Aeroelasticity

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Index

Static Aeroelasticity

Static Aeroelasticity

- Definition ⋯ interaction of aerodynamic and elastic force

  intensive to rate and acceleration of the structural deflection

- Two class of problems

  1. Effects of elastic deformation on the airloads

     ⋯ normal operating condition → performance, handling qualities, structural load distribution

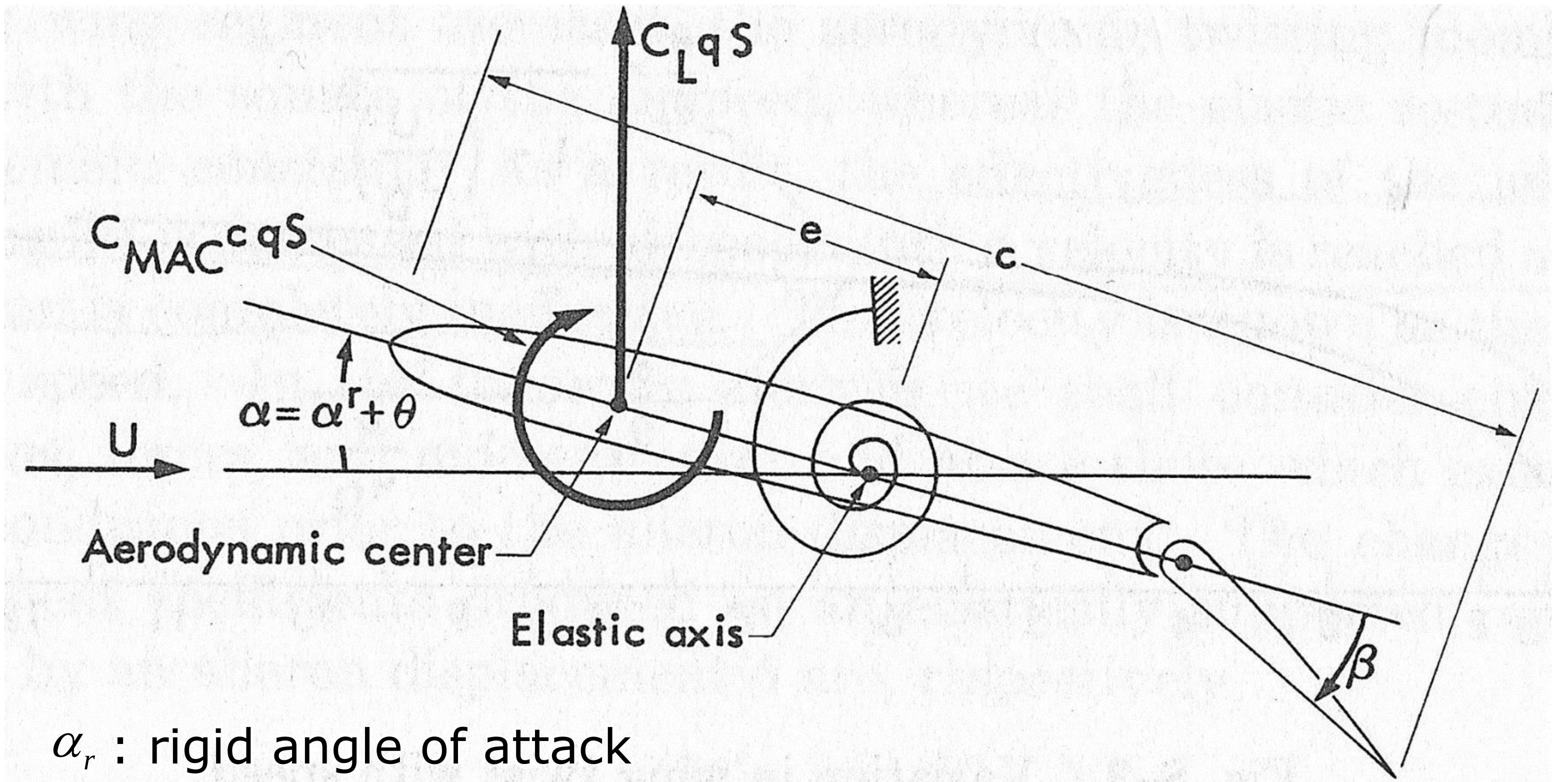
  2. Static stability ⋯ divergence

     [Note] Instability ⋯ tendency to move away from equilibrium

     static ⋯ $e^{(a+iw)t}, a > 0, w = 0$

Typical section of aircraft wing

··· consider a rigid airfoil mounted on an elastic support (in a wind tunnel)



α_r : rigid angle of attack

θ : elastic angle of attack

$\alpha = \alpha_r + \theta$

Typical section of aircraft wing

$\alpha_{i,}\alpha$: ↻ (+)

L: lift, ↑ (+), $L = \frac{1}{2}\rho U^2 S C_L$, where U: flow speed, ρ: flow density

W : weight

q : $\frac{1}{2}\rho U^2$, dynamic pressure

S : reference area = $c \cdot 1$

C_L: lift coefficient

M_{ac} : $qScC_{M_{ac}}$,where : pitching moment coefficient at a.c.

[Note] $C_L = fn.(\alpha, M, \text{airfoil shape})$

$$C_{M_{ac}} = fn.(M, \text{airfoil shape})$$

Typical section of aircraft wing

From a Taylor series

$$C_L = C_{L0} + \frac{\partial C_L}{\partial \alpha}\alpha + h.o.t.$$

$$= C_{L0} + \frac{\partial C_L}{\partial \alpha}\alpha$$

$$C_{M_{ac}} = C_{M_{ac0}} + \frac{\partial C_{M_{ac}}}{\partial \alpha}\alpha + h.o.t.$$

$$= C_{M_{ac0}} + \frac{\partial C_{M_{ac}}}{\partial \alpha}\alpha$$

[Note] for a flat plate

$$C_{L_\alpha} = \frac{\partial C_L}{\partial \alpha} = 2\pi$$

$$C_{M_{ac_\alpha}} = \frac{\partial C_{M_{ac}}}{\partial \alpha} = 0$$

Typical section of aircraft wing

For a rigid support

$$L_{rigid} = \frac{1}{2}\rho U^2 S C_{L_\alpha} \cdot \alpha_r$$

However, for an elastic support

$$L_{elastic} = L = qSC_{L_\alpha} \cdot \alpha$$

$$\alpha(\alpha_r + \alpha_e)$$

[Note]

$$L_{rigid} \neq L_{elastic}$$

$$L_{rigid} < L_{elastic}$$

Consider the system in equilibrium, all applied moments at support point must equal the torsional reaction at the point

Torsional divergence

Consider the system in equilibrium, all applied moments at support point must equal the torsional reaction at the point

$$L(x_{ea}-x_{ac})-W(x_{ea}-x_{cg})+M_{ac}=K_{\alpha}\alpha_e$$

$$qSC_{L\alpha}(\alpha_r+\alpha_e)(x_{ea}-x_{cg})-W(x_{ea}-x_{cg})+qScC_{M_{ac_\alpha}}=K_{\alpha}\alpha_e$$

$$\rightarrow\ \alpha_e=\frac{-W(x_{ea}-x_{cg})+qScC_{M_{ac_\alpha}}+qSC_{L\alpha}\alpha_r(x_{ea}-x_{cg})}{K_{\alpha}-qScC_{M_{ac}}(x_{ea}-x_{cg})}$$

0 ⋯ torsional divergence

For a given q and α_r, we can evaluate L

Torsional divergence

[Note] equilibrium eqn.

$$\underbrace{(K - qSC_{L\alpha} \cdot e)}_{A}\underbrace{\alpha_e}_{x} = \underbrace{qScC_{Mac_\alpha} + qSC_{L_\alpha}\alpha_r e - W \cdot d}_{B}$$

Ax=B → Kx=F

The stability is associated with the homogeneous part of the eqn.
: Ax=0, A ≡ 0 → divergence condition

Torsional Divergence (BAH)

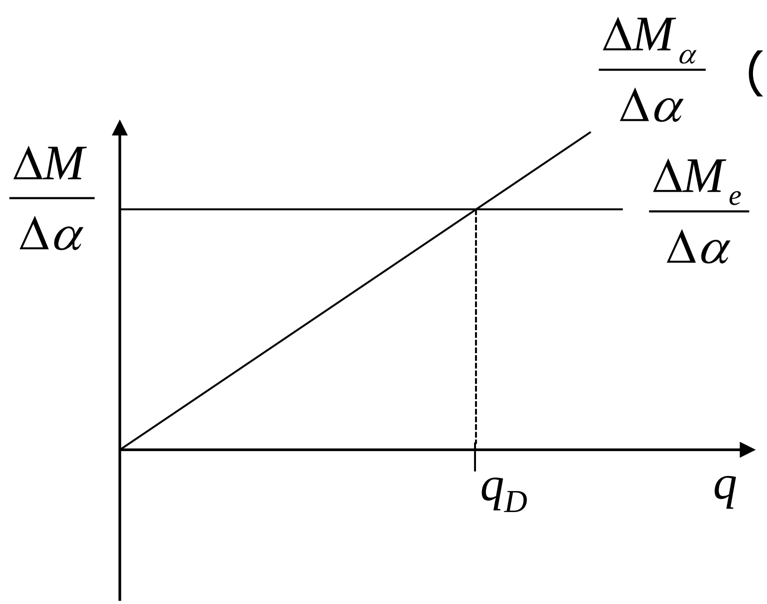
$$1 - \underbrace{C^{\theta\theta}}_{1/K_\alpha} \frac{\partial C_L}{\partial \alpha} qSe = 0$$

If e=0 (a.c. falls on e.a.)

or aft of it, wing is stable at all speeds.

- Divergence: independent of α_r (initial a.o.a) and airfoil camber $C_{M_{ac}}$
  the increase in aerodynamic moment about the elastic axis due to an arbitrary change in a.o.a is equal to the corresponding increase in elastic restoring moment

Torsional Divergence (BAH)



$$\Delta M_e = \frac{\Delta \alpha}{C^{\theta\theta}}$$

$$\Delta M_a = \frac{\partial C_L}{\partial \alpha} \Delta \alpha q S e$$