Supplementary document for lecture 07: Analyzing 1st order reactions in series

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$

when C_{B0} = 0 & C_{C0} = 0

Firstly, obtain a solution for C_A.

$$\frac{dC_A}{dt} = -k_1 C_A$$

$$\frac{dC_A}{C_A} = -k_1 dt$$

$$\int_{C_{A0}}^{C_A} \frac{dC_A}{C_A} = -k_1 \int_0^t dt$$

$$\underline{\therefore C_A = C_{A0} e^{-k_1 t}}$$

Now, applying the solution for C_B rate expression:

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B$$

$$\frac{dC_B}{dt} + k_2 C_B = k_1 C_{A0} e^{-k_1 t}$$

Recall your knowledge on engineering mathematics!

Using integrating factor to solve linear first-order differential equations For $\frac{dy}{dx} + p(x) y = r(x)$ $y = e^{-\int_0^x p(x)dx} \cdot \left[\int_0^x e^{\int_0^x p(x)dx} r(x) + const. \right]$

$$C_B = e^{-\int_0^t k_2 dt} \cdot \left[\int_0^t e^{\int_0^t k_2 dt} k_1 C_{A0} e^{-k_1 t} dt + const. \right]$$

$$= \frac{k_1}{k_2 - k_1} C_{A0} \left(e^{-k_1 t} - e^{-k_2 t} \right) + const. \cdot e^{-k_2 t}$$

since $C_B = 0$ at $t = 0$, $const. = 0$

$$\therefore C_B = \frac{k_1}{k_2 - k_1} C_{A0}\left(e^{-k_1 t} - e^{-k_2 t}\right)$$

For C_C, you can easily get the solution by mass balance:

$C_{A0} = C_A + C_B + C_C$ (at all t)

$$C_C = C_{A0} - C_A - C_B = C_{A0} - k_1 C_{A0} e^{-k_1 t} - \frac{k_1}{k_2 - k_1} C_{A0}\left(e^{-k_1 t} - e^{-k_2 t}\right)$$

$$\therefore C_C = C_{A0} + \frac{C_{A0}}{k_2 - k_1}\left(k_1 e^{-k_2 t} - k_2 e^{-k_1 t}\right)$$