Modeling suspended growth processes – derivation of main equations

1) Biomass mass balance

$$V\frac{dX_a}{dt}=0-\left[(Q-Q^w)X_a^e\right]-Q^wX_a^r+r_XV$$

$$V\frac{dX_a}{dt}=0 \text{ by steady state assumption}$$

$$\frac{(Q-Q^w)X_a^e+Q^wX_a^r}{VX_a}=\frac{r_X}{X_a}=\frac{Yr_{su}-bX_a}{X_a}$$

i) The left hand side of the equation equals to 1/SRT.

ii) Substrate utilization rate of the system at steady state (constant influent substrate concentration S^0 and constant effluent substrate concentration S) should be:

$$r_{su}=\frac{S^0-S}{\tau}\qquad (\tau = \text{hydraulic retention time})$$

So,

$$\frac{1}{SRT}=\frac{Y(S^0-S)}{X_a}-b$$

$$X_a=\left(\frac{SRT}{\tau}\right)\left[\frac{Y(S^0-S)}{1+b\cdot SRT}\right]$$

2) Substrate mass balance

$$V\frac{dS}{dt}=QS^0-QS+r_{su}V$$

$$V\frac{dS}{dt}=0 \text{ by steady state assumption}$$

$$S^0-S=\frac{V}{Q}\cdot r_{su}=\tau\cdot\frac{kX_aS}{K_s+S}=\tau\cdot\frac{kS}{K_s+S}\cdot\left(\frac{SRT}{\tau}\right)\left[\frac{Y(S^0-S)}{1+b\cdot SRT}\right]$$

$$1=\frac{kS}{K_s+S}\cdot\frac{Y\cdot SRT}{1+b\cdot SRT}$$

$$S=\frac{K_s+(1+b\cdot SRT)}{SRT(Yk-b)-1}$$