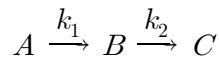


Supplementary document for lecture 07:  
Analyzing 1<sup>st</sup> order reactions in series



when  $C_{B0} = 0$  &  $C_{C0} = 0$

Firstly, obtain a solution for  $C_A$ .

$$\frac{dC_A}{dt} = -k_1 C_A$$

$$\frac{dC_A}{C_A} = -k_1 dt$$

$$\int_{C_{A0}}^{C_A} \frac{dC_A}{C_A} = -k_1 \int_0^t dt$$

$$\therefore C_A = C_{A0} e^{-k_1 t}$$

Now, applying the solution for  $C_B$  rate expression:

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B$$

$$\frac{dC_B}{dt} + k_2 C_B = k_1 C_{A0} e^{-k_1 t}$$

Recall your knowledge on engineering mathematics!

Using integrating factor to solve linear first-order differential equations

For  $\frac{dy}{dx} + p(x)y = r(x)$

$$y = e^{-\int_0^x p(x)dx} \cdot \left[ \int_0^x e^{\int_0^x p(x)dx} r(x) + const. \right]$$

$$C_B = e^{-\int_0^t k_2 dt} \cdot \left[ \int_0^t e^{\int_0^t k_2 dt} k_1 C_{A0} e^{-k_1 t} dt + const. \right]$$

$$= \frac{k_1}{k_2 - k_1} C_{A0} (e^{-k_1 t} - e^{-k_2 t}) + const. \cdot e^{-k_2 t}$$

since  $C_B = 0$  at  $t = 0$ ,  $const. = 0$

$$\therefore C_B = \frac{k_1}{k_2 - k_1} C_{A0} (e^{-k_1 t} - e^{-k_2 t})$$

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For  $C_C$ , you can easily get the solution by mass balance:

$$C_{A0} = C_A + C_B + C_C \quad (\text{at all } t)$$

$$C_C = C_{A0} - C_A - C_B = C_{A0} - k_1 C_{A0} e^{-k_1 t} - \frac{k_1}{k_2 - k_1} C_{A0} (e^{-k_1 t} - e^{-k_2 t})$$

$$\therefore C_C = C_{A0} + \frac{C_{A0}}{k_2 - k_1} (k_1 e^{-k_2 t} - k_2 e^{-k_1 t})$$

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