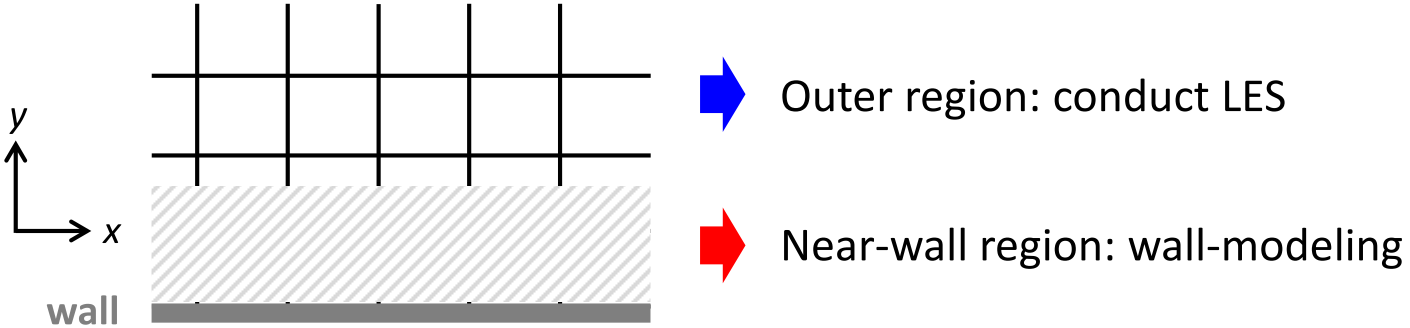
Wall-modeled LES

(or Hybrid LES/RANS)

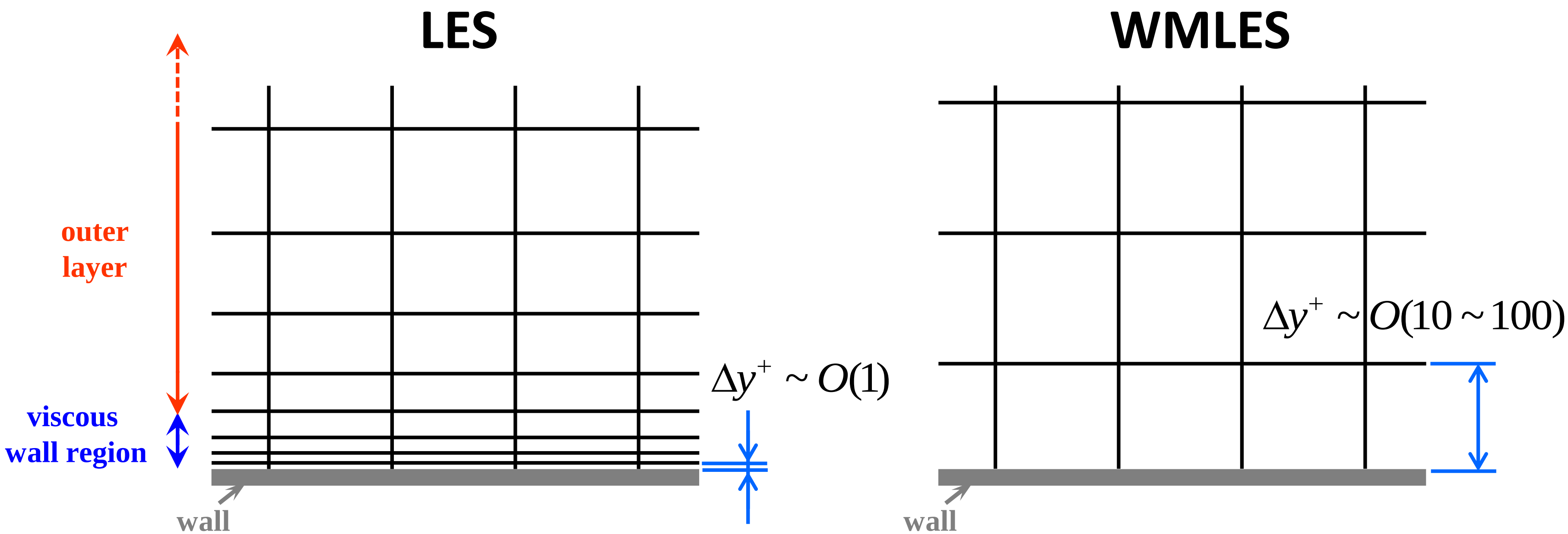
Wall-modeled LES

- Despite recent progresses in turbulent-flow predictions using Large Eddy Simulation (LES) and supercomputing technologies, an accurate and efficient prediction of turbulent flow at high Reynolds number in an engineering environment is still very difficult to achieve.
  - ► LES requires too many grid points near the wall.

- To resolve this problem, wall-modeled large eddy simulation (WMLES) techniques which model the near-wall dynamics of turbulent flow have been suggested.



Wall-modeled LES



- Grid-point requirements for LES & WMLES

	Chapman (AIAA J, 1979)	Choi & Moin (PoF, 2011)
LES	$N \sim Re^{1.8}$	$N \sim Re^{13/7}$
WMLES	$N \sim Re^{0.4}$	$N \sim Re^{1}$

➡ **WMLES is an efficient tool for the prediction of high Reynolds number flows.**

Wall-modeled LES

- **WMLES methods**

1. Detached eddy simulation (DES)

2. Wall shear stress model using outer layer information

3. Wall shear stress model using zonal approach

Wall-modeled LES

1. Detached eddy simulation (DES)

▶ Instead of using two separated turbulence models for hybrid LES/RANS, a single turbulence model acts as RANS and LES models for near-wall and detached regions, respectively, by adjusting the wall distance function (Spalart et al., 1997)

$$\nu_T = \tilde{\nu} f_{v1} \quad \text{where } f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}, \; \chi \equiv \frac{\nu}{\nu}$$

Spalart-Allmaras model (1992)

$$\frac{D\tilde{\nu}}{Dt} = \underbrace{C_{b1}\tilde{S}\tilde{\nu}}_{\text{production}} + \underbrace{\frac{1}{\sigma}\left\{\nabla\cdot\left[(\nu+\tilde{\nu})\nabla\tilde{\nu}\right] + C_{b2}\left(\nabla\tilde{\nu}\right)^2\right\}}_{\text{diffusion}} - \underbrace{C_{w1} f_w \left[\frac{\tilde{\nu}}{\tilde{d}}\right]^2}_{\text{destruction}}$$

d : wall distance

$$\tilde{d} \equiv \min(d_{RANS}, d_{LES}) \quad \left\langle \begin{array}{ll} d_{RANS} = d, & d = \text{wall distance} \\ d_{LES} = C_{DES}\Delta, & \Delta = \max(\Delta x, \Delta y, \Delta z),\; C_{DES} = 0.65 \end{array} \right.$$

Wall-modeled LES

- DES based on shear stress transport (SST, a two-equation RANS model) model (Travin et al., 2000)

$$\nu_T = \frac{a_1 k}{\max(a_1\omega, \Omega F_2)}$$

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho U_i k)}{\partial x_i} = \tilde{P}_k - \beta^* \rho k \omega + \frac{\partial}{\partial x_i}\left[(\mu + \sigma_k \mu_t)\frac{\partial k}{\partial x_i}\right]$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho U_i \omega)}{\partial x_i} = \alpha \rho S^2 + -\beta \rho \omega^2 + \frac{\partial}{\partial x_i}\left[(\mu + \sigma_\omega \mu_t)\frac{\partial \omega}{\partial x_i}\right] + 2(1-F_1)\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$

$$F_1 = \tanh\left\{\left\{\min\left[\max\left(\frac{\sqrt{k}}{\beta^*\omega\tilde{d}}, \frac{500\nu}{\tilde{d}^2\omega}\right), \frac{4\rho\sigma_{\omega 2}k}{CD_{k\omega}\tilde{d}^2}\right]\right\}^4\right\}, F_2 = \tanh\left\{\left[\max\left(\frac{2\sqrt{k}}{\beta^*\omega\tilde{d}}, \frac{500\nu}{\omega\tilde{d}^2}\right)\right]^2\right\}$$

$$\tilde{d} \equiv \min(d_{RANS}, d_{LES})$$

$$d_{RANS} = k^{0.5} / \left(C_\mu \omega\right)$$

$$d_{LES} = C_{DES}\Delta$$

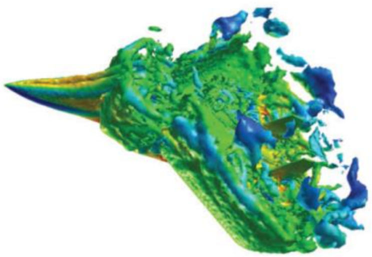
$$C_{DES} = F_1 C_{DES}^{k-\omega} + (1-F_1)F_1 C_{DES}^{k-\varepsilon}$$

$$C_{DES}^{k-\omega} = 0.78, \quad C_{DES}^{k-\varepsilon} = 0.61$$

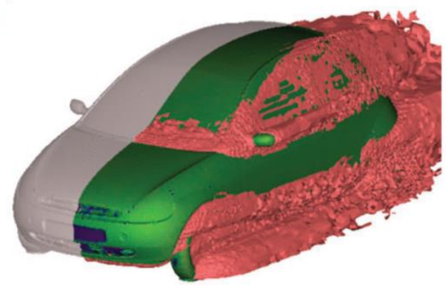
$$\Delta = \max(\Delta x, \Delta y, \Delta z)$$

Wall-modeled LES

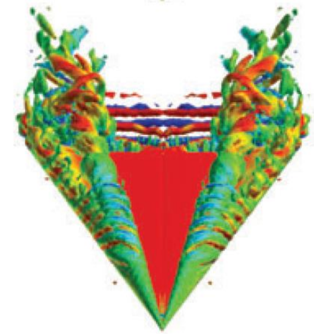
- DES is the most widely used WMLES technique for engineering applications because of the simplicity of the model implementation.



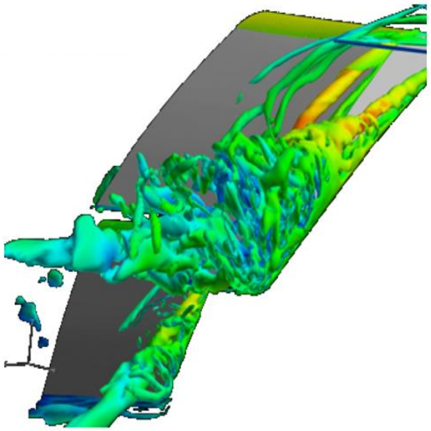
F-15 jet at a 65˚ AOA
(Forsythe *et al.* 2004)



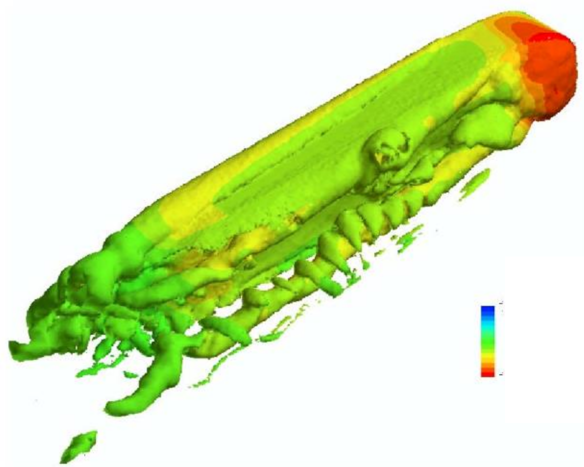
Ford automobile
(Mendonca et al. 2002)



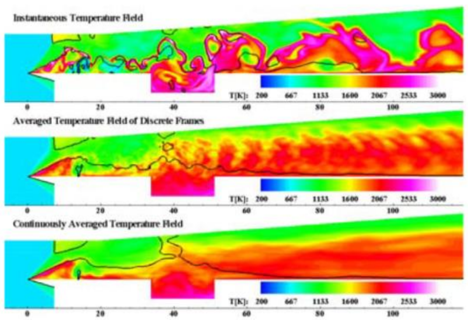
A sharp-edged delta wing
at 27˚ AOA (Morton, 2003)



Flap edge
(Langtry *et al.* 2009)



Ground transportation system
(Maddox *et al.* 2004)



Scramjet combustor
(Choi *et al.* 2007)

Wall-modeled LES

- **Issues on DES**
  - The switching location between RAN and LES is *a priori* determined by the given grid regardless of the flow characteristics. This property may induce wrong separation called grid induced separation (GIS).
  - Delayed detached eddy simulation (DDES) approach has been proposed to resolve this issue by considering flow dependent variables in determining the wall distance function (Spalart et al., 2006).

$$\tilde{d} \equiv d_{RANS} - f_d \max(0, d_{RANS} - C_{DES}\Delta)$$

$$f_d = 1 - \tanh\left(\left[8 r_d\right]^3\right)$$

$$r_d = \frac{\tilde{\nu}}{\sqrt{U_{i,j}U_{i,j}}\kappa^2 d^2}$$

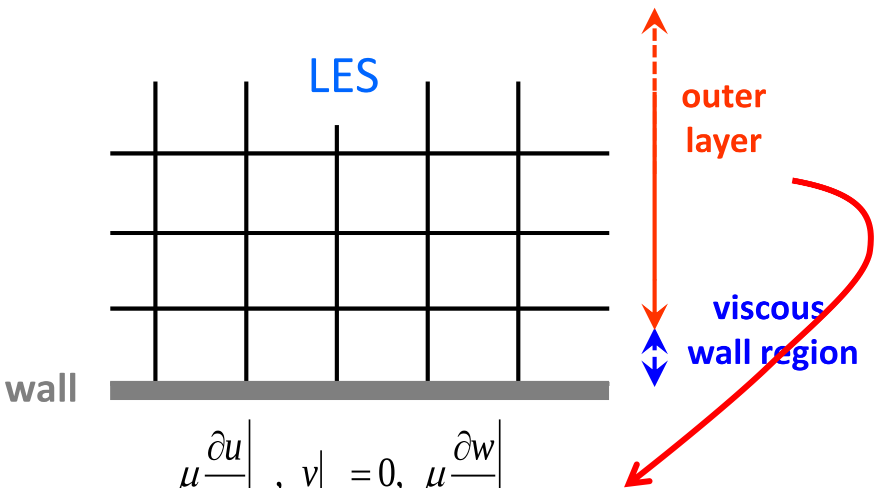
$$\Delta = \max(\Delta x, \Delta y, \Delta z),\ C_{DES} = 0.65$$

  - Definition of △: $\Delta = (\Delta x \Delta y \Delta z)^{1/3},\ \Delta = \sqrt{N_x^2 \Delta y \Delta z + N_y^2 \Delta x \Delta z + N_x^2 \Delta x \Delta y}$
  - C_{DES} = 0.65 is from isotropic turbulence. Is this the best?
  - DES requires $\Delta y^+ = O(1)$ for the first grid size above the wall to provide accurate wall shear stress, which requires a significant amount of grids in the wall-normal direction.

Wall-modeled LES

2. Wall shear stress model using outer layer information

▶ provides the instantaneous wall shear stresses (rather than no-slip b.c.) as the wall boundary condition without resolving near the wall (Schumann, 1975).



$$\mu \frac{\partial u}{\partial y}\bigg|_w, \quad v\big|_w = 0, \quad \mu \frac{\partial w}{\partial y}\bigg|_w$$

- Physical intuition (Schumann, 1975; Piomelli, 1989)
- Suboptimal control theory (Nicoud et al., 2001)

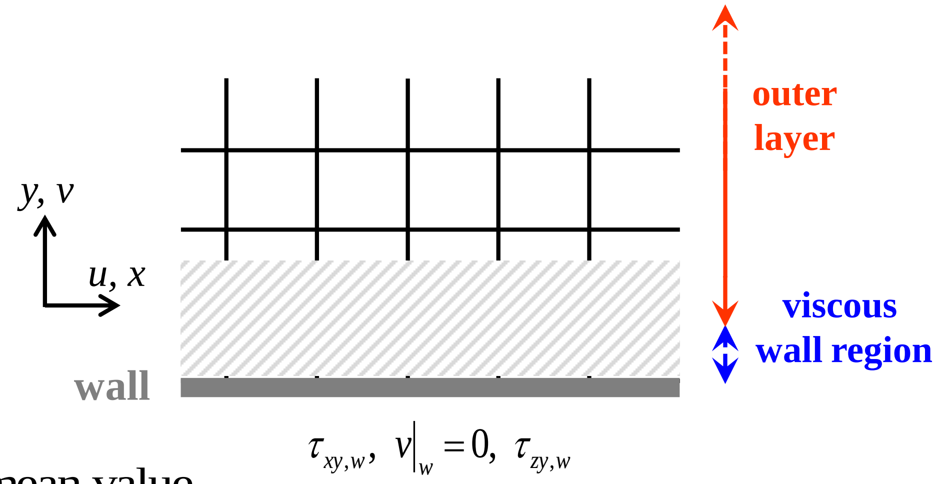
Wall-modeled LES

- Schumann (1975): Channel flow

$$\tau_{xy,w} = \mu \frac{\partial u}{\partial y}\bigg|_w = \frac{u}{\langle u \rangle}\langle \tau_w \rangle$$

$$v_w = 0$$

$$\tau_{zy,w} = \mu \frac{\partial w}{\partial y}\bigg|_w = \frac{w}{\langle u \rangle}\langle \tau_w \rangle$$



$\langle \bullet \rangle$: mean value

$\langle \tau_w \rangle$ is obtained from the pressure gradient in case of channel flow

- Grötzbach (1987) obtained the mean wall shear stress $\langle \tau_w \rangle$ from the log-law.

$$\tau_{xy,w} = \frac{u}{\langle u \rangle}\langle \tau_w \rangle$$

$$v_w = 0$$

$$\tau_{zy,w} = \frac{w}{\langle u \rangle}\langle \tau_w \rangle$$

⇦ Obtain u_τ (or τ_w) from $\dfrac{\langle u \rangle}{u_\tau} = \dfrac{1}{\kappa}\ln\left(\dfrac{y u_\tau}{\nu}\right) + B.$

Wall-modeled LES

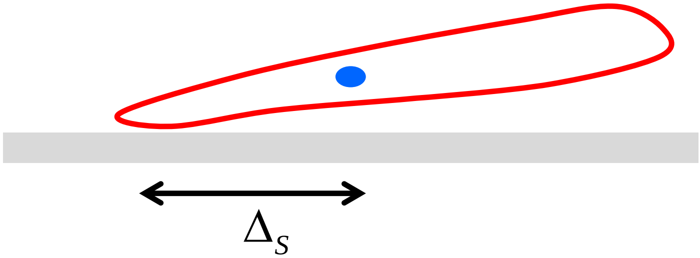
- Piomelli et al. (1989): a modified Schumann's approach

$$\tau_{xy,w}(x,z) = \frac{u(x+\Delta_s, z)}{\langle u \rangle}\langle \tau_w \rangle$$

$$v_w = 0$$

$$\tau_{zy,w}(x,z) = \frac{w(x+\Delta_s, z)}{\langle u \rangle}\langle \tau_w \rangle$$

Δ_s : downstream position



- Nicoud et al. (2001), Templeton et al. (2012): suboptimal control theory

➡ These approaches show fairly good predictions of the mean velocity profile in the outer layer.

WMLES using wall shear stress model

- Q: How about providing accurate mean (rather than instantaneous) wall shear stress as the boundary condition for WMLES?

$$\tau_{xy,w} = \frac{u}{\langle u \rangle}\langle \tau_w \rangle$$
$$v_w = 0$$
$$\tau_{zy,w} = \frac{w}{\langle u \rangle}\langle \tau_w \rangle$$

⇒

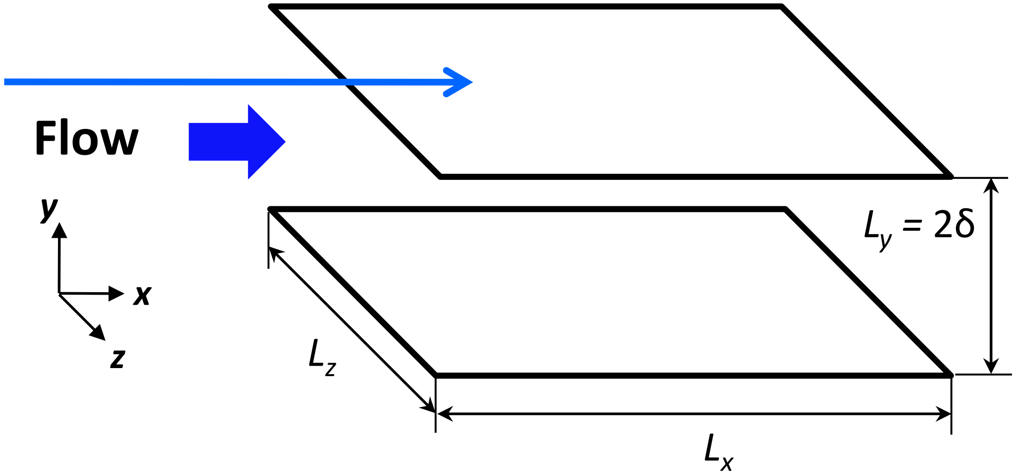
$$\tau_{xy,w} = \langle \tau_w \rangle$$
$$v_w = 0$$
$$w_w = 0$$

▶ In the framework of finite volume method, providing accurate amount of mean wall shear stress may be sufficient for the momentum transport near the wall.

WMLES of Turbulent Channel Flow

▶ Two different boundary conditions

1. $u_w = v_w = w_w = 0$
2. $\left.\frac{\partial u}{\partial y}\right|_w = \left.\frac{d\bar{u}}{dy}\right|_{Re_\tau}, v_w = w_w = 0$



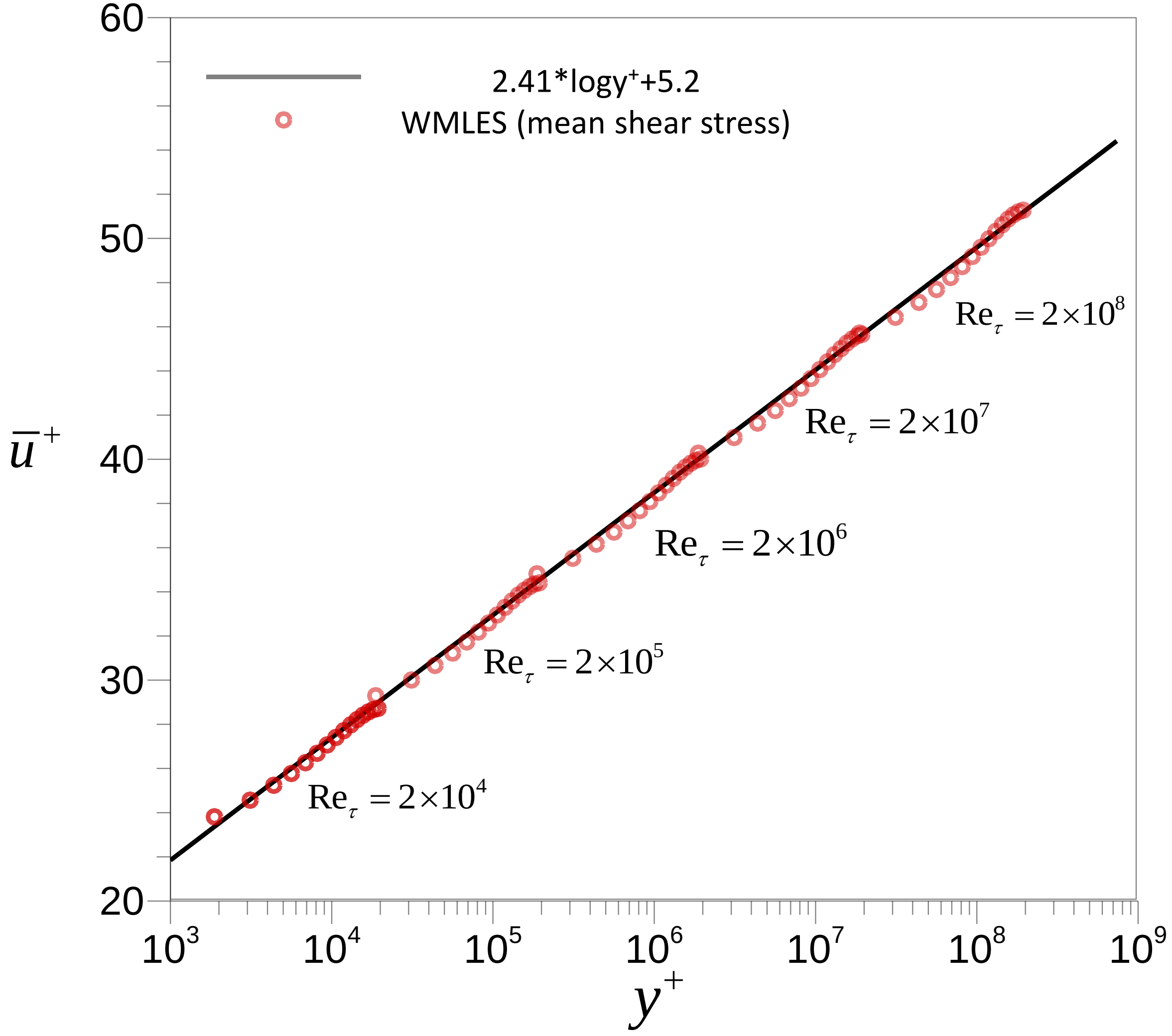
- $Re_\tau = u_\tau \delta / \nu = 2\times10^4 \sim 2\times10^8$
- Subgrid-scale (SGS) model: dynamic global model

  (Park et al. 2006, PoF; Lee et al. 2010, PoF)
- Uniform grids: 32(*x*)*33(*y*)*32(*z*)
- Domain size: $L_x=2\pi\delta$, $L_z=2\pi\delta/3$

Re_τ	Δx^+	Δy^+	Δz^+
20,000	3,927	1,250	1,309

WMLES of Turbulent Channel Flow

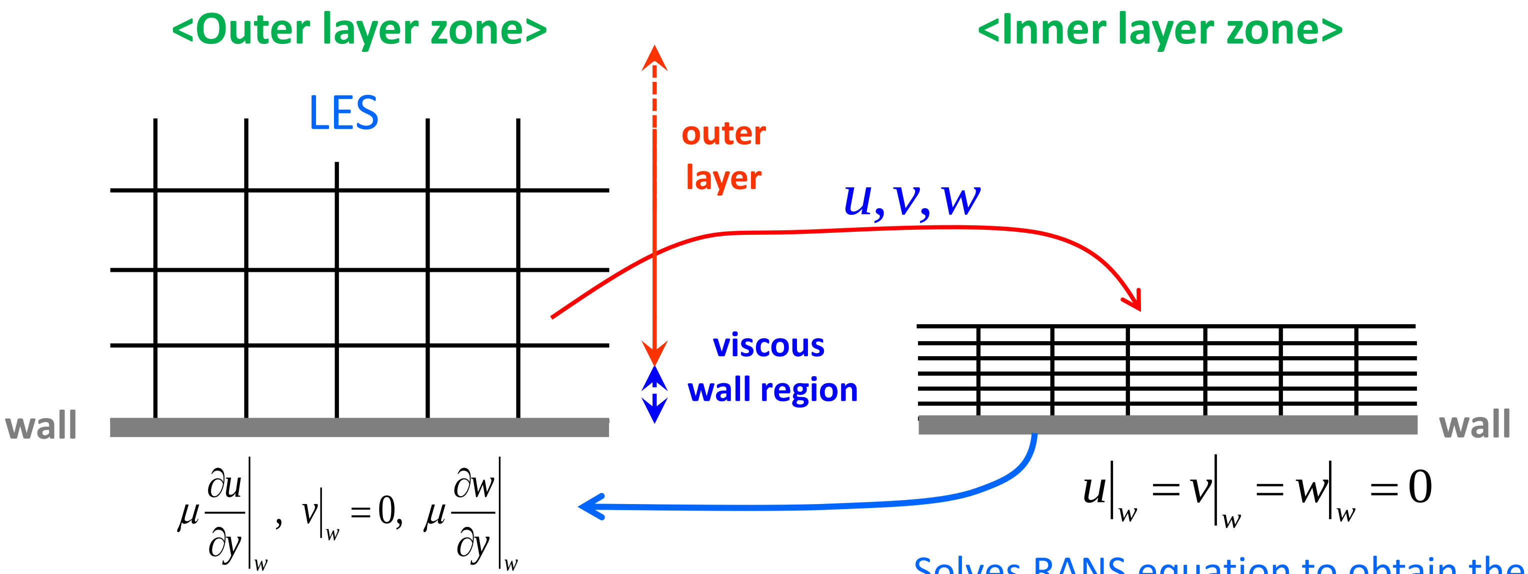
- **Mean streamwise velocity** $\mathrm{Re}_\tau = 2\times10^4 \sim 2\times10^8$ 32(*x*)*33(*y*)*32(*z*)



Wall-modeled LES

3. Wall shear stress model using zonal approach

▶ adopts resolved grid in the inner layer zone and obtains instantaneous wall shear stresses by solving additional equation (RANS) there (Balaras, Benocci & Piomelli, 1996).



Solves RANS equation to obtain the instantaneous wall shear stress:

- Mixing-length model
- Spalart-Allmaras model (1992)
- Thin boundary layer equation (Wang & Moin, 2002)

  .....

Comparison of WMLES techniques

	Wall shear stress model using outer layer information	Wall shear stress model using zonal approach	Detached eddy simulation
Grid requirement	$\Delta x^+, \Delta z^+ \sim O(100 \sim 1000)$ $\Delta y^+_{near\ wall} \sim O(100 \sim 1000)$	Outer layer: $\Delta x^+, \Delta y^+, \Delta z^+ \sim O(100 \sim 1000)$ Inner layer: $\Delta x^+, \Delta z^+ \sim O(100 \sim 1000)$ $\Delta y^+_{near\ wall} \sim O(1)$	$\Delta x^+, \Delta z^+ \sim O(100 \sim 1000)$ $\Delta y^+_{near\ wall} \sim O(1)$
Cost	less		
Current use in engineering applications		more	
Remarks	• has firm theoretical and mathematical background. • shows good performances in predicting canonical flows. • applicability to more complicated flow should be explored.	• the efficiency is better than DES because the inner layer is separately solved. • shows good results for some separated flows.	• easy to implement for flow over complex geometries. • has been widely adopted for practical engineering flows. • DES becomes more and more complicated when applied to complex flows

Landmarks on turbulence simulation

DNS
- (1972) DNS of isotropic turbulence
- (1987) DNS of turbulent channel flow
- (1988) DNS of turbulent boundary layer flow
- (1997) DNS of backward -facing step
- (2006) DNS of turbulent channel flow at Re_τ=2003

RANS
- (1925) Prandtl's mixing length model
- (1972) k-ε model
- (1975) Reynolds stress model (RSM)
- (1992) Spalart-Allmaras model
- (1994) SST k-ω model

LES
- (1963) Smagorinsky model
- (1991) Dynamic Smagorinsky model
- (2004) Vreman model
- (2006,2010) Dynamic global model???

WM LES
- (1975) Wall shear stress model
- (1979) Chapman's estimation on grid requirement
- (1996) Zonal hybrid RANS-LES
- (1997) Detached eddy simulation
- (2012) Choi & Moin's estimation on grid requirement

1920 1960 1970 1980 1990 2000 2010

Future perspectives on turbulence simulation

DNS

- Ever increasing supercomputing power will still make DNS a powerful research tool for the researches on canonical turbulent flows.

LES

- LES will be widely used for turbulent flow over/inside complex geometries at moderate Reynolds number, hopefully together with the dynamic global model.

WMLES

- Even with current supercomputing power, LES of high Reynolds number flows is still a formidable job, and therefore WMLES will play an important role for those problems.
- Much more studies are still required to make WMLES robust and accurate.