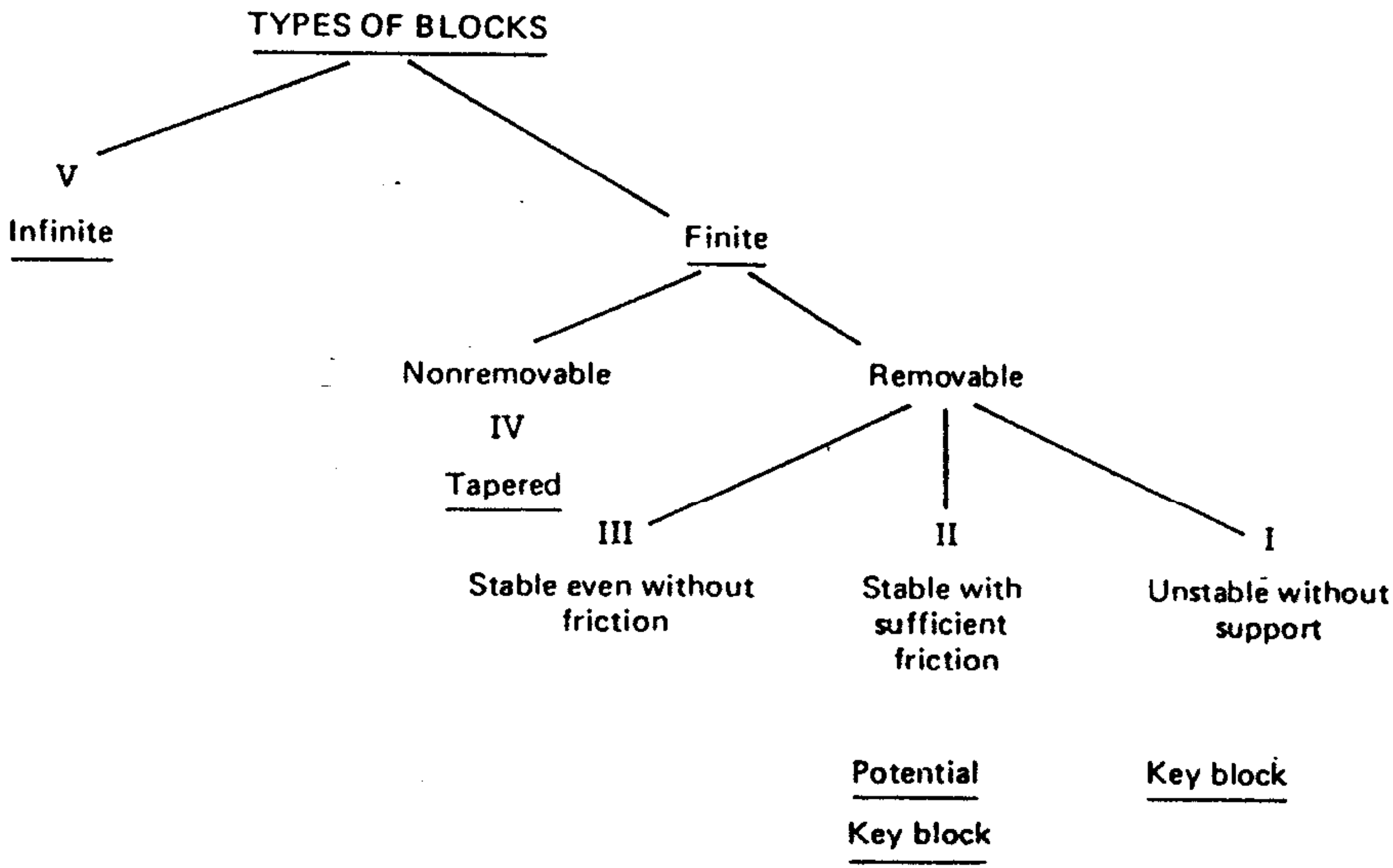
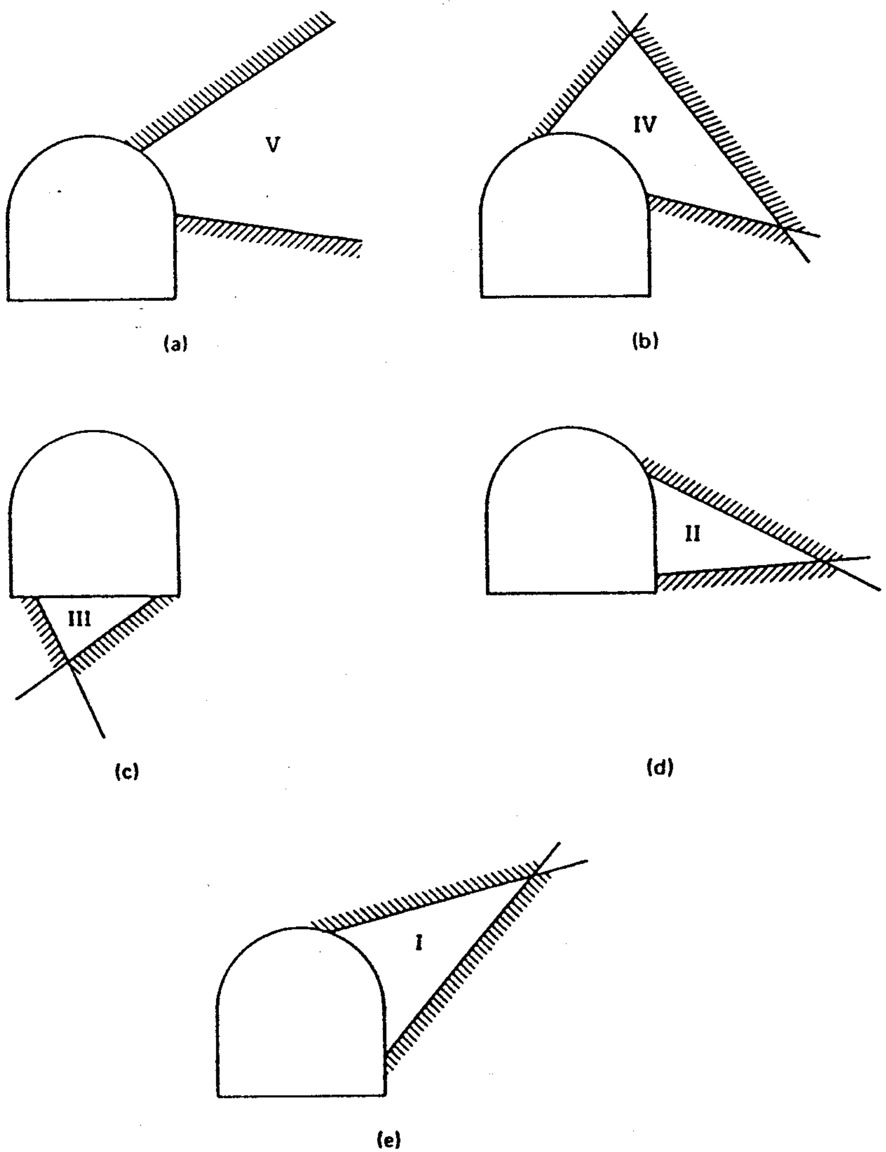
4. The removability of blocks

1) Types of blocks

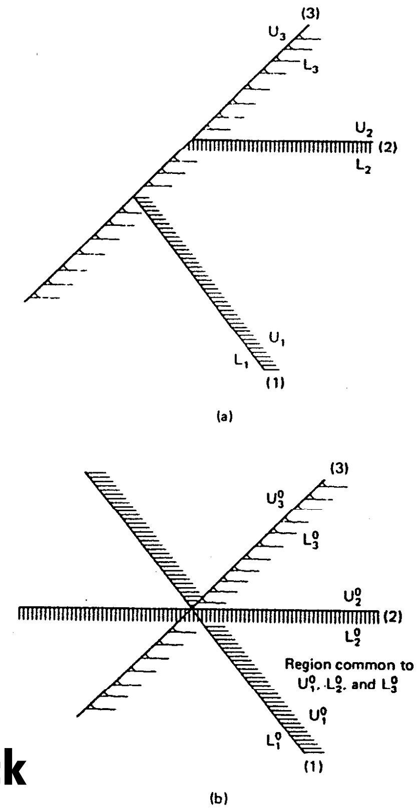


1) Types of blocks

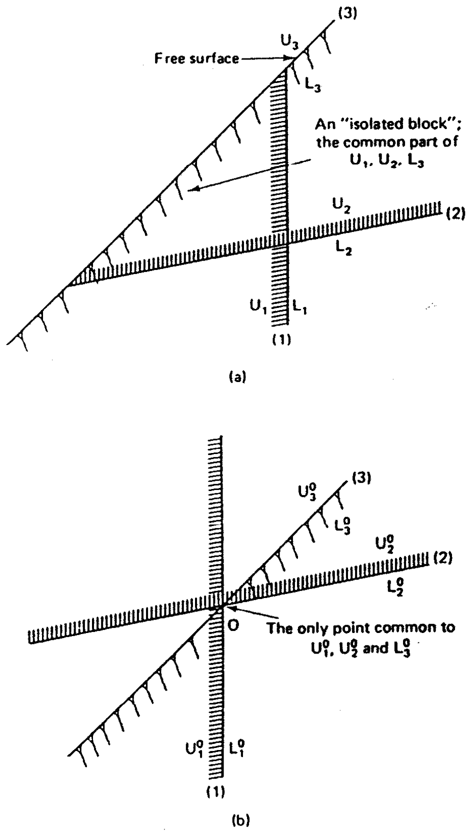


2) Theorem of finiteness

A convex block is finite if its block pyramid is empty. Conversely, a convex block is infinite if its block pyramid is not empty.



Infinite block



Finite block

2) Theorem of finiteness

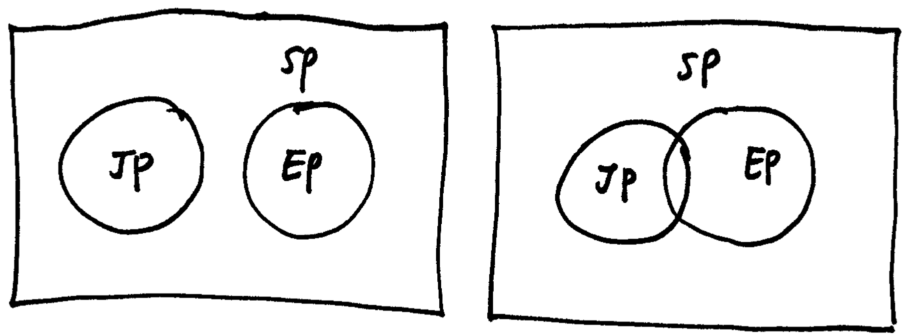
$BP = JP \cap EP$

Finiteness theorem $\rightarrow$ Finite block has BP of $\varnothing$

$$JP \cap EP = \varnothing$$

$$\sim EP = SP$$

$$\therefore JP \subset SP$$

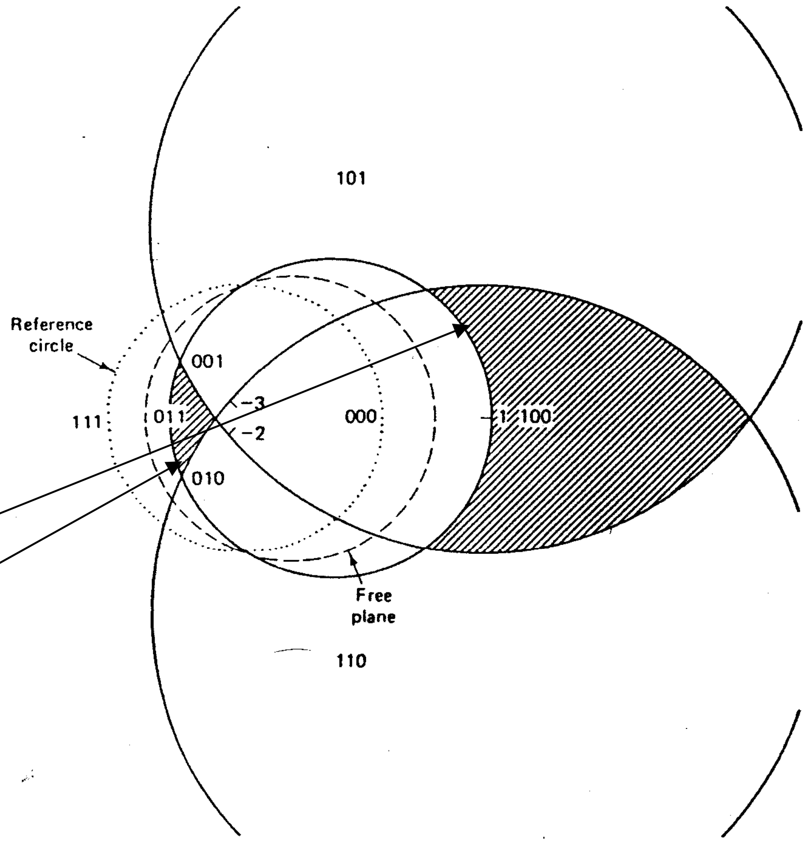


2) Theorem of finiteness

- Finiteness theorem applied to the stereographic projection

Dip/Dip direction

1. 30/090
2. 65/050
3. 65/130
4. 15/090 (free plane)



Free plane

1) Roof
2) Floor

2) Theorem of finiteness

- Mathematical proofs

① Half space is convex.

- A convex block is a region where a line between any two points in the region is contained completely.

When (x_1, y_1, z_1) and (x_2, y_2, z_2) belong to the half space

$Ax + By + Cz \geq D$ ··············· half space

$Ax_1 + By_1 + Cz_1 \geq D$ and $Ax_2 + By_2 + Cz_2 \geq D$.

A line connecting the two points, $(x_1, y_1, z_1) - (x_2, y_2, z_2)$:

$$x = x_1 + (x_2 - x_1)t, \quad y = y_1 + (y_2 - y_1)t, \quad z = z_1 + (z_2 - z_1)t, \quad 0 \leq t \leq 1$$

$$\rightarrow A\left(x_1 + (x_2 - x_1)t\right) + B\left(y_1 + (y_2 - y_1)t\right) + C\left(z_1 + (z_2 - z_1)t\right)$$

$$= Ax_1(1-t) + By_1(1-t) + Cz_1(1-t) + Ax_2t + By_2t + Cz_2t \geq D$$

$$\because Ax_1(1-t) + By_1(1-t) + Cz_1(1-t) \geq D(1-t), \quad Ax_2t + By_2t + Cz_2t \geq Dt$$

2) Theorem of finiteness

② The intersection of any two convex blocks is also a convex block.

Since any two points in the intersection are in both blocks and each block is convex, the line between these points is within each block and therefore is in the intersection. Thus the intersection is convex.

2) Theorem of finiteness

③ BP of a finite block is empty.

Definition of a block:

$$\begin{aligned} \mathrm{A}_1 x + B_1 y + C_1 z &\geq D_1 \\ &\vdots \\ \mathrm{A}_\mathrm{m} x + B_\mathrm{m} y + C_\mathrm{m} z &\geq D_\mathrm{m} \quad \cdots\cdots \quad (a) \end{aligned}$$

Definition of BP:

$$\begin{aligned} \mathrm{A}_1 x + B_1 y + C_1 z &\geq 0 \\ &\vdots \\ \mathrm{A}_\mathrm{m} x + B_\mathrm{m} y + C_\mathrm{m} z &\geq 0 \quad \cdots\cdots \quad (b) \end{aligned}$$

Suppose that the block is finite but its BP is not empty. Then,

$(x_0, y_0, z_0) \neq (0,0,0)$ satisfying (b) should exist,

$A_i x_0 t + B_i y_0 t + C_i z_0 t \geq 0, \quad 1 \leq i \leq m, \quad t > 0,$ and

$(x_1, y_1, z_1) \neq (0,0,0)$ satisfying (a) exists.

Combining the two sets of inequalities,

$A_i x_1 + B_i y_1 + C_i z_1 + A_i x_0 t + B_i y_0 t + C_i z_0 t \geq D_i$

Then the block becomes infinite because $(x_1 + x_0 t, y_1 + y_0 t, z_1 + z_0 t)$ exist innumerously.

$\rightarrow$ The above assumption is not true.

2) Theorem of finiteness

④ If BP is empty the block is finite.

Definition of a block:

$$A_1 x + B_1 y + C_1 z \geq D_1$$
$$\vdots$$
$$A_m x + B_m y + C_m z \geq D_m \quad \cdots\cdots \quad (a)$$

Definition of BP:

$$A_1 x + B_1 y + C_1 z \geq 0$$
$$\vdots$$
$$A_m x + B_m y + C_m z \geq 0 \quad \cdots\cdots \quad (b)$$

Suppose that BP is empty but the block is infinite. Then,

$\left(x_0, y_0, z_0\right)$ satisfying (b) should alwasys be $\left(0,0,0\right)$, and

$\left(x_1 + n_x t, y_1 + n_y t, z_1 + n_z t\right)$ for $t \geq 0$ belonging to the block exist

where $\left(n_x, n_y, n_z\right)$ is a direction vector $\left(\neq \left(0,0,0\right)\right)$.

Applying this point to (a):

$$A_i\left(x_1 + n_x t\right) + B_i\left(y_1 + n_y t\right) + C_i\left(z_1 + n_z t\right) \geq D_i$$

Since $A_i x_1 + B_i y_1 + C_i z_1 \geq D_i$, $A_i n_x t + B_i n_y t + C_i n_z t$ should be equal to or greater than 0

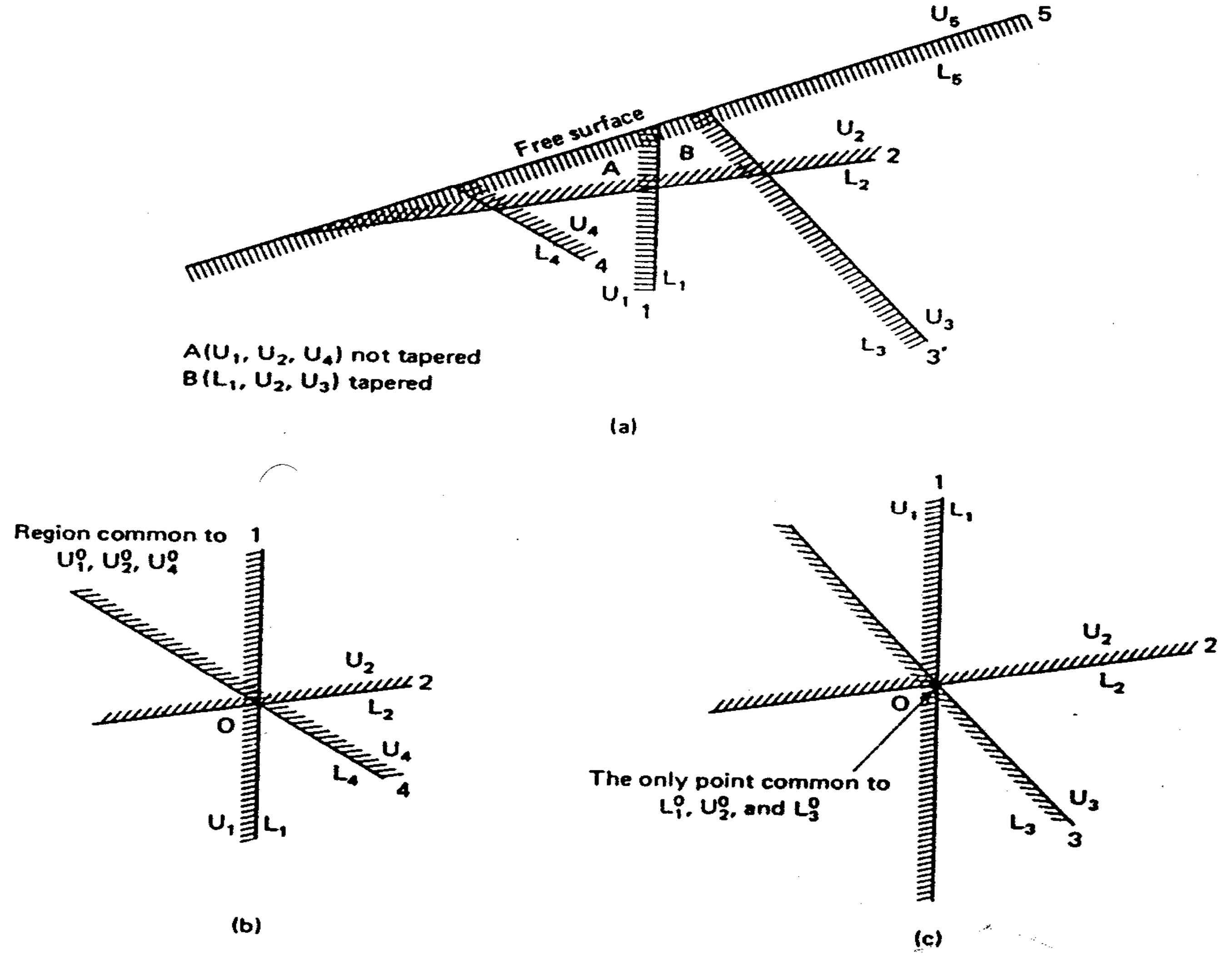
Then $\left(n_x t, n_y t, n_z t\right)$ satisfies (b), which contradicts the assumption that BP is empty.

3) Theorem on removability of a finite convex block

A convex block is removable if its block pyramid is empty and its joint pyramid is not empty.

A convex block is not removable (tapered) if its block pyramid is empty and its joint pyramid is also empty.

3) Theorem of removability of a finite convex block

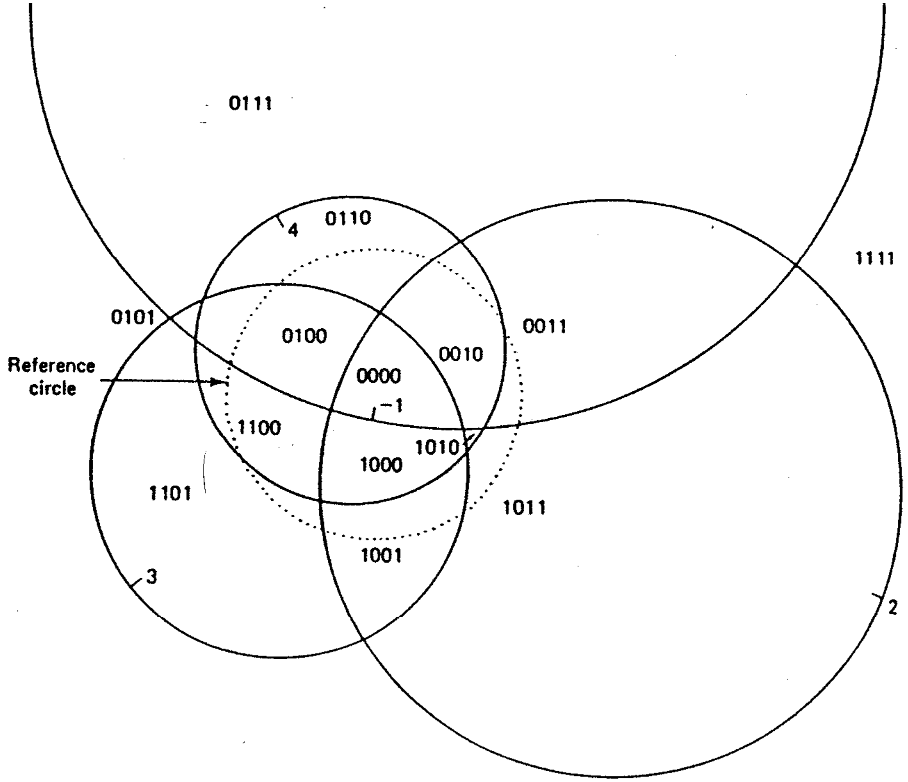


3) Theorem of removability of a finite convex block

- Removability theorem applied to the stereographic projection

- No. of tapered blocks with 3 or 4 joint sets

- Removable blocks with 4 joint sets



n	2^n	$n^2 - n+2$	No. of tapered
2	4	4	0
3	8	8	0
4	16	14	2
5	32	22	10

3) Theorem on removability of a finite convex block

- Mathematical meaning of removability theorem

- $\hat{x}_0$ satisfying $\hat{n}_i \cdot \hat{x}_0 \geq 0$ exists when $\hat{n}_i$ is an inward-pointing joint normal.

$$
\begin{gathered}
\mathrm{A}_1 x_0 + B_1 y_0 + C_1 z_0 \geq 0 \\
\vdots \\
\mathrm{A}_\mathrm{n} x_0 + B_\mathrm{n} y_0 + C_\mathrm{n} z_0 \geq 0
\end{gathered}
$$

where $\left(x_0, y_0, z_0\right) \neq \left(0,0,0\right)$.

- If BP is empty and JP is not empty $\left(x_0, y_0, z_0\right) \neq \left(0,0,0\right)$ and $\left(x_1, y_1, z_1\right) \neq \left(0,0,0\right)$ exist.

$A_\mathrm{i} x_1 + B_\mathrm{i} y_1 + C_\mathrm{i} z_1 \geq D_i, \quad 1 \leq i \leq n+k$

- If the point $\left(x_1, y_1, z_1\right)$ moves to $\left(x_1 + x_0 t, y_1 + y_0 t, z_1 + z_0 t\right)$ for $t \geq 0$

$A_\mathrm{i} x_1 + \mathrm{B}_\mathrm{i} y_1 + \mathrm{C}_\mathrm{i} z_1 + t\left(A_i x_0 + B_i y_0 + C_i z_0\right) \geq D_i$ for $1 \leq i \leq n$

$\rightarrow$ the new point $\left(x_1 + x_0 t, y_1 + y_0 t, z_1 + z_0 t\right)$ belongs to removable space.

4) Shi's theorem for the removability of non-convex blocks

- United blocks: union of convex blocks (p.121, 123)

$$JP = \bigcap_{i=1}^{h} JP(A_i)$$

$$EP = \bigcup_{i=1}^{h} EP(A_i)$$

-Necessary condition of removability of a united block

$$JP \cap EP = \varnothing$$

$$JP \neq \varnothing$$

- Removable direction belongs to JP (p.124)

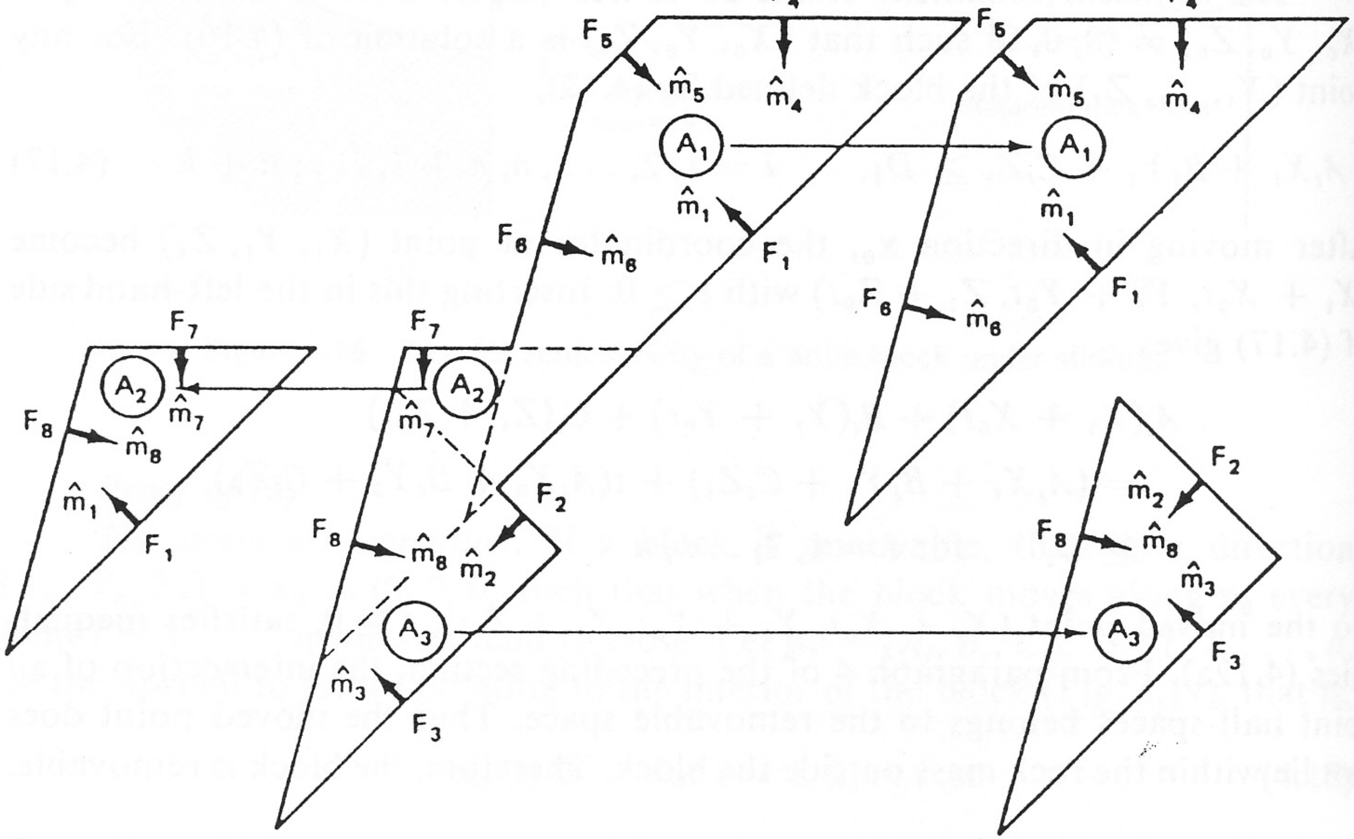


Figure 4.18 Decomposition of a united block.

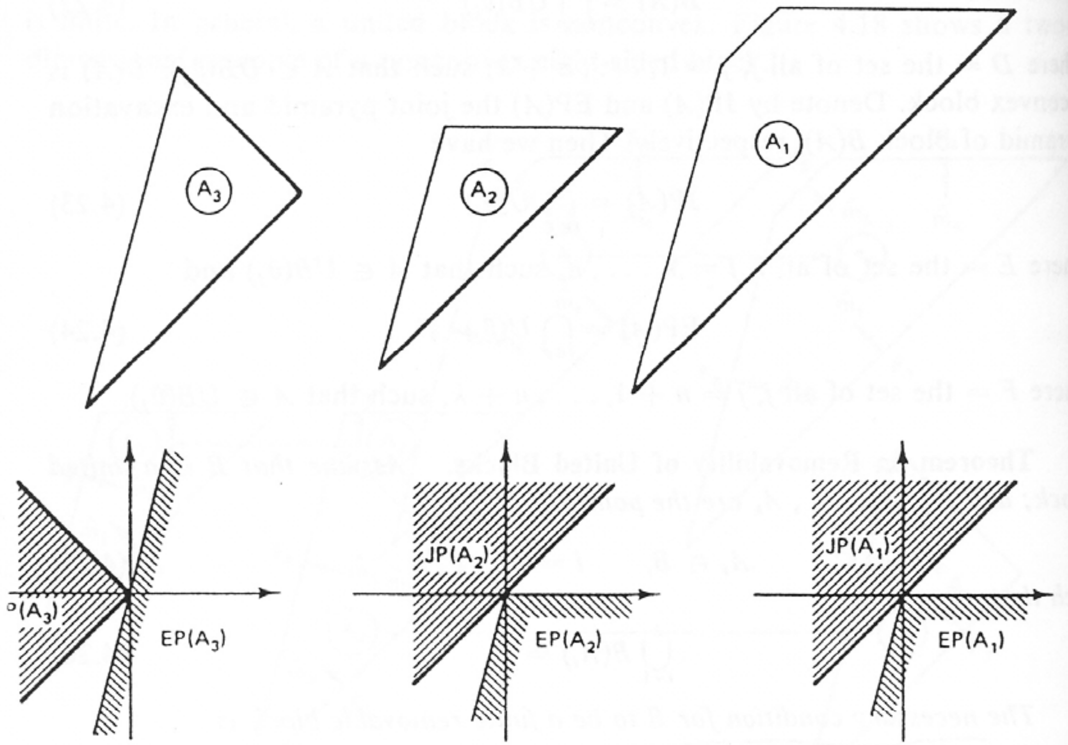


Figure 4.19 JP and EP for each component of the united block.

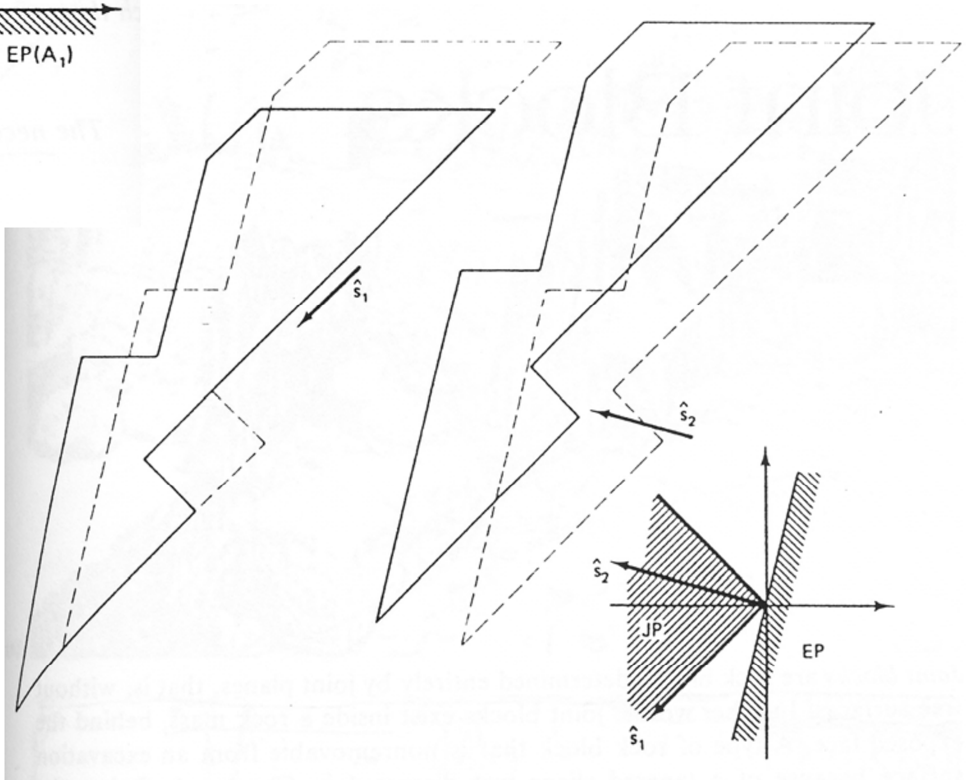


Figure 4.20 Application of Shi's theorem for a united block.