4. Discontinuity frequency

▶ Frequency
- Linear frequency λ_l
- Areal frequency λ_a
- Volumetric frequency λ_V

① Linear frequency

The normal linear frequency (λ_L) has a relationship with the linear frequency (λ_l) as follow.

$$\lambda_L = \frac{\lambda_l}{\cos\theta}$$

where θ is an acute angle between a scanline and a joint normal.

a) Determination of the normal linear frequency of a joint set using s scanlines non-parallel to each other:

$$\lambda_L = \frac{\sum_{j=1}^{s} \lambda_{Lj}\, l_j}{\sum_{j=1}^{s} l_j} \quad \text{- weighted mean of the normal linear frequencies}$$

b) Determination of the normal linear frequencies of D joint sets using total linear frequencies from s scanlines non-parallel to each other (Karzulovic & Goodman (1985)):

$$\lambda_{Sj} = \sum_{i=1}^{D} \lambda_{Li} \cos\theta_{ij} \quad \text{- total linear frequency from the j}^{\text{th}} \text{ scanline}$$

Setting λ_{Sj} a theoretical total linear frequency and λ_{Sj}^* an observed one the error of frequency in the j[th] scanline (ϵ_j) and the sum of squared errors (ϵ_t) are as follows.

$$\epsilon_j = \lambda_{Sj}^* - \lambda_{Sj}, \quad \epsilon_t = \sum_{j=1}^{s} \epsilon_j^2$$

which can be solved using tensor notation as below.

Representing $\lambda_{Sj}^* \rightarrow \lambda_j^*$, $\quad \lambda_{Sj} \rightarrow \lambda_j{}'$, $\quad \lambda_{Li} \rightarrow \lambda_i$

$$\lambda_j{}' = \lambda_i \cos\theta_{ij} = \lambda_i C_{ij}$$

$$\begin{aligned} \epsilon_t = \epsilon_j\, \epsilon_j &= (\lambda_j^* - \lambda_i C_{ij})(\lambda_j^* - \lambda_k C_{kj}) \\ &= \lambda_j^* \lambda_j^* - \lambda_j^* \lambda_k C_{kj} - \lambda_j^* \lambda_i C_{ij} + \lambda_i C_{ij} \lambda_k C_{kj} \\ &= \lambda_j^* \lambda_j^* - 2\lambda_j^* \lambda_k C_{kj} + \lambda_i C_{ij} \lambda_k C_{kj} \end{aligned}$$

Because ϵ_t is a function of λ_k (or λ_i), the minimum error can be obtained by partial differentiation of it with respect to λ_m:

$$\frac{\partial \epsilon_t}{\partial \lambda_m} = \epsilon_{t,m} = \; -2\lambda_j^* C_{mj} + C_{mj}\lambda_k C_{kj} + C_{mj}\lambda_i C_{ij} = 0$$

$$\therefore C_{mj} C_{kj} \lambda_k = \lambda_j^* C_{mj}$$

which can be expressed in matrix form as follow.

$[C] \cdot \lambda^* = [C] \cdot [C^T] \cdot \lambda$ (refer to p.100)

② Areal frequency

- Definition: the number of joint trace centers in unit area of plane (rock exposure)
- Setting an acute angle between a sampling window and a joint plane ϕ, the relationship between normal areal frequency (λ_A) and areal frequency (λ_a) is as follow.

$$\lambda_A = \frac{\lambda_a}{\sin \phi}$$

- Sampling windows in practical survey have finite size and it is frequently hard to judge whether the trace centers are in the window or not. The end-point estimator (Mauldon, 1998) is very useful in these cases.

$$\lambda_a = \frac{N - N_T + N_C}{2A}$$

This estimator defines the areal frequency as a sum of contained traces and a half of dissecting traces divided by sampling area. Lyman (2003) proved that the end-point estimator is an unbiased maximum likelihood estimator and at the same time it is an optimal estimator having the least variance.

③ Volumetric frequency

- Necessary for joint 3D modelling
- Direct measurement is almost impossible. → Indirect estimation is needed.
- Difficult to find out joint centers →

  Total discontinuity area / rock mass volume is alternatively adopted.
- λ_L, λ_A, μ_S (mean diameter) or size (diameter) distribution are required to estimate the volumetric frequency.

a) Calculation of λ_V using λ_L

When θ is an acute angle between a scanline and a joint normal, L is the length of scanline and r is a joint radius the number of joint intersections in a scanline can be calculated as below.

Combining $N_r = \lambda_{Vr}\, L\, \pi r^2 \cos\theta$ and $\lambda_{Lr} = N_r / L\cos\theta$ makes

$$\lambda_{Vr} = \frac{\lambda_{Lr}}{\pi r^2},$$

To calculate the total number of joints having various radii, set

$\lambda_{Vr} \rightarrow \lambda_V f(r)dr$ and $N_r \rightarrow dN_r$, which makes

$$N = \int_{r=0}^{r=r_{\max}} dN_r = \lambda_V \pi\, L \cos\theta \int_{r=0}^{r=r_{\max}} r^2 f(r)\, dr \quad , \quad \lambda_V = \frac{\lambda_L}{\pi E(r^2)} = \frac{\lambda_L}{\pi M_{r2}} = \frac{4\lambda_L}{\pi M_{s2}}$$

b) Calculation of λ_V using λ_A

When ϕ is an acute angle between a sampling window and a joint plane, A is an area of the sampling window and s is joint diameter, the number of joints whose trace center is located in the sampling window is as follow

$$N_s = \lambda_{a,s} A = s\ A\ \sin\phi \lambda_{Vs}$$

For various s,

$\lambda_{Vs} = \lambda_V f(s)ds$, and

$$N = \int_{s=0}^{s=s_{\max}} N_s = A\sin\phi \int_{s=0}^{s=s_{\max}} s\lambda_{Vs} = A\sin\phi\lambda_V \int_{s=0}^{s=s_{\max}} s\ f(s)ds = A\sin\phi\lambda_V\, \mu_s = \lambda_a A$$

$$\rightarrow \lambda_V = \frac{\lambda_A}{\mu_s}$$

▶ Discontinuity Frequency Extrema

- Meaning: Maxima + minima of the total linear frequency for all possible orientations of scanlines
- Assumption: Each joint set consists of parallel joints.
- Terminology:

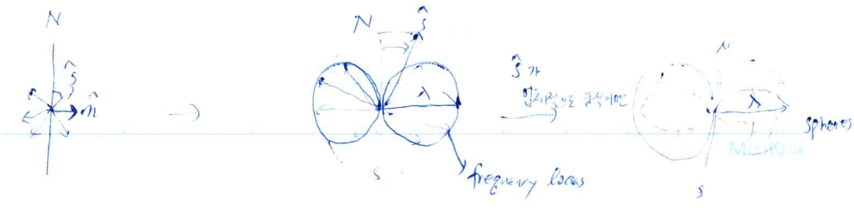
  λ_S: total linear frequency, λ_i: normal linear frequency of the i$^{\text{th}}$ set,

  $\hat{n_i}$: normal vector of the i$^{\text{th}}$ set, $\hat{s}$: orientation vector, D: number of sets
- Mathematical expression

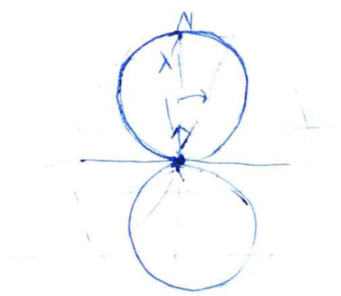
$$\lambda_S = \sum_{i=1}^{D} \lambda_i \cos\theta_i = \sum_{i=1}^{D} \left| \lambda_i \hat{n}_i \cdot \hat{s} \right| = \sum_{i=1}^{D} \left| \overrightarrow{m_i} \cdot \hat{s} \right| = \left(\sum_{i=1}^{D} \overrightarrow{m_i} \right) \cdot \hat{s} = \overrightarrow{M} \cdot \hat{s}$$

where the rightmost equation is applied to the case $\hat{n}_i$ is adjusted to make an acute angle with $\hat{s}$.

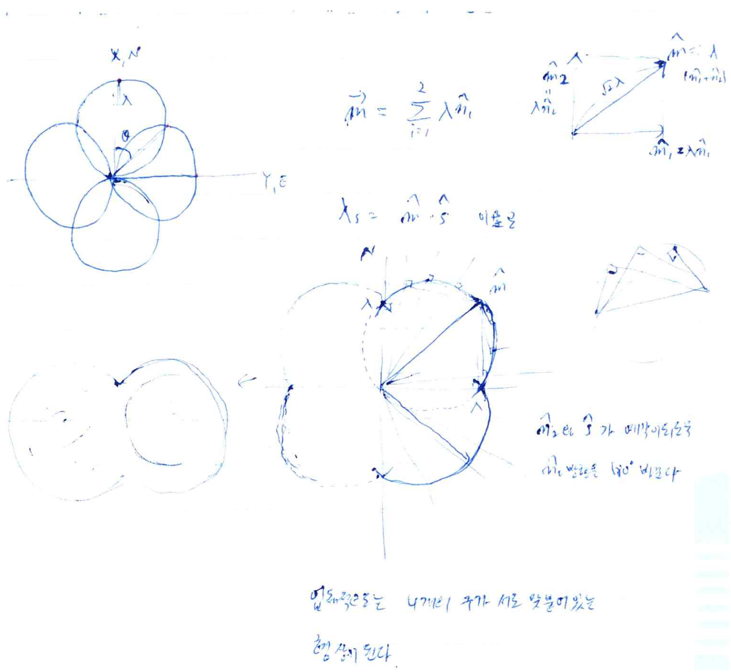
- Frequency locus when D=1 and joints have strike/dip as NS/90:



- Frequency locus when D=1 and joints have strike/dip as EW/90:



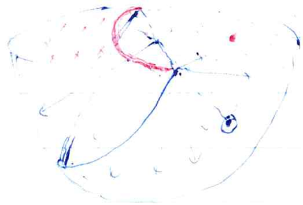
- Frequency locus when D=2 and each joint set has strike/dip as NS/90 and EW/90, respectively:



- Frequency locus when D=3 and all joint sets are vertical and their strikes are 60° apart each other: Home work

- General case of joint set orientation

  1) Single joint set



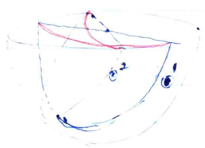
: $\hat{s}\cdot\hat{n} > 0$

: $\hat{s}\cdot\hat{n} < 0 \rightarrow \hat{s}\cdot(-\hat{n})$

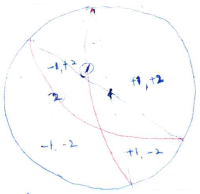
$\lambda_s = \hat{m}\cdot\hat{s}$

  2) Two 3) three... joint sets

② two joint sets



위에서 보면 →



4가지 영역으로 나뉨

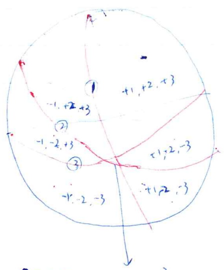
(+1,+2)와 (-1,-2), 그리고 (-1,+2)와(+1,-2)

는 서로 전체 set direction이 부호가 완전히 반대.

→ local max., min. 이 서로 같다.

→ 2중에 하나만 해석의 의미가 있다.

③ three joint sets



7가지 영역+1 -1,+2,-3

Inter-cusp zone : Block theory 의

Joint pyramid에 해당.

D 개의 절리군이 존재할 때

D^2-D+2 개의 Joint pyramid가

존재함. 단 3개 이상의 jt 평면이 하나의 직선에서 교차하지 않음을 전제

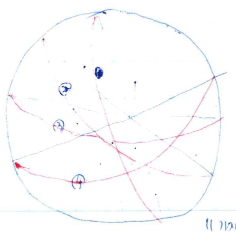
$$2+\sum_{i=2}^{D}2(i-1) = 2-2(D-1)+2\sum_{i=2}^{D} i$$

$$= 4-2D+(D-1)(D+2) = D^2-D+2$$

D: 1 2 3 4

J.P: 2 4 8 14

④ four joint sets



11가지영역 +3

여기에서는 의미있는 J.P.가 절반이 되므로

D=2 → 2

=3 → 4

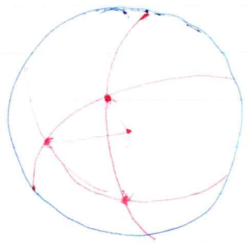
=4 → 7 개가 된다

- Local(global) minimum

– local minimum frequency

각 inter-cusp 즉 joint pyramid 의 cusp (꼭지점) 의 방향에 scanline 이 존재할때의 frequency 값을 계산한다

$$\hat{\lambda}_{12} = \hat{n}_1 \times \hat{n}_2$$



local minimum frequency 들을 비교해서 가장 작은 값 → global minimum 이 된다

- Study of Priest & Samaniego (1983)

Joint traces belonging to three sets were generated in a virtual sampling plane. The normal linear frequency of each set is taken to be 0.8, 2.8, 1.6 (m^{-1}) and the trace length is set to follow negative exponential distributions whose mean length is 2m, 6m and 4m, respectively. The trace orientation is assumed to obey normal distributions whose standard deviation is 0° or 15°. Variation of total linear frequency according to scanline orientation is shown in Fig. 4.7.

▶ Discontinuity occurrence

-Even though the linear frequency of each joint set does not obey Poisson distribution the total linear frequency of a few joint sets usually obey Poisson distribution. When the total linear frequency of all the joint sets obey Poisson distribution, the total areal frequency and total volumetric frequency can be also considered to obey Poisson distributions.