds_1 and ds_2 are functions of z and the curvatures)

Then the magnitude of a differential length element is given by

$$(ds)^2 = d\vec{R}\cdot d\vec{R} = (d\vec{r} + zd\vec{n} + dz\vec{n})\cdot(d\vec{r} + zd\vec{n} + dz\vec{n}) =$$
$$= A_1^2(1 + z/R_1)^2(d\alpha_1)^2 + A_2^2(1 + z/R_2)^2(d\alpha_2)^2 + (dz)^2 \qquad (15)$$

The verification of this equation is shown in ref. [10].

From Fig. 5, and the definition of $(ds)^2$, it is clear that if a shell element of thickness dz at an altitude z from the middle surface is isolated, the lengths of the edges of the element are given by

$$ds_1(z) = A_1(1 + z/R_1)(d\alpha_1) \qquad (16)$$
$$ds_2(z) = A_2(1 + z/R_2)(d\alpha_2) \qquad (17)$$

and the corresponding area elements of the faces are

$$d\Sigma_1(z) = A_1(1 + z/R_1)(d\alpha_1)(dz) \qquad (18)$$
$$d\Sigma_2(z) = A_2(1 + z/R_2)(d\alpha_2)(dz) \qquad (19)$$

Constitutive Relation in Shell Coordination

Equation (10) is the stress-strain relation in the cartesian coordinate system. In the orthogonal curvilinear coordinates, α_1 and α_2, the relationship is given by

$$\begin{Bmatrix}\sigma_{x11}\\ \sigma_{x22}\\ \sigma_{x12}\end{Bmatrix} = [C]^*\begin{Bmatrix}\varepsilon_{x11}\\ \varepsilon_{x22}\\ \varepsilon_{x12}\end{Bmatrix} + \begin{Bmatrix}\overset{*}{\sigma}_{x11}\\ \overset{*}{\sigma}_{x22}\\ \overset{*}{\sigma}_{x12}\end{Bmatrix} \qquad (20)$$

Strain-Displacement Relations

The formulation begins with first defining a displacement vector in the shell coordinate system:

$$\vec{u}(\alpha_1, \alpha_2, z) = U_1(\alpha_1, \alpha_2, z)\vec{t}_1 + U_2(\alpha_1, \alpha_2, z)\vec{t}_2 + W_2(\alpha_1, \alpha_2, z)\vec{n} \qquad (21)$$

where U_1 and U_2 are the magnitudes of the projections on the unit tangent vectors of the displacement of a point P.

Now in an orthogonal curvilinear system, the normal and shearing components of strain are

$$\varepsilon_{x_{ii}} = \frac{\partial}{\partial\alpha_i}\left(\frac{U_i}{\sqrt{g_i}}\right) + \frac{1}{2g_i}\sum_{k=1}^{3}\frac{\partial g_i}{\partial\alpha_k}\frac{U_k}{\sqrt{g_k}} \qquad (22)$$

$$\varepsilon_{x_{ij}} = \frac{1}{\sqrt{g_i g_j}}\left[g_i\frac{\partial}{\partial\alpha_j}\left(\frac{U_i}{\sqrt{g_i}}\right) + g_j\frac{\partial}{\partial\alpha_i}\left(\frac{U_i}{\sqrt{g_j}}\right)\right] \qquad (23)$$

where i or j = 1, 2, 3, and the sum is shown explicitly. The entities are correspondent to the quantities in the shell theory as

$$\alpha_1 = \alpha_1,\ \alpha_2 = \alpha_2,\ \alpha_3 = z,\ U_1 = U_1,\ U_2 = U_2,\ U_3 = W, \qquad (24)$$
$$g_1 = A_1^2(1 + z/R_1)^2,\ g_2 = A_2^2(1 + z/R_2)^2,\ g_3 = 1.$$

Then it follows that

$$\varepsilon_{x_{11}} = \frac{1}{A_1(1 + z/R_1)}\left(\frac{\partial U_1}{\partial\alpha_1} + \frac{U_2}{A_2}\frac{\partial A_1}{\partial\alpha_2} + \frac{A_1 W}{R_1}\right)$$

$$\varepsilon_{x_{22}} = \frac{1}{A_2(1 + z/R_2)}\left(\frac{\partial U_2}{\partial\alpha_2} + \frac{U_1}{A_1}\frac{\partial A_2}{\partial\alpha_1} + \frac{A_2 W}{R_2}\right)$$

$$\varepsilon_{x_3} = \frac{\partial W}{\partial z}$$

$$\varepsilon_{x12} = \frac{A_2(1 + z/R_2)}{A_1(1 + z/R_1)}\frac{\partial}{\partial\alpha_1}\left[\frac{U_2}{A_2(1 + z/R_2)}\right] + \frac{A_1(1 + z/R_1)}{A_2(1 + z/R_2)}\frac{\partial}{\partial\alpha_2}\left[\frac{U_1}{A_1(1 + z/R_1)}\right] \qquad (25)$$

$$\varepsilon_{x_1n} = \frac{1}{A_1(1 + z/R_1)}\frac{\partial W}{\partial\alpha_1} + A_1(1 + z/R_1)\frac{\partial}{\partial z}\left[\frac{U_1}{A_1(1 + z/R_1)}\right]$$

$$\varepsilon_{x_2n} = \frac{1}{A_2(1 + z/R_2)}\frac{\partial W}{\partial\alpha_2} + A_2(1 + z/R_2)\frac{\partial}{\partial z}\left[\frac{U_2}{A_2(1 + z/R_2)}\right]$$

Note that within the framework of infinitesimal elasticity, these relations are exact.

The displacement distribution is assumed to be linear in the thickness direction:

$$U_1(\alpha_1, \alpha_2, z) = u_1^0(\alpha_1, \alpha_2) + z\beta_1(\alpha_1, \alpha_2)$$

$$U_2(\alpha_1, \alpha_2, z) = u_2^0(\alpha_1, \alpha_2) + z\beta_2(\alpha_1, \alpha_2) \qquad (26)$$

$$W(\alpha_1, \alpha_2, z) = w(\alpha_1, \alpha_2)$$

Substituting for U_1, U_2, W from Eq. (26) into the last two equations of Eq. (25) yields:

$$\varepsilon_{\alpha_1 n} = \frac{1}{A_1(1+z/R_1)}\frac{\partial W}{\partial \alpha_1} + A_1(1+z/R_1)\frac{\partial}{\partial z}\left[\frac{u_1^0}{A_1(1+z/R_1)} + \frac{z\beta_1}{A_1(1+z/R_1)}\right] = 0$$

$$\varepsilon_{\alpha_2 n} = \frac{1}{A_2(1+z/R_2)}\frac{\partial W}{\partial \alpha_2} + A_2(1+z/R_2)\frac{\partial}{\partial z}\left[\frac{u_2^0}{A_2(1+z/R_2)} + \frac{z\beta_1}{A_2(1+z/R_2)}\right] = 0$$

so that we obtain

$$\beta_1 = \frac{u_{\alpha_1}^0}{R_1} - \frac{1}{A_1}\frac{\partial w}{\partial \alpha_1} \tag{27}$$

$$\beta_2 = \frac{u_{\alpha_2}^0}{R_2} - \frac{1}{A_2}\frac{\partial w}{\partial \alpha_2} \tag{28}$$

For the convenience of manipulation, an operator matrix is used to relate β's and u's and w :

$$\begin{bmatrix} u_{\alpha_1}^0 \\ u_{\alpha_2}^0 \\ w \\ \beta_1 \\ \beta_2 \end{bmatrix} = [\Theta]\begin{bmatrix} u_{\alpha_1}^0 \\ u_{\alpha_2}^0 \\ w \end{bmatrix} \tag{29}$$

where

$$[\Theta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{R_1} & 0 & \frac{1}{A_1}\frac{\partial}{\partial \alpha_1} \\ 0 & \frac{1}{R_2} & \frac{1}{A_2}\frac{\partial}{\partial \alpha_2} \end{bmatrix}$$

Substitution of the displacement assumption Eqs.(26) in the exact strain-displacement relations yields:

$$\varepsilon_{\alpha_{11}} = \frac{1}{(1+z/R_1)}(\varepsilon_{\alpha_{11}}^0 + z\kappa_1) \tag{30}$$

$$\varepsilon_{\alpha_{22}} = \frac{1}{(1+z/R_2)}(\varepsilon_{\alpha_{22}}^0 + z\kappa_2) \tag{31}$$

$$\varepsilon_{\alpha_{12}} = \frac{1}{(1+z/R_1)}(\omega_1 + z\tau_1) + \frac{1}{(1+z/R_2)}(\omega_2 + z\tau_2) \tag{32}$$

$$\varepsilon_n = \varepsilon_{\alpha_1 n} = \varepsilon_{\alpha_2 n} = 0 \tag{33}$$

where

$$\varepsilon_{\alpha_{11}}^0 = \frac{1}{A_1}\frac{\partial u_{\alpha_1}^0}{\partial \alpha_1} + \frac{u_2}{A_1A_2}\frac{\partial A_1}{\partial \alpha_{\alpha_2}^0} + \frac{w}{R_1} \tag{34}$$

$$\varepsilon_{\alpha_{22}}^0 = \frac{1}{A_2}\frac{\partial u_{\alpha_2}^0}{\partial \alpha_2} + \frac{u_{\alpha_1}^0}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} + \frac{w}{R_2} \tag{35}$$

$$\kappa_1 = \frac{1}{A_1}\frac{\partial \beta_1}{\partial \alpha_1} + \frac{\beta_2}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} \tag{36}$$

$$\kappa_2 = \frac{1}{A_2}\frac{\partial \beta_2}{\partial \alpha_2} + \frac{\beta_1}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} \tag{37}$$

$$\omega_1 = \frac{1}{A_1}\frac{\partial u_{\alpha_2}^0}{\partial \alpha_1} - \frac{u_{\alpha_1}^0}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} \tag{38}$$

$$\omega_2 = \frac{1}{A_2}\frac{\partial u_{\alpha_1}^0}{\partial \alpha_2} - \frac{u_{\alpha_2}^0}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} \tag{39}$$

$$\tau_1 = \frac{1}{A_1}\frac{\partial \beta_2}{\partial \alpha_1} - \frac{\beta_1}{A_1A_2}\frac{\partial A_1}{\partial \alpha_2} \tag{40}$$

$$\tau_2 = \frac{1}{A_2}\frac{\partial \beta_1}{\partial \alpha_2} - \frac{\beta_2}{A_1A_2}\frac{\partial A_2}{\partial \alpha_1} \tag{41}$$

To express the strains in terms of the mid-surface displacement, we have the following expression involving a differential operator:

$$\begin{bmatrix} \varepsilon_{\alpha_{11}}^0 \\ \varepsilon_{\alpha_{22}}^0 \\ \omega_1 \\ \omega_2 \\ \kappa_1 \\ \kappa_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} = [\Omega]\begin{bmatrix} u_{\alpha_1}^0 \\ u_{\alpha_2}^0 \\ w \\ \beta_1 \\ \beta_2 \end{bmatrix} \tag{42}$$

Definition of $[\Omega]$ is given in the APPENDIX.

However, we can express Eqs. (30-32) in matrix form,

$$\begin{bmatrix} \varepsilon_{\alpha 11} \\ \varepsilon_{\alpha 22} \\ \varepsilon_{\alpha 12} \end{bmatrix} = [\Delta]\begin{bmatrix} \varepsilon_{\alpha_{11}}^0 \\ \varepsilon_{\alpha_{22}}^0 \\ \omega_1 \\ \omega_2 \\ \kappa_1 \\ \kappa_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} \tag{43}$$

where

$$[\Delta] = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_2 & 0 & 0 & 0 & z\phi_2 & 0 & 0 \\ 0 & 0 & \phi_1 & \phi_2 & 0 & 0 & z\phi_1 & z\phi_2 \end{bmatrix}$$

and

$$\phi_1 = \frac{1}{1+\frac{z}{R_1}}$$

$$\phi_2 = \frac{1}{1+\frac{z}{R_2}}$$

After doing this, the stress-strain relation, Eq. (20), can be rewritten in terms of the strains of the reference surface:

$$\begin{Bmatrix} \sigma_{\alpha 11} \\ \sigma_{\alpha 22} \\ \sigma_{\alpha 12} \end{Bmatrix} = [C]^*[\Delta]\begin{bmatrix} \varepsilon_{\alpha_{11}}^0 \\ \varepsilon_{\alpha_{22}}^0 \\ \omega_1 \\ \omega_2 \\ \kappa_1 \\ \kappa_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} + \begin{Bmatrix} \sigma_{\alpha 11}^* \\ \sigma_{\alpha 22}^* \\ \sigma_{\alpha 12}^* \end{Bmatrix} \tag{44}$$

For conciseness we can name

$$[\Gamma]^* = [C]^*[\Delta] \tag{45}$$

Then

$$\begin{Bmatrix} \sigma_{\alpha 11} \\ \sigma_{\alpha 22} \\ \sigma_{\alpha 12} \end{Bmatrix} = [\Gamma]^*\begin{bmatrix} \varepsilon_{\alpha_{11}}^0 \\ \varepsilon_{\alpha_{22}}^0 \\ \omega_1 \\ \omega_2 \\ \kappa_1 \\ \kappa_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} + \begin{Bmatrix} \sigma_{\alpha 11}^* \\ \sigma_{\alpha 22}^* \\ \sigma_{\alpha 12}^* \end{Bmatrix} \tag{46}$$

Stress Resultants

The stress and strain resultants are defined as follows:

$$(N_{11}, M_{11}) = \int_{-h/2}^{h/2} \sigma_{x_{11}}^{(k)}(1 + z/R_2)(1, z)dz$$

$$(N_{22}, M_{22}) = \int_{-h/2}^{h/2} \sigma_{x_{22}}^{(k)}(1 + z/R_1)(1, z)dz$$

$$(N_{12}, M_{12}) = \int_{-h/2}^{h/2} \sigma_{x_{12}}^{(k)}(1 + z/R_2)(1, z)dz$$

$$(N_{21}, M_{21}) = \int_{-h/2}^{h/2} \sigma_{21}^{(k)}(1 + z/R_1)(1, z)dz$$

Note that although $\sigma_{12} = \sigma_{21}$, from the equations above we see that $N_{12} \neq N_{12}$ and $M_{12} \neq M_{21}$. Substituting Eq. 46 into the above definition of the resultants, and expressing the force and moment resultants in the matrix form yields:

$$\begin{bmatrix} N_{11} \\ N_{22} \\ N_{12} \\ N_{21} \\ M_{11} \\ M_{12} \\ M_{12} \\ M_{21} \end{bmatrix} = [\Psi]^* \begin{bmatrix} \varepsilon_{x_{11}}^o \\ \varepsilon_{x_{22}}^o \\ \omega_1 \\ \omega_2 \\ \kappa_1 \\ \kappa_2 \\ \tau_1 \\ \tau_2 \end{bmatrix} + \begin{bmatrix} N_{11}^* \\ N_{22}^* \\ N_{12}^* \\ N_{21}^* \\ M_{11}^* \\ M_{22}^* \\ M_{12}^* \\ M_{21}^* \end{bmatrix} \tag{47}$$

The definition of $[\Psi]^*$ is in the APPENDIX, and forcing terms are

$$(N_1^*, M_1^*) = \int_{-h/2}^{h/2} \sigma_{x_{11}}^{*(k)}(1 + z/R_2)(1, z)dz$$

$$(N_2^*, M_2^*) = \int_{-h/2}^{h/2} \sigma_{x_{22}}^{*(k)}(1 + z/R_1)(1, z)dz$$

$$(N_{12}^*, M_{12}^*) = \int_{-h/2}^{h/2} \sigma_{x_{12}}^{*(k)}(1 + z/R_2)(1, z)dz$$

$$(N_{21}^*, M_{21}^*) = \int_{-h/2}^{h/2} \sigma_{x_{21}}^{*(k)}(1 + z/R_1)(1, z)dz$$

Substituting Eq.(29) into Eq.(42), then the result into Eq. (47), we have the force and moment resultants expressed in terms of the displacement:

$$\begin{bmatrix} N_{11} \\ N_{22} \\ N_{12} \\ N_{21} \\ M_{11} \\ M_{22} \\ M_{12} \\ M_{21} \end{bmatrix} = [\Psi]^* [\Omega][\Theta] \begin{bmatrix} u_{x_1}^o \\ u_{x_2}^o \\ w \end{bmatrix} + \begin{bmatrix} N_{11}^* \\ N_{22}^* \\ N_{12}^* \\ N_{21}^* \\ M_{11}^* \\ M_{22}^* \\ M_{12}^* \\ M_{21}^* \end{bmatrix} \tag{48}$$

To further simplify, we define

$$[\Phi] = [\Psi]^* [\Omega][\Theta] \tag{49}$$

which is an 8 x 3 differential operator matrix.

Equations of Motion

Under the assumptions we made above, the equations of motion of a shell can be written as

$$\frac{\partial(N_{11}A_2)}{\partial\alpha_1} + \frac{\partial(N_{21}A_1)}{\partial\alpha_2} + N_{12}\frac{\partial A_1}{\partial\alpha_2} - N_{22}\frac{\partial A_2}{\partial\alpha_1} + A_1A_2\frac{Q_{13}}{R_1} + A_1A_2F_1 = A_1A_2\rho h u_{x_1}^o \tag{50}$$

$$\frac{\partial(N_{12}A_2)}{\partial\alpha_1} + \frac{\partial(N_{22}A_1)}{\partial\alpha_2} + N_{21}\frac{\partial A_2}{\partial\alpha_1} - N_{11}\frac{\partial A_1}{\partial\alpha_2} + 2A_1A_2\frac{Q_{23}}{R_2} + A_1A_2F_2 = A_1A_2\rho h u_{x_2}^o \tag{51}$$

$$\frac{\partial(Q_{13}A_2)}{\partial\alpha_1} + \frac{\partial(Q_{23}A_1)}{\partial\alpha_2} - A_1A_2\left(\frac{N_{11}}{R_1} + \frac{N_{22}}{R_2}\right) + A_1A_2F_3 = A_1A_2\rho h w \tag{52}$$

where Q_{13} and Q_{23} are defined by

$$Q_{13}A_1A_2 = \frac{\partial(M_{11}A_2)}{\partial\alpha_1} + \frac{\partial(M_{21}A_1)}{\partial\alpha_2} + M_{12}\frac{\partial A_1}{\partial\alpha_2} - M_{22}\frac{\partial A_2}{\partial\alpha_1} \tag{53}$$

$$Q_{23}A_1A_2 = \frac{\partial M_{12}A_2}{\partial\alpha_1} + \frac{\partial M_{22}A_1}{\partial\alpha_2} + M_{21}\frac{\partial A_2}{\partial\alpha_1} - M_{11}\frac{\partial A_1}{\partial\alpha_2} \tag{54}$$

The N's and M's can be expressed in terms of the displacements of the reference surface plus a nonlinear recovery force term induced by the activation of the SMA actuator. Therefore, we have three equations and three unknowns, and the recovery force terms make the equations of motion nonlinear.

Substitution of Eqs. (47, 48) into Eqs.(49-52) yields the final form of the equations of motion in terms of the reference surface displacements:

$$\Lambda_{11}u_{x_1}^o + \Lambda_{12}u_{x_2}^o + \Lambda_{13}w + f_{11} = A_1A_2h\rho\, u_{x_1}^o \tag{55}$$

$$\Lambda_{21}u_{x_1}^o + \Lambda_{22}u_{x_2}^o + \Lambda_{23}w + f_{22} = A_1A_2h\rho\, u_{x_2}^o \tag{56}$$

$$\Lambda_{31}u_{x_1}^o + \Lambda_{32}u_{x_2}^o + \Lambda_{33}w + f_{33} = A_1A_2h\rho\, \ddot{w} \tag{57}$$

where Λ_{ij} are differential operators, and f_{ij} are forcing functions, including both the recovery force and external force. Their definitions are

$$\Lambda_{1j} = \frac{\partial(\Phi_{1j}A_2)}{\partial\alpha_1} + \frac{\partial(\Phi_{4j}A_1)}{\partial\alpha_2} + \Phi_{3j}\frac{\partial A_1}{\partial\alpha_2} - \Phi_{2j}\frac{\partial A_2}{\partial\alpha_1} + \frac{1}{R_1}\left(\frac{\partial(\Phi_{5j}A_2)}{\partial\alpha_1} + \frac{\partial(\Phi_{8j}A_1)}{\partial\alpha_2} + \Phi_{7j}\frac{\partial A_1}{\partial\alpha_2} - \Phi_{6j}\frac{\partial A_2}{\partial\alpha_1}\right) \tag{58}$$

$$\Lambda_{2j} = \frac{\partial(\Phi_{3j}A_2)}{\partial\alpha_1} + \frac{\partial(\Phi_{2j}A_1)}{\partial\alpha_2} + \Phi_{3j}\frac{\partial A_2}{\partial\alpha_1} - \Phi_{1j}\frac{\partial A_1}{\partial\alpha_2} + \frac{1}{R_2}\left(\frac{\partial(\Phi_{7j}A_2)}{\partial\alpha_1} + \frac{\partial(\Phi_{6j}A_1)}{\partial\alpha_2} + \Phi_{8j}\frac{\partial A_2}{\partial\alpha_1} - \Phi_{5j}\frac{\partial A_1}{\partial\alpha_2}\right) \tag{59}$$

$$\Lambda_{3j} = -A_1A_2\left(\frac{N_{1j}}{R_1} + \frac{N_{2j}}{R_2}\right) +$$

$$\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\left(\frac{\partial(A_2\Phi_{5j})}{\partial\alpha_1}+\frac{\partial(A_1\Phi_{8j})}{\partial\alpha_2}+\Phi_{7j}\frac{\partial A_1}{\partial\alpha_2}-\Phi_{6j}\frac{\partial A_2}{\partial\alpha_1}\right)\right)+$$

$$\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\left(\frac{\partial(A_2\Phi_{7j})}{\partial\alpha_1}+\frac{\partial(A_1\Phi_{6j})}{\partial\alpha_2}+\Phi_{8j}\frac{\partial A_2}{\partial\alpha_1}-\Phi_{5j}\frac{\partial A_1}{\partial\alpha_2}\right)\right) \quad (60)$$

$$f_{11}=\frac{\partial(A_1N^*_{11})}{\partial\alpha_1}+\frac{\partial(A_2N^*_{21})}{\partial\alpha_2}+N^*_{12}\frac{\partial A_1}{\partial\alpha_2}-N^*_{22}\frac{\partial A_2}{\partial\alpha_1}+$$

$$\frac{1}{R_1}\left(\frac{\partial(A_2M^*_{11})}{\partial\alpha_1}+\frac{\partial(A_1N^*_{21})}{\partial\alpha_2}+M^*_{12}\frac{\partial A_1}{\partial\alpha_2}-M^*_{22}\frac{\partial A_2}{\partial\alpha_1}\right)+A_1A_2F_1 \quad (61)$$

$$f_{22}=\frac{\partial(A_2N^*_{12})}{\partial\alpha_1}+\frac{\partial(A_1N^*_{22})}{\partial\alpha_2}+N^*_{21}\frac{\partial A_2}{\partial\alpha_1}-N^*_{11}\frac{\partial A_1}{\partial\alpha_2}+$$

$$\frac{1}{R_2}\left(\frac{\partial(A_2M^*_{12})}{\partial\alpha_1}+\frac{\partial(A_1M^*_{22})}{\partial\alpha_2}+M^*_{21}\frac{\partial A_2}{\partial\alpha_1}-M^*_{11}\frac{\partial A_1}{\partial\alpha_2}\right)+A_1A_2F_2 \quad (62)$$

$$f_{33}=-A_1A_2\left(\frac{N^*_{11}}{R_1}+\frac{N^*_{22}}{R_2}\right)+$$

$$\frac{\partial}{\partial\alpha_1}\left(\frac{1}{A_1}\left(\frac{\partial(A_2M^*_{11})}{\partial\alpha_1}+\frac{\partial(A_1M^*_{21})}{\partial\alpha_2}+M^*_{12}\frac{\partial A_1}{\partial\alpha_2}-M^*_{22}\frac{\partial A_2}{\partial\alpha_1}\right)\right)+$$

$$\frac{\partial}{\partial\alpha_2}\left(\frac{1}{A_2}\left(\frac{\partial(A_2M^*_{12})}{\partial\alpha_1}+\frac{\partial(A_1M^*_{21})}{\partial\alpha_2}+M^*_{21}\frac{\partial A_2}{\partial\alpha_1}-M^*_{11}\frac{\partial A_1}{\partial\alpha_2}\right)\right)+A_{A2}F_3 \quad (63)$$

DISCUSSION

Embedding SMA fiber actuators within composite structures such as shells creates several new possibilities for active control. The nature of the control may be to reduce vibration; stiffen or otherwise change the mechanical behavior; modify the structural acoustic behavior; prevent, reduce or arrest damage; or possibly change the geometric configuration of the structure. All of the control scenarios can be accomplished by the two actuation techniques briefly described above in reference to active modal modification and active strain energy tuning. The SMA fiber actuators can be used simply to change the material stiffness, $[C]^*$, without inducing and restoring stress, σ_r. The actuator fibers may also be plastically deformed and embedded prior to consolidation of the laminate, resulting in both a change of stiffness and a distributed transverse load throughout the structure. These control parameters, although very useful, are highly nonlinear and further complicate an already fairly complex shell theory.

In the above formulation, we see that the forcing functions, f_{11}, f_{22}, and f_{33}, are functions of the actuation forces and moments, N^*'s and M^*'s. Theoretically, f_{11}, f_{22}, and f_{33} can be controlled in an open loop manner; however, as can be seen in Figs. 1 and 3, the explicit behavior and electro-thermo-mechanical coupling is very complex and not yet fully understood. Various active/adaptive control strategies show much promise in controlling the behavior and response of SMA composites. By controlling the forcing functions, the displacements, natural frequencies, mode shapes, stiffness, strength, and many physical properties can be tuned by 1) varying the electrical power (or more precisely the temperature) used to activate the SMA fibers, 2) varying the quantity of SMA fibers activated, and 3) selecting the location and orientation of the actuator fibers. SMA composites are extremely versatile for performing active control.

There is a cost associated with this versatility of control scenarios. One of the costs is the result of the actuator's highly nonlinear behavior, which greatly complicates the analysis and synthesis of SMA composite structures. To fully understand the complexity of the problem, the behavior of the SMA actuator must be understood first. Figure 1 shows the recovery stress vs. initial plastic strain under various activation temperatures. Notice that the initial plastic strains may often be in the range of 1%-10% with an 8% strain resulting in the highest restoring stress. However, for general elastic engineering materials we normally limit our analysis to the linear-elastic range, which is typically less than 0.2% strain. Even though the model presented above involves first plastically deforming the fibers by up to 8% strain, because the elastic portion of the strain is recovered before embedding in a composite structure, as shown in Fig. 2, it is assumed that the strain induced in the actuator fibers will not further plastically deform the structure. This assumption, that the actuator fibers will always remain in the elastic range, is the only linear relationship that is assumed in the model. One of the nonlinear aspects of this problem is illustrated in Fig. 6. A vertical line a-b is drawn in Fig. 6 to show that when the fibers are actuated, the restoring stress creates a relatively small change in the actuator strains, resulting in a change in the restoring stress. It is also obvious that if the elastic strains are small compared to the initial strains in the SMA fiber, any deformation of the shell will cause a small recovery strain change in the actuators.

Figure 7 is the recovery stress vs. strain for a fixed temperature. The initial plastic strain of the actuator simply displaces the origin of the stress-strain curve and does not

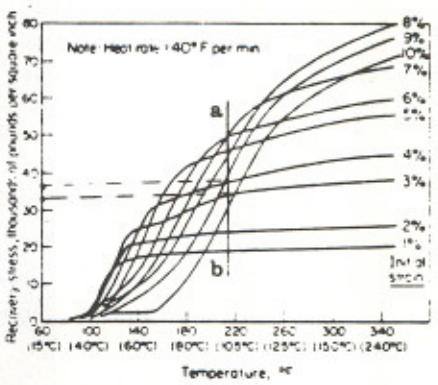

Fig. 6 Recovery Stress vs. Initial Strain.

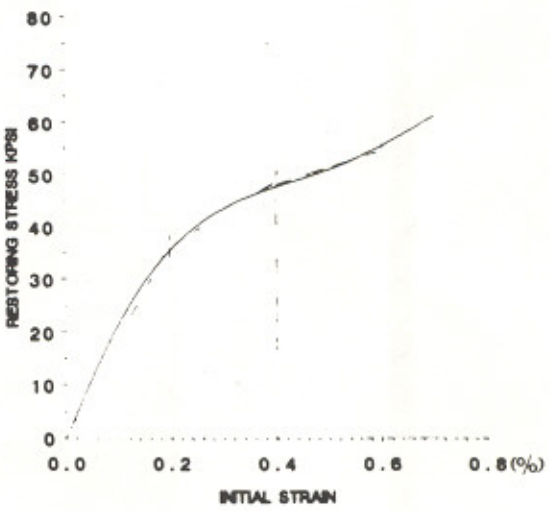

Fig. 7 Recovery Stress vs. Initial Strain at Temperature T.

change the Young's modulus a significant amount. If the problem described above is restricted to small strains, say less than 0.2% as is common in linear-elasticity, then the recovery stress vs. strain relationship can be linearized. This linearization allows for a constant "stiffness" or recovery stress to strain constant to be used in Eq. 5, greatly simplifying the problem.

To summarize, we can say that if we confine the problem to small strains, we can use a fixed recovery stress and stiffness for the SMA fiber actuators at a constant activation temperature.

CONCLUSIONS

In this paper, the formulation of a SMA reinforced laminated shell theory has been presented. The model of SMA fiber actuators may also be simplified to model embedded piezoelectric actuators. Including SMA actuators in the laminated shell theory creates a nonlinear problem. Reducing the computational effort needed to evaluate the nonlinear response of the laminate by restricting the use of the model to conditions that allow for some linearization of the material response will allow this model to be implemented in a finite element code. Once the model has been validated, the model will be invaluable for predicting the structural response of shells to force, acoustic, and displacement inputs and will allow various control techniques to be investigated to control or modify the post-buckling behavior of laminated plates.

ACKNOWLEDGEMENTS

The authors respectfully acknowledge the support for this research effort by the Office of Naval Research Young Investigator Program.

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APPENDIX

Definition of the matrix $[\Omega]$,

$$[\Omega_{11}, \ldots, \Omega_{15}] = \left[\frac{1}{A_1}\frac{\partial}{\partial \alpha_1}, \ \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}, \ \frac{1}{R_1}, \ 0, \ 0 \right];$$

$$[\Omega_{21}, \ldots, \Omega_{25}] = \left[\frac{1}{A_2}\frac{\partial}{\partial \alpha_2}, \ \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}, \ \frac{1}{R_2}, \ 0, \ 0 \right];$$

$$[\Omega_{31}, \ldots, \Omega_{35}] = \left[-\frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_2}, \ \frac{1}{A_1}\frac{\partial}{\partial \alpha_2}, \ 0, \ 0, \ 0 \right];$$

$$[\Omega_{41}, \ldots, \Omega_{45}] = \left[\frac{1}{A_2}\frac{\partial}{\partial \alpha_2}, \ -\frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}, \ 0, \ 0, \ 0 \right];$$

$$[\Omega_{51}, \ldots, \Omega_{55}] = \left[0, \ 0, \ 0, \ \frac{1}{A_1}\frac{\partial}{\partial \alpha_1}, \ \frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2} \right];$$

$$[\Omega_{61}, \ldots, \Omega_{65}] = \left[0, \ 0, \ 0, \ \frac{1}{A_1 A_2}\frac{\partial A_2}{\partial \alpha_1}, \ \frac{1}{A_2}\frac{\partial}{\partial \alpha_2} \right];$$

$$[\Omega_{71}, \ldots, \Omega_{75}] = \left[0, \ 0, \ 0, \ -\frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_2}, \ \frac{1}{A_1}\frac{\partial}{\partial \alpha_1} \right];$$

$$[\Omega_{81}, \ldots, \Omega_{85}] = \left[0, \ 0, \ 0, \ \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}, \ -\frac{1}{A_1 A_2}\frac{\partial A_1}{\partial \alpha_1} \right];$$

The definition of $[\Psi]^*$ is, where j = 1, ... 8,

$$\Psi^*_{1j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{1j}(1 + z/R_2)dz$$

$$\Psi^*_{2j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{2j}(1 + z/R_1)dz$$

$$\Psi^*_{3j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{3j}(1 + z/R_2)dz$$

$$\Psi^*_{4j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{3j}(1 + z/R_1)dz$$

$$\Psi^*_{5j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{1j}(1 + z/R_2)zdz$$

$$\Psi^*_{6j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{2j}(1 + z/R_1)zdz$$

$$\Psi^*_{7j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{3j}(1 + z/R_2)z\,dz$$

$$\Psi^*_{8j} = \int_{-h/2}^{h/2} \Gamma^{*(k)}_{3j}(1 + z/R_1)z\,dz$$