Optimal Design of Energy Systems (M2794.003400)

Chapter 5. MODELING THERMAL EQUIPMENT

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Chapter 5. Modeling Thermal Equipment

5.1 Using physical insight

- Major concerns of this chapter : actual thermal equipment

Heat exchanger :

It is important to select the type of heat exchanger and calculate how a certain heat exchanger will perform

Distillation separator :

Understanding of separation of binary mixtures expands the horizons of applications of the simulation and optimization

Turbomachinery :

Studying the turbomachinery shows how the use of dimensionless group can simplify the equation

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5.2 Selecting vs. simulating a heat-exchanger

- **Selecting** the heat exchanger:

① Choosing type of the heat exchanger (Shell & tube, Finned, compact, etc.)
② Specifying the details (number of tubes, tube diameter, core size, etc.)
③ Heat transfer duty is specified already

- **Simulating** the heat exchanger:

① Heat exchanger already exists, either in actual hardware or specific design
② Simulation of a heat exchanger consists of predicting outlet conditions
③ Performance charicteristics of the heat exchanger are available
(such as the area and overall heat transfer coefficients)

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5.3 Counterflow heat exchanger

- Most favorable ΔT is achieved with a counterflow arrangement

(Hot side fluid)

$$q = w_h c_{ph}(t_{h,i} - t_{h,o})$$

(Cold side fluid)

$$q = w_c c_{pc}(t_{c,o} - t_{c,i})$$

(Heat transfer rate)

$$q = UA\Delta T_{lm}$$

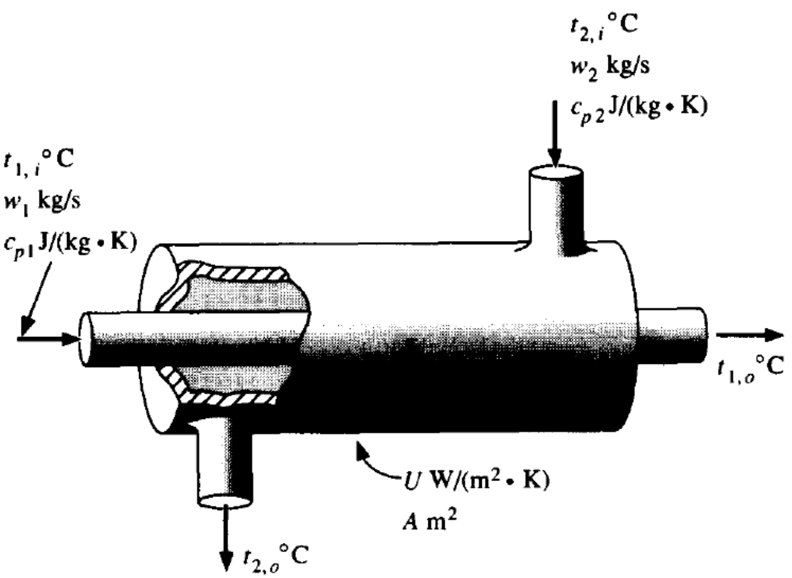

Fig. Typical counterflow heat exchanger

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5.3.1 LMTD (Log Mean Temperature Difference) method for a counter arrangement

(Hot side fluid) $dq = -w_h c_{ph} dT_h = -W_h dT_h$

(Cold side fluid) $dq = -w_c c_{pc} dT_c = -W_c dT_c$

(Heat transfer) $dq = UdA(T_h - T_c) = UdA\Delta T$

$$\Rightarrow \quad d(\Delta T) = d(T_h - T_c) = dT_h - dT_c = -dq\left(\frac{1}{W_h} - \frac{1}{W_c}\right)$$

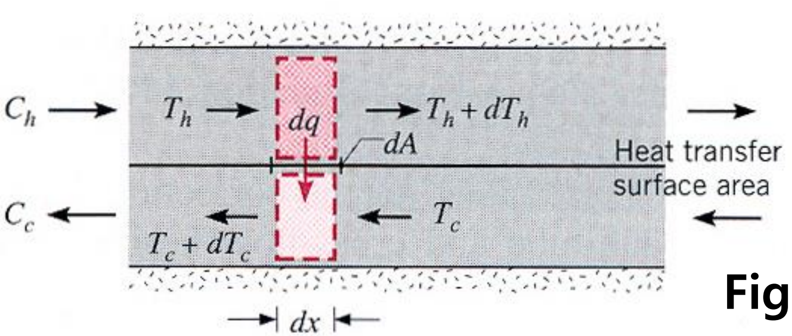

Fig. Heat exchange at counter flow HX

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5.3.1 LMTD method for a counter arrangement

- represent the heat transfer rate as

$$dq = \frac{-d(\Delta T)}{1/W_h - 1/W_c} = UdA\Delta T \quad \rightarrow \quad \frac{d(\Delta T)}{\Delta T} = -UdA(\frac{1}{W_h} - \frac{1}{W_c})$$

- integrating on both side,

$$\int \frac{d(\Delta T)}{\Delta T} = \ln\left(\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}}\right) = -\frac{UA}{q}(T_{h,i} - T_{h,o} - T_{c,o} + T_{c,i})$$

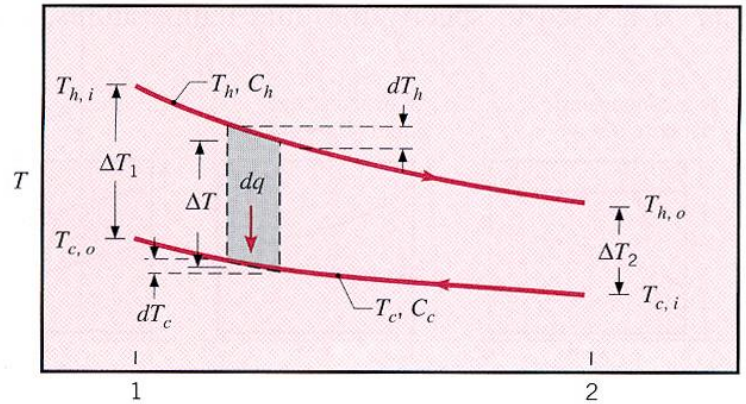

Fig. Heat exchange at counter flow HX

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5.3.1 LMTD method for a counter arrangement

- finally, heat transfer rate at the counter flow hx is represented as

$$q = UA\Delta T_{lm} = UA\frac{\left(\left(T_{h,o} - T_{c,i}\right) - \left(T_{h,i} - T_{c,o}\right)\right)}{\ln\left((T_{h,o} - T_{c,i})/(T_{h,i} - T_{c,o})\right)} = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

- When '**select**' the heat exchanger under a certain fluid condition, LMTD method is a good way to specify the required UA

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5.3.2 ε-NTU method for a counter flow HX

- **Effectiveness, ε** :

$$\varepsilon = \frac{q}{q_{max}} \quad (0 < \varepsilon < 1)$$

$$q = \varepsilon q_{max} = \varepsilon W_{min}(T_{h,i} - T_{c,i})$$

- **Number of Transfer unit, NTU** :

$$NTU = \frac{UA}{W_{min}}$$

- **Heat capacity ratio, W_r** :

$$W_r = \frac{W_{min}}{W_{max}}$$

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5.3.2 ε-NTU method for a counter flow HX

- It is possible to represent ε as a function of NTU and heat capacity ratio for all the types of heat exchanger

$$\varepsilon = f(NTU, W_r)$$

- To '**simulate**' the existing heat exchanger, ε-NTU method is a useful way to obtain heat transfer rate of the heat exchanger

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5.3.2 ε-NTU method for a counter flow HX

- To get an ε-NTU relation for a counter flow HX ($W_{min} = W_h$) , effectiveness is given as

$$\boldsymbol{\varepsilon} = \frac{q}{q_{max}} = \frac{W_h(T_{h,i} - T_{h,o})}{W_{min}(T_{h,i} - T_{c,i})} = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$$

- In a fact that heat transfer rate of each side is same, heat capacity ratio is represented as

$$q = W_h\left(T_{h,i} - T_{h,o}\right) = W_c(T_{c,o} - T_{c,i})$$

$$\boldsymbol{W_r} = \frac{W_{min}}{W_{max}} = \frac{W_h}{W_c} = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{h,o}}$$

5.3.2 ε-NTU method for a counter flow HX

- Meanwhile, rearranging the relation for heat transfer rate and LMTD yields

$$q = UA\frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \quad \Rightarrow \quad \frac{\Delta T_2}{\Delta T_1} = \exp\left[\frac{UA}{q}(\Delta T_2 - \Delta T_1)\right]$$

$$\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \exp\left[\frac{UA}{q}\left[\left(T_{h,o} - T_{c,i}\right) - \left(T_{h,i} - T_{c,o}\right)\right]\right]$$

$$= \exp\left[-UA\left(\frac{\left(T_{h,i} - T_{h,o}\right)}{q} - \frac{\left(T_{c,o} - T_{c,i}\right)}{q}\right)\right] = \exp\left[-UA(\frac{1}{W_{min}} - \frac{1}{W_{max}})\right]$$

- Right hand side of the equation is represented as

$$\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \exp\left[\frac{UA}{W_{min}}\left(1 - \frac{W_{min}}{W_{max}}\right)\right] = \exp[-\mathrm{NTU}(1 - W_r)]$$

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5.3.2 ε-NTU method for a counter flow HX

- To eliminate the outlet temperature of the left hand side, following sequence is needed.

$$\frac{T_{h,o}-T_{c,i}}{T_{h,i}-T_{c,o}}=\frac{\left(T_{h,o}-T_{h,i}\right)+\left(T_{h,i}-T_{c,i}\right)}{\left(T_{h,i}-T_{c,i}\right)+\left(T_{c,i}-T_{c,o}\right)}=\frac{1-\dfrac{\left(T_{h,i}-T_{h,o}\right)}{\left(T_{h,i}-T_{c,i}\right)}}{1-\dfrac{\left(T_{c,o}-T_{c,i}\right)}{\left(T_{h,i}-T_{c,i}\right)}}=\frac{1-\dfrac{\left(T_{h,i}-T_{h,o}\right)}{\left(T_{h,i}-T_{c,i}\right)}}{1-\dfrac{\left(T_{h,i}-T_{h,o}\right)}{\left(T_{h,i}-T_{c,i}\right)}\dfrac{\left(T_{c,o}-T_{c,i}\right)}{\left(T_{h,i}-T_{h,o}\right)}}$$

- In a fact that $\boldsymbol{W_r}=\frac{T_{c,o}-T_{c,i}}{T_{h,i}-T_{h,o}}$ and $\boldsymbol{\varepsilon}=\frac{T_{h,i}-T_{h,o}}{T_{h,i}-T_{c,i}}$

$$\frac{T_{h,o}-T_{c,i}}{T_{h,i}-T_{c,o}}=\frac{1-\varepsilon}{1-\varepsilon W_r}$$

5.3.2 ε-NTU method for a counter flow HX

- Finally, by reconnecting the left hand side and right hand side, the relation for heat transfer rate obtained from the LMTD method is represented as

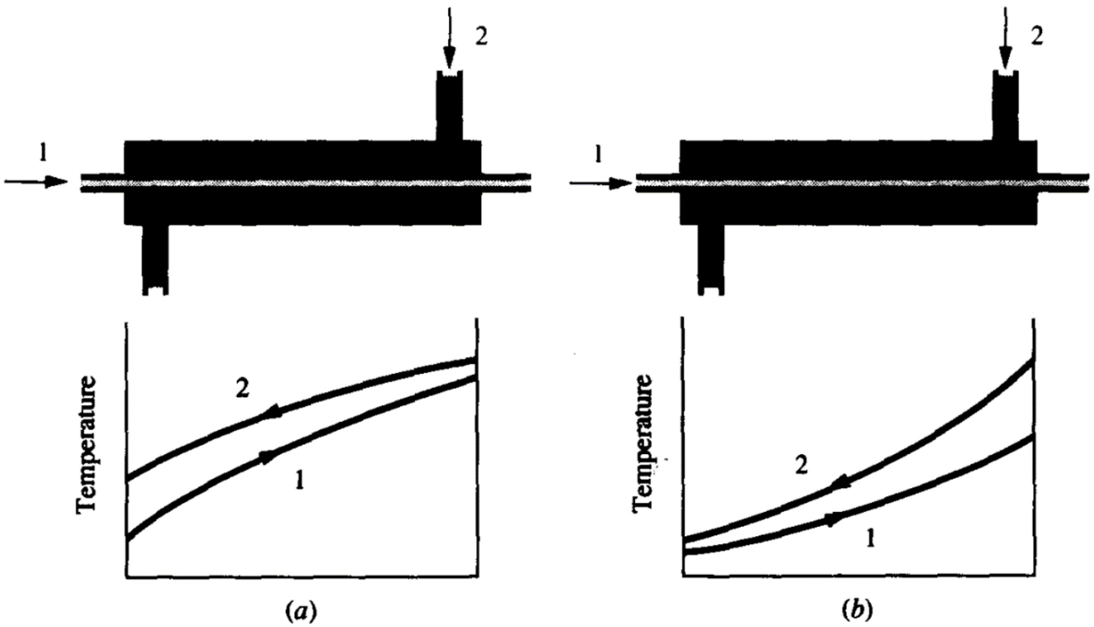
$$\frac{\Delta T_2}{\Delta T_1} = \exp\left[\frac{UA}{q}(\Delta T_2 - \Delta T_1)\right] \quad \Rightarrow \quad \frac{1-\varepsilon}{1-\varepsilon W_r} = \exp[-\mathrm{NTU}(1+W_r)]$$

- Thus, it is obvious that LMTD relation and ε-NTU relation are two different form of one heat transfer system. Rearranging the relation for ε yields

$$\varepsilon = \frac{1 - exp[-NTU(1+W_r)]}{1 - exp[-NTU(1-W_r)]}$$

5.3.2 ε-NTU method for a counter flow HX

- It is possible to get same relation when $W_{min} = W_h$



$W_{min} = (wc_p)_c$ $\quad$ $W_{min} = (wc_p)_h$

Fig. Temperature profiles in a counterflow heat exchanger

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5.5 Evaporator and Condensers

Liquid → Vapor (Evaporator) **Vapor → Liquid** (Condensers)

One of the fluid changes phase, and no superheating or subcooling

→ **Its temperature or pressure remains constant**

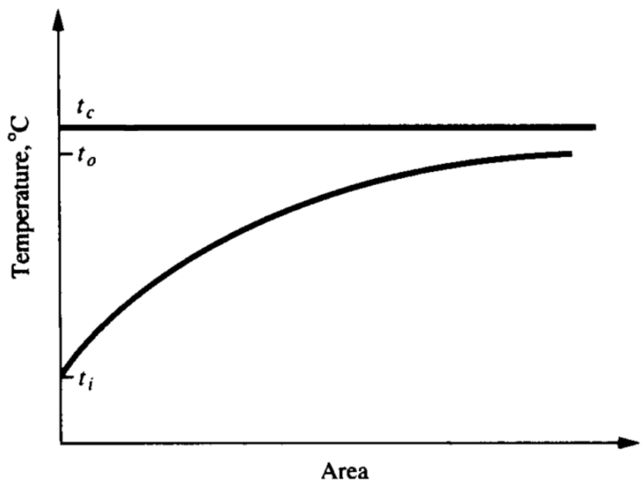

Fig. Temperature distribution in fluids in a condenser

5.5 Evaporator and Condensers

- When secondary fluid(hot side) is at a two phase state, temperature is at a constant state.

$$q = UA \frac{(t_{h,o} - t_c) - (t_{h,i} - t_c)}{\ln[(t_{h,o} - t_c)/(t_{h,i} - t_c)]} \quad \rightarrow \quad \frac{(t_{h,o} - t_c)}{(t_{h,i} - t_c)} = \exp[\frac{UA}{q}(t_{h,i} - t_{h,o})]$$

- Thus, $t_{h,o}$ is represented as

$$t_{h,o} = t_{h,i} - (t_{h,i} - t_{c,i})(1 - e^{-NTU})$$

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5.5 Evaporator and Condensers

- ε-NTU relation for the case is represented as

$$\frac{t_{h,i} - t_{h,o}}{t_{h,i} - t_{c,i}} = \varepsilon = 1 - e^{-NTU}$$

- Or as an alternative form

$$NTU = -\ln(1 - \varepsilon)$$

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5.6 ε-NTU method for several cases

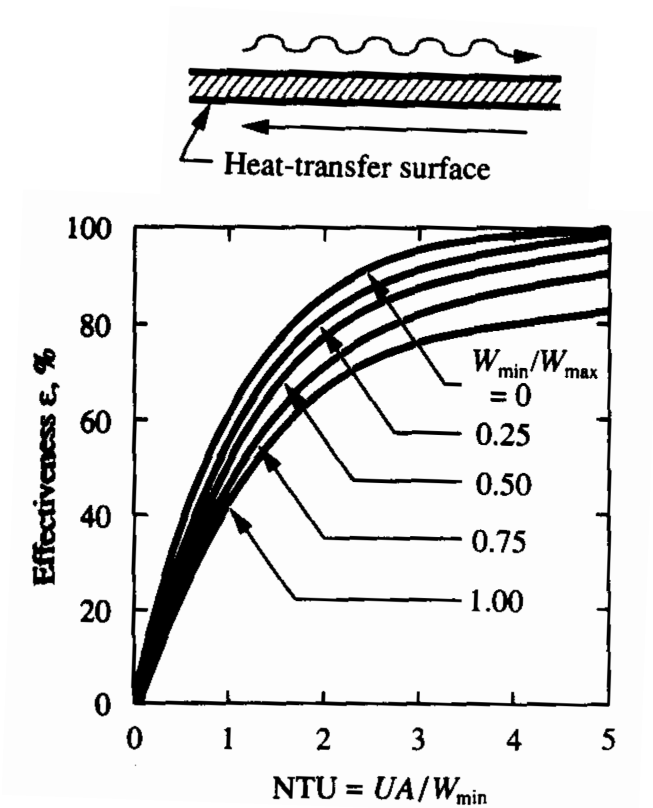

Fig. Effectiveness of counter HX

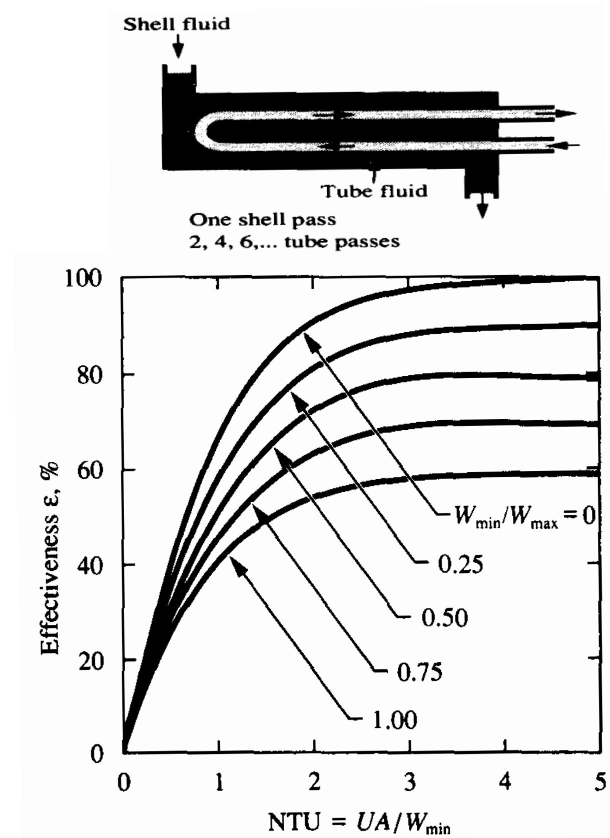

Fig. Effectiveness of parallel HX

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5.6 ε-NTU method for several cases

Table. ε-NTU relation (for NTU)

Flow Arrangement	Relation	
Parallel ow	$\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$	(11.28b)
Counterow	$\mathrm{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$	
	$\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon} \quad (C_r = 1)$	(11.29b)
Shell-and-tube		
One shell pass (2, 4, . . . tube passes)	$(\mathrm{NTU})_1 = -(1+C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$	(11.30b)
	$E = \dfrac{2/\varepsilon_1 - (1+C_r)}{(1+C_r^2)^{1/2}}$	(11.30c)
n shell passes ($2n$, $4n$, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$	(11.31b, c, d)
Cross-ow (single pass)		
C_{max} (mixed), C_{min} (unmixed)	$\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$	(11.33b)
C_{min} (mixed), C_{max} (unmixed)	$\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r\ln(1-\varepsilon)+1]$	(11.34b)
All exchangers ($C_r = 0$)	$\mathrm{NTU} = -\ln(1-\varepsilon)$	(11.35b)

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5.6 ε-NTU method for several cases

Table. ε-NTU relation (for ε)

Flow Arrangement	Relation		
Parallel ow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$		(11.28a)
Counterow	$\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]}$	$(C_r < 1)$	
	$\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$	$(C_r = 1)$	(11.29a)
Shell-and-tube			
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$		(11.30a)
n shell passes ($2n$, $4n$, . . . tube passes)	$\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$		(11.31a)
Cross-ow (single pass)			
Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\}\right]$		(11.32)
C_{max} (mixed), C_{min} (unmixed)	$\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$		(11.33a)
C_{min} (mixed), C_{max} (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$		(11.34a)
All exchangers ($C_r = 0$)	$\varepsilon = 1 - \exp(-\mathrm{NTU})$		(11.35a)

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5.10 Binary solutions

- Mass fraction of A : $x_A = \dfrac{m_A}{m_A + m_B}$

- Mole fraction of A : $y_A = \dfrac{N_A}{N_A + N_B} = \dfrac{m_A/M_A}{m_A/M_A + m_B/M_B}$

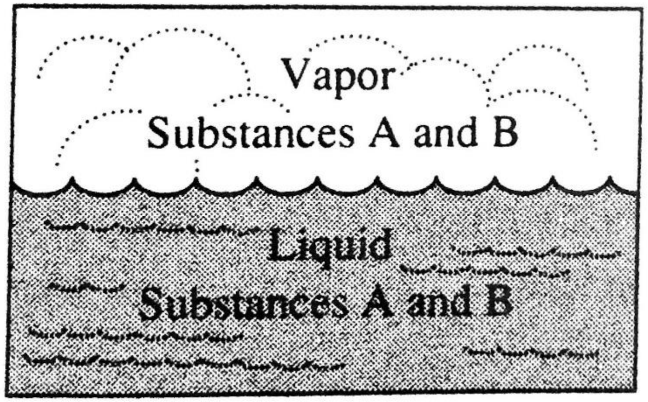

Fig. Typical binary solution

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5.11 Temperature-concentration-pressure characteristics

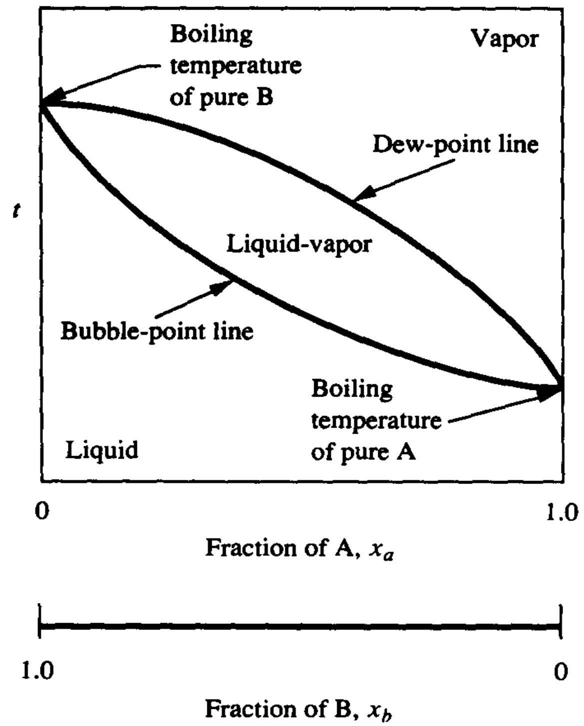

Fig. Temperature-concentration diagram at a constant pressure

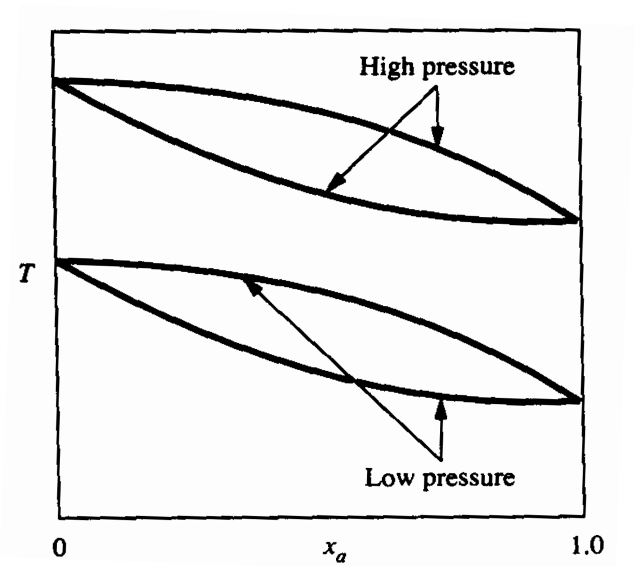

Fig. Temperature-concentration diagram for two different pressure

5.12 Develiping a T vs x diagram

- There exist three tools to develop the binary properties

(Saturation pressure-temperature relation)

$$\ln P = C + \frac{D}{T}$$

Saturation preussre

constant.

Temperature

(Raoults' law)

$$P_a = x_a P_{sat,a}$$

sat. P of pure A

vapor pressure in mixture

mole fraction of A
in the liq. phase

(Partial pressure)

$$P = P_a + P_b$$

$$P_a = y_a P$$

total P

$$P_b = y_b P$$

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5.13 Condensation of a binary mixture

- A pure substance condenses at constant pressure, the temperature remains constant
- On the other hand, temperature of a binary mixture changes progressively

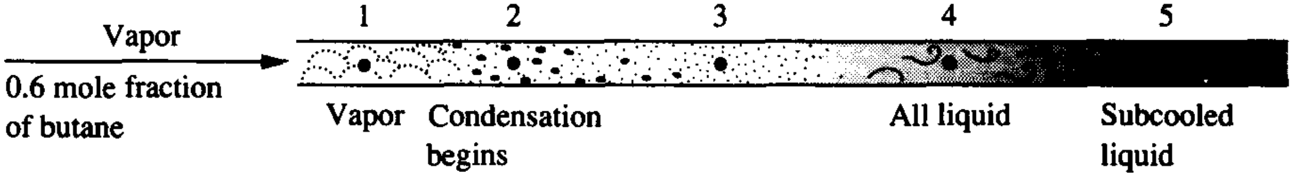

Fig. Condensation of a binary mixture

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5.14 Single stage distillation

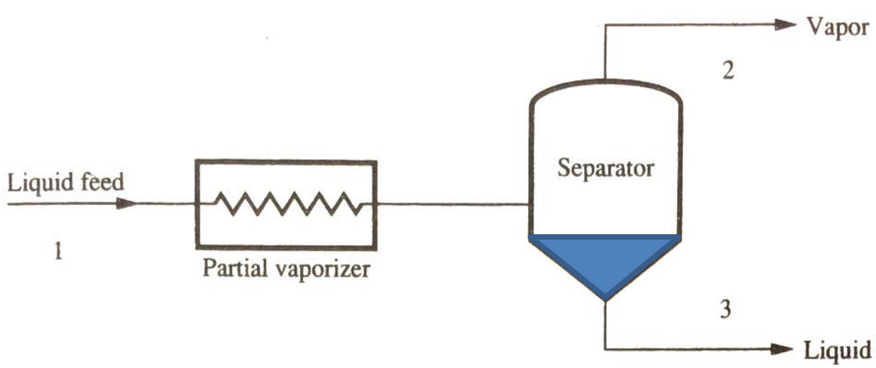

Fig. Single-stage still

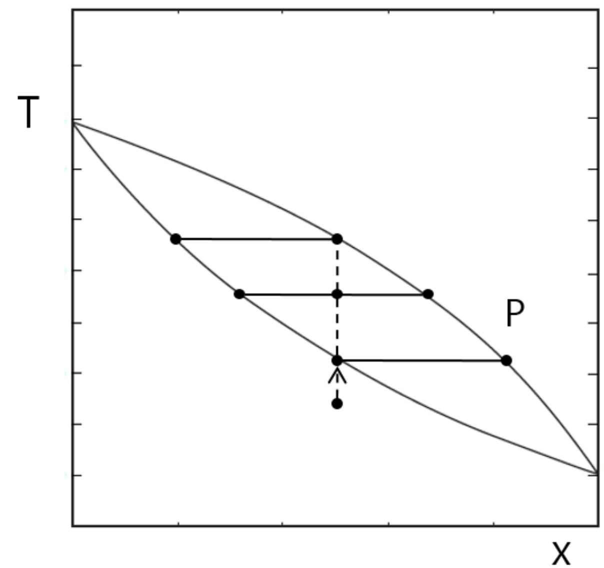

Fig. Some possible outlet conditions

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5.15 Rectification

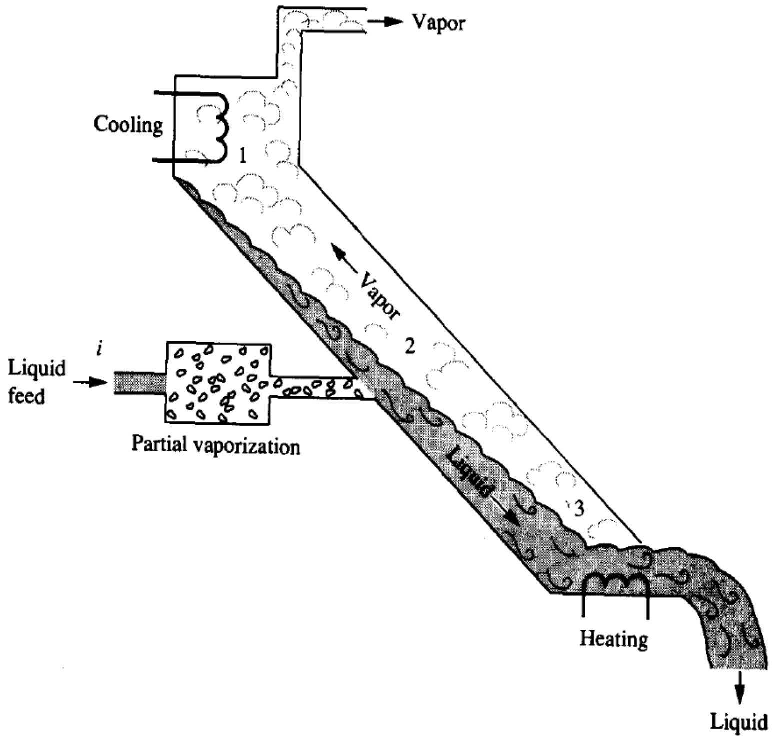

Fig. A rectification column

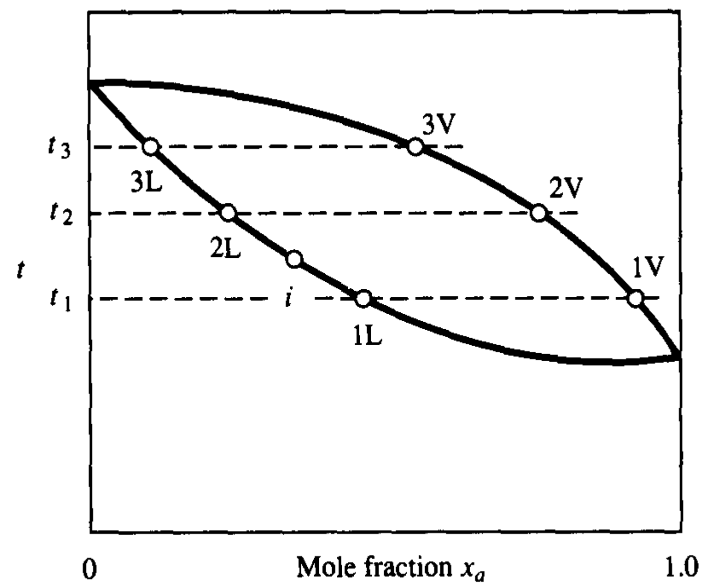

Fig. States of binary system in rectification column

5.16 Enthalpy

- Enthalpy values of binary solutions and mixtures of vapor are necessary
- For system simulation, the enthalpy data would be most convenient in equation form
- More frequently the enthalpy data appear in graphic form as shown in the Figure below

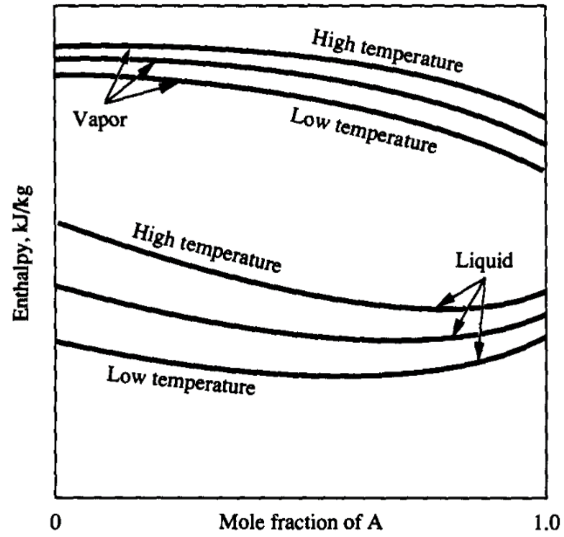

Fig. Form of an enthalpy-concentration diagram

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5.17 Pressure drop and pumping

- Pressure drop of an incompressible fluid :

(C : constant, w : mass of flow, n : 1.8 ~ 2.0)

$$\Delta p = C(w^n)$$

- Power required incompressible fluid :

$$Power = \eta_{pump} T\varpi = \Delta p Q = C(w^n) \cdot \frac{w}{\rho} = \frac{C}{\rho} w^{n+1}$$

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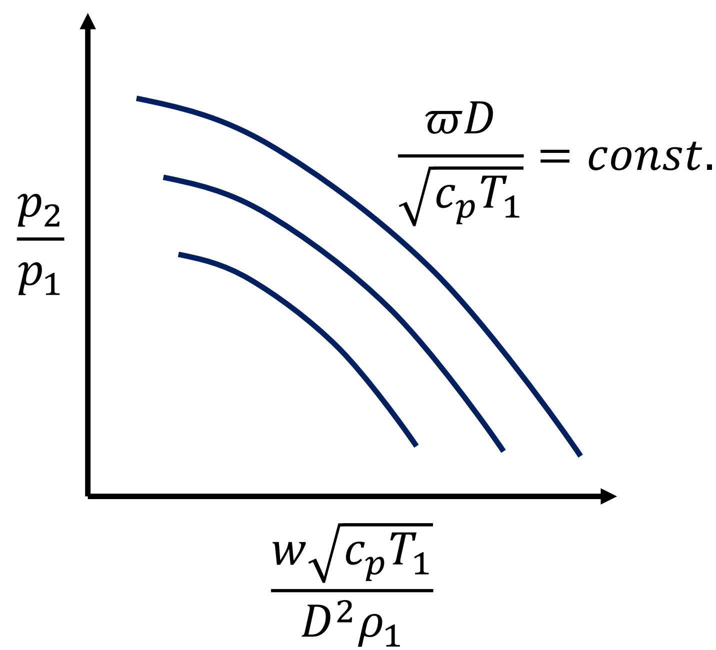
5.18 Turbomachinery

- Dimensional form : $f(p_1, c_p, T_1, \varpi, w, D) = p_2$
- Non-dimensional form : $f\left(\dfrac{w\sqrt{c_p T_1}}{D^2 \rho_1}, \dfrac{\varpi D}{\sqrt{c_p T_1}}\right) = \dfrac{p_2}{p_1}$

using Pi theorem

$$\Pi_1 = \frac{w\sqrt{c_p T_1}}{D^2\rho_1}, \Pi_2 = \frac{\varpi D}{\sqrt{c_p T_1}}, \Pi_3 = \frac{p_2}{p_1}$$



p_1	pressure input
p_2	pressure output
w	mass flow rate
D	impeller diameter
c_p	heat capacity
T_1	input temperature
ρ_1	input density