

NFO Supplement:

Dispersion

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yunchan@snu.ac.kr

Dispersion in Optical Fibers

Types of dispersion:

Chromatic dispersion:

Material dispersion

Waveguide dispersion: usually *smaller* than material dispersion

Short wavelength: The effective index is close to n_{core} .

Long wavelength: The effective index is close to $n_{cladding}$.

Modal dispersion:

Pulse spreading in a multimode fiber

*Dispersion is a problem in fiber communications, limiting the **bandwidth** of the fiber.*

Material Dispersion (1)

White light that is a mixture of colors can be separated into different wavelengths.

Natural Dispersion:
RAINBOW

Refractive index n is inherently a function of wavelength.

Recall: Snell's law

$$n_i \sin \theta_i = n_t \sin \theta_t$$

Material Dispersion (2)

Nonlocality in time:

$\mathbf{D}(\mathbf{x}, \omega) = \varepsilon(\omega)\mathbf{E}(\mathbf{x}, \omega)$ → For a monochromatic input

$$\begin{aligned}\rightarrow \mathbf{D}(\mathbf{x}, t) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \varepsilon(\omega)\mathbf{E}(\mathbf{x}, \omega)e^{-i\omega t} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \varepsilon(\omega) \left(\int_{-\infty}^{\infty} \mathbf{E}(\mathbf{x}, t')e^{i\omega t'} dt' \right) e^{-i\omega t} d\omega\end{aligned}$$

$$\leftarrow \varepsilon(\omega) = \varepsilon_0 [1 + \chi_e(\omega)]$$

$$\leftarrow G(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\varepsilon(\omega)/\varepsilon_0 - 1] e^{-i\omega\tau} d\omega$$

$$\rightarrow \mathbf{D}(\mathbf{x}, t) = \varepsilon_0 [\mathbf{E}(\mathbf{x}, t) + \int_{-\infty}^{\infty} G(\tau)\mathbf{E}(\mathbf{x}, t - \tau) d\tau]$$

Causality and analyticity domain of $\varepsilon(\omega)$:

$$\rightarrow \mathbf{D}(\mathbf{x}, t) = \varepsilon_0 [\mathbf{E}(\mathbf{x}, t) + \int_0^{\infty} G(\tau)\mathbf{E}(\mathbf{x}, t - \tau) d\tau]$$

$$\rightarrow \varepsilon(\omega)/\varepsilon_0 = 1 + \int_0^{\infty} G(\tau)e^{i\omega\tau} d\tau \rightarrow \varepsilon(-\omega)/\varepsilon_0 = \varepsilon^*(\omega^*)/\varepsilon_0$$

Material Dispersion (3)

Analyticity domain of $\epsilon(\omega)$:

$$\epsilon(z)/\epsilon_0 = 1 + \frac{1}{2\pi i} \oint_C \frac{[\epsilon(\omega')/\epsilon_0 - 1]}{\omega' - z} d\omega' \quad \leftarrow \text{Cauchy's theorem}$$

$$\epsilon(\omega)/\epsilon_0 = 1 + \frac{1}{2\pi i} \lim_{\delta \rightarrow 0} \left\{ \int_{-\infty}^{\omega-\delta} \frac{[\epsilon(\omega')/\epsilon_0 - 1]}{\omega' - \omega} d\omega' + \int_{\omega+\delta}^{\infty} \frac{[\epsilon(\omega')/\epsilon_0 - 1]}{\omega' - \omega} d\omega' + \pi i [\epsilon(\omega)/\epsilon_0 - 1] \right\}$$

Lower half-circle
detouring around ω

Principal part

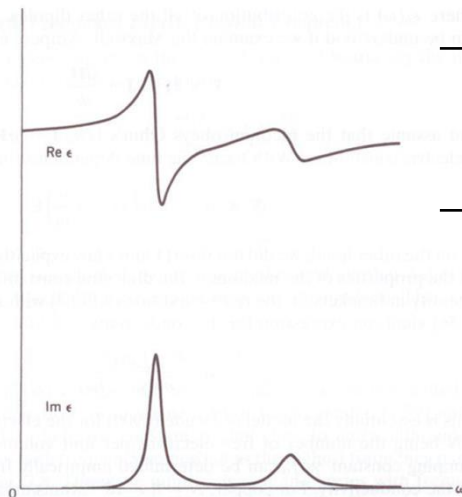
$$\rightarrow \epsilon(\omega)/\epsilon_0 = 1 + \frac{1}{\pi i} P \int_{-\infty}^{\infty} \frac{[\epsilon(\omega')/\epsilon_0 - 1]}{\omega' - \omega} d\omega'$$

$$\rightarrow \text{Re}[\epsilon(\omega)/\epsilon_0] = 1 + \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Im}[\epsilon(\omega')/\epsilon_0]}{\omega' - \omega} d\omega'$$

$$\rightarrow \text{Im}[\epsilon(\omega)/\epsilon_0] = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\text{Re}[\epsilon(\omega')/\epsilon_0 - 1]}{\omega' - \omega} d\omega'$$

\rightarrow Kramers-Kronig relations:

The real and imaginary parts are related to each other!



Material Dispersion (4)

Sellmeier equation:

$$n^2(\omega) = 1 + \sum_{j=1}^m \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2}$$

The origin of chromatic dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons.

← Recall: Kramers-Kronig relations