Ch. 15 Kinematics of Rigid Bodies

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15.11 Plane Motion of a Particle Relative to a Rotating Frame

Fig. 15.27. ---- two frames of ref, in the plane of the figure

fixed frame OXY, rotating frame Oxy

$(\dot{\vec{r}})_{OXY}$: rate of change of $\vec{r}$, w.r.t. a fixed frame

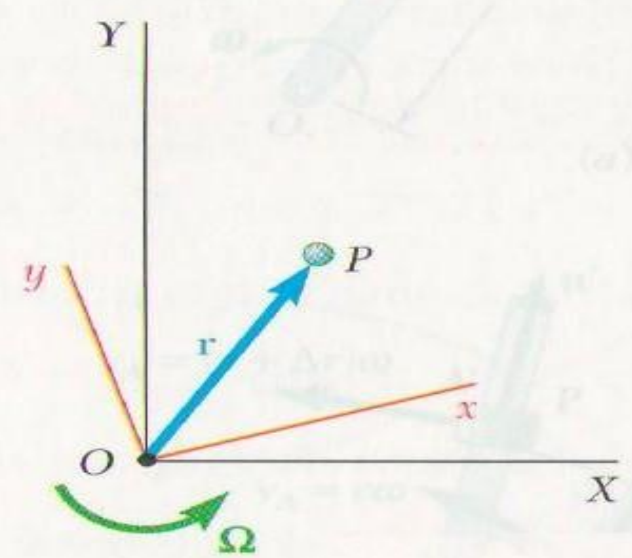

Fig. 15.27

15.11 Plane Motion of a Particle Relative to a Rotating Frame

$(\vec{r})_{OXY}$: w. r. t the rotating frame OXY

$\vec{\Omega}$: angular velocity of the frame Oxy w. r. t OXY

$$\vec{v}_P = (\dot{\vec{r}})_{OXY} = \underbrace{\vec{\Omega} \times \vec{r}}_{\vec{v}_{P'}} + \underbrace{(\dot{\vec{r}})_{Oxy}}_{\vec{v}_{P/F}} \qquad (15.32)$$

F : rotating frame

$\vec{v}_{P'}$: P' velocity in the slab which coincides with P at the instant

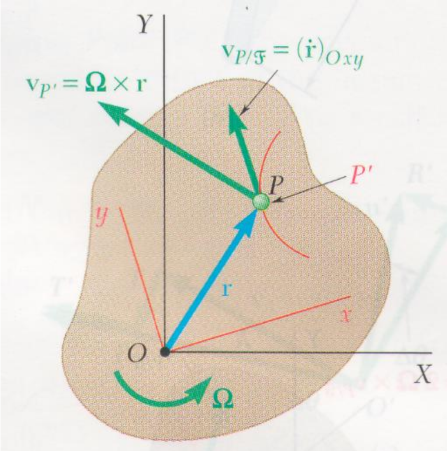

Fig. 15.28

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F} \qquad (15.33)$$

$\vec{v}_P$: Abs. velocity of P

$\vec{v}_{P'}$: vel. of P' of moving frame F coinciding with P

$\vec{v}_{P/F}$: vel. Of P relative to F

15.11 Plane Motion of a Particle Relative to a Rotating Frame

Absolute acceleration --- rate of change of $\vec{v}_P$, w, r, t OXY

$$\vec{a}_P = \dot{\vec{v}}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times \dot{\vec{r}} + \underbrace{\frac{d}{dt}[(\dot{\vec{r}})_{Oxy}]}_{(\ddot{\vec{r}})_{Oxy} + \vec{\Omega} \times (\dot{\vec{r}})_{Oxy}} \quad (15.34)$$

$$\dot{\vec{r}} = \vec{v}_P$$

$$\boxed{= \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Oxy} \quad by\ Eq.(15.32)}$$

$$= \underbrace{\dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})}_{\vec{a}_{P'}} + \underbrace{2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy}}_{\vec{a}_C} + \underbrace{(\ddot{\vec{r}})_{Oxy}}_{\vec{a}_{P/F}}$$

$\vec{a}_C$: complementary / Coriolis acceleration

(15.36)

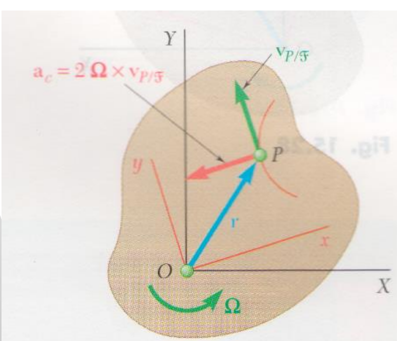

Fig. 15.29

$$\overrightarrow{a_P} = \overrightarrow{a_{P'}} + \overrightarrow{a_{P/F}} + \overrightarrow{a_C}$$

$\overrightarrow{a_P}$ → Abs. Accel of P

$\overrightarrow{a_{P'}}$ → Accel. of P′ of moving frame F coinciding with P

$\overrightarrow{a_{P/F}}$ → Accel. of P′ of relative to F

$\overrightarrow{a_C}$ → $2\vec{\Omega} \times \overrightarrow{v_{P/F}}$

Compared with Eq. (15.21) $\overrightarrow{a_P} = \overrightarrow{a_P} + \overrightarrow{a_{B/A}}$

$\overrightarrow{a_{B/A}}$: accel. w. r. t a frame in translation

→ if the frame is rotating, necessary to include Coriolis' acceleration $\overrightarrow{a_C}$

Direction of $\overrightarrow{a_C}$

$\left|\overrightarrow{a_C}\right| = 2\Omega v_{P/F}$, rotating $\overrightarrow{v_{P/F}}$ through 90°

in the sense of rotation of the moving frame (Fig. 15.29)

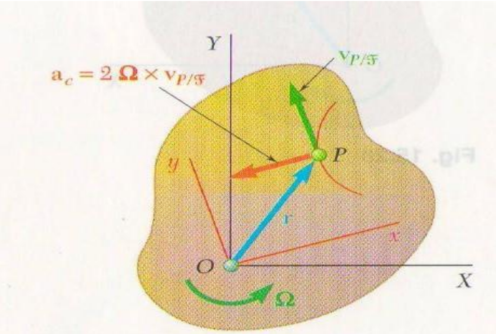

Fig. 15.29

Significance of $\overrightarrow{a_C}$

Abs. velocity of P at time t and $t + \Delta t$ (Fig. 15.30(b))

At t, velocity components $\overrightarrow{u}$, $\overrightarrow{v_A}$, at $t + \Delta t$ $\overrightarrow{u'}$,$\overrightarrow{v_{A'}}$

- Fig 15.30(c) , change in velocity during $\Delta t \rightarrow \overrightarrow{RR'}$, $\overrightarrow{TT''}$, $\overrightarrow{T''T'}$

- $\overrightarrow{TT''}$ ---- change in the direction of $\overrightarrow{v_A}$, $\overrightarrow{TT''}/\Delta t$ represents

$$\overrightarrow{a_A} \quad as \quad \Delta t \rightarrow 0$$

$$\lim_{t\to 0}\frac{TT''}{\Delta t} = \lim_{t\to 0} v_A \frac{\Delta\theta}{\Delta t} = r\omega\omega = r\omega^2 = a_A$$

$\overrightarrow{RR'}$ --- change in direction of $\vec{u}$ due to the rotation

$\overrightarrow{T''T'}$ ---change in magnitude of $\overrightarrow{v_A}$ due to the motion of P along the rod

"combined effect of the relative motion of P and of the rotation of the rod

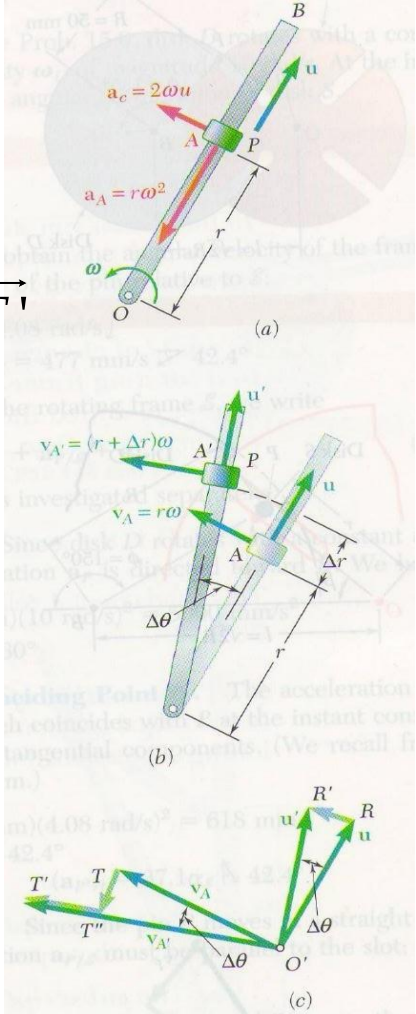

Fig. 15.30

15.11 Plane Motion of a Particle Relative to a Rotating Frame

- sum of these two → $\overrightarrow{a_C}$

$$\overrightarrow{RR'} = u\Delta\theta, \; T''T' = v_{A'} - v_A = (r+\Delta r)\omega - r\omega = \omega\Delta r$$

$$\lim_{t\to 0}\left(\frac{RR'}{\Delta t} + \frac{T''T'}{\Delta t}\right) = \lim_{t\to 0}\left(u\frac{\Delta\theta}{\Delta t} + \omega\frac{\Delta r}{\Delta t}\right) = u\omega + \omega u = 2\omega u$$

Eqs. (15.33.),(15.36) → mechanism which contain parts sliding on each other

abs. and relative motions of sliding pins and collars.

$\overrightarrow{a_C}$ ---- useful in long-range projectiles, appreciablby affected by the earth rotation.

* system of axes attached to the earth--- not truly a Newtonian frame.

→ rotating frame of ref., formulas derived in this section facilitate the study of the motion w.r.t. axes attached to the earth.