Wireless Channel Model

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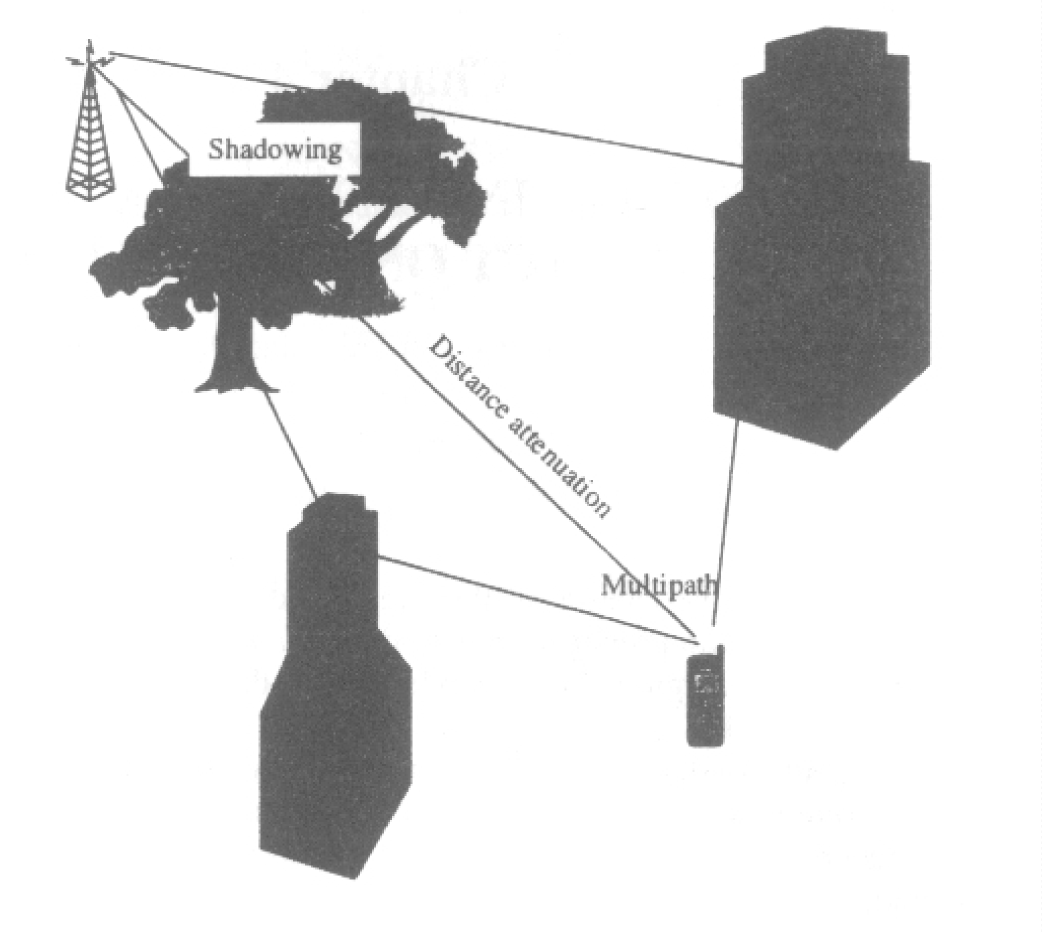
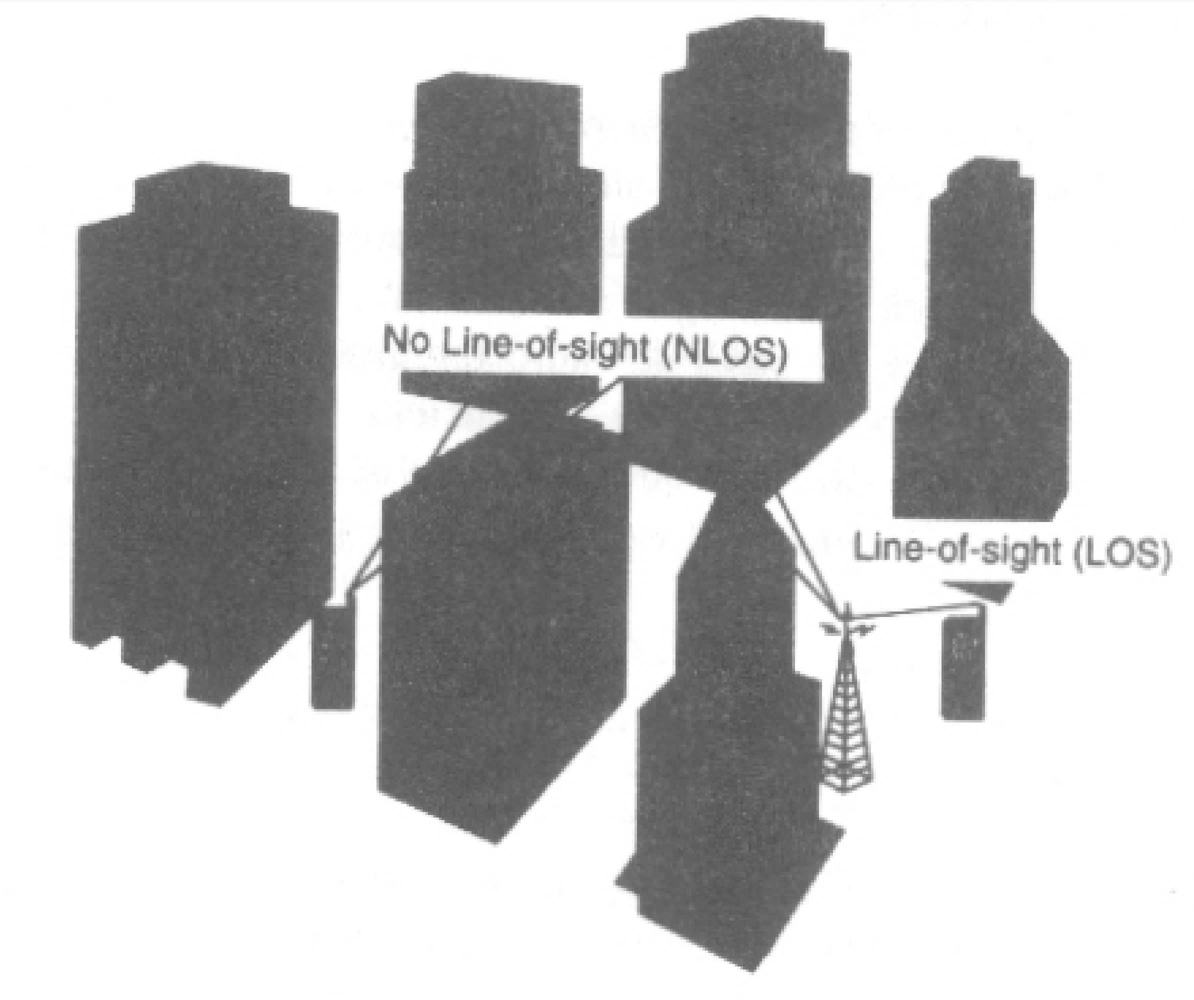
Signal Fading on Radio Channel

- Path Loss
  - large-scale
- Shadowing
  - medium-scale
  - slow varying
- Multipath
  - small-scale
  - fast varying



LOS Path vs. NLOS Path



Path Loss & Shadowing

- Path Loss
  - caused by dissipation of the power radiated by the transmitter
  - depends on the distance between transmitter and receiver
- Shadowing
  - caused by obstacles between the transmitter and receiver that absorb power.

Path Loss Modeling

- Maxwell’s equations
  - Complex and impractical
- Free space path loss model
  - Too simple
- Ray tracing models
  - Requires site-specific information
- Empirical Models
  - Don’t always generalize to other environments
- Simplified power falloff models
  - Main characteristics: good for high-level analysis

Preliminary

- QPSK(Quadrature PSK) modulation

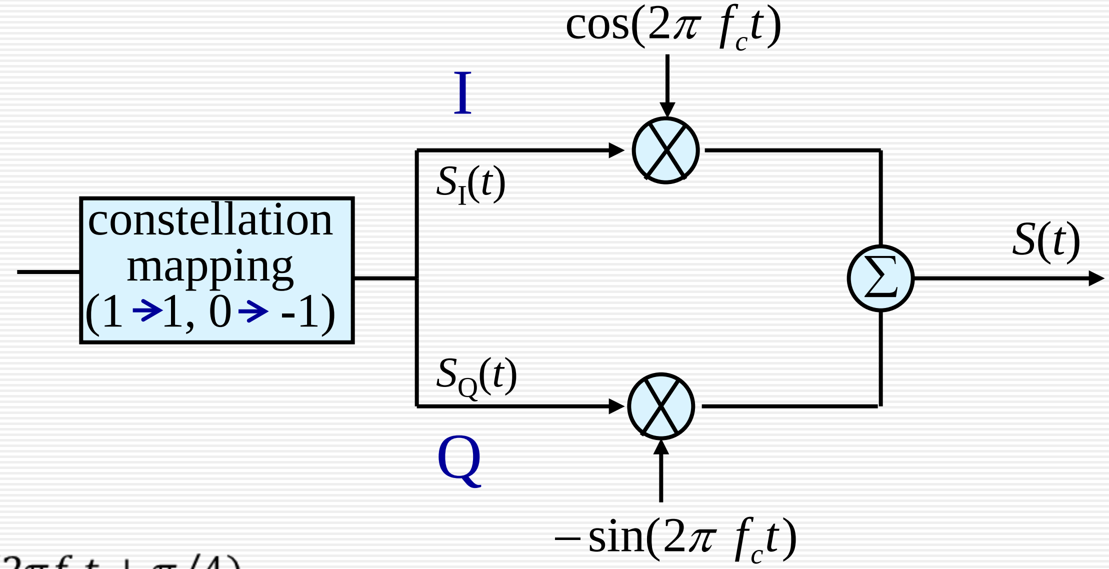
I Q phase

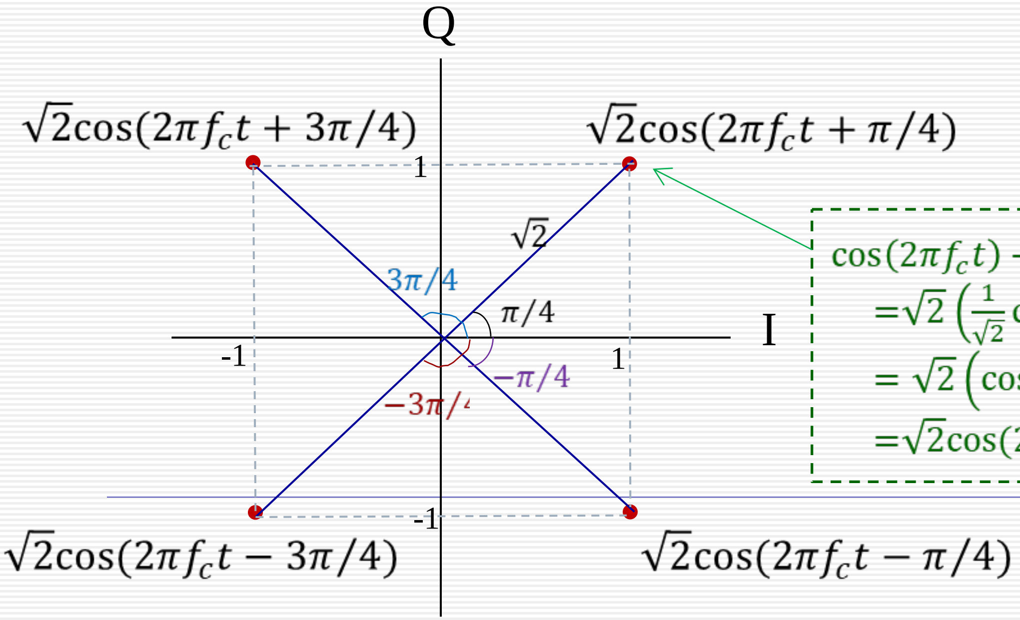
1 1: $\pi/4$ (+1+j)

0 1: $3\pi/4$ (-1+j)

0 0: $-3\pi/4$ (-1-j)

1 0: $-\pi/4$ (+1-j)





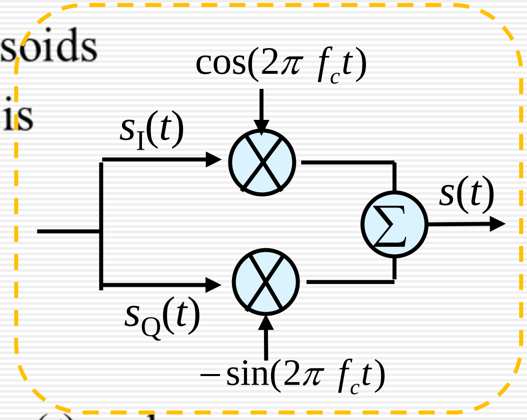
$$\cos(2\pi f_c t) - \sin(2\pi f_c t)$$
$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin(2\pi f_c t)\right)$$
$$= \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)\cos(2\pi f_c t) - \sin(\frac{\pi}{4})\sin(2\pi f_c t)\right)$$
$$= \sqrt{2}\cos(2\pi f_c t + \frac{\pi}{4})$$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

Complex Baseband Signal Representation

- Complex baseband representation
  - The transmitted or received signals are actually real sinusoids
  - The complex representations are used to facilitate analysis

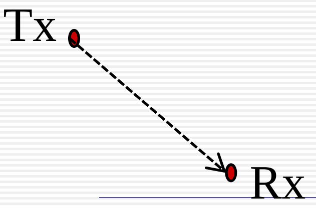


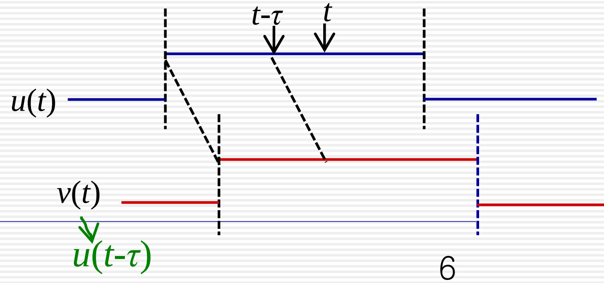
- Transmitted signal
  - $s(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$
  - $u(t) = s_I(t) + js_Q(t)$; $e^{j2\pi f_c t} = \cos(2\pi f_c t) + j\sin(2\pi f_c t)$
    - complex baseband signal with in-phase component $s_I(t)$ and quadrature component $s_Q(t)$

$$u(t)\,e^{j2\pi f_c t} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$
$$+j(\,s_I(t)\sin(2\pi f_c t) + s_Q(t)\,cos(2\pi f_c t)\,)$$

- Received signal
  - $r(t) = \mathrm{Re}\{\alpha\, u(t-\tau)e^{j2\pi f_c(t-\tau)}\}$

    $= Re\{\alpha\, u(t)e^{-j2\pi f_c\tau}e^{j2\pi f_c t}\}$ for narrowband signal: $u(t) \approx u(t-\tau)$
    - Free space: $f_c\tau = {}^{d}/_{\lambda}$, $\alpha = (\lambda\sqrt{G_l})/(4\pi d)$





Free Space Path Loss

- LOS channel
  - no obstructions between the transmitter and receiver
  - signal propagates along a straight line
- Received signal
  - phase shift

$$r(t) = \mathrm{Re}\left\{ \frac{\lambda\sqrt{G_l}\, e^{-j2\pi d/\lambda}}{4\pi d} u(t) e^{j2\pi f_c t} \right\} = \mathrm{Re}\left\{ \frac{\lambda\sqrt{G_l}}{4\pi d} u(t) e^{j(2\pi f_c t - 2\pi d/\lambda)} \right\}$$

    - λ: wavelength, G_l: antenna gain, *d*: distance between transmitter and receiver
  - Ratio of the received power to the transmit power:

$$\frac{P_r}{P_t} = \left[\frac{\sqrt{G_l}\,\lambda}{4\pi d}\right]^2 = \left(\frac{\sqrt{G_l}\,\lambda}{4\pi}\right)^2 d^{-2}$$

path loss exponent

    - power falls off in inverse proportion to the square of the distance
    - As carrier frequency increases, the received power decreases

Ray Tracing

- Two Ray Model
  - predicts signal variation resulting from a ground reflection interfering with the LOS path
  - when a single ground reflection dominates the multipath effect
  - characterizes signal propagation in isolated areas with few reflectors (rural roads or highway)

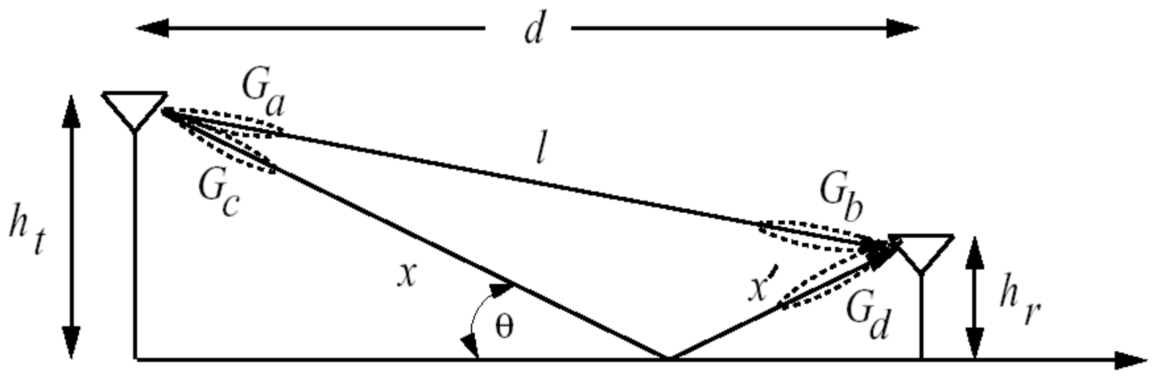


Figure 2.4: Two-Ray Model.

Two Ray Model

- Received signal
  - $$r(t)=\mathrm{Re}\left\{\frac{\lambda}{4\pi}\left[\frac{\sqrt{G_l}u(t)e^{-j2\pi l/\lambda}}{l}+\frac{R\sqrt{G_r}u(t-\tau)e^{-j2\pi(x+x^{'})/\lambda}}{x+x^{'}}\right]e^{j2\pi f_c t}\right\}$$
    - R : ground reflection coefficient
    - $G_l=G_aG_b$, $G_r=G_cG_d$
- Received Power
  - Narrowband signal: $u(t)\approx u(t-\tau)$
    - $$\frac{P_r}{P_t}=\left[\frac{\lambda}{4\pi}\right]^2\left|\frac{\sqrt{G_l}}{l}+\frac{R\sqrt{G_r}e^{-j\Delta\phi}}{x+x^{'}}\right|^2$$ where $\Delta\phi=\frac{2\pi(x+x^{'}-l)}{\lambda}\approx\frac{4\pi h_t h_r}{\lambda d}$
  - For large d, $x+x^{'}\approx l\approx d$, $\theta\approx 0$, $G_l\approx G_r$, $R\approx -1$, $e^{-x}\approx 1-x$
    - $$\frac{P_r}{P_t}\approx\left[\frac{\sqrt{G_l}h_t h_r}{d^2}\right]^2=\left(\sqrt{G_l}h_t h_r\right)^2 d^{-4}$$

Empirical Path Loss Models (1)

- Empirical Path Loss for the given environment (e.g. city, suburban)
  - The average of local mean attenuation (LMA) measurements at distance *d* averaged over all available measurements
- Okumura's Model
  - large urban macrocell, base station heights are 30-100 m
- Hata's Model
  - empirical formulation of the graphical path loss data provided by Okumura
  - closed form formula
- COST 231 Extension
  - extended Hata model for higher frequency (2 GHz)

Empirical Path Loss Models (2)

- Piecewise Linear Model

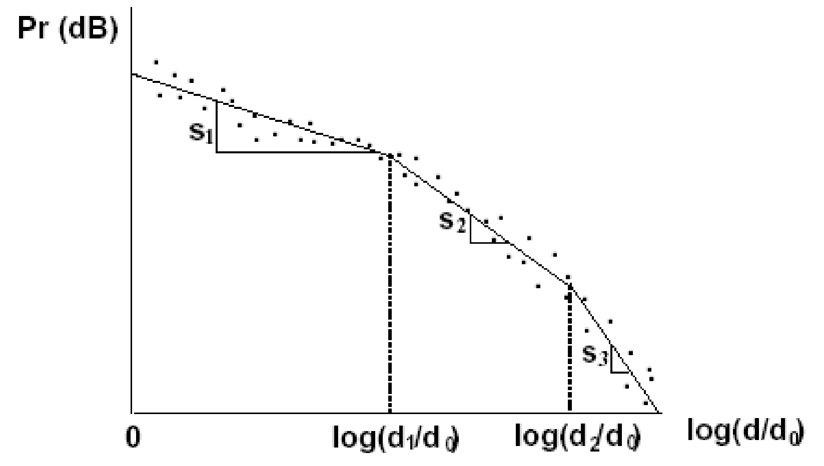


Figure 2.9: Piecewise Linear Model for Path Loss.

- Indoor Attenuation Factors
  - partition loss

Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

  - floor loss
  - the building penetration loss
  - It is difficult to find generic models

Simplified Pass Loss Model

- Path loss as a function of distance: $P_r = P_t K \left[d / d_0 \right]^{-\gamma}$
  - sometimes simple model can captures the essence of signal propagation without resorting to complicated path loss models.
  - Free-space, two-ray, Hata, COST extension to Hata are all of the same form
  - K : constant which depends on antenna characteristics and the average channel attenuation
    - Free space path gain at distance d_0 assuming omni-directional antennas
    - Empirical measurements at d_0
  - d_0: reference distance
    - generally valid only at $d > d_0$
    - d_0 : 1-10 m (indoor), 10-100 m (outdoor)
  - γ : path loss exponent
    - at higher frequencies tend to be higher and at higher antenna heights tend to be lower

Shadowing (1)

- Statistical models
  - The transmitted signal experiences random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects.
- Log-normal shadowing: $\psi = P_r / P_t$
  - Distribution of ψ_{dB} (the dB value of ψ) is Gaussian with mean $\mu_{\psi_{dB}}$ and standard deviation $\sigma_{\psi_{dB}}$

$\psi_{dB} = 10 \log \psi$

$$\psi_{dB} = \frac{10}{\ln 10} \ln \psi$$

normal

log-normal

$$\frac{d\psi_{dB}}{d\psi} = \frac{10}{\ln 10} \frac{1}{\psi}$$

$$f(\psi_{dB}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}} \exp\left[-\frac{(\psi_{dB} - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}} \right]$$

  - Ratio of transmit-to-receive power , ψ , is a random variable with a log-normal distribution

$$f(\psi_{dB})\Delta\psi_{dB} = f(\psi)\Delta\psi$$

$$f(\psi) = \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{\psi_{dB}}\psi} \exp\left[-\frac{(10\log_{10}\psi - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}} \right]$$

$$f(\psi) = f(\psi_{dB}) \frac{d\psi_{dB}}{d\psi} = \frac{10}{\ln 10} \frac{1}{\psi} f(\psi_{dB})$$

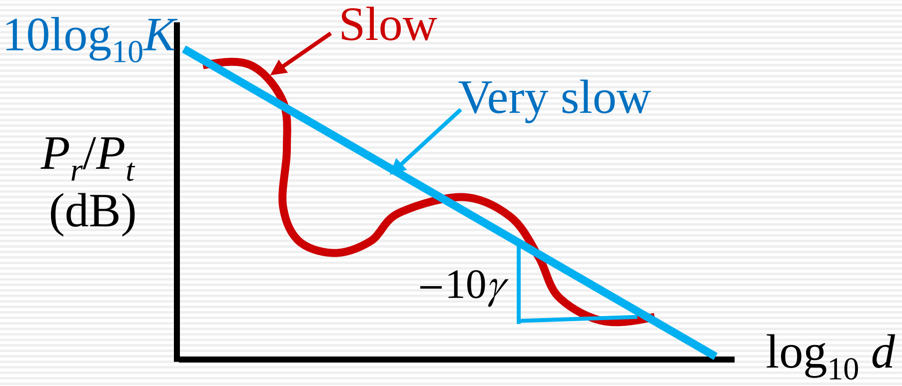
Shadowing (2)

- Justification for the Gaussian model as the distribution of ψ_{dB}
  - when shadowing is dominated by the attenuation from blocking object
  - attenuation of a signal as traveling through an obstacle with depth d
    - $s(d) = e^{-\alpha d}$, where α is an attenuation constant.
  - attenuation of a signal as it propagates through the region $d_t = \sum d_i$
    - $s(d_t) = e^{-\alpha \sum d_i} = e^{-\alpha d_t} \longrightarrow d_t = \frac{-1}{\alpha} \ln s(d_t) = \frac{-1}{\alpha \log_{10} e} \log_{10} s(d_t)$
  - d_t : Gussian r.v. (by the Central Limit Theorem)
  - $\log_{10} s(d_t)$: Gussian r.v.

- Decorrelation distance:
  - the distance at which autocovariance equals 1/e of its maximum value
  - on the order of the size of the blocking objects or clusters of objects

Combined Path Loss and Shadowing (1)

- Combined model : $P_r = G_{PL} G_{SH} P_t$
  - average path loss: from the path loss model ($G_{PL} = K(d/d_0)^{-\gamma}$)
  - shadow fading with mean of 0 dB : variations about the path loss
- Simplified path loss with log-normal shadowing
  - $$\boxed{\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma \log_{10} \frac{d}{d_0} + \psi_{dB}}$$
  - ψ_{dB} : a Guassian r.v. with mean zero and variance $\sigma_{\psi_{dB}}$
  - $d_0 = 1$



Combined Path Loss and Shadowing (2)

- **Outage Probability** under the path loss and shadowing
  - $P_{\min}$: the minimum received power level
  - outage probability: $p_{out}(P_{\min}, d)$
    - the probability that the received power at a given distance *d* falls below $P_{\min}$ (dB)
    - $p_{out}(P_{\min}, d) = p(P_r(d) \text{ in dB} < P_{\min})$
    - for the combined path loss and shadowing

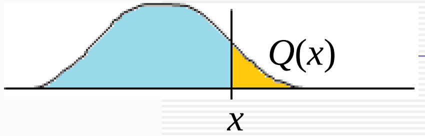
$$p(P_r(d) < P_{\min}) = 1 - Q\left(\frac{P_{\min} - (P_t + 10\log_{10} K - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right)$$

$$P_{r,dB} = P_{t,dB} + 10\log_{10} K - 10\gamma \log_{10}(d/d_0) + \psi_{dB} < P_{\min}$$

$$\Longrightarrow \psi_{dB} < P_{\min} - (P_{t,dB} + 10\log_{10} K - 10\gamma \log_{10}(d/d_0))$$

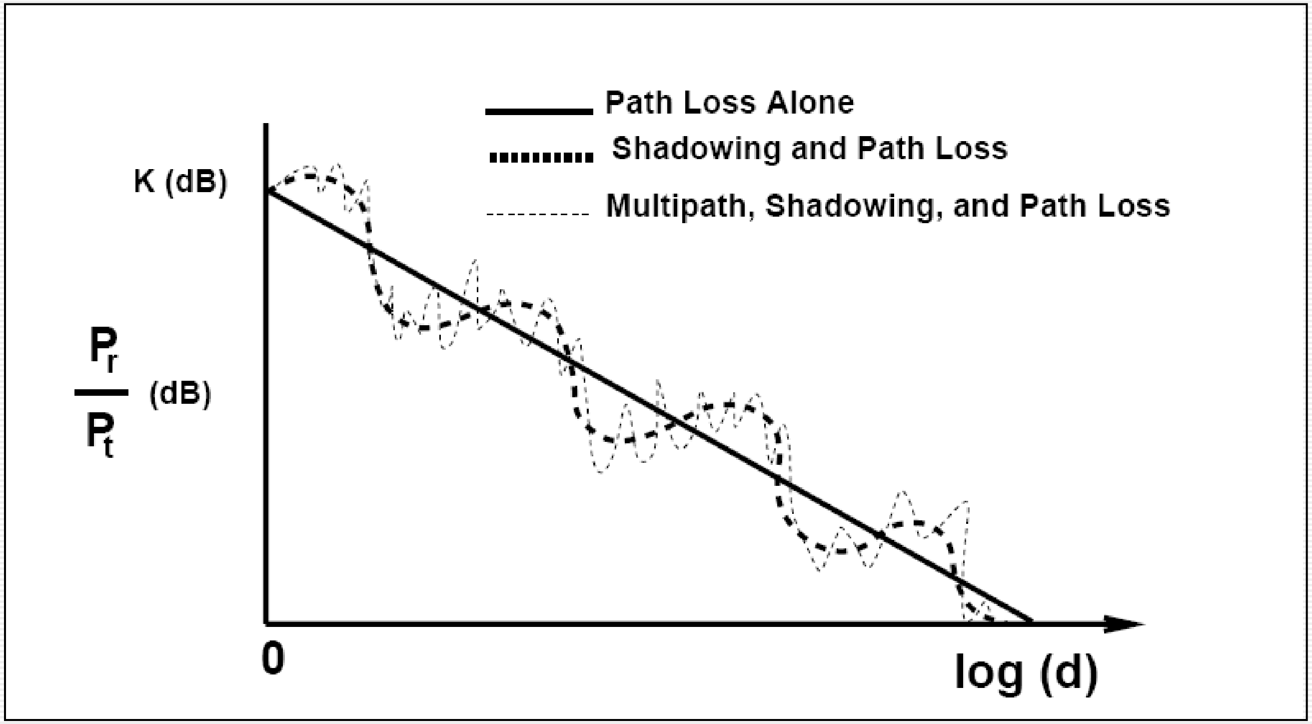
$$\frac{\psi_{dB} - 0}{\sigma_{\psi_{dB}}} < \frac{P_{min} - (P_{t,dB} + 10\log_{10} K - 10\gamma \log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}$$

Standard normal distribution



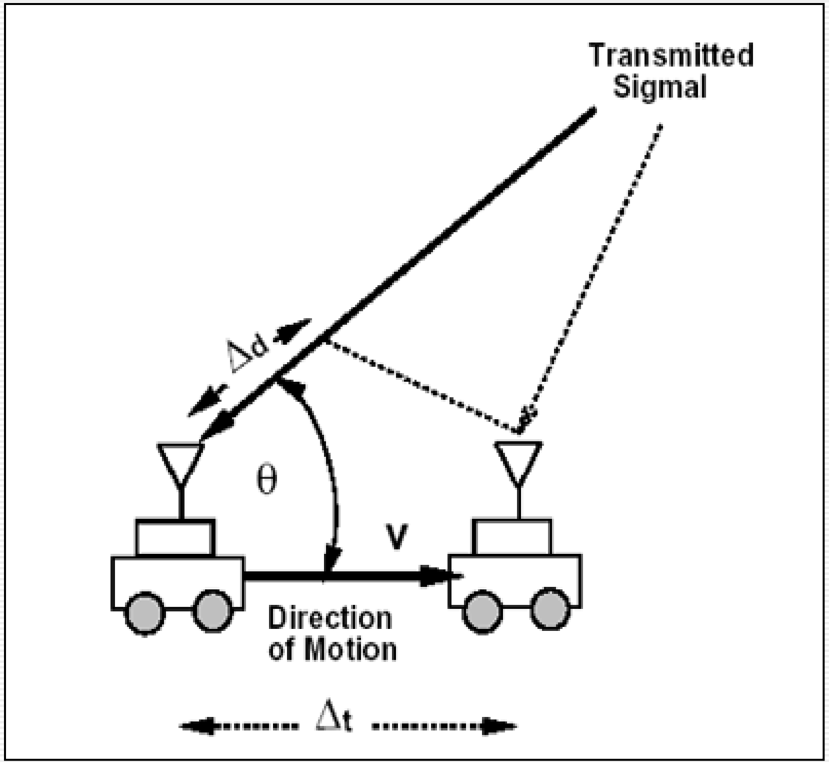
Multipath fading

- short-term fluctuation of the received signal caused by multipath propagation
- when mobile is moving
- Fading becomes fast as a mobile moves faster



Doppler Shift

- When the transmitter is moving, the received signal has a Doppler shift
- $$f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$
  $$\phi_D = 2\pi f_D$$
- Doppler effect is on the order of 100 Hz for typical vehicle speed (75km/hr) and frequencies (about 1GHz)



$$\Delta\emptyset = 2\pi\frac{\Delta d}{\lambda} = 2\pi\frac{v\Delta t\ cos\theta}{\lambda}$$

Multipath Component

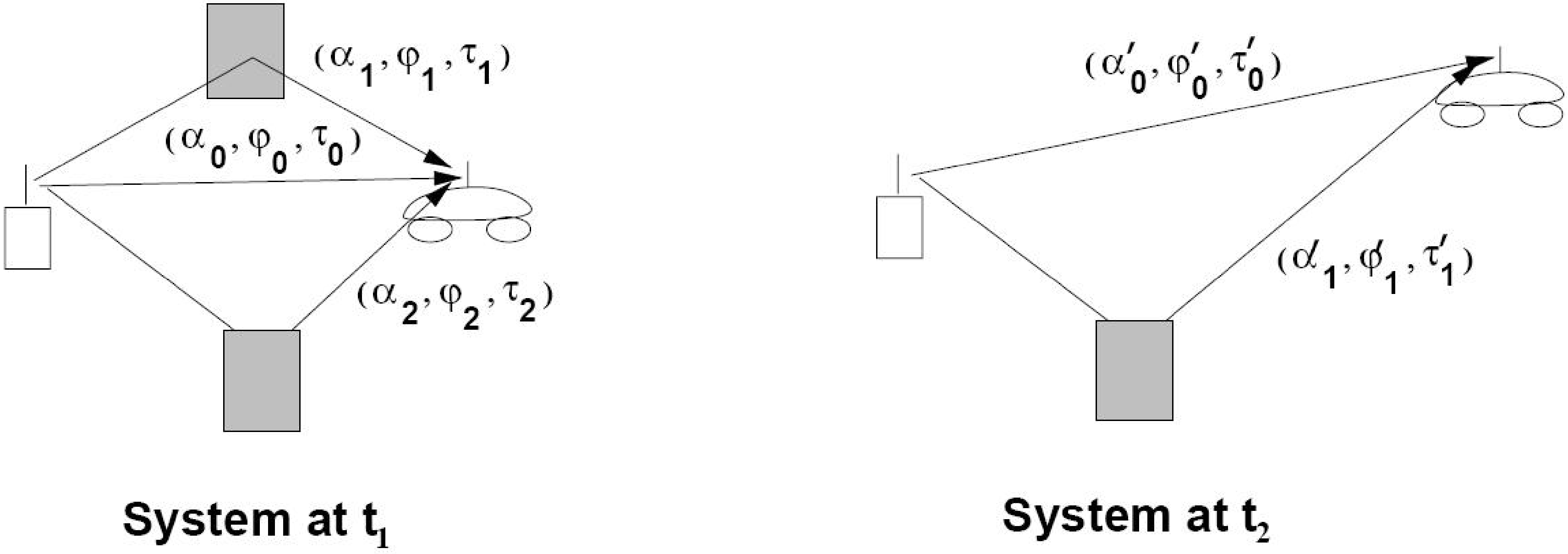


Figure 3.2: System Multipath at Two Different Measurement Times.

Transmit & Receive Signal Models (1)

- Complex baseband representation
  - The transmitted or received signals are actually real sinusoids
  - The complex representations are used to facilitate analysis
- Transmitted signal
  - $s(t) = \mathrm{Re}\left\{u(t)e^{j2\pi f_c t}\right\} = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$
  - $u(t) = s_I(t) + js_Q(t)$
    - complex baseband signal with in-phase component $s_I(t)$ and quadrature component $s_Q(t)$
- Received signal

$$r(t) = \mathrm{Re}\left\{\sum_{n=0}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j(2\pi f_c(t-\tau_n(t))+\phi_D(t))}\right\} \text{ where } N(t) \text{ is the number of NLOS paths}$$

$$= \mathrm{Re}\left\{\sum_{n=0}^{N(t)} \alpha_n(t)u(t-\tau_n(t))e^{j2\pi f_c t}e^{-j(2\pi f_c \tau_n(t)-\phi_D(t))}\right\}$$

$$= \mathrm{Re}\left\{\left(\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}u(t-\tau_n(t))\right)e^{j2\pi f_c t}\right\} \quad \text{where } \phi_n(t) = 2\pi f_c \tau_n(t) - \phi_D(t)$$

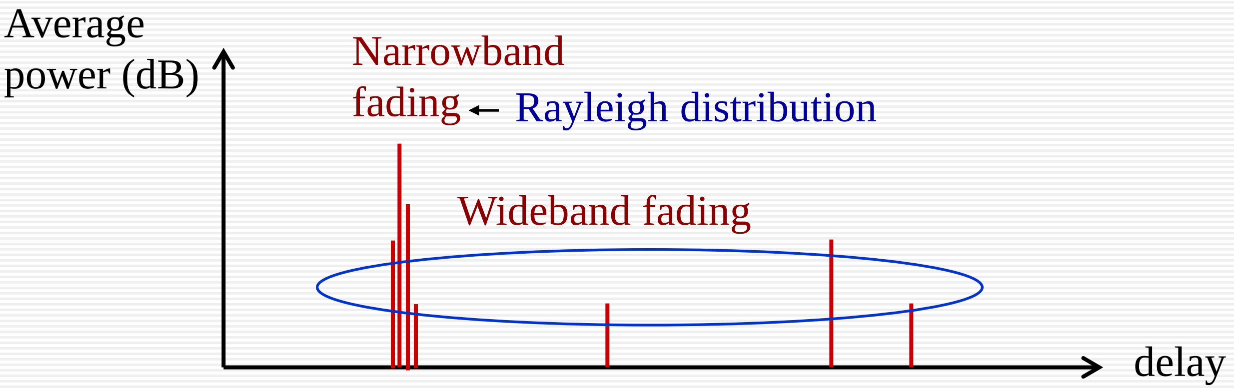
Transmit & Receive Signal Models (2)

- In order to characterize the random scale factor caused by the multipath, $u(t)$=1
  - Received signal:

$$r(t) = \mathrm{Re}\left\{ \overbrace{\left(\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \right)}^{\text{channel characteristic}} e^{j2\pi f_c t} \right\}$$

- *n*th path component at time t : $\alpha_n(t),\ \phi_n(t), \tau_n(t)$

- Delay power profile : characteristics of multipath channel



Resolvable/Nonresolvable Multipath Component

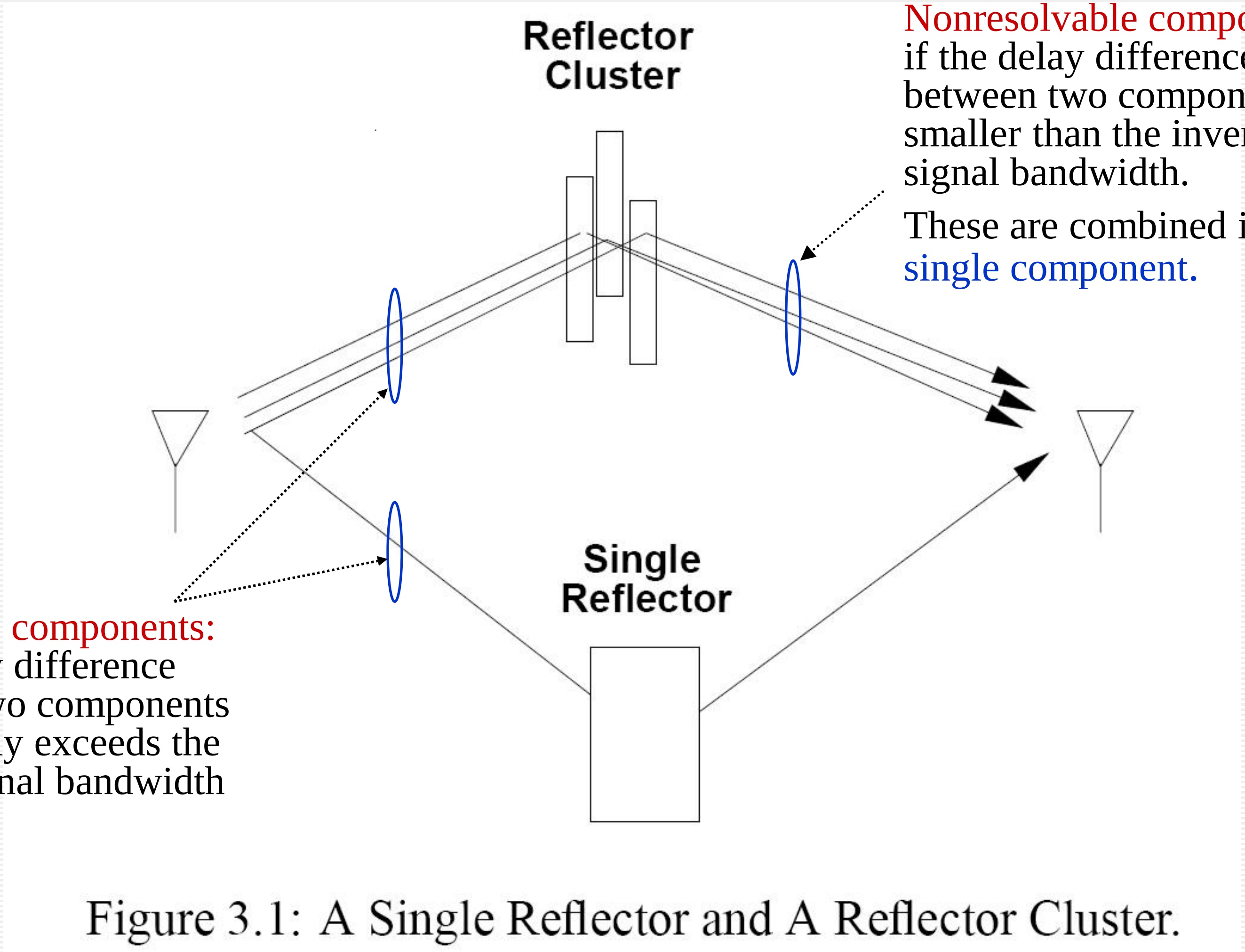


Figure 3.1: A Single Reflector and A Reflector Cluster.

Nonresolvable components: if the delay difference between two components is smaller than the inverse signal bandwidth.

These are combined into a single component.

Resolvable components: if the delay difference between two components significantly exceeds the inverse signal bandwidth

Intersymbol Interference

- Two multipath components with delay τ_1 and τ_2 are resolvable if $|\tau_1 - \tau_2| >> 1/B_u$

- Narrowband fading:
  delay spread $T_m << T$
  - There is little interference with a subsequently transmitted pulse.

- Wideband fading: $T_m >> T$
  - The resolvable multipath components interfere with subsequently transmitted pulses:
    intersymbol interference (ISI)

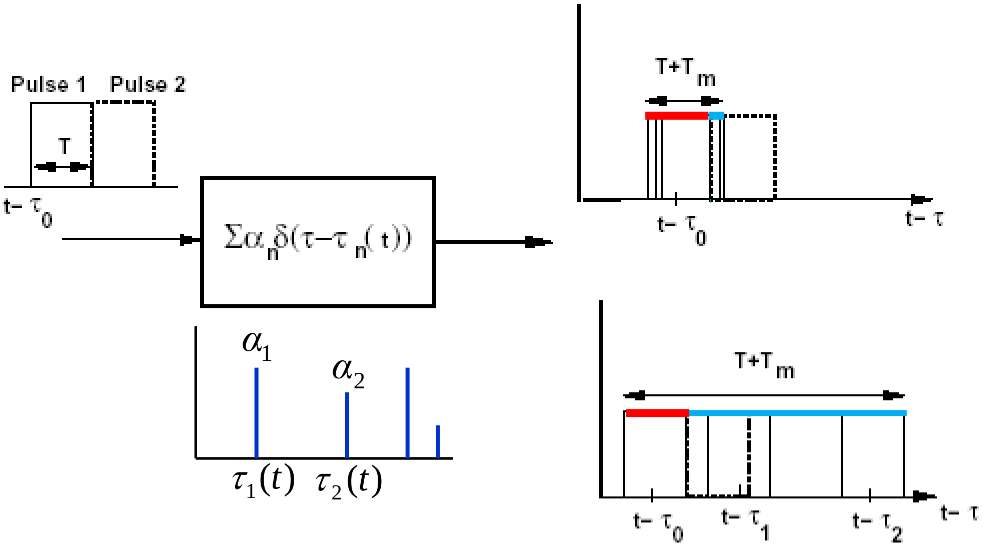


Figure 3.11: Multipath Resolution.

Narrowband Fading Model (1)

- Narrowband fading assumption
  - Delay spread: $T_m << 1/B$
  - The LOS and all multipath components are typically nonresolvable.
- In order to characterize the random scale factor caused by the multipath, $u(t)=1$
  - Transmitted signal: $s(t) = \mathrm{Re}\left\{e^{j2\pi f_c t}\right\} = \cos 2\pi f_c t$
  - Received signal:

nonresolvable

$$r(t) = \mathrm{Re}\left\{\left(\sum_{n=0}^{N(t)} \alpha_n(t)\, e^{-j\phi_n(t)}\right) e^{j2\pi f_c t}\right\}$$

$$= \sum_{n=0}^{N(t)} \alpha_n(t) \cos\phi_n(t) \cos 2\pi f_c t + \sum_{n=0}^{N(t)} \alpha_n(t) \sin\phi_n(t) \sin 2\pi f_c t$$

$$= r_I(t) \cos 2\pi f_c t + r_Q(t) \sin 2\pi f_c t$$

Narrowband Fading Model (2)

- Received Signal
  - $r(t) = r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t$
  - in-phase component: $r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\cos\phi_n(t)$
  - quadrature component: $r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\sin\phi_n(t)$

- $r_I(t)$ and $r_Q(t)$ can be approximated as independent Gaussian process with the same autocorrelation, a mean of zero, and a crosscorrelation of zero

Narrowband Fading: Envelope & Power Distribution of NLOS Multipath

$$r(t) = r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t$$

$$= \sqrt{r_I{}^2(t) + r_Q{}^2(t)}\cos(2\pi f_c t + \theta)$$

- Signal envelope
  - $z(t) = |r(t)| = \sqrt{r_I{}^2(t) + r_Q{}^2(t)}$
  - r_I and r_Q are Gaussian random variables with mean zero and variance σ^2
  - z(t) is Rayleigh distributed
    - $f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}}, \quad z \ge 0$
- Power
  - $z^2(t) = |r(t)|^2$
  - The received signal power is exponentially distributed with mean $2\sigma^2$
    - $f_{Z^2}(x) = \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}}, \quad \mathrm{x} \ge 0$

$X = Z^2 \rightarrow f_X(x)\Delta x = f_Z(z)\Delta z$

$f_X(x) = f_Z(z)\frac{dz}{dx} = f_Z(z)\,\frac{1}{2z}$

Narrowband Fading: Signal Envelope over Channel having a LOS

- Signal envelope
  - The received signal equals the superposition of a LOS component and a complex Gaussian component
  - The signal envelope z(t) has a Rician distribution. $Y = \sqrt{(s_1 + X_1)^2 + (s_2 + X_2)^2}$

$$f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{(z^2+s^2)}{2\sigma^2}} I_0\left(\frac{zs}{\sigma^2}\right), \qquad z \ge 0$$

* I_0: the modified Bessel function

- Average received power
  - $\overline{P_r} = \int_0^\infty x^2 f_Z(x)dx = s^2 + 2\sigma^2$

LOS component power

NLOS component power

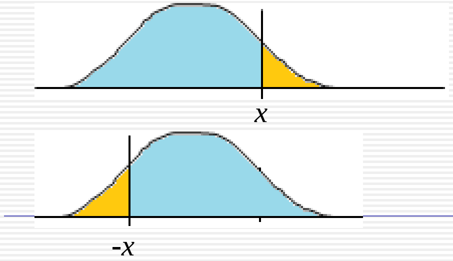
  - Fading parameter : $K = \frac{s^2}{2\sigma^2}$
    - K = 0: Rayleigh fading
    - $K = \infty$: no fading (only a LOS component)
    - the smaller K, the severer fading

Combined PL, Shadowing, and Rayleigh Fading

- Combined model
  - $P_r(d) = P_t K \left({d}/{d_0} \right)^{-\gamma} G$ $G_{SH} G_{MP}$
  - G is a composite lognormal-exponential r.v , which is a lognormal r.v.
  - $G_{dB} = 10 \log_{10} G$: Gaussian r.v. with mean $\breve{\mu}$ and variance $\breve{\sigma}^2$
    - $\breve{\mu} = \mu_{SH,dB} - 2.5$ and $\breve{\sigma}^2 = \sigma^2_{SH,dB} + 5.57^2$, where $\mu_{SH,dB}$ and $\sigma_{SH,dB}$ are mean and sd of lognormal shadowing gain in dB
  - $P_{r,dB}(d) = P_{t,dB} + K_{dB} - 10\gamma \log_{10}(d/d_0) + G_{dB}$
- Outage Probability
  - $p(P_r(d) < P_{\min}) = p(P_{r,dB}(d) < P_{\min,dB})$

$$= p\left(G_{dB} < P_{\min,dB} - P_{t,dB} - K_{dB} + 10\gamma \log_{10}\left(d/d_0\right)\right)$$

$$= Q\left(\frac{\breve{\mu} - \left(P_{\min,dB} - P_{t,dB} - K_{dB} + 10\gamma \log_{10}\left(d/d_0\right)\right)}{\breve{\sigma}} \right)$$



1-Q(x) = Q(-x)