Supplement to Problem Set D

- **Example for the behavior of computed solutions by changing order-of-polynomial approximation (*N*) and mesh size (*K*)**
  - $\frac{\partial q}{\partial t} + a\frac{\partial q}{\partial x} = 0$ with $x \in [0, 2\pi]$, $a = -2\pi$, $q_0(x) = q(x,0) = \sin(\frac{2\pi}{\lambda}x)$

    - With periodic BC and a fixed time step for all cases, computed results shows

      i) $\|q - q_h\|_2 = O(h^{N+1}) \le C(T)h^{N+1} \simeq (c_1 + c_2 T)h^{N+1}$ with T = target time

      ii) computational cost $= C(T)K(N+1)^2$ with $h = \frac{2\pi}{K}$, N = order-of-approximation

    - Comparison of computed results reveals that higer-order approximation is beneficial for the cases requiring i) highly accurate solutions, ii) long-time integrations

1. Error analysis and convergence rate

- For a specific target time

N \ K	2	4	8	16	32	64	Convergence rate
1	-	4.0E-01	9.1E-02	2.3E-02	**5.7E-03**	1.4E-03	2.0
2	2.0E-01	4.3E-02	**6.3E-03**	8.0E-04	1.0E-04	1.3E-05	3.0
4	**3.3E-03**	3.1E-04	9.9E-06	3.2E-07	1.0E-08	3.3E-10	5.0
8	2.1E-07	2.5E-09	4.8E-12	2.2E-13	5.0E-13	6.6E-13	≈ 9.0

< Global L^2-errors >

Supplement to Problem Set D

- **2. Scaled computational cost**
  - Computational costs need to be checked during the computations for prob. 1.

N \ K	2	4	8	16	32	64
1	1.00	2.19	3.50	8.13	**19.6**	54.3
2	2.00	3.75	**7.31**	15.3	38.4	110.
4	**4.88**	8.94	20.0	45.0	115.	327.
8	15.1	32.0	68.3	163.	665.	1271.

< Example for scaled computation cost >

3. Errors for long-time integrations

- Select at least three combinations of (N, K) for similar $(N+1)\times K$ value.
- Check the L^2-errors for longer target times.

Final time (T)	π	10π	100π	1000π	2000π
$(N, K) = (2, 4)$	4.3E-02	7.8E-02	5.6E-01	>1	>1
$(N, K) = (4, 2)$	3.3E-03	4.4E-03	2.8E-02	2.6E-01	4.8E-01
$(N, K) = (4, 4)$	3.1E-04	3.3E-04	3.4E-04	7.7E-04	1.4E-03

< Example for global L^2-errors as a function of final time T >