Optimal Design of Energy Systems

Chapter 14 Steady-State Simulation of Large system

Min Soo KIM

Department of Mechanical and Aerospace Engineering
Seoul National University

Chapter 14. Steady-State Simulation

14.1 Developments in system simulation

Moderate size simulation (chap.6)

- Successive substitution
- Newton-Raphson technique

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For large system

- Time, memory problems
- Need to refine methods

Example 14.1 : Convergence & Divergence in successive substitution

Using ***Gauss-Seidel method***, solve for the x's in the following linear equation.

$$A : 4x_1 - 3x_2 + x_3 = 12$$
$$B : x_1 - 2x_2 + 2x_3 = 6$$
$$C : 2x_1 + x_2 + 3x_3 = 6$$

(Solution)

Trial value

$x_2=0,\ x_3=0$

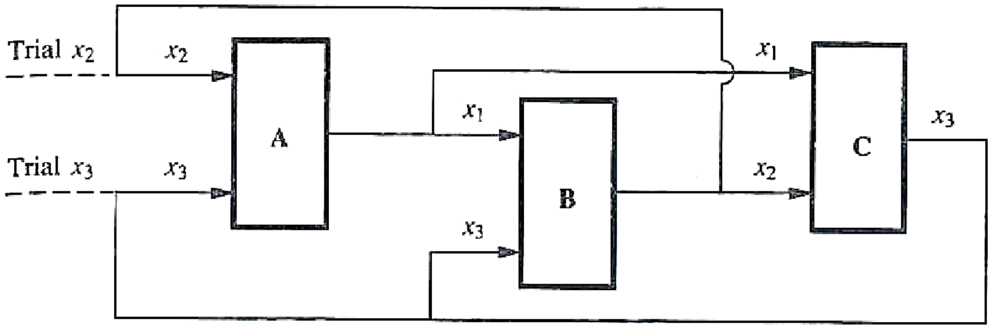

FIGURE 14-1
The Gauss-Seidel method as a successive substitution process.

Example 14.1 : Convergence & Divergence in successive substitution

Result : Table 14.1

Cycle	x_1	x_2	x_3
1	3.0	-1.5	0.5
2	1.75	-1.625	1.375
10	2.045	-1.021	0.977
∞	2	-1	1

Convergent

$x_1 = 2, \quad x_2 = -1, \quad x_3 = 1$

Example 14.1 : Convergence & Divergence in successive substitution

With different order

$$A : x_1 = (12 + 3x_2 - x_3)/4$$
$$C : x_2 = 6 - 2x_1 + 3x_3$$
$$B : x_3 = (6 - x_1 + 2x_2)/2$$

Result : Table 14.2

Cycle	x_1	x_2	x_3
1	3	0	1.5
2	2.625	-3.75	-2.0625
3	0.703	10.78	13.43
4	7.29	-49.75	-50.61

Diverge

Example 14.1 : Convergence & Divergence in successive substitution

Method 1

A-B-C, convergent

$$\begin{bmatrix} 4 & -3 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

A-C-B, divergent

$$\begin{bmatrix} 4 & -3 & 1 \\ 2 & 1 & 3 \\ 1 & -2 & 2 \end{bmatrix}$$

For convergent case, <u>large-magnitude</u> coefficient in <u>diagonal position</u>

Example 14.1 : Convergence & Divergence in successive substitution

Method 2

Multiply lower-triangular by λ

A-B-C, convergent

$$\begin{vmatrix} 4\lambda & -3 & 1 \\ \lambda & -2\lambda & 2 \\ 2\lambda & \lambda & 3\lambda \end{vmatrix} = 0$$

$\lambda = 0, 0.125 + 0.696i, 0.125 - 0.696i$

$all\ |\lambda| < 1, converge$

A-C-B, divergent

$$\begin{vmatrix} 4\lambda & -3 & 1 \\ 2\lambda & \lambda & 3 \\ \lambda & -2\lambda & 2\lambda \end{vmatrix} = 0$$

$\lambda = 0, 0.2713, -4.146$

$not\ all\ |\lambda| < 1, diverge$

14.3 Partial substitution in successive substitution

$$x_{j,i+1} = \beta x_{j,i+1^*} + (1-\beta)x_{j,i}$$

$$\beta = partial\ substitution\ factor$$
$$x_j = variable\ being\ computed$$
$$i = indicating\ previous\ value$$
$$i+1 = indicating\ new\ value$$
$$i+1^* = indicating\ new\ value\ computed\ directly\ from\ the\ equation$$

$\beta = 1$: Successive substitution (Gauss-Seidel method)

$reduce\ \beta$: Toward a more convergent process

14.4 Evaluation of Newton-Raphson technique

Successive substitution

- Straightforward to program
- Sparing PC memory

Newton-Raphson

- More reliable
- Fast convergence

14.5 Some characteristics of the Newton-Raphson technique

- Sequence of equations is not important
- Probability of convergence is high
- Diverge if trial values are too far from the final solution
- Variables may change a large amount on the first iteration (Fig. 14-6)

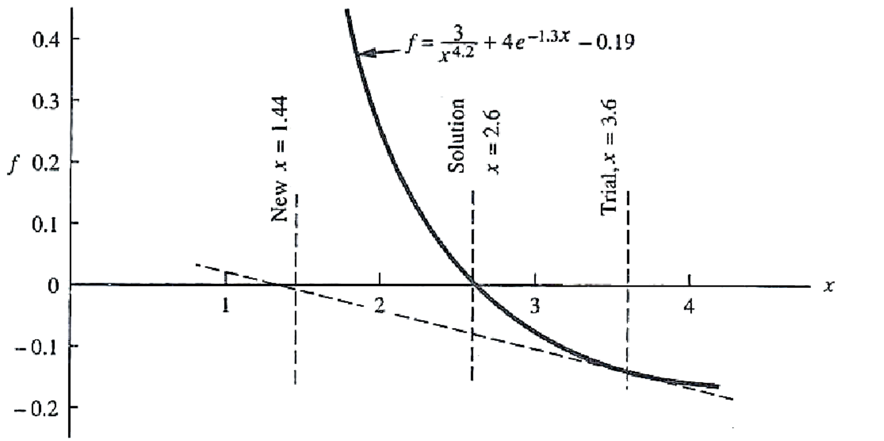

FIGURE 14-6
First Newton-Raphson iteration may move the values of the variables further from the solution than the trial values.

Damp the first iteration by changing the variables if failure occurs

14.8 Quasi-Newton method

Newton Raphson method : The corrections of the variables are found by solution of the set of linear equations whose coefficient matrix J is composed of the partial derivatives as like below

$$f_1(x_{1,t},x_{2,t},x_{3,t}) \approx f_1(x_{1,c},x_{2,c},x_{3,c}) + \frac{\partial f_1(x_{1,t},x_{2,t},x_{3,t})}{\partial x_1}(x_{1,t}-x_{1,c}) + \frac{\partial f_1(x_{1,t},x_{2,t},x_{3,t})}{\partial x_2}(x_{2,t}-x_{2,c}) + \frac{\partial f_1(x_{1,t},x_{2,t},x_{3,t})}{\partial x_3}(x_{3,t}-x_{3,c})$$

$$\Rightarrow \quad \boldsymbol{JX} = \boldsymbol{F} \ : \ \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} x_{1,t}-x_{1,c} \\ x_{2,t}-x_{2,c} \\ x_{3,t}-x_{3,c} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

14.8 Quasi-Newton method

Quasi-Newton method: In the quasi-Newton technique with the Broyden update, equation is changed as below (inversed *J* is applied)

$$\boldsymbol{JX=F} \ : \ \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \begin{bmatrix} x_{1,t}-x_{1,c} \\ x_{2,t}-x_{2,c} \\ x_{3,t}-x_{3,c} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \Rightarrow \boldsymbol{X=-HF}$$

Further, H is updated for each iteration by an operation expressed symbolically as

$$H_{k+1}=H_k+\frac{(X_k-H_kY_k)X_k^TH_k}{X_k^TH_kY_k}$$

$X = \text{the } \textbf{negative} \text{ of the } \Delta x\text{'s used in chapter 6}$

$H = \text{the inverse of the partial derevative matrix} = J^{-1}$

$Y_k = F_{k+1}-F_k$

14.8 Quasi-Newton method

Newton-Raphson vs Quasi-Newton

(Example) Volume flow rate and pressure relation of duct and fan is presented as like below. Find operation point by using Newton-Raphson method and Quasi-Newton method

duct $f_1 = 0.0625 + 0.653Q^{1.8} - P$

fan $f_2 = 0.3 - 0.2Q^2 - P$

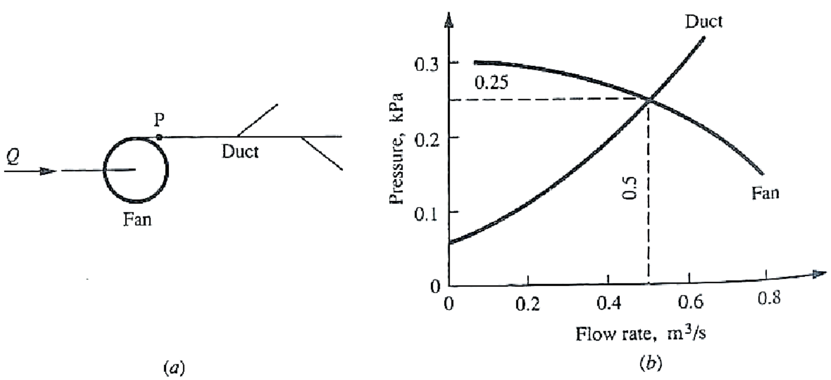

FIGURE 14-11
(*a*) A fan-duct system, and (*b*) the pressure-flow characteristics.

14.8 Quasi-Newton method

Newton-Raphson vs Quasi-Newton

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial P} & \frac{\partial f_1}{\partial Q} \\ \frac{\partial f_2}{\partial P} & \frac{\partial f_2}{\partial Q} \end{bmatrix} \quad X = \begin{bmatrix} P_t - P_c \\ Q_t - Q_c \end{bmatrix} \quad F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$J = \begin{bmatrix} -1 & 1.1759 \\ -1 & -0.4 \end{bmatrix} \quad F = \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix}$$

➡

$$J^{-1} = \begin{bmatrix} -0.2539 & -0.7461 \\ 0.6345 & -0.6345 \end{bmatrix} \quad X = \begin{bmatrix} -0.156 \\ 0.392 \end{bmatrix}, \quad P_t = 0.256, \quad Q_t = 0.608$$

14.8 Quasi-Newton method

Newton-Raphson vs Quasi-Newton

Iteration	Variables	Matrix of Partial Derivatives		Inverse of Matrix	
At trial values	P=0.1	-1.0	1.1759	-0.2539	-0.7461
	Q=1.0	-1.0	-0.4000	0.6345	-0.6345
1	P=0.256	-1.0	0.7902	-0.2356	-0.7644
	Q=0.608	-1.0	-0.2435	0.9674	-0.9674
2	P=0.250	-1.0	0.6836	-0.2291	-0.7709
	Q=0.508	-1.0	-0.2032	1.1276	-1.1276
3	P=0.250	-1.0	0.6748	-0.2285	-0.7715
	Q=0.500	-1.0	-0.1999	1.1432	-1.1432
4	P=0.250	-1.0			
	Q=0.500	-1.0			

$$P = 0.25\ \text{kPa}$$

$$Q = 0.5\ \text{m}^3/\text{s}$$

14.8 Quasi-Newton method

Newton-Raphson vs Quasi-Newton

$$F_k = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix}$$

$$X_k = \begin{bmatrix} P_{1,c} - P_{1,t} \\ Q_{1,c} - Q_{1,t} \end{bmatrix} = -H_k F_k = -\begin{bmatrix} -0.25392 & -0.74608 \\ 0.63449 & -0.63449 \end{bmatrix} \begin{bmatrix} 0.6155 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.15629 \\ -0.39053 \end{bmatrix}$$

$$V_{k+1} = \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix} + \begin{bmatrix} 0.15629 \\ -0.39053 \end{bmatrix} = \begin{bmatrix} 0.25629 \\ 0.60947 \end{bmatrix}$$

$$F_{k+1} = \begin{bmatrix} 0.07402 \\ -0.03058 \end{bmatrix}, Y_k = F_{k+1} - F_k = \begin{bmatrix} -0.54148 \\ -0.03058 \end{bmatrix}$$

$$H_{k+1} = H_k + \frac{(X_k - H_k Y_k) X_k^T H_k}{X_k^T H_k Y_k} = \begin{bmatrix} -0.24630 & -0.74956 \\ 0.76030 & -0.69190 \end{bmatrix}$$

14.8 Quasi-Newton method

Newton-Raphson vs Quasi-Newton

Iteration	Variables, V	Functions, F	Inverse, H	
1	0.25629	0.074024	-0.24630	-0.74956
	0.90647	-0.030579	0.76030	-0.69190
2	0.25160	0.020603	-0.23200	-0.76370
	0.53203	-0.008211	1.04286	-0.97139
3	0.25011	0.001656	-0.22850	-0.76713
	0.50257	-0.000624	1.13130	-1.05802
4	0.25001	0.000042	-0.22744	-0.76816
	0.50004	-0.000016	1.16053	-1.08636

Newton-Raphson	Quasi-Newton
Fast convergence if the trial values are good	Wider convergence range than NR method