Nonlinear Static Analysis Procedure

ATC -40 : seismic evaluation and retrofit of concrete buildings

Disadvantages of equivalent linear analysis procedure using design inelastic response spectrum

Inelastic behavior cannot be directly predicted.

No direct relationship between the design earthquake load and the performance of structures such as strength and ductility

Thus, it is easy but not accurate

Disadvantages of nonlinear time history analysis

Technically there are too large uncertainties and difficulties.

Uncertainties in time history ground motions of earthquakes

Technically, it is difficult for engineers to perform nonlinear time history analysis.

Thus, it is accurate but difficult and uncertainty

Compromise is required between the two methods.

---- nonlinear static analysis method.

Nonlinear static analysis method - Nonlinear analysis but static

Similar methods

Capacity spectrum method

Equal displacement method

Direct displacement design

Capacity –based design

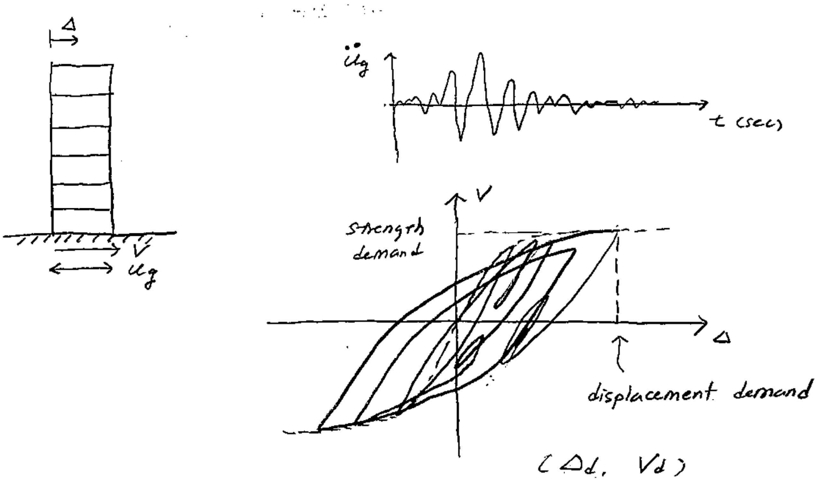
Secant method

Displacement coefficient method

How can we relate the demand to the capacity of structure when nonlinear static analysis is used?

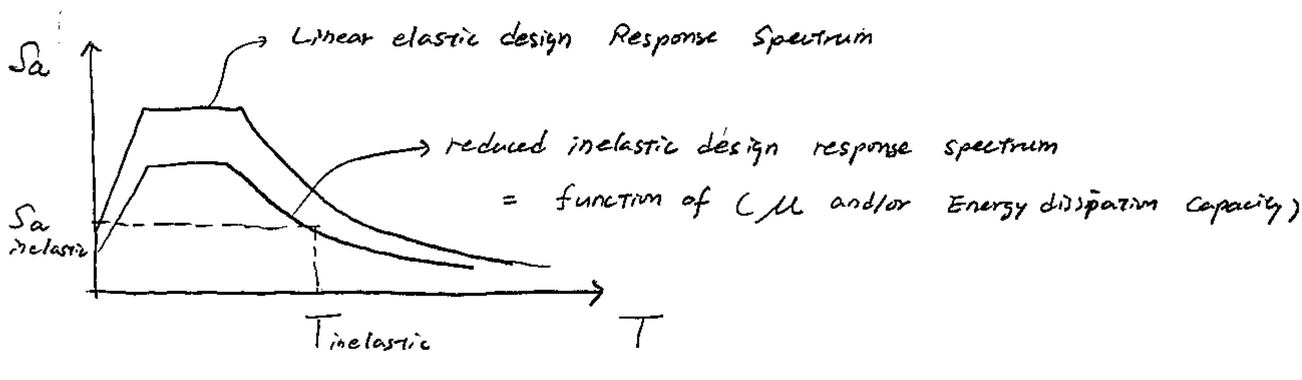
In actual behavior of structures,

Structures are subjected to repeated cyclic loading, showing large inelastic deformation (or ductility) and energy dissipation.



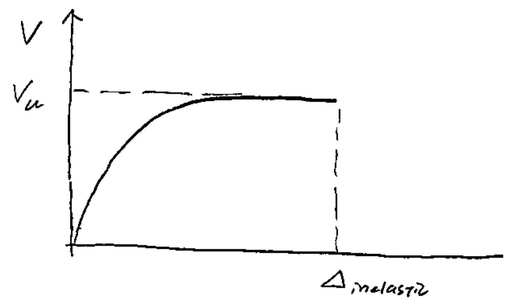
In static analysis,

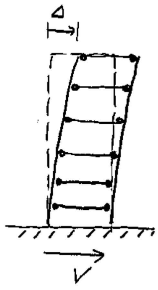
Demand is defined as the function of the dynamic period of structures, and can be reduced by considering the ductility μ or energy dissipation capacity of structures.



Capacity is defined as the relationship of load-carrying capacity and deformation of structures.

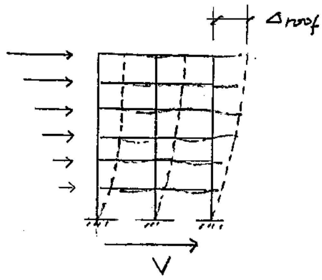
Capacity of a structure can be predicted by performing nonlinear static analysis (Pushover analysis).

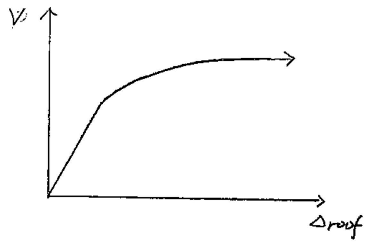




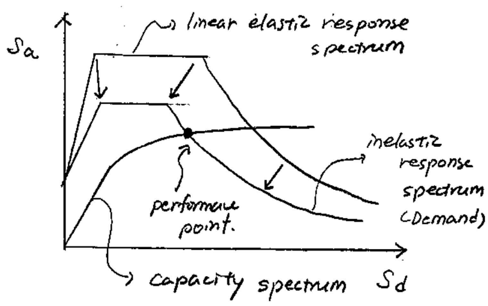
Procedure of Capacity Spectrum Method

1) Construct inelastic V(base-shear)-Δ_{roof} curve performing nonlinear static analysis (usually by computer program). In general, the lateral load distribution is not known. Thus, multiple lateral load distributions are considered, such as triangular distribution and uniform distribution.





2) Convert the V-Δ_{roof} curve to $Sa - Sd$ curve to produce the capacity spectrum

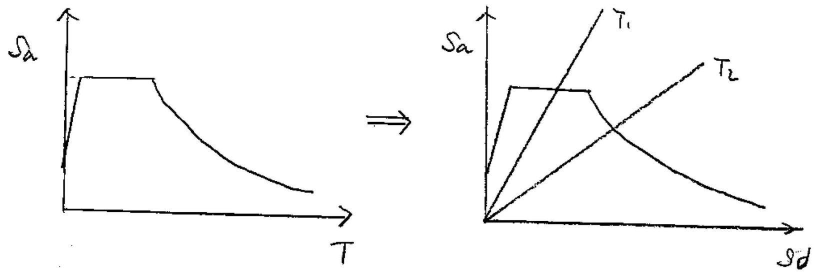


V - Δ roof curve
is converted to Sa - Sd curve

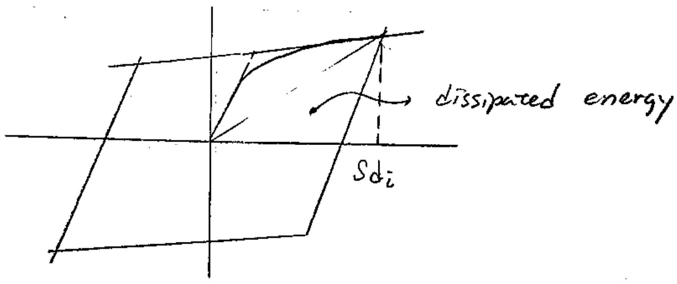
$\begin{cases} Sa : \text{Spectral acceleration} \\ Sd : \text{Spectral displacement} \end{cases}$

$\begin{cases} Sa : \text{spectral acceleration} \\ Sd : \text{spectral displacement} \end{cases}$

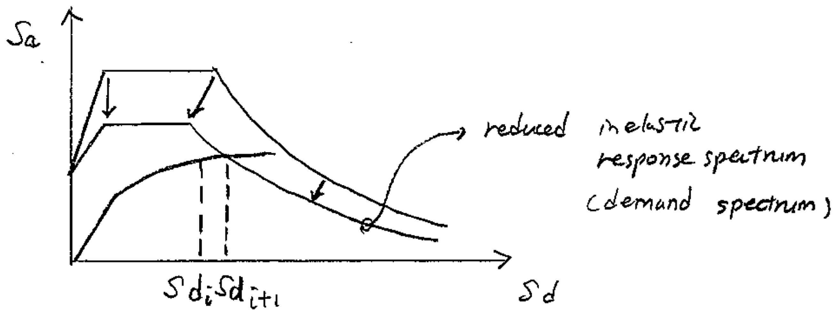
3) Construct Sa- Sd Linear elastic response spectrum (5% damping) using code-specified Sa-T relation.



4) Assuming a maximum deformation value corresponding to an assumed performance point, estimate effective damping ratio βeff corresponding to the energy dissipated during inelastic cyclic behavior



5) Using the βeff value, make the inelastic response spectrum (demand spectrum) reduced from the elastic response spectrum



6) Draw the demand spectrum and the capacity spectrum in the same $Sa - Sd$ plane.

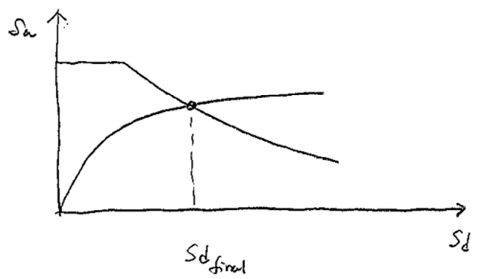
When the capacity spectrum and the demand spectrum curves have the same βeff value, the intersection point can be defined as the performance point.

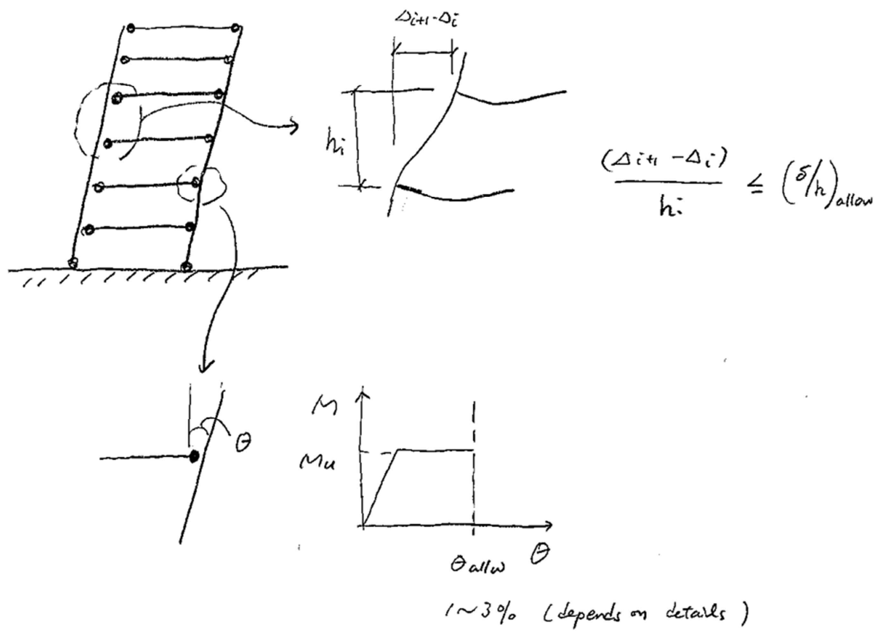
In the capacity curve, the βeff value is the function of the deformation. Thus, iterative calculations are required assuming a deformation.

I.e. If $Sd\mathrm{i} \approx Sdi_{+1}$, Sa, Sd ⇒performance point

If $Sd\mathrm{i} \neq Sdi_{+1}$, New $Sd\mathrm{i}$ is assumed and repeat 4) and 5)

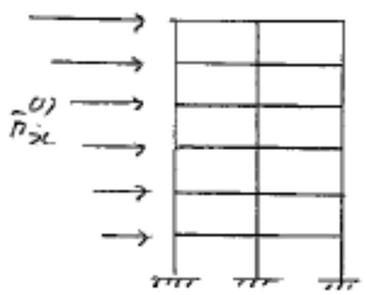
7) Check safety of overall structure and its members, particularly comparing the deformation demand with the deformation capacity.



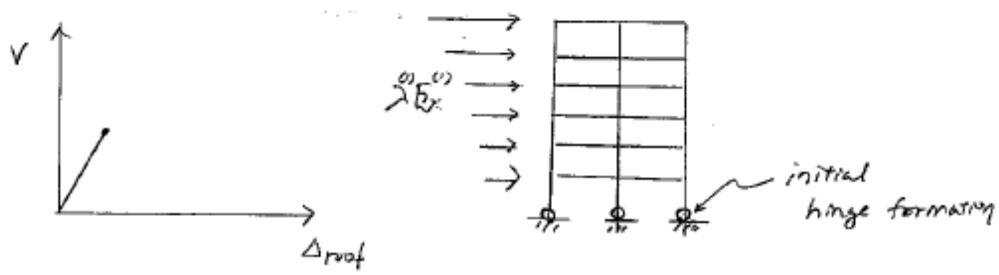


Procedure to determine capacity curve (V-Δ curve)

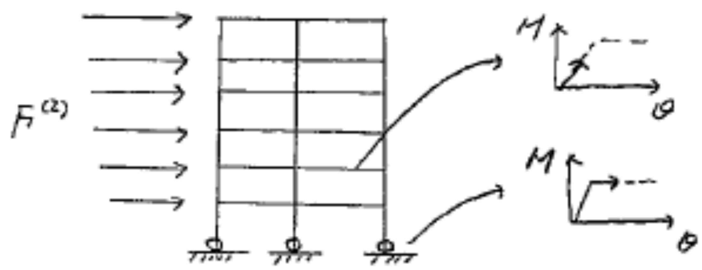
1. Create a structural model using numerical nonlinear analysis programs (Drain 2D, MiDas, SAP, Perform 3D, etc)
2. Multiple lateral load distributions are assumed. Generally, triangular and uniform distributions are used.
3. For a given lateral load distribution, structural analysis is performed.



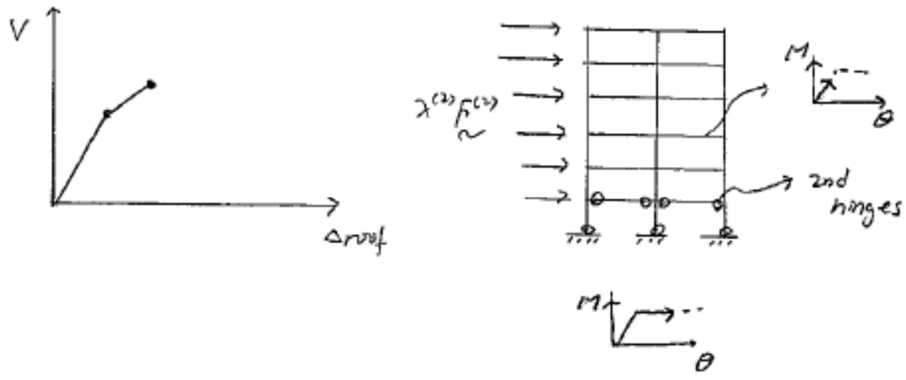
4. Until first yielding, linear elastic behavior is obtained.



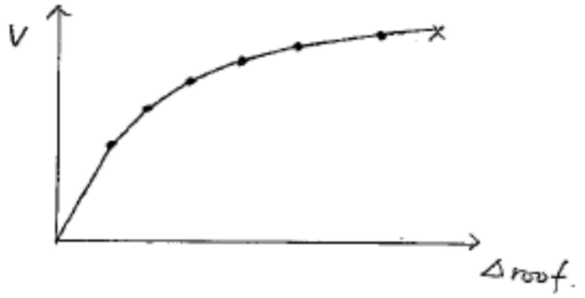
5. After first yielding, the stiffness at each plastic hinge is reduced.



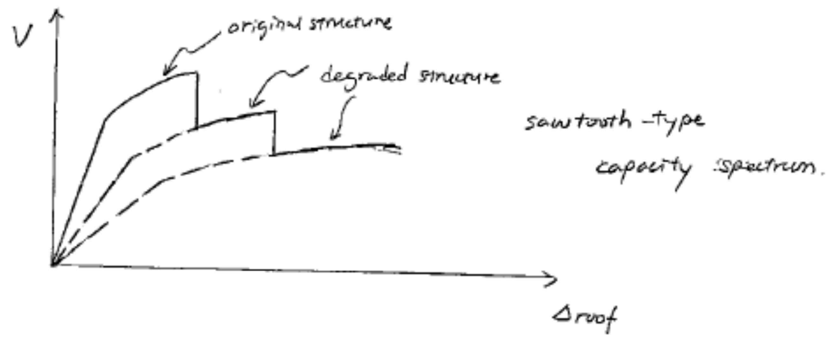
6. At the second yielding, again, the stiffness of each new plastic hinge is reduced.



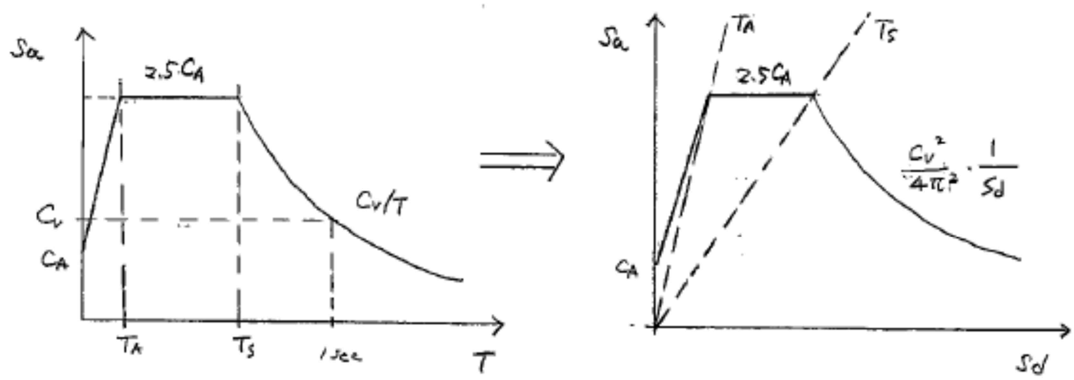
7. Repeat structural analysis until the target displacement is reached.



8. If the structure has brittle elements or degraded elements, the overall behavior shows step-down curves.



Construction of Linear Elastic Response Spectrum (5% damping)



Traditional Sa - T spectrum

Sa - Sd response spectrum (ADRS)

C_A : seismic coefficient based on peak ground acceleration

C_V : seismic coefficient based on peak ground velocity

$$\left.\begin{array}{l} T_S = C_V / 2.5C_A \\ T_A = 0.2T_S \end{array}\right\} \text{control periods}$$

Sa : spectral acceleration (Non dimension)

Sd : spectral displacement

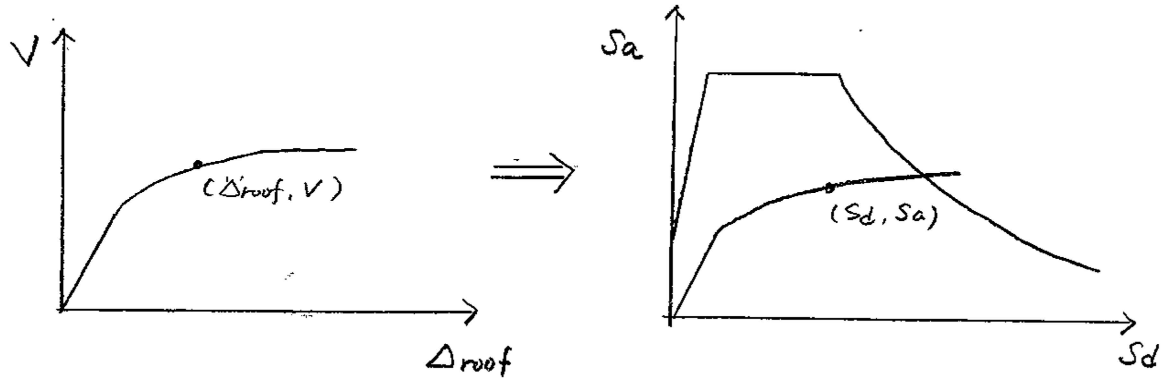
$$Sd = \frac{1}{\omega^2} Sa = \frac{T^2}{4\pi^2} Sa$$

$$Sa = C_V / T$$

$$Sa^2 = C_V{}^2 / T^2 = C_V{}^2 Sa / (4\pi^2 Sd)$$

$$Sa = \frac{C_V{}^2}{4\pi^2} \frac{1}{Sd}$$

Conversion of Capacity V-Δ curve to Capacity Sa-Sd curve



We need to set up the relations between V and Sa, and between Δ_{roof} and Sd

Δ_{roof} - Sd **relationship**

Dynamic Equilibrium equation of a system subjected to EQ movement

$$\underset{\sim}{M}\ddot{\underset{\sim}{U}} + \underset{\sim}{C}\dot{\underset{\sim}{U}} + \underset{\sim}{K}\underset{\sim}{U} = -\underset{\sim}{M}\underset{\sim}{r}\ddot{\underset{\sim}{U}}g$$

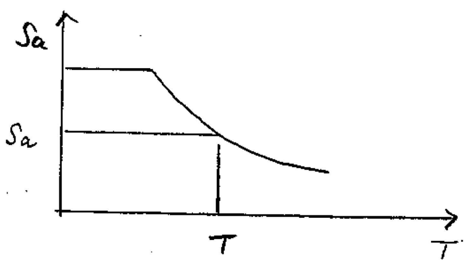
This Equation is decomposed into n – one dof systems

1st mode

$$\underset{\sim}{\phi}_1^T \underset{\sim}{M}\underset{\sim}{\phi}_1 \ddot{Y}_1 + \underset{\sim}{\phi}_1^T \underset{\sim}{C}\underset{\sim}{\phi}_1 \dot{Y}_1 + \underset{\sim}{\phi}_1^T \underset{\sim}{K}\underset{\sim}{\phi}_1 \underset{\sim}{Y}_1 = -\underset{\sim}{\phi}_1^T \underset{\sim}{M}\underset{\sim}{r}\ddot{\underset{\sim}{U}}g$$

$$\ddot{Y}_1 + 2\xi\omega_1\dot{Y}_1 + \omega_1^2 Y_{11} = -\underline{\left(\frac{\underset{\sim}{\phi}_1^T \underset{\sim}{M}\underset{\sim}{r}}{\underset{\sim}{\phi}_1^T \underset{\sim}{M}\underset{\sim}{\phi}_1}\right)}\ddot{\underset{\sim}{U}}g = -(PF_1)\ddot{\underset{\sim}{U}}g$$

$$\begin{aligned} Y_{1(\max)} &= Sd(PF_1) \\ &= (\frac{1}{\omega^2}Sa)(PF_1) \\ &= (\frac{T^2}{4\pi^2}Sa)(PF_1) \end{aligned}$$



PF_1 : participation factor of mode 1

$$\underset{\sim}{u} = Y_1 \underset{\sim}{\phi}_1 + Y_2 \underset{\sim}{\phi}_2 + \cdots$$

If the first mode governs the response,

$$\underset{\sim}{u} \approx Y_1 \underset{\sim}{\phi}_1 = Sd(PF_1)\underset{\sim}{\phi}_1$$

$$u_{roof} = \Delta_{roof} = Sd(PF_1) \cdot \phi_{1,roof}$$

$$Sd = \frac{\Delta_{roof}}{(PF_1) \cdot \phi_{1,roof}}$$

V - Sa **relationship**

Story Force vector dominated by 1st mode

$$\underset{\sim}{F} \approx \underset{\sim}{M} \ddot{Y}_{1(\max)} \phi_1$$

$$\ddot{Y}_{1(\max)} = Sa(PF_1)$$

$$\underset{\sim}{F} = Sa(PF_1)\underset{\sim}{M}\underset{\sim}{\phi}_1$$

$$V = \underset{\sim}{r}^T \underset{\sim}{F} = Sa(PF_1)\underset{\sim}{r}^T \underset{\sim}{M}\underset{\sim}{\phi}_1$$

Thus, $Sa = \dfrac{V}{(PF_1)\underset{\sim}{r}^T \underset{\sim}{M}\underset{\sim}{\phi}_1}$ or $= \left(\dfrac{V/W}{\dfrac{(PF_1)\underset{\sim}{r}^T \underset{\sim}{M}\underset{\sim}{\phi}_1}{W}} \right) = \dfrac{V/W}{M_{eff}/W} = \dfrac{V/W}{\alpha_1}$

M_{eff} = effective mass

α_1 = effective mass coefficient

In other expressions

Participation factor

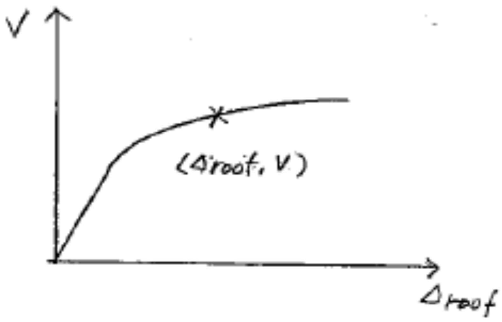
$$(PF_1) = \left(\frac{\underset{\sim}{\phi}_1^T \underset{\sim}{M} \underset{\sim}{r}}{\underset{\sim}{\phi}_1^T \underset{\sim}{M} \underset{\sim}{\phi}_1} \right) = \frac{\sum_{i=1}^{N} w_i \phi_{1i} / \mathrm{g}}{\sum_{i=1}^{N} w_i \phi_{1\,i}^2 / \mathrm{g}} \qquad \begin{cases} N: \ no\ of\ story \\ w: \ story\ \mathrm{weight} \end{cases}$$

Effective mass

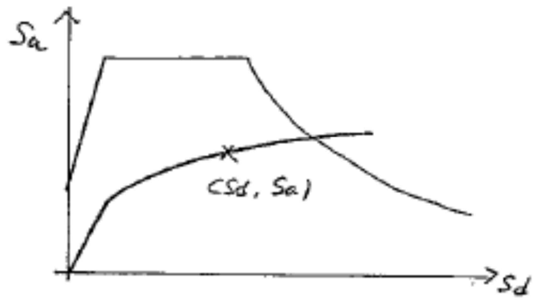
$$(M_{\mathrm{eff}_1}) = (PF_1)\, \underset{\sim}{\mathrm{r}}^T \, \underset{\sim}{\mathrm{M}} \underset{\sim}{\phi}_1 = \frac{(\underset{\sim}{\phi}_1^T \underset{\sim}{\mathrm{M}} \underset{\sim}{r})^2}{\underset{\sim}{\phi}_1^T \underset{\sim}{\mathrm{M}} \underset{\sim}{\phi}_1} = \frac{(\sum^{N} w_i \phi_{1i} / g)^2}{\sum^{N} w_i \phi_{1\,i}^2 / g}$$

Effective mass coefficient: α_1

$$\alpha_1 = \frac{M_{eff_1}}{W} = \frac{(\sum^{N} w_i \phi_{1i} / g)^2}{(\sum^{N} w_1 / g (\sum^{N} (w_i \phi_{i\,1}^2) / g)}$$



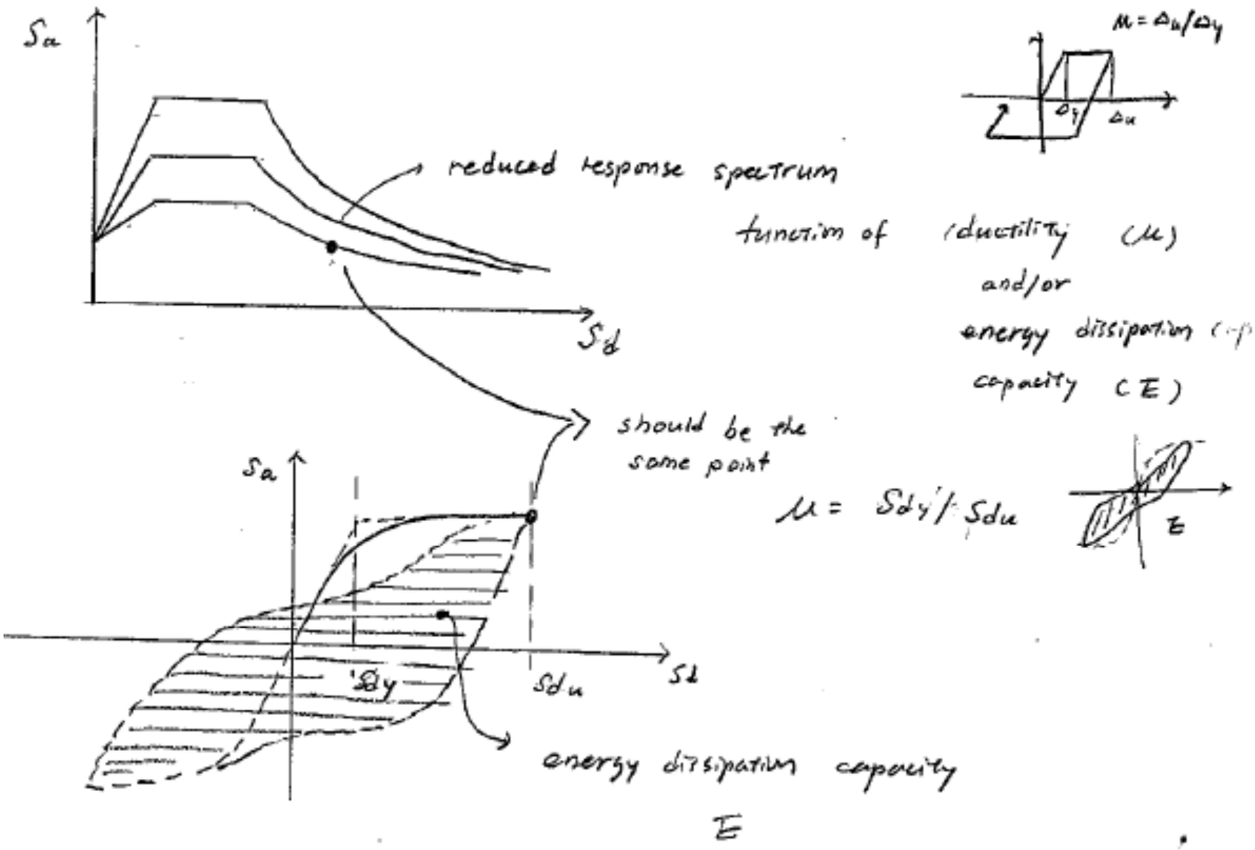
capacity curve



capacity spectrum

$$\begin{cases} Sa = \dfrac{V / W}{\alpha_1} = \dfrac{V}{(PF_1) \underline{r}^T \underline{M} \underline{\phi}_1} \\ Sd = \dfrac{\Delta_{roof}}{(PF_1) \phi_{1roof}} \end{cases}$$

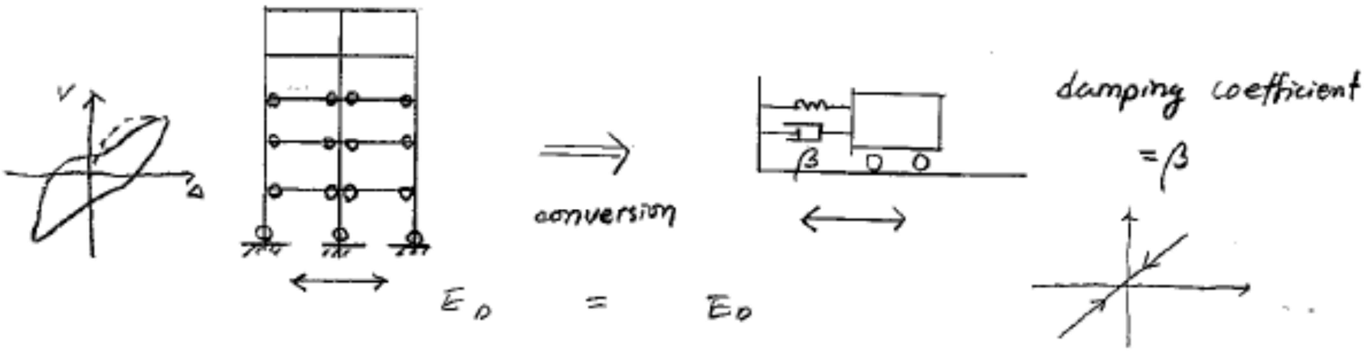
Determination of performance point (intersection point between the capacity and demand)



Reduced response spectrum (demand spectrum)

The cyclic hysteretic response of actual system is converted to the response of an equivalent linear elastic damped system with the same capacity of energy dissipation.

(only when linear system is used, the force spectrum is the same as displacement spectrum.)



A single degree of freedom system

$$M\ddot{U} + C\dot{U} + KU = P(t) = P_0 \sin \omega t$$

The energy dissipated by viscous damping in one cycle of harmonic vibration

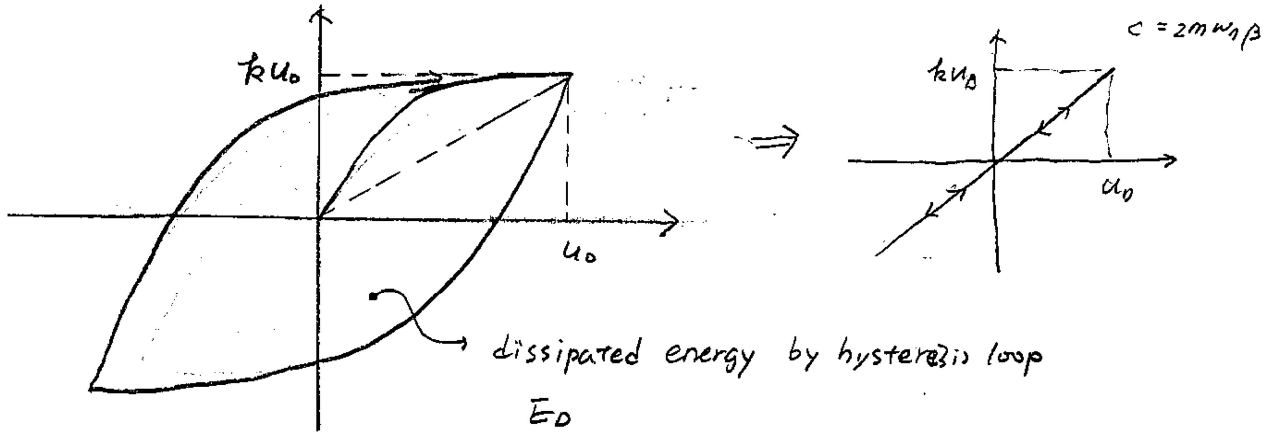
$$E_D = \int f_D du = \int_0^{2\pi/\omega} (c\dot{u})\dot{u}dt = \int_0^{2\pi/\omega} c\dot{u}^2 dt$$

$$= c\int_0^{2\pi/w} [\omega u_0 \cos(\omega t - \phi)]^2 dt = \pi c \omega u_0{}^2$$

$$= 2\pi\xi \frac{\omega}{\omega_n} k u_0{}^2 \qquad \omega_n = \sqrt{\frac{k}{m}} \qquad \xi = \frac{c}{2m\omega_n} = \beta$$

If $\omega \approx \omega_{n,}$ $E_D = 2\pi\beta k u_0{}^2 = 4\pi\beta \frac{k u_0^2}{2}$

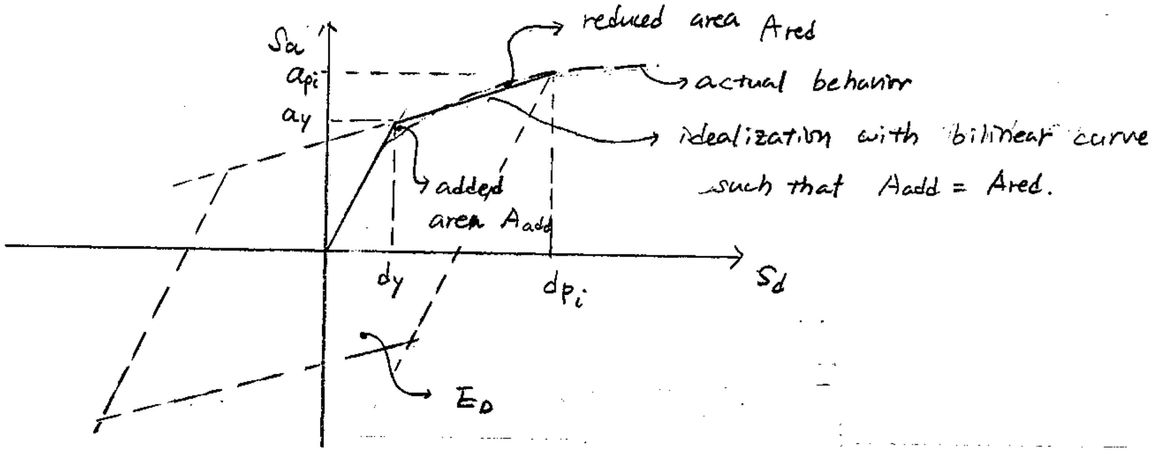
Set $E_{s0}(\text{strain energy}) = \frac{ku_0^2}{2}$, $E_D = 4\pi\beta E_{s0}$



With the known value of E_D, we can calculate the damping coefficient ξ or β

$$\beta = \frac{1}{4\pi}\frac{E_D}{E_{s0}}$$

Estimation of Damping coefficient $\beta_0, \beta_{eg}, \beta_{eff}$



E_D = Energy dissipated by damping

= Area of enclosed by hysteresis loop

= Area of parallelogram

E_{S0} = Maximum strain energy

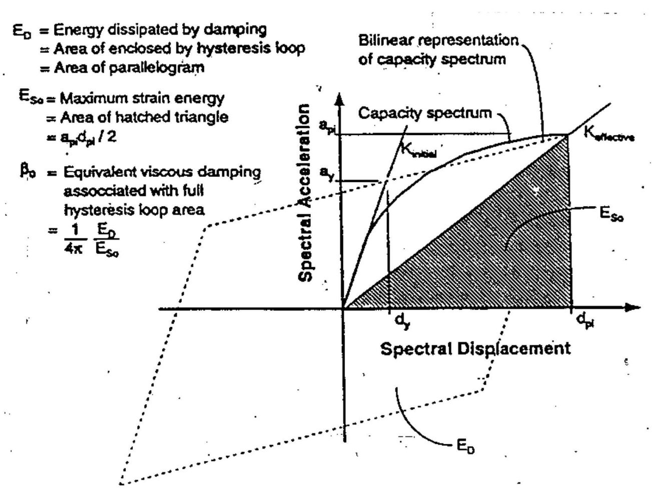
= Area of hatched triangle

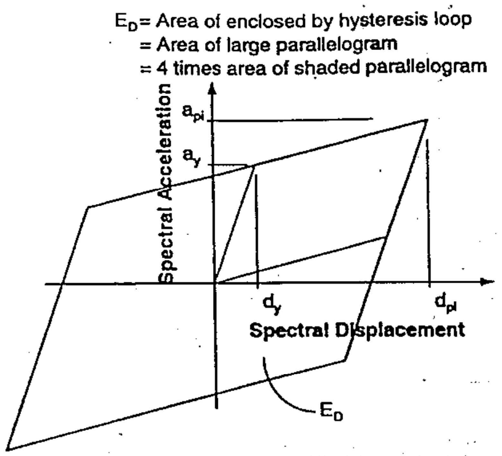
$= a_{pi} d_{pi} / 2$

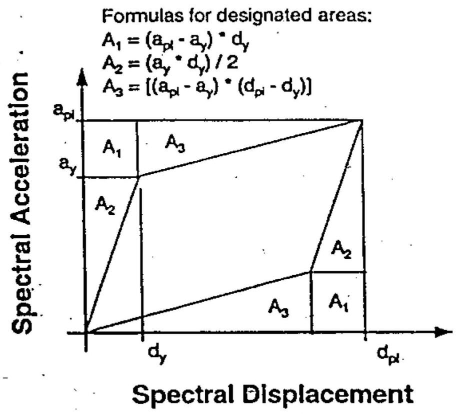
β_0 = Equivalent viscous damping associated

with full hysteresis loop area

= $\dfrac{1}{4\pi} \dfrac{E_D}{E_{S0}}$







E_D = Area of enclosed by hysteresis loop

= Area of large parallelogram

= 4 times area of shaded parallelogram

Formulas for designated areas:

$$A_1 = (a_{pi} \cdot a_y) \cdot d_y$$
$$A_2 = (a_y \cdot d_y)/2$$
$$A_3 = [(a_{pi} - a_y) \cdot (d_{pi} - d_y)]$$

$$E_D = 4(a_{pi} d_{pi} - 2A_1 - 2A_2 - 2A_3)$$

$$= 4[a_{pi} d_{pi} - a_y d_y - (d_{pi} - d_y)(a_{pi} - a_y) - 2d_y (a_{pi} - a_y)]$$

$$= 4(a_y d_{pi} - d_y a_{pi})$$

$$E_{so} = a_{pi} d_{pi}/2$$

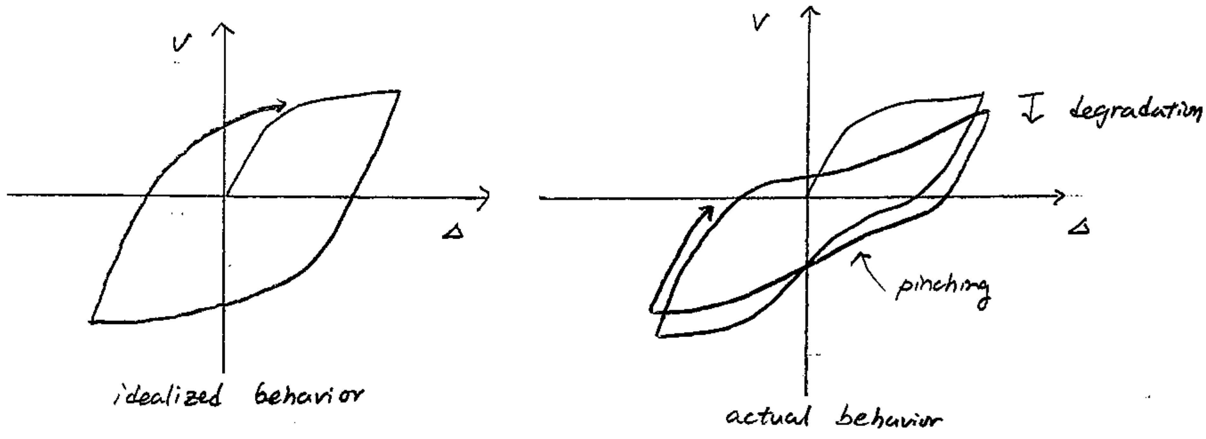
$$\beta_0 = \frac{1}{4\pi}\frac{E_D}{E_{so}} = \frac{1}{4\pi}\frac{4(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}/2} = \frac{2}{\pi}\frac{a_y d_{pi} - d_y a_{pi}}{a_{pi} d_{pi}}$$

$$= \frac{0.637(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}}$$

$$= \frac{63.7(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}}$$ (in percentage)

$\beta_{eq} = \beta_0 + 5$ (equivalent viscous damping including 5% inherent in the structure)

Modification of damping coefficient for degraded and/or pinched cyclic behavior



The idealized hysteresis loop overestimates realistic levels of damping. Therefore we introduce effective viscous damping $\beta_{eff.}$

$$\beta_{eff} = \kappa\beta_0 + 5 = \frac{63.7\kappa(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}} + 5$$

κ : damping modification factor

Table 8-1. Values for Damping Modification Factor, κ

Structural Behavior Type[1]	β_0 (percent)	κ
Type A[2]	≤16.25	1.0
	>16.25	$1.13 - \frac{0.51(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}}$
Type B	≤25	0.67
	>25	$0.845 - \frac{0.446(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}}$
Type C	Any Value	0.33

1. See Table 8-4 for structural behavior types.
2. The formulas are derived from Tables of spectrum reduction factors. B(or B1). Specified for the design of

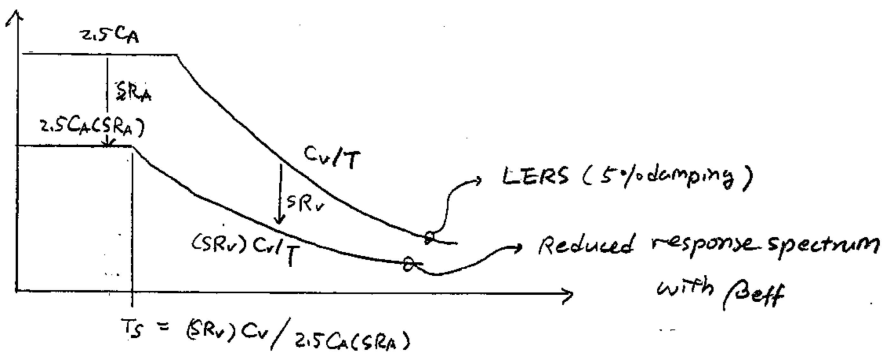
base isolated buildings in the 1991 UBC, 1994 UBC and 1994 NEHRP Provisions. The formulas created for this document give the same results as are in the Tables in the other documents.

Table 8-4. Structural Behavior Types

Shaking Duration[1]	Essentially New Building[2]	Average Existing Building[3]	Poor Existing Building[4]
Short	Type A	Type B	Type C
Long	Type B	Type C	Type C

1. See Section 4.5.2 for criteria.
2. Buildings whose primary elements make up an essentially new lateral system and little strength or stiffness is contributed by noncomplying elements.
3. Buildings whose primary elements are combinations of existing and new elements, or better than average existing systems.
4. Buildings whose primary elements make up noncomplying lateral force systems with poor or unreliable hysteretic behavior.

Construction of reduced Response Spectrum



$$SR_A = \frac{1}{B_S} \approx \frac{3.21 - 0.68\ln(\beta_{eff})}{2.12} = \frac{3.21 - 0.68\ln\left[\frac{63.7\kappa(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}} + 5\right]}{2.12} \tag{8-9}$$

$\geq$ Value in Table 8-2

$$SR_V = \frac{1}{B_L} \approx \frac{2.31 - 0.41\ln(\beta_{eff})}{1.65} = \frac{2.31 - 0.41\ln\left[\frac{63.7\kappa(a_y d_{pi} - d_y a_{pi})}{a_{pi} d_{pi}} + 5\right]}{1.65} \tag{8-10}$$

$\geq$ Value in Table 8-2

Table 8-2. Minimum Allowable SR_A and SR_V Values[1]

Structural Behavior Type[2]	SR_A	SR_V
Type A[2]	0.33	0.50
Type B	0.44	0.56
Type C	0.56	0.67

1. Values for SR_A and SR_V shall not be less than those shown in this Table
2. See Table 8-4 for structural behavior types.

Table 8-3. Spectral Reduction Factors, SR_A = $1/B_S$ and SR_V = $1/B_L$

	Behavior Type A [1]			Behavior Type B [1]			Behavior Type C [1]		
β_0 (percent)	β_{eff}	SRA (1/Bs)	SRv (1/BL)	β_{eff}	SRA (1/Bs)	SRv (1/BL)	β_{eff}	SRA (1/Bs)	SRv (1/BL)
0	5	1.00	1.00	5	1.00	1.00	5	1.00	1.00
5	10	0.78	0.83	8	0.83	0.87	7	0.91	0.93
15	20	0.55	0.66	15	0.64	0.73	10	0.78	0.83
25	28	0.44	0.57	22	0.53	0.63	13	0.69	0.76
35	35	0.38	0.52	26	0.47	0.59	17	0.61	0.70
≥45	40	0.33	0.50[2]	29	0.44	0.56	20	0.56	0.67[2]

1. Structural behavior type, see Table 8-4.
2. Controlled by minimum allowable value for SRv, see Table 8.2

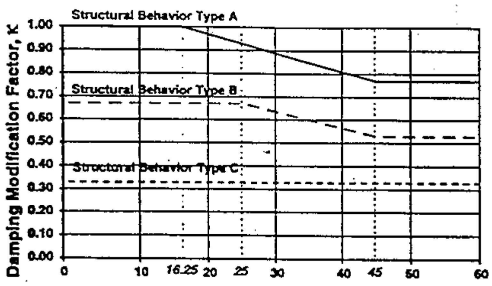


Figure 8-15. Damping Modification Factor, κ, for Structural Behavior Types A, B and C

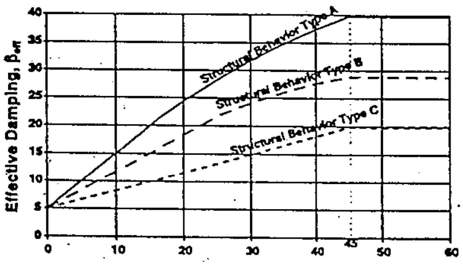


Figure 8-16. Effective Damping, β_{eff}, for Structural Behavior Types A, B and C

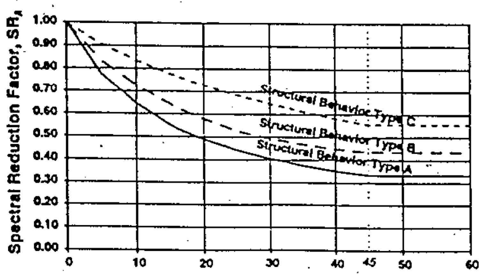


Figure 8-17. Spectral Reduction Factor, SR_A, for Structural Behavior Types A, B and C

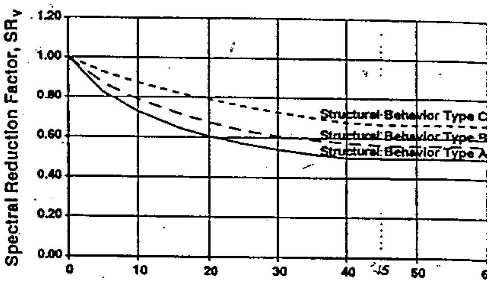


Figure 8-18. Spectral Reduction Factor, SR_V, for Structural Behavior Types A, B and C

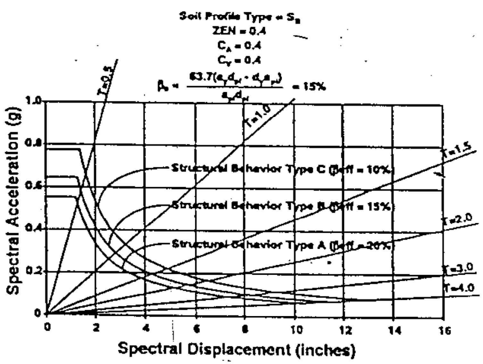
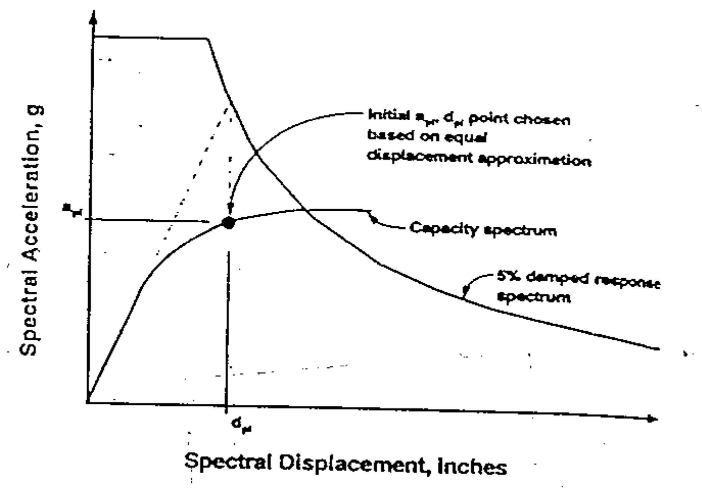


Figure 8-19. Example ADRS Response Spectra for Structural Behavior Types A, B and C

Calculation of Performance Point

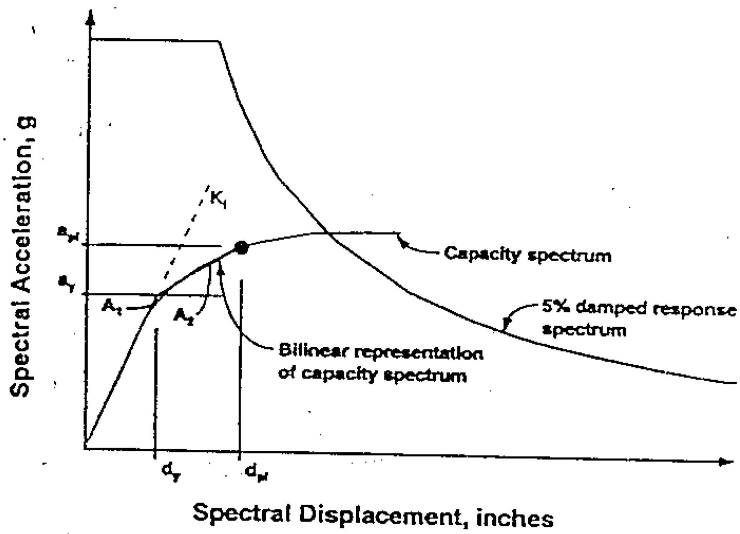
1) select a trial performance point, a_{pi}, d_{pi}

A first choice of point a_{pi}, d_{pi} could be the displacement approximation



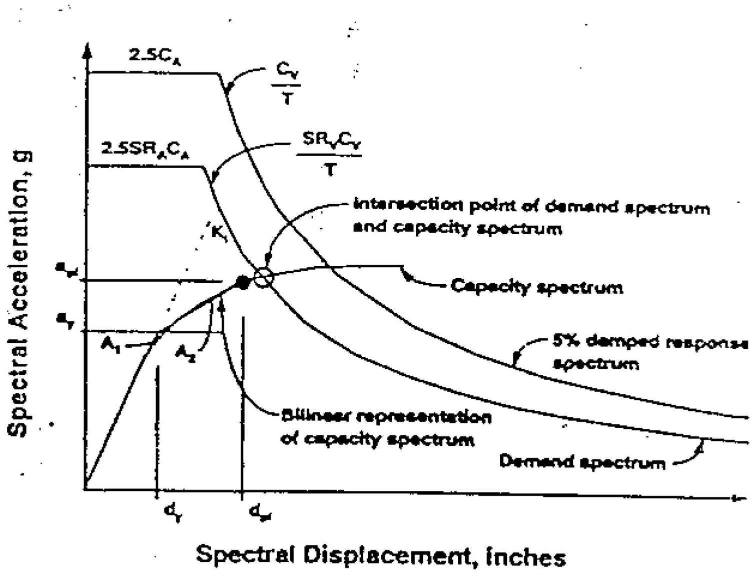
2) Develop a bilinear representation of the capacity spectrum.

Calculate effective damping $\beta eff = K\beta_0 + 5$



3) Calculate the spectral reduction factors.

Draw the demand spectrum.



4) If the displacement at which the demand spectrum intersects the capacity spectrum, d_i, is within acceptable range(eg. $0.95d_{pi} \le d_i \le 1.05d_{pi}$),

then the trial performance point a_{pi}, d_{pi} is determined as the point $a_P d_P$.

and the displacement, d_P, represents the maximum structural displacement expected for the demand EQ.

5) If the demand spectrum does not intersect the capacity spectrum within acceptable tolerance, the $a_{Pi} = a_{i,}$ $d_{Pi} = d_i$ repeat 1) to 4)

Advantages of capacity Spectrum Method

1. Nonlinear Response of the structure against Earthquake Excitation can be estimated with reasonable precision.

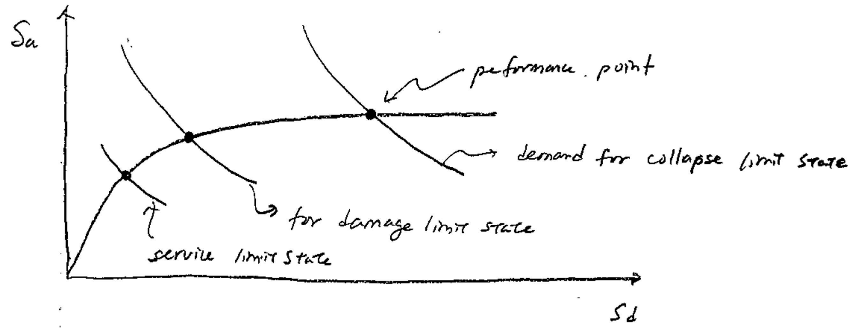
   => No Specified R factor

2. Failures of structure and members can be examined, directly.

   -overall ductility of structure

   -plastic rotation at each plastic hinge

3. Responses of the structure with respect to multi-level performance-requirements can be evaluated at the same time.



4. Effect of overstrength due to material, Design method, etc. can be directly addressed.

Uncertainties in Capacity Spectrum Method

1. Assumed lateral load profile (high mode effect) is used. In general, the higher mode effect is not considered. - multiple lateral load distributions should be used.

2. Demand spectrum is constructed by using an equivalent linear system which is different from the actual energy dissipation mechanism.

3. It is difficult to accurately estimate the energy dissipation capacity without performing cyclic load analysis.