$\nu_x^2 \quad \frac{S_\zeta^{*2}}{\;-1 \quad I_\zeta^* M_x^*}\Big]$

$\lesssim 1$/rev. The solution near $\omega =$

$^* + M_x^*) \; \frac{S_\zeta^{*2}}{I_\zeta^* M_y^* M_x^*}\Big]$

ion, and also for the $\omega = \nu_\zeta - 1$
limit $\omega = 0$, for which the char-
0, which gives $\nu_\zeta - 1 = 0$. With
olution defines where the roots
speed for which $\nu_\zeta = 1$/rev. In
$\zeta - 1$ that intercepts the Ω-axis
he low frequency lag mode root
l values of S_ζ.

case of no damping can be pre-
leman diagram, which is a plot
of the characteristic equation)
imensional solution for the un-
ζ, plus the corresponding nega-
(The negative solutions for ω
ns, and so need not be plotted.)
hed using the above results for
and $\Omega = \infty$, plus the knowledge
ever cross. The character of the
ly on the lag frequency $\nu_\zeta^2 =$
o 12-13 present the Coleman
ted rotor ($K_1 = 0$ and $K_2 < 1$),
$K_2 < 1$), and a stiff in-plane
are sketches of typical results
ume that the nonrotating lag
the hingeless rotors. The un-
rizontal lines at $\omega = \omega_x$ and
otor are the low and high fre-
approach $\nu_{NR} = \sqrt{K_1}$ at low

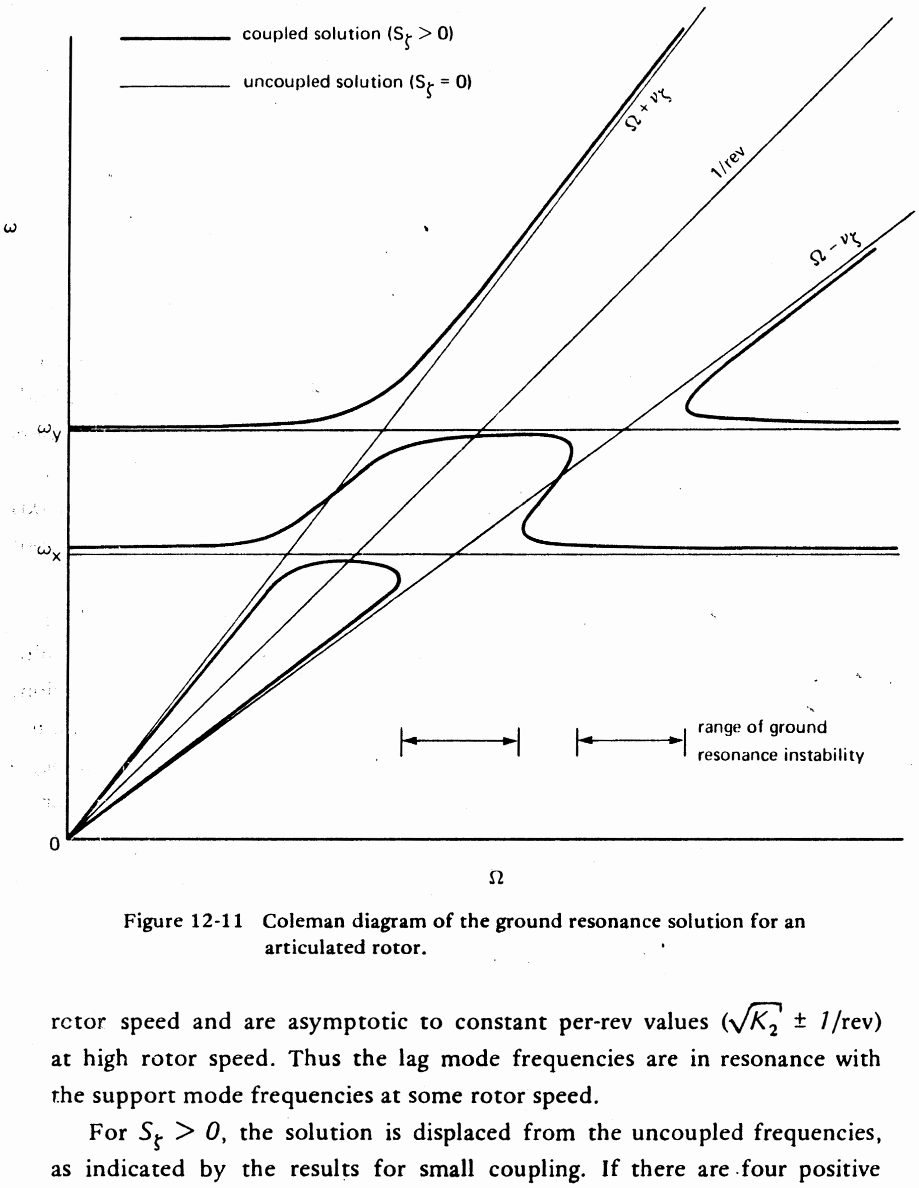

Figure 12-11 Coleman diagram of the ground resonance solution for an articulated rotor.

rotor speed and are asymptotic to constant per-rev values ($\sqrt{K_2} \pm 1$/rev) at high rotor speed. Thus the lag mode frequencies are in resonance with the support mode frequencies at some rotor speed.

For $S_\zeta > 0$, the solution is displaced from the uncoupled frequencies, as indicated by the results for small coupling. If there are four positive solutions for ω at a given rotor speed, then the system is stable (neutrally stable for this case of zero damping). For the articulated and soft in-plane hingeless rotors (Figs. 12-11 and 12-12), however, there are ranges of Ω where only two positive real solutions for ω exist, occurring around the

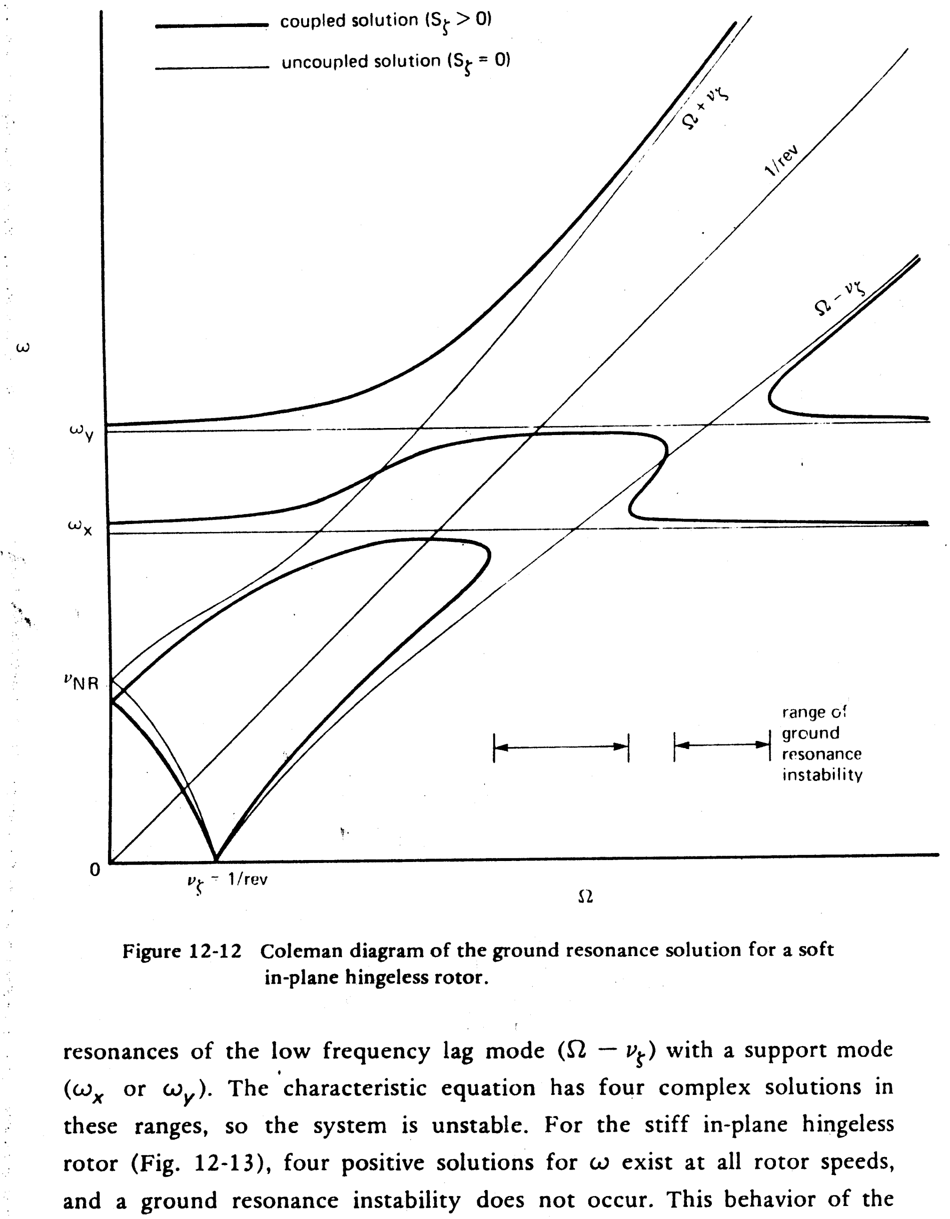

Figure 12-12 Coleman diagram of the ground resonance solution for a soft in-plane hingeless rotor.

resonances of the low frequency lag mode ($\Omega - \nu_\zeta$) with a support mode (ω_x or ω_y). The characteristic equation has four complex solutions in these ranges, so the system is unstable. For the stiff in-plane hingeless rotor (Fig. 12-13), four positive solutions for ω exist at all rotor speeds, and a ground resonance instability does not occur. This behavior of the ground resonance solution is determined by the direction the roots are shifted when $S_\zeta > 0$, which depends on whether $\nu_\zeta < 1$/rev or $\nu_\zeta > 1$/rev at the resonance of the low frequency lag mode with a support mode.

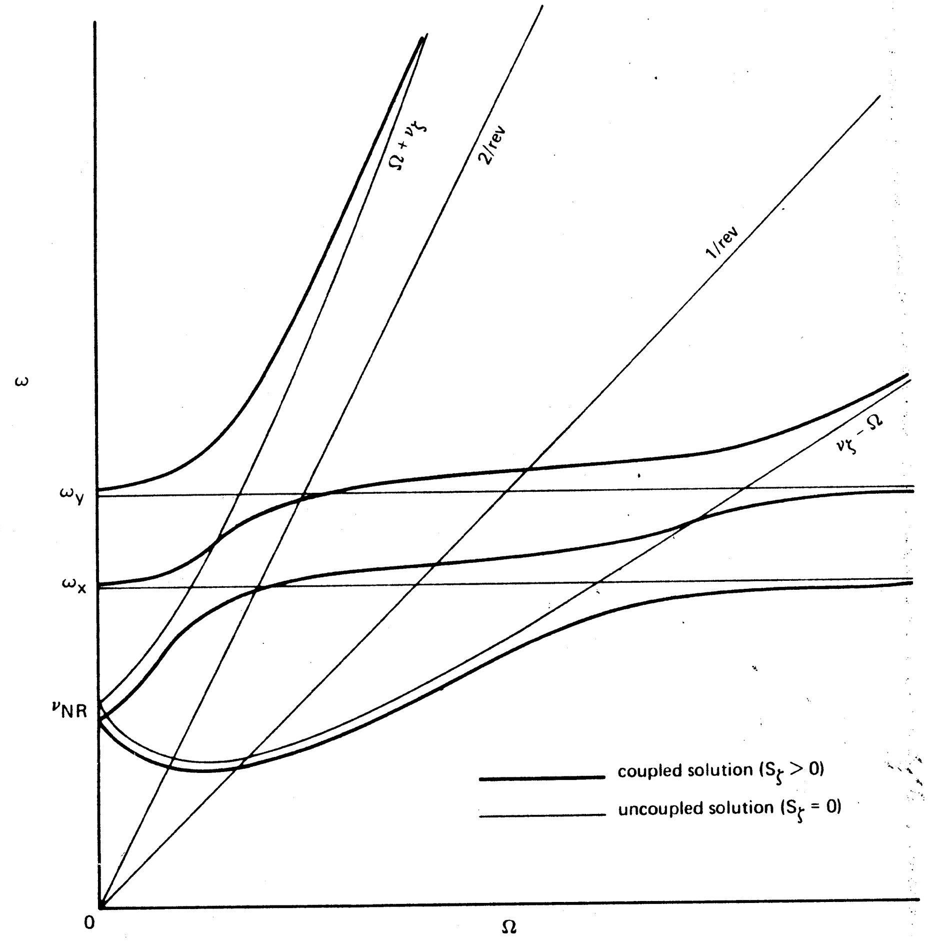

Figure 12-13 Coleman diagram of the ground resonance solution for a stiff in-plane hingeless rotor.

In conclusion, a ground resonance instability can occur at a resonance of a rotor mode and a support mode. The resonances of the high frequency lag mode ($\omega = 1 + \nu_\zeta$) are always stable, but resonances of the low frequency lag mode ($\omega = 1 - \nu_\zeta$) will be unstable if the rotating natural frequency ν_ζ is below 1/rev, as for articulated and soft in-plane hingeless rotors. Thus the placement of the rotor lag frequency determines whether or not a ground resonance instability can occur.

three roots corresponding to $\omega = \nu_\zeta$ and $\omega_y \pm 1$ the largest and smallest are increased, while the middle root is decreased. From this behavior the solution for $S_\zeta > 0$ can be sketched. Figs. 12-14 and 12-15 present typical Coleman diagrams for articulated (soft in-plane) and stiff in-plane two-bladed rotors. As in the case of three or more blades, a ground resonance instability appears with soft in-plane rotors ($\nu_\zeta < 1$/rev) at the resonance of the support and the low frequency lag mode – which in the rotating frame means $\nu_\zeta = \Omega - \omega_y$.

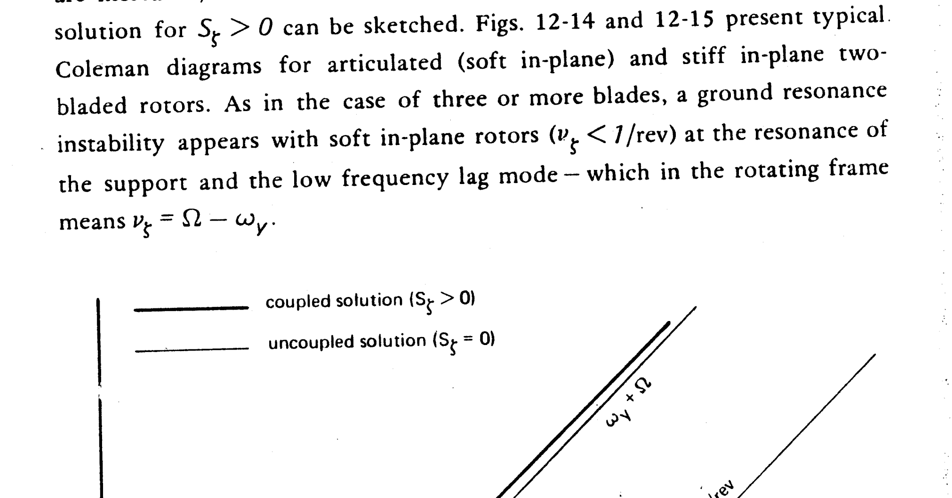

Figure 12-14 Coleman diagram of the ground resonance solution for a two-bladed articulated rotor ($\nu_\zeta < 1$/rev) on an isotropic support.

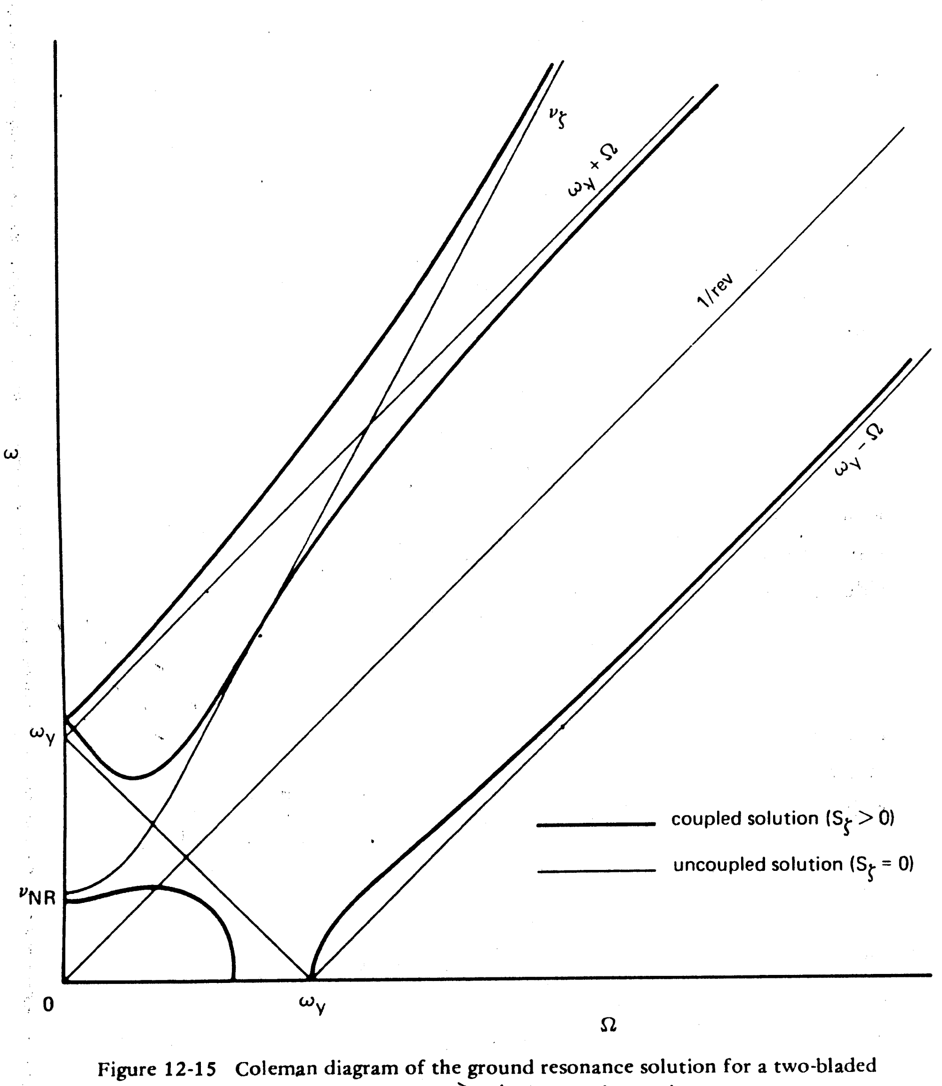

Figure 12-15 Coleman diagram of the ground resonance solution for a two-bladed stiff in-plane rotor ($\nu_\zeta > 1$/rev) on an isotropic support.

Note that for $N = 2$ the center of the gound resonance instability range is shifted to a rotor speed above the uncoupled resonance, in contrast to the $N \geqslant 3$ case, for which the instability range remains centered about the resonance. This suggests that for large enough coupling the instability region might be shifted above the rotor operating range. To examine this