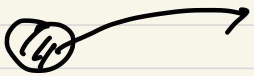
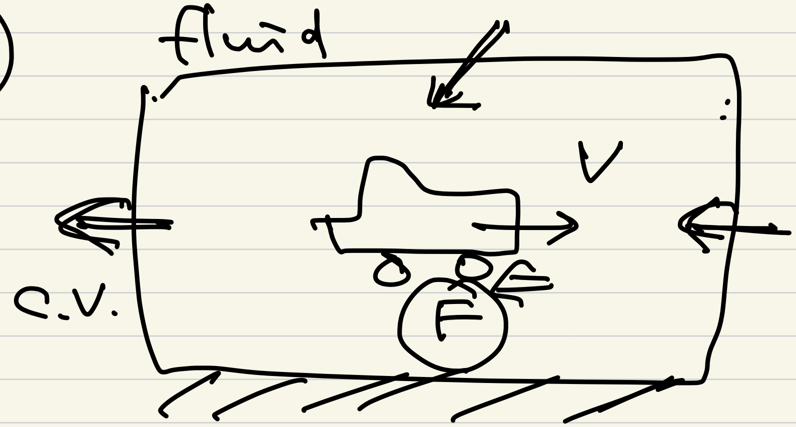
Ch. 3 Integral relations for a control volume

System vs. control volume

mass



$F = ma$



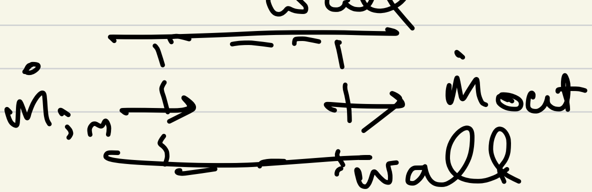
⊙ Reynolds transport theorem

$$\boxed{\frac{d}{dt}B_{sys} = \frac{d}{dt}\int_{c.v}\beta\rho\,d\forall + \int_{c.s}\rho\beta(\underline{V}_r\cdot\underline{n})\,dA}$$

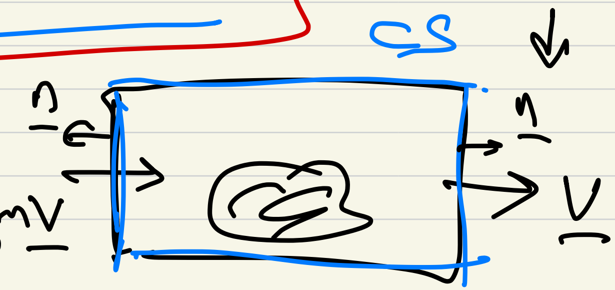
$$\beta = dB/dm\,,\quad \underline{V}_r = \underline{V} - \underline{V}_c$$

• mass conservation : $B_{sys} = m\,,\ \beta = \dfrac{dm}{dm} = 1$

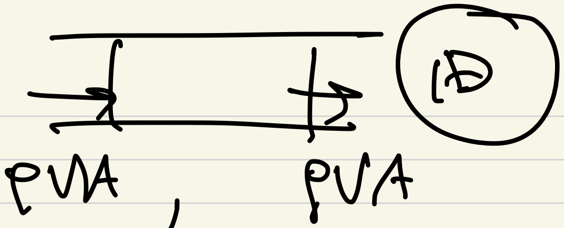
$$\Rightarrow \frac{dm}{dt} = \boxed{0 = \frac{d}{dt}\int_{cv}\rho\,d\forall + \underbrace{\int_{cs}\rho(\underline{V}_r\cdot\underline{n})\,dA}_{(\rho VA)_{out} - (\rho VA)_{in}}}$$

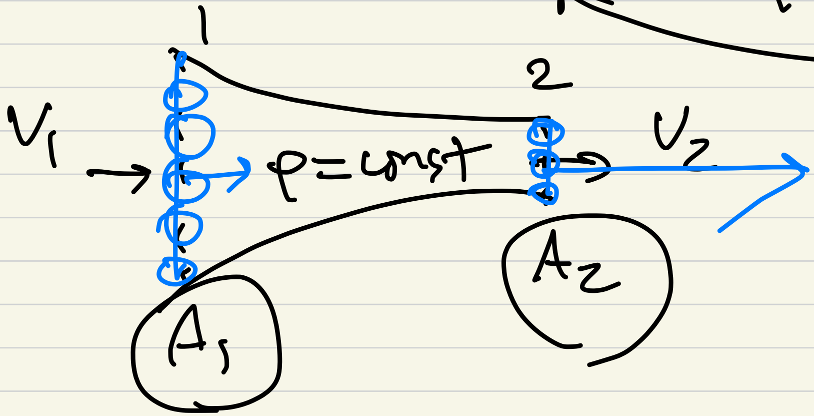


$\dot m_{out} = \dot m_{in}$



$\rightarrow \rho V A = \text{const}$



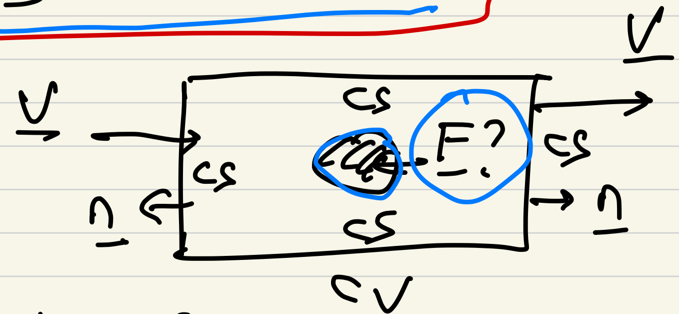


$\rho V_1 A_1 = \rho V_2 A_2$

$\rightarrow V_2 = V_1 \dfrac{A_1}{A_2} > V_1$

- mtm conservation ($B_{sys} = m\underline{V}$; $\beta = \dfrac{dB}{dm} = \underline{V}$)

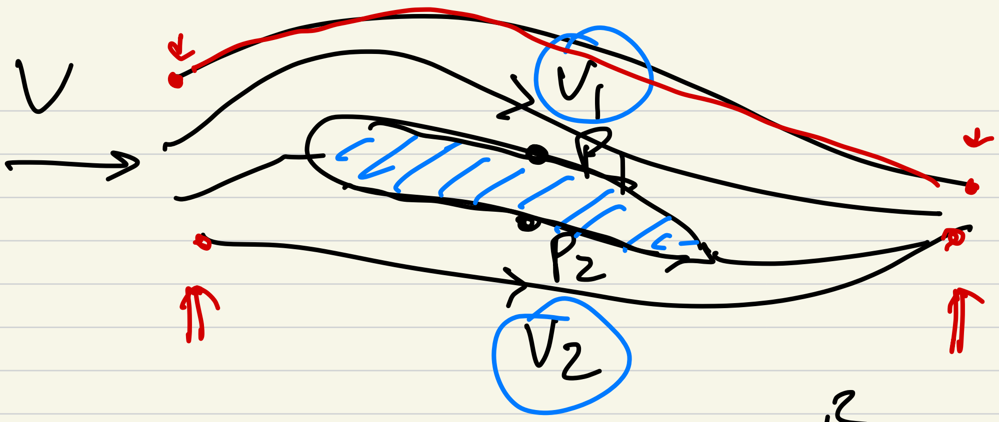
$$\rightarrow \frac{d}{dt}(m\underline{V}) = \boxed{\Sigma \underline{F} = \frac{d}{dt}\int_{CV} \rho \underline{V}\, d\forall + \int_{CS} \rho \underline{V} (\underline{V}_r \cdot \underline{n})\, dA}$$



$\Downarrow$

Bernoulli eq.
$$\int_1^2 \frac{\partial V}{\partial t} ds + \int_1^2 \frac{1}{\rho} dp + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1) = 0$$

unsteady frictionless flow along a streamline



$$V_1 > V_2 \xrightarrow{B} P_1 < P_2$$

$\Uparrow$ why? $\downarrow$ lift $\Uparrow$

Ch.8

steady : $\dfrac{P}{\rho} + \dfrac{V^2}{2} + gz = \text{const}$

- Angular mtm conservation : $\underline{B} = m(\underline{r} \times \underline{V})$, $\beta = \underline{r} \times \underline{V}$

$$\frac{d}{dt}\Sigma(\underline{r}\times\underline{V})\,dm = \Sigma M_o = \underbrace{\frac{d}{dt}\int_{cv}\rho(\underline{r}\times\underline{V})\,d\forall}_{} + \underbrace{\int_{cs}\rho(\underline{r}\times\underline{V})(\underline{V_r}\cdot\mathbf{n})\,dA}_{}$$

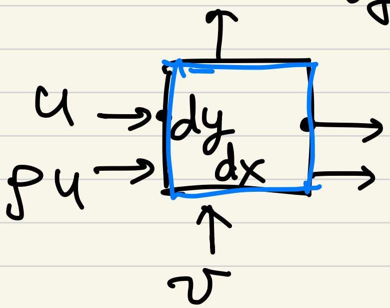
$e = e_{internal} + e_{kinetic} + e_{potential} + \cdots$

- energy conservation

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt} = \frac{d}{dt}\int_{cv}\rho e\,d\forall + \int_{cs}\rho e(\underline{V_r}\cdot\mathbf{n})\,dA$$

Ch.4 Diff'l relations for fluid flow

$v + \frac{\partial v}{\partial y} dy + \frac{1}{2}\frac{\partial^2 v}{\partial y^2} dy^2 + \cdots$



$u + \frac{\partial u}{\partial x} dx + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} dx^2 + \cdots$

$\rho u + \frac{\partial}{\partial x}(\rho u) dx + \frac{1}{2}\frac{\partial^2}{\partial x^2}(\rho u) dx^2 + \cdots$

mass conser. : $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \underline{V}) = 0$ continuity eq.

mtm conser : $\boxed{\frac{\partial}{\partial t}(\rho \underline{V}) + \nabla\cdot(\rho \underline{V}\,\underline{V}) = \rho \underline{g} - \nabla p + \nabla\cdot(\mu \nabla \underline{V})}$

Navier-Stokes equation

$\underline{V} = (u, v, w)$

$$\left\{\begin{aligned}
&\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u u) + \frac{\partial}{\partial y}(\rho u v) + \frac{\partial}{\partial z}(\rho u w) = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)\\
&\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}(\rho v v) + \frac{\partial}{\partial z}(\rho v w) = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)\\
&\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho u w) + \frac{\partial}{\partial y}(\rho v w) + \frac{\partial}{\partial z}(\rho w w) = \rho g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned}\right.$$

coupled, unsteady, nonlinear, 2nd-order PDE, three-dimensional, unknowns: ρ, u, v, w, P, T 6

6 eqs.

$P = \rho R T$ $\Leftarrow$

$\frac{\partial}{\partial t}(\rho c_p T) + \cdots$ energy eq.

· stream ft. : $\nabla \cdot \underline{u} = 0$ continuity eq. for ρ = const
유선 incomp. flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ 3D → ~~stream ft~~

2D : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$\rightarrow u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$

ψ : stream ft.

ψ = const : streamline

· vorticity : $\underline{\zeta} = \nabla \times \underline{V} \neq 0$: rotational flow
와도

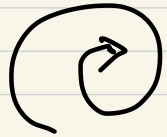
$= 0$: irrotational flow

$$\nabla \times \underline{V} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

i_z : $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \zeta_z$

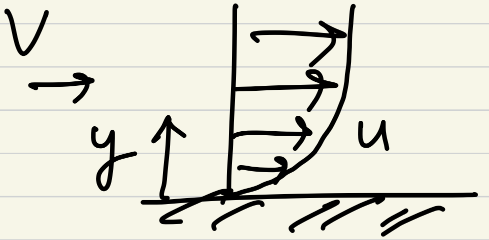
i_x : $\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \zeta_x$

vorticity vs. vortex (와류)



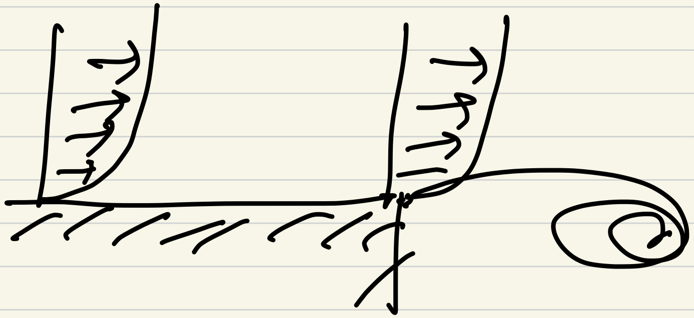
$\underline{\xi} = \nabla \times \underline{V}$ $\neq$ ~~수학적 정의~~

수학적 정의 물리적 정의

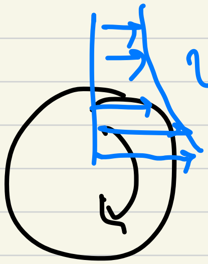


$v = 0$

$\frac{\partial u}{\partial y} \neq 0 \rightarrow \underline{\xi} \neq 0$



no vortex



$v \sim \frac{1}{r} \rightarrow \underline{\xi} = 0$

zero vorticity

vortex

- frictionless irrotational flow $\rightarrow$ Bernoulli eq. every where

$$\underline{\xi} = 0$$
$$\parallel$$
$$\nabla \times \underline{V} = 0 \rightarrow \underline{V} = \nabla \phi$$

ϕ : velocity potential

$\phi = \text{const} \perp \psi = \text{const}$

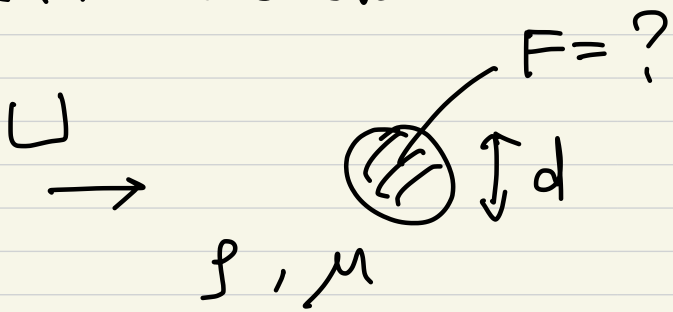
Ch. 5 Dimensional analysis and similarity

- Principle of dimensional homogeneity

$$A = B + C + D$$

$$W = mg \longrightarrow W = 9.81\ m \quad \times$$

- Pi (π) theorem

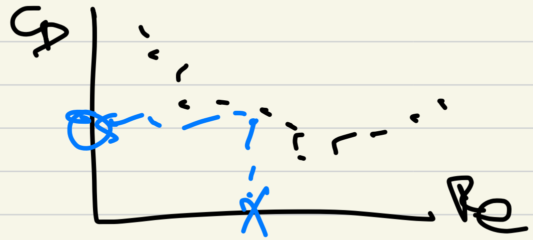


$$F = F(\overset{L}{d}, \overset{T}{U}, \overset{M}{\rho}, \mu) \Leftarrow$$

$$\Pi_1 = F d^{\alpha} U^{\beta} \rho^{\gamma} = [L^0, T^0, M^0]$$

$$\Pi_2 = \mu d^{\alpha'} U^{\beta'} \rho^{\gamma'} = [L^0, T^0, M^0]$$

$$\rightarrow \boxed{\overset{C_D \,\parallel}{\frac{F}{\rho U^2 d^2}} = g\left(\frac{\rho U d}{\mu}\right)} = g(Re)$$

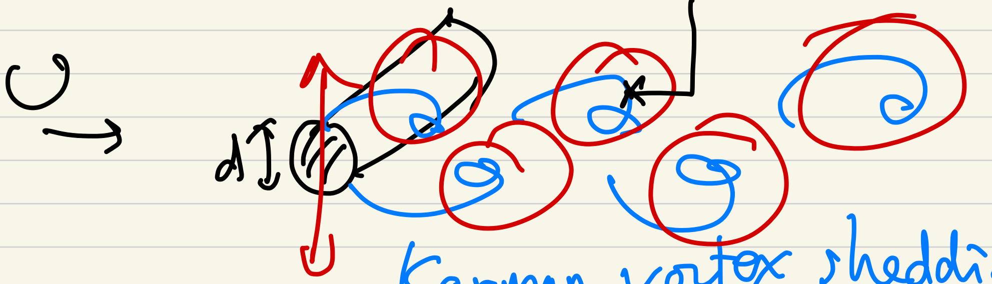


- Non-dimensionalization of governing eqs.

$$\underline{v}^* = \underline{v}/U \qquad \rightarrow \qquad \nabla^* \cdot \underline{v}^* = 0$$

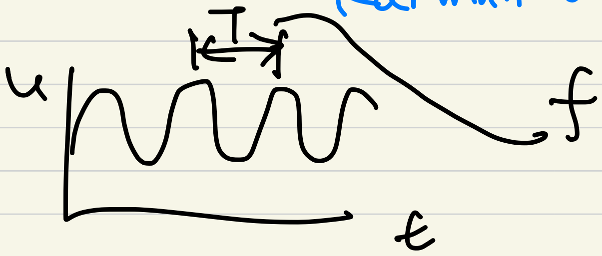
$$x^* = x/L \qquad \frac{\partial \underline{v}^*}{\partial t^*} + (\underline{v}^* \cdot \nabla^*)\, \underline{v}^* = -\nabla^* p^* + \frac{1}{Re} \nabla^{*2} \underline{v}^*$$

$Re = \frac{\rho U d}{\mu}$, Fr, We, St - - - - .



$Re = \frac{\rho U d}{\mu}$

Karman vortex shedding occurs at $Re \geq 47$.



$St = \frac{fd}{U}$

strouhal #

$d = 0.15\,m$

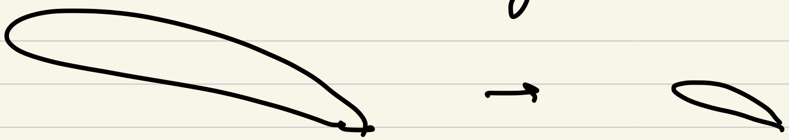
air $\nu = 1.5 \times 10^{-5}\,m^2/s$

$Re = \frac{\rho U d}{\mu} = \frac{Ud}{\nu} = 47$

$\rightarrow U = \frac{47 \times 1.5 \times 10^{-5}}{0.15} = 47 \times 10^{-4}\,m/s$

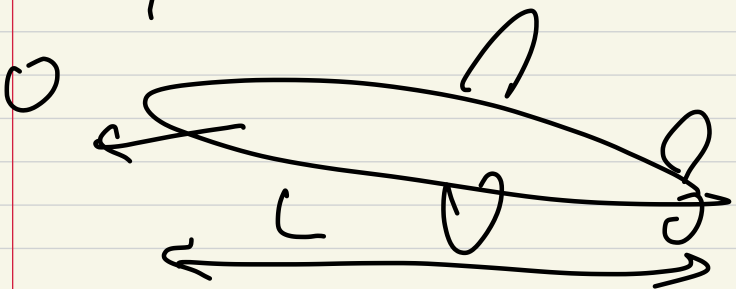
$= 4.7 \times 10^{-3}\,m/s = 4.7\,mm/s$

- Geometric similarity [L]



Kinematic " [L, T]

Dynamic " [M, L, T] $\rightarrow Re = \frac{\rho U L}{\mu}$



wind tunnel

$L \Rightarrow \frac{1}{10} L$

$U \Rightarrow 10U$

not easy

$\boxed{CFD}$