Topics in Ship Structures

02 Low Cycle Fatigue for Base Material

Reference : Fundamentals of Metal Fatigue Analysis Ch. 2 Strain – Life

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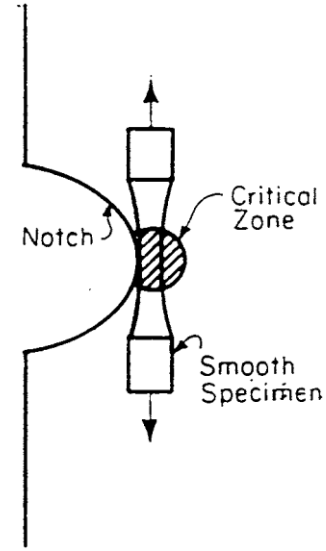
High Cycle Fatigue vs Low Cycle Fatigue

- Each failure occurs by apparently different physical mechanisms
- High cycle fatigue
- *Low cycle fatigue*
  - ✓ Significant plastic strain occurs during at least some of the loading cycles.
  - ✓ Relatively short fatigue lives between 10~100,000 cycles
  - ✓ Ductility and resistance to plastic flow are important
  - ✓ Post welding treatment and high tensile material are not effective.
  - ✓ Engineering Structures are designed such that the nominal loads remain elastic.
  - ✓ However, stress concentrations often cause plastic strains to develop in the vicinity of notches.
  - ✓ Crack initiation life is estimated.



Basic Definitions

- Engineering Stress and Strain

$$S = engineering\ stress = \frac{P}{A_0}$$

$$e = engineering\ strain = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$$

- The true stress is defined as the ratio of the applied load to the instantaneous cross sectional area

$$\sigma = true\ stress = \frac{P}{A}$$

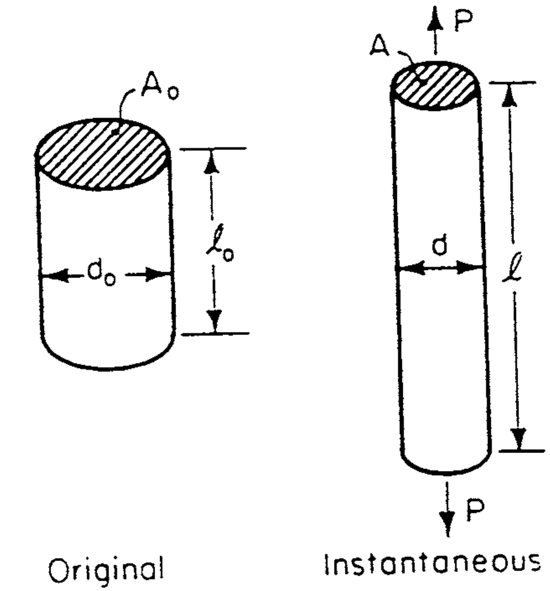
- The true strain is defined as the sum of all the instantaneous engineering strains.

$$\varepsilon = true\ strain = \int_{l_0}^{l} \frac{dl}{l} = \ln \frac{l}{l_0}$$

- Incase of engineering strain.

$$e = \text{engineering strain} = \int_{l_0}^{l} \frac{dl}{l_0} = \frac{l - l_0}{l_0}$$

P = applied load
l_0 = original length
d_0 = original diameter
A_0 = original area
l = instantaneous length
d = instantaneous diameter



Original and deformed configuration

True and engineering stress-strain

True and Engineering Stress-Strain Relationship (valid up to necking)

The instantaneous length : $l = l_0 + \Delta l$

The true strain : $\varepsilon = \ln \frac{l_0 + \Delta l}{l_0} = \ln\left(1 + \frac{\Delta l}{l_0}\right) = \boxed{\ln(1+e)}$

The volume remains constant up to necking

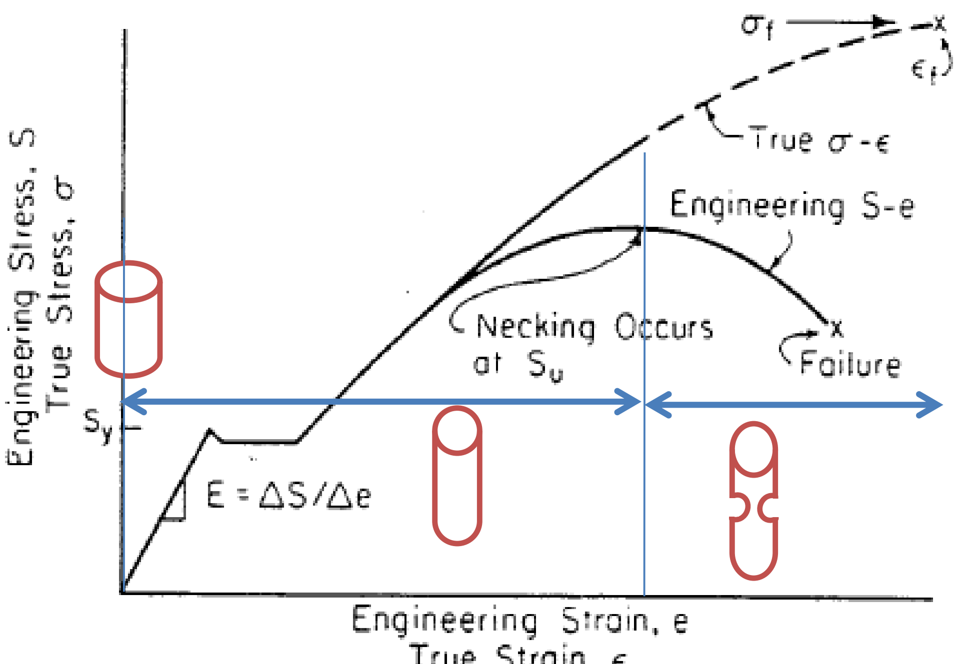
$$A_0 l_0 = Al \Rightarrow \frac{A_0}{A} = \frac{l}{l_0}$$

$$\boxed{\varepsilon = \ln \frac{l}{l_0} = \ln \frac{A_0}{A}}$$

$$P = SA_0 \qquad \sigma = \frac{P}{A} \Rightarrow \sigma = S\frac{A_0}{A}$$

$$\Rightarrow \boxed{\varepsilon = \ln(1+e) = \ln \frac{A_0}{A}} \Rightarrow \frac{A_0}{A} = 1 + e$$

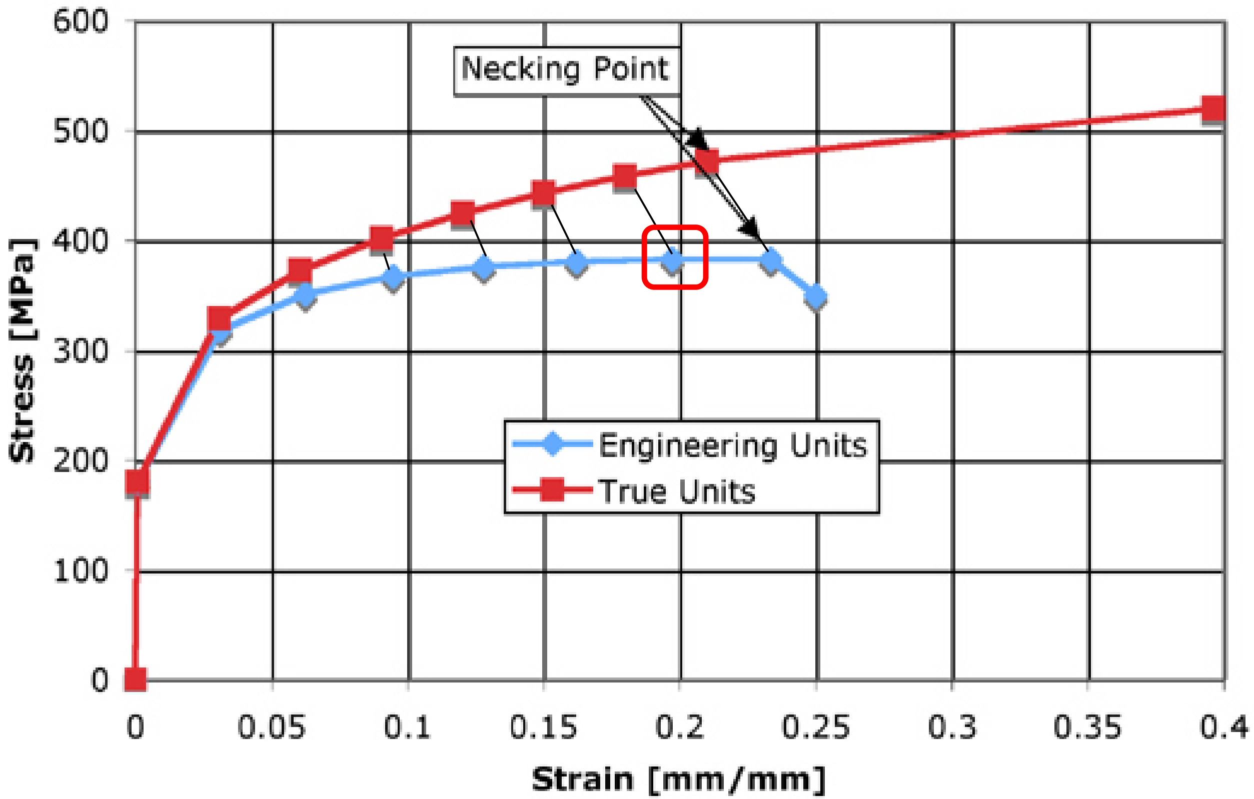
$$\sigma = S(1+e) \qquad \varepsilon = \ln(1+e)$$



Comparison engineering and true stress-strain

True and engineering stress-strain

$(e,S)=(0.2,380)$ $\rightarrow$ $\boxed{\begin{array}{c}\varepsilon=\ln(1+e)\\ \sigma=S(1+e)\end{array}}$ $\rightarrow$ $(\varepsilon, \sigma)=(0.182, 456)$



Comparison engineering and true stress-strain

Stress-Strain relationship

- Total true strain (ε_t) = Linear elastic strain (ε_e)+ plastic strain (ε_p)

$$\varepsilon_t = \varepsilon_e + \varepsilon_p$$

- For most metals a log-log plot of true stress versus true plastic strain is modeled as a straight line.

$$\sigma = K(\varepsilon_p)^n \qquad \varepsilon_p = \left(\frac{\sigma}{K}\right)^{1/n}$$

K : strength coefficient, n : strain hardening exponent.

- True fracture strength $\sigma_f = \dfrac{P_f}{A_f}$

A_f : area at fracture, P_f : load at fracture.

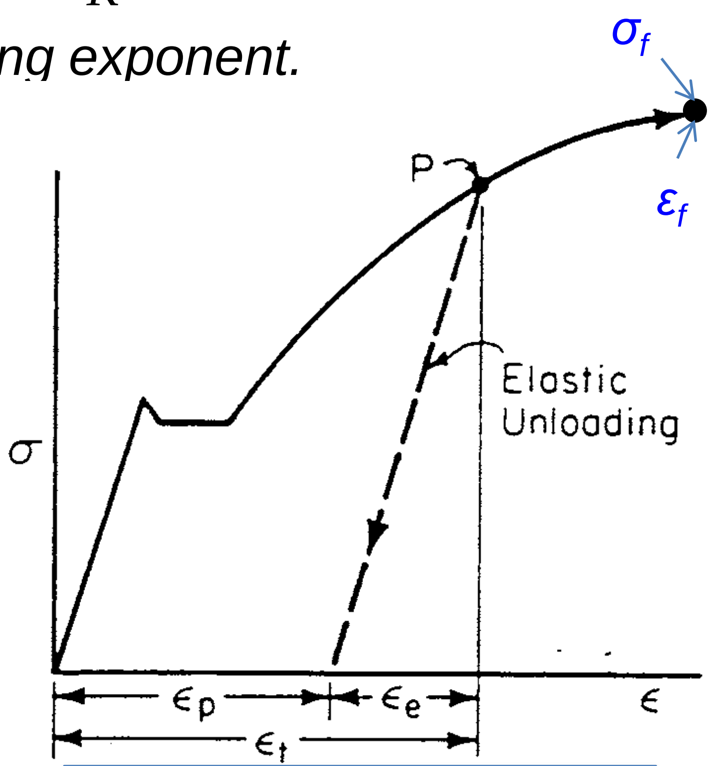
- True fracture ductility, true strain at final fracture.

$$\varepsilon_f = \ln\frac{A_0}{A_f} = \ln\frac{1}{1-RA}, \quad RA = \frac{A_0 - A_f}{A_0}$$

RA : Reduction in area

- K can be defined in terms of σ_f and ε_f .

$$\sigma_f = K(\varepsilon_f)^n \qquad K = \frac{\sigma_f}{\varepsilon_f^n}$$



Elastic and plastic strain

Stress-Strain relationship

- Plastic strain can be defined in terms of these quantities.

$$\boxed{\varepsilon_p = \left(\frac{\sigma}{K}\right)^{1/n}} \quad \boxed{K = \frac{\sigma_f}{\varepsilon_f^n}} \quad \Rightarrow \quad \varepsilon_p = \left(\frac{\sigma}{\sigma_f / \varepsilon_f^n}\right)^{1/n} = \left(\frac{\sigma \varepsilon_f^n}{\sigma_f}\right)^{1/n} = \varepsilon_f \left(\frac{\sigma}{\sigma_f}\right)^{1/n}$$

- Total strain can be expresses as

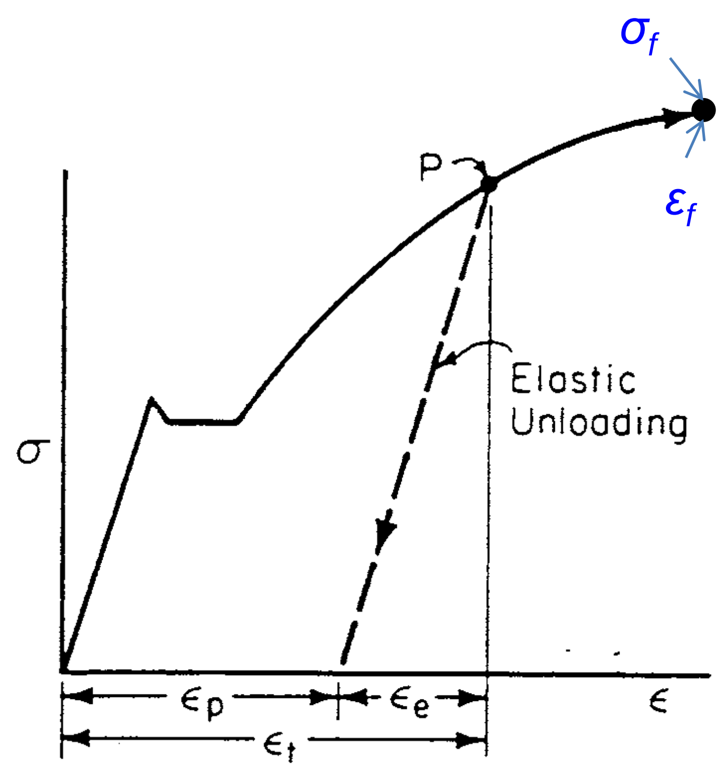
$$\varepsilon_t = \varepsilon_e + \varepsilon_p$$

- Elastic strain

$$\varepsilon_e = \frac{\sigma}{E}$$

- Total strain can be rewritten as

$$\boxed{\varepsilon_t = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/n}}$$

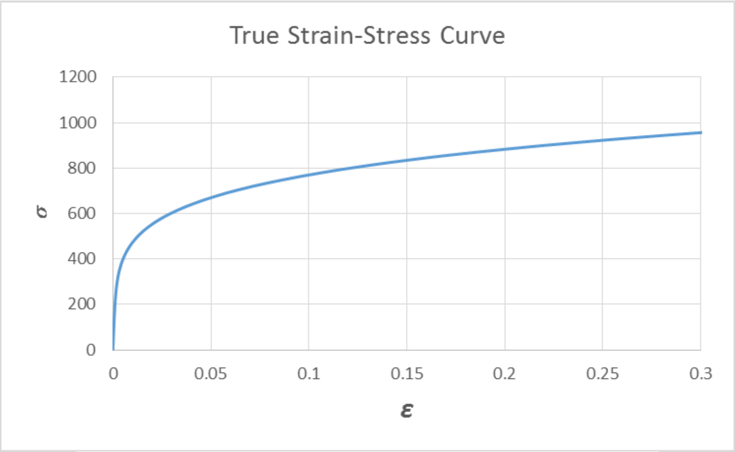


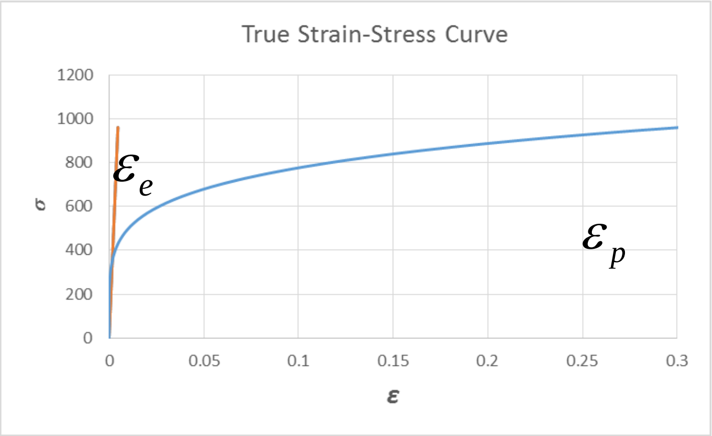
Elastic and plastic strain

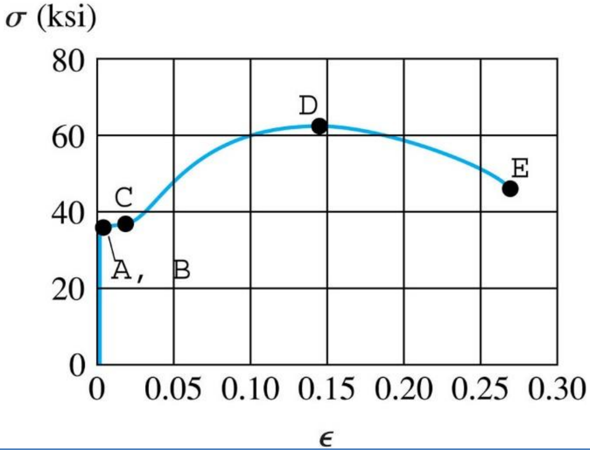
True and engineering stress-strain

❖ Example of True Stress-Strain Curve

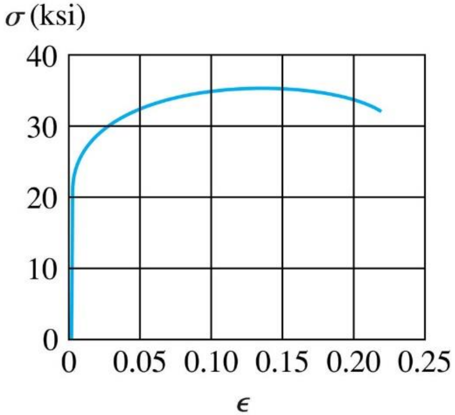
- *E = modulus of elasticity = 20600 MPa*
- *n = cyclic strain hardening exponent =0.193*
- *K = cyclic strength coefficient = 1210 MPa*





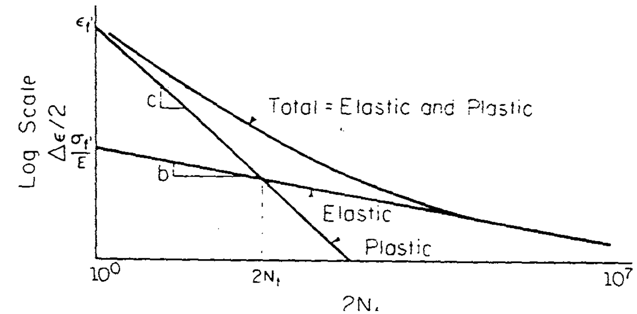


Engineering stress-strain curve (steel)



Engineering stress-strain curve (aluminum)

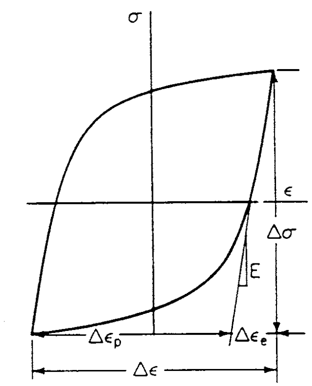
Low Cycle Fatigue Calculation Procedure



Strain-life curve

Low Cycle Fatigue

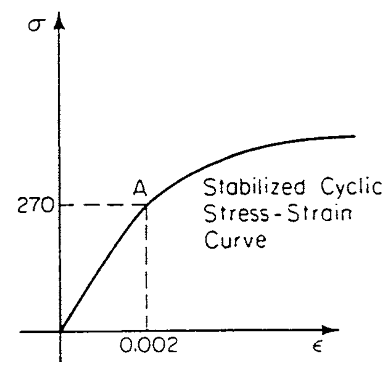
True Strain Amplitude



True Strain-Stress Relations under inelastic loading :
Hysteresis Curve

Massing's hypothesis

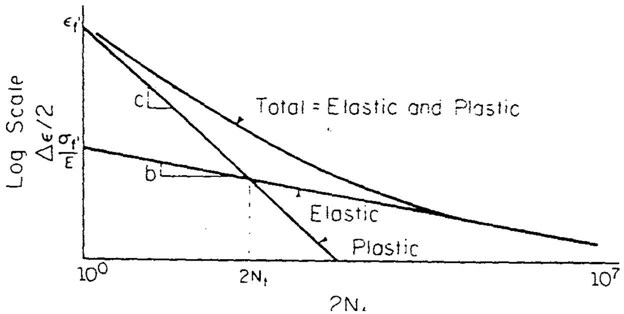
Stabilized Cyclic Strain-Stress Curve



1) Companion Sample
2) Incremental Step Test

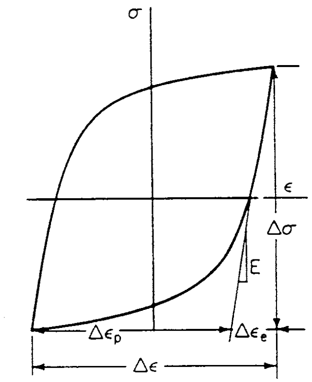
Low Cycle Fatigue Calculation Procedure

Low Cycle Fatigue



Strain-life curve

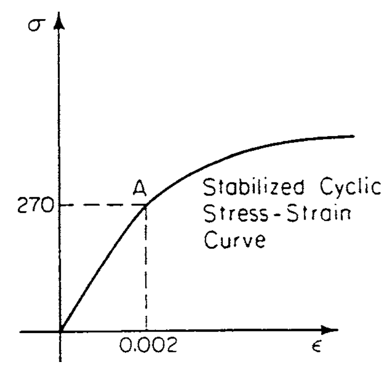
True Strain Amplitude



True Strain-Stress Relations under inelastic loading :

Hysteresis Curve

Massing's hypothesis



Stabilized Cyclic Strain-Stress Curve

1) Companion Sample
2) Incremental Step Test

2.2.2 Cyclic Stress-Strain Behavior – Hysteresis loop

- Cyclic stress-strain curves are useful for assessing the durability of structures and components subjected to repeated loading.
- Hysteresis loop : the response of a material subjected to inelastic loading.
- The area within the loop : plastic deformation work done on the material
- Total strain range.

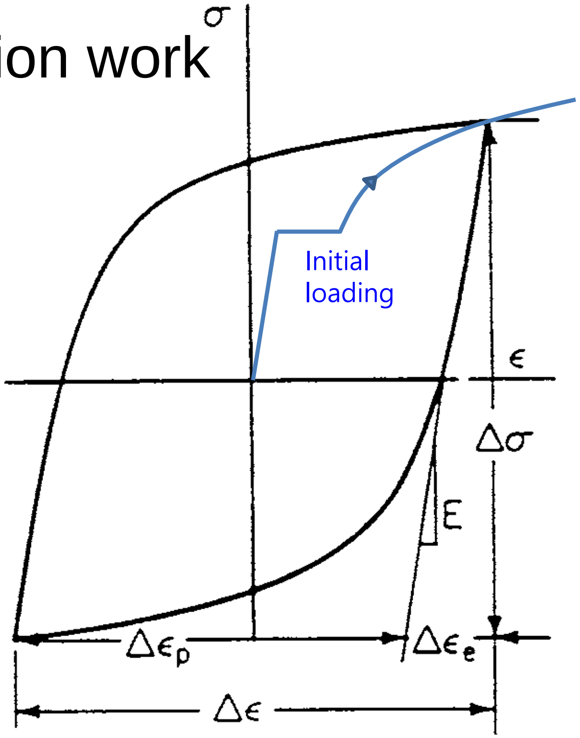
$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p$$

- Total strain amplitude

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$$

the elastic term may be replaced

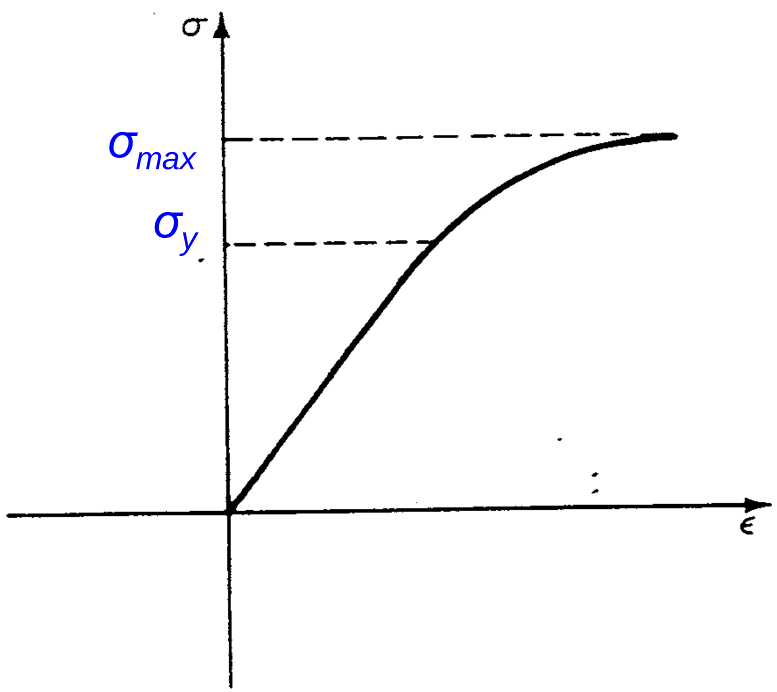
$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\varepsilon_p}{2}$$



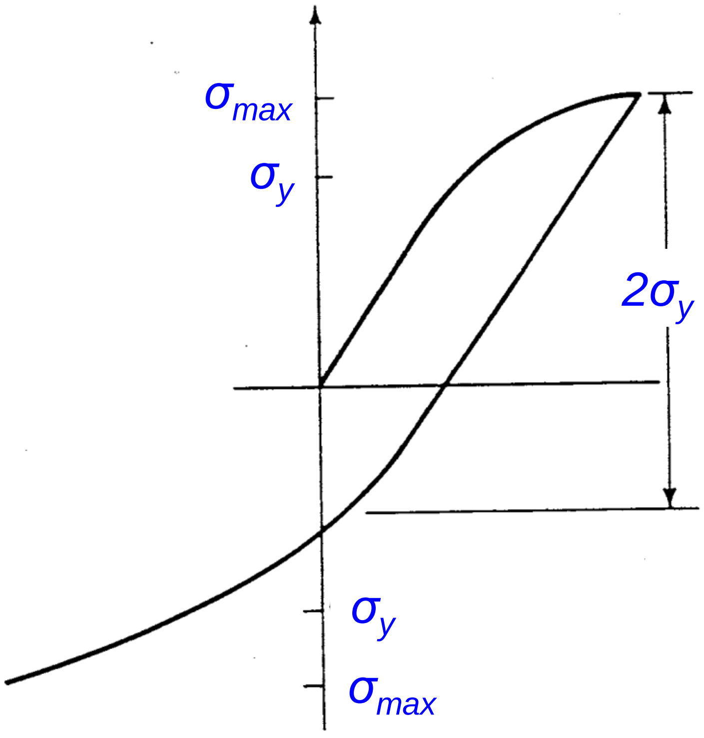
Hysteresis Loop

2.2.2 Cyclic Stress-Strain Behavior – Baushinger effect

1. Tensional loading : past the yield strength, σ_y, to some value σ_{max}



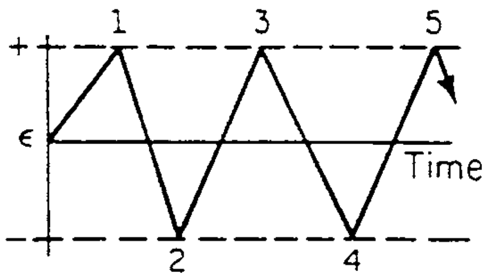
2. Compressive loading : inelastic (plastic) strains develop before $-\sigma_y$ is reached.



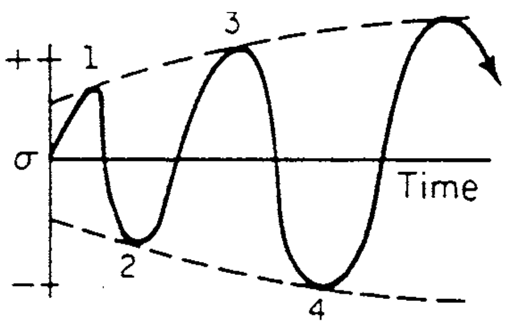
(b)

2.2.3 Transient Behavior : Cyclic Strain Hardening

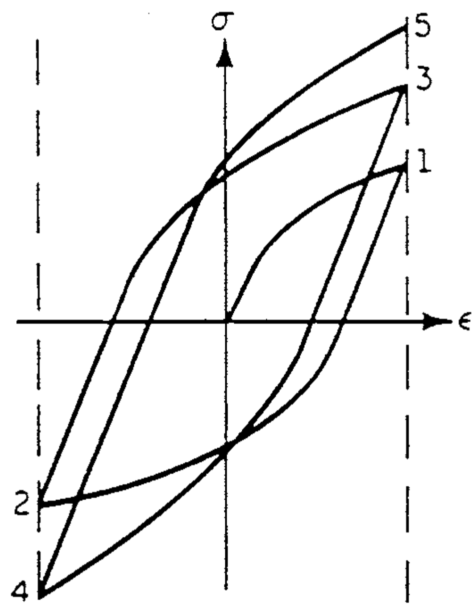
- The stress-strain response of metals is often drastically altered due to repeated loading.
  1. Cyclically harden : maximum stress increases with each cycle of strain.
     → requires more load to keep imposing the constant strain.



Constant strain amplitude



Stress response



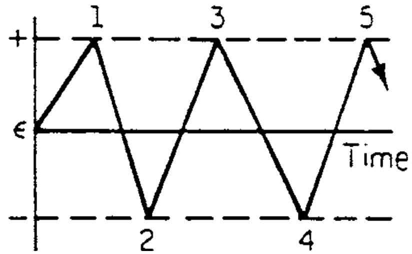
Cyclic stress-strain response

Cyclic Hardening

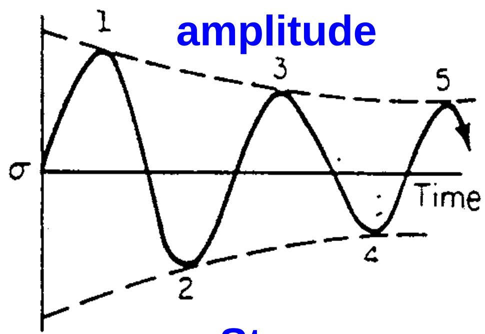
2.2.3 Transient Behavior : Cyclic Softening

2. Cyclically soften : maximum stress increases with each cycle of strain

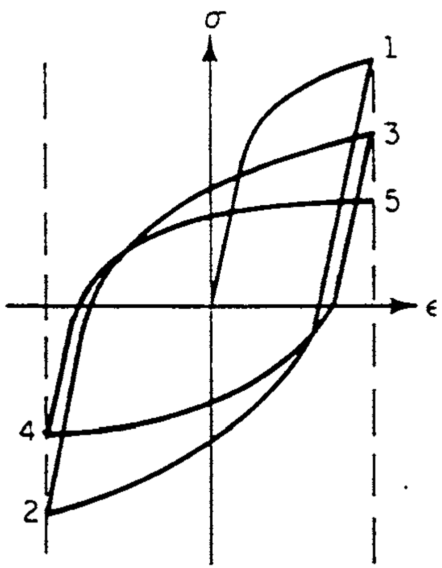
→ requires less load to keep imposing the constant strain.



Constant strain amplitude



Stress response



Cyclic stress-strain response

Cyclic Softening

3. Be cyclically stable : requires the same load

4. Have mixed behavior(soften or harden depending on strain range)

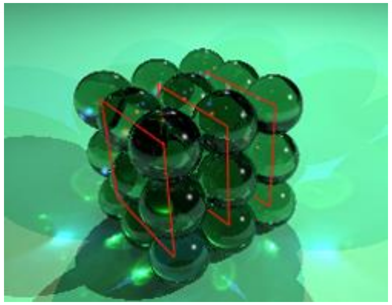
What is dislocation?

- **Dislocation** : a crystallographic(결정학상의) defect, or irregularity, within a crystal structure.

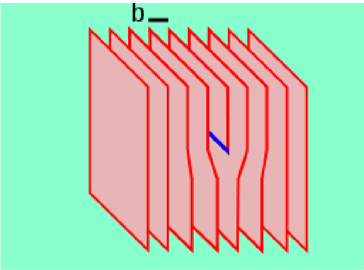
- A crystalline material : consists of a regular array of atoms, arranged into lattice planes.

- **An edge dislocation** : a defect where an extra half-plane of atoms is introduced mid way through the crystal, distorting nearby planes of atoms.

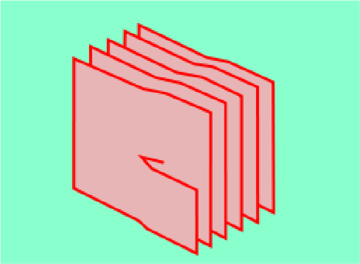
- **A screw dislocation :** Imagine cutting a crystal along a plane and **slipping one half across** the other.



Crystal lattice showing atoms and lattice planes



Edge dislocation



Screw dislocation

2.2.3 Transient Behavior : Cyclic Hardening and Softening

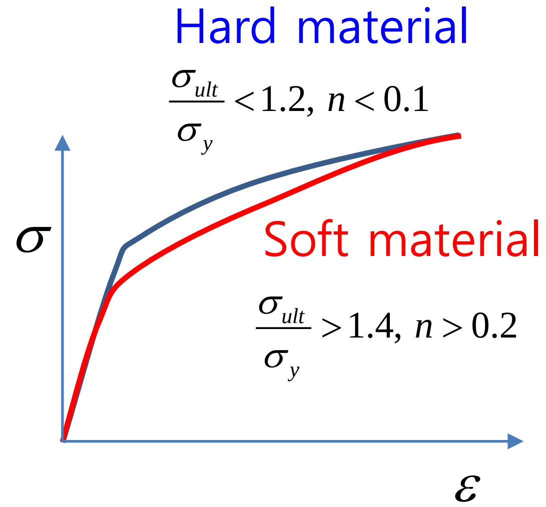
❖ The reason of materials soften or harden

- **For soft material** : initially the dislocation density is low. The density rapidly increases due to cyclic plastic straining contributing to significant cyclic strain hardening

- **For hard material** : subsequent strain cycling causes a rearrangement of dislocations which offers less resistance to deformation and the material cyclically softens.

$\dfrac{\sigma_{ult}}{\sigma_y} > 1.4$: the material will cyclically harden

$\dfrac{\sigma_{ult}}{\sigma_y} < 1.2$: the material will cyclically soften



2.2.3 Transient Behavior : Cyclic Hardening and Softening

- Between 1.2 and 1.4, small change in cyclic response.
- Monotonic strain hardening exponent, *n*, can be used to predict the material's cyclic behavior.
  - ✓ n> 0.20 the material will cyclically harden
  - ✓ n< 0.10 the material will cyclically soften

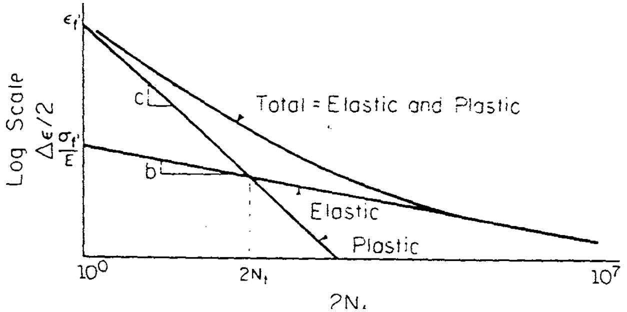
$$\sigma = K(\varepsilon_p)^n$$

- Cyclically stable condition reaches after 20~40% of the fatigue life.
- Fatigue properties are usually specified at 50% of fatigue life when the material response is stabilized.

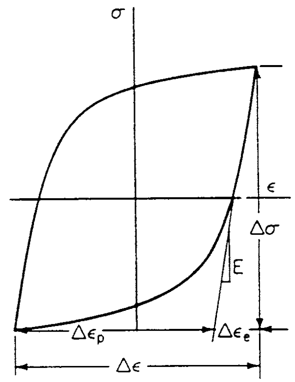
Low Cycle Fatigue Calculation Procedure

Low Cycle Fatigue

True Strain Amplitude



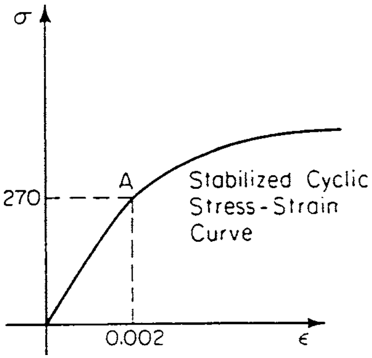
Strain-life curve



True Strain-Stress Relations under inelastic loading :
Hysteresis Curve

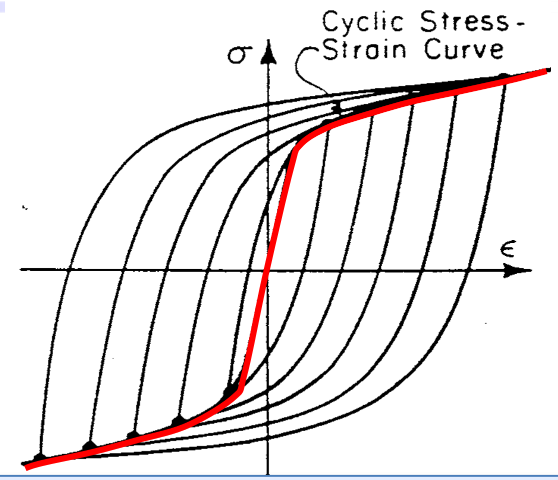
Massing's hypothesis

Stabilized Cyclic Strain-Stress Curve
using
1) Companion Sample
2) Incremental Step Test



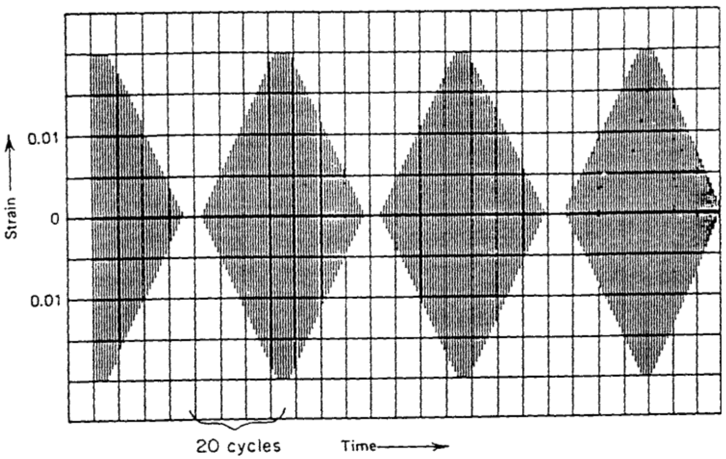
2.2.4 Cyclic Stress-Strain Curve Determination

1. **Companion samples** : A series of samples are tested at various strain levels and the stabilized hysteresis loops are superimposed and the tips of the loops are connected. Time consuming.



Companion samples

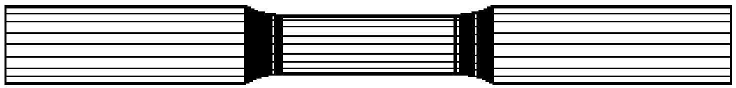
2. **Incremental step test** : widely accepted since quick and good results. The response stabilizes after 3-4 blocks and fails after about 20 blocks. The tips of the stabilized hysteresis loops are connected → Cyclic Stress-Strain Curve

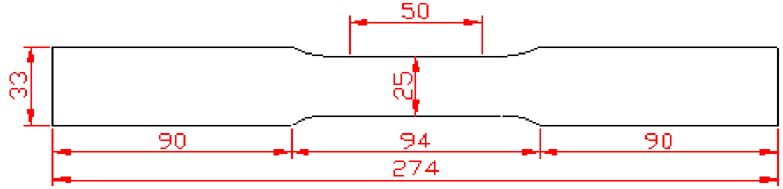


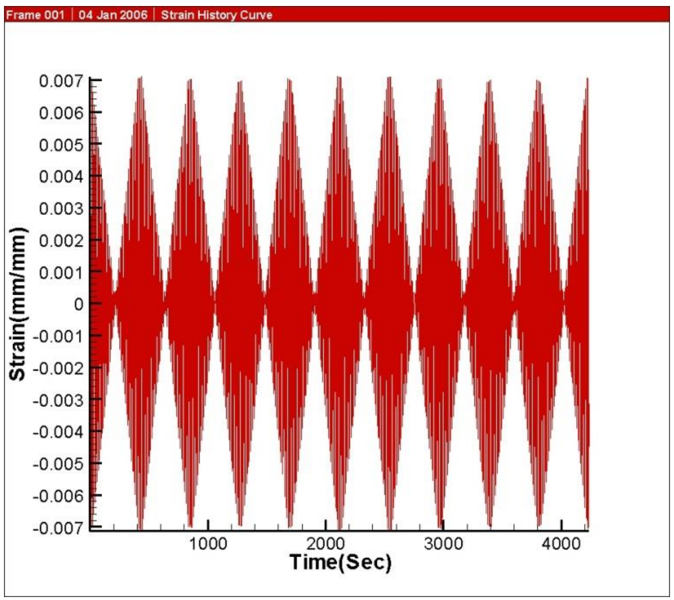
Incremental step test

2.2.4 Cyclic Stress-Strain Curve Determination

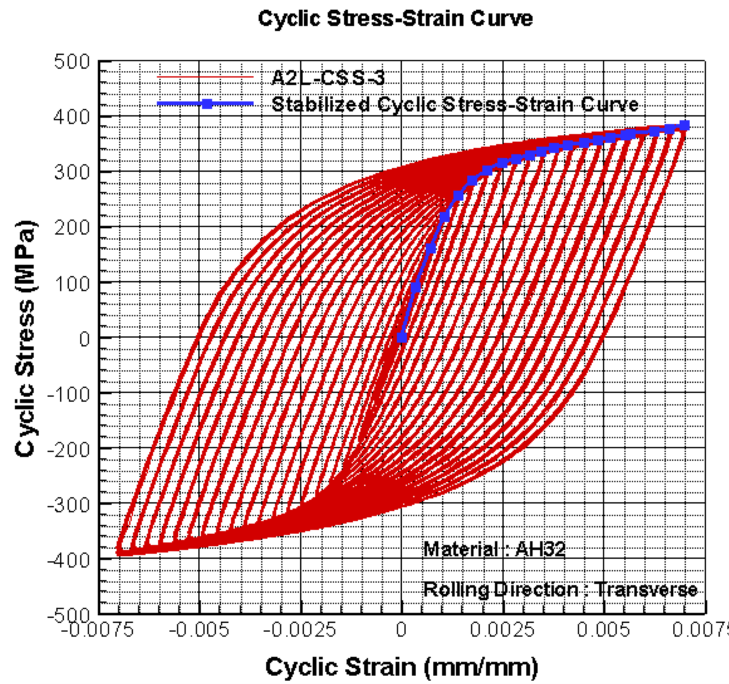
- **An example of Incremental step test**







Strain History



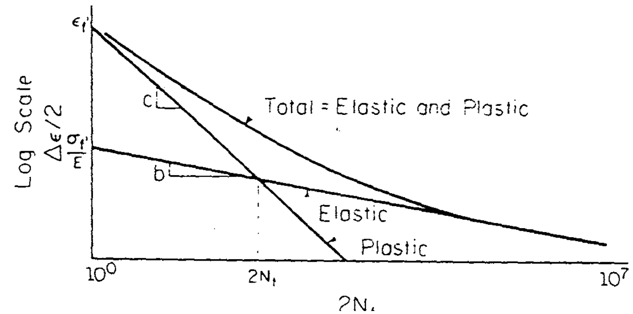
Hysteresis loop & Cyclic Stress-Strain Curve

Low Cycle Fatigue Calculation Procedure

Low Cycle Fatigue

↑

True Strain Amplitude



Strain-life curve

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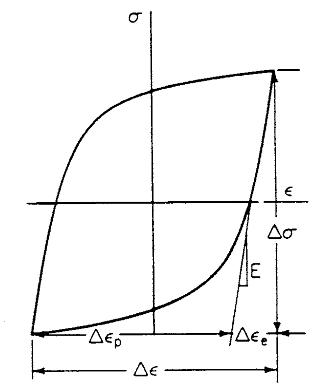
True Strain-Stress Relations under inelastic loading :
Hysteresis Curve



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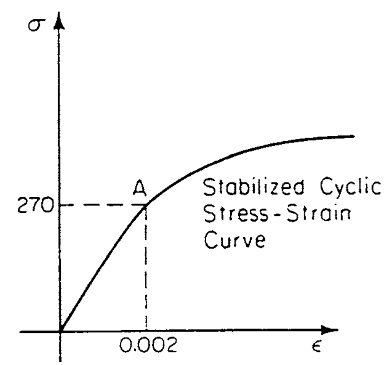
Massing's hypothesis

Stabilized **Cyclic Strain-Stress Curve**



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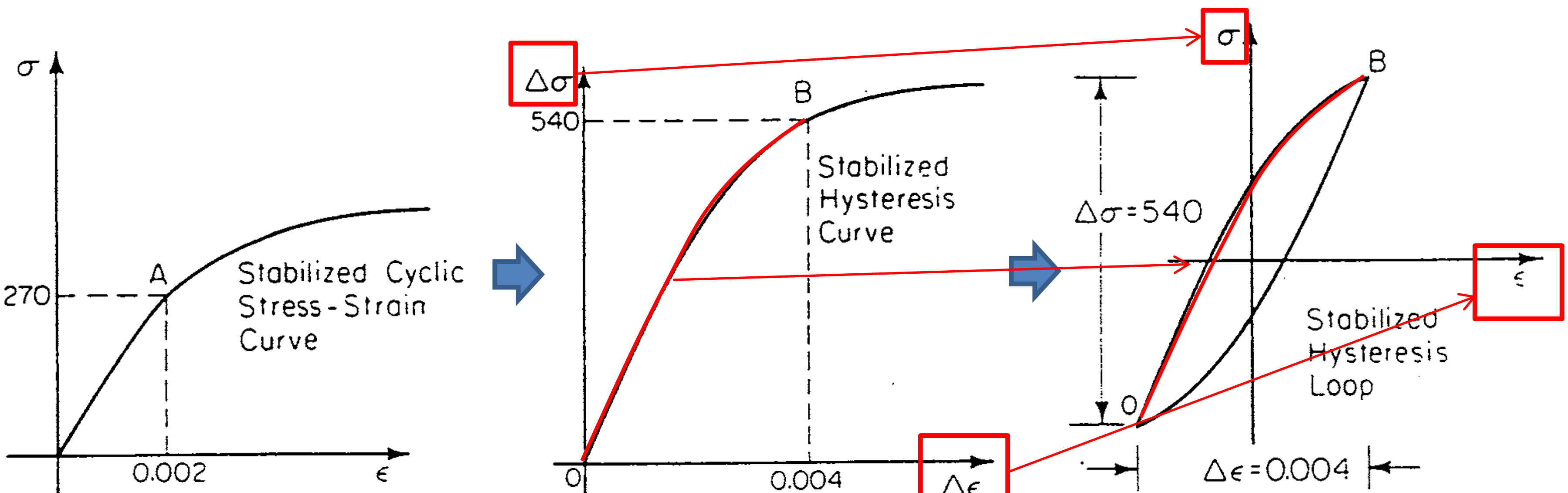
1) Companion Sample
2) Incremental Step Test

2.2.4 Cyclic Stress-Strain Curve Determination

- After the incremental step test, if the specimen is pulled to failure, the stress-strain curve will be nearly identical to the one obtained by connecting the loop.
- **Massing's hypothesis** : the stabilized hysteresis loop may be obtained by doubling the cyclic stress-strain curve.



Stabilized Cyclic Stress-Strain Curve

Doubling

Hysteresis Loop

Cyclic true stress versus plastic strain

- Log-log plot of the completely reversed stabilized cyclic true stress versus true plastic strain

$$\sigma = K'(\varepsilon_p)^{n'} \Rightarrow \varepsilon_p = \left(\frac{\sigma}{K'}\right)^{1/n'}$$

Where, σ = *cyclically stable stress amplitude*

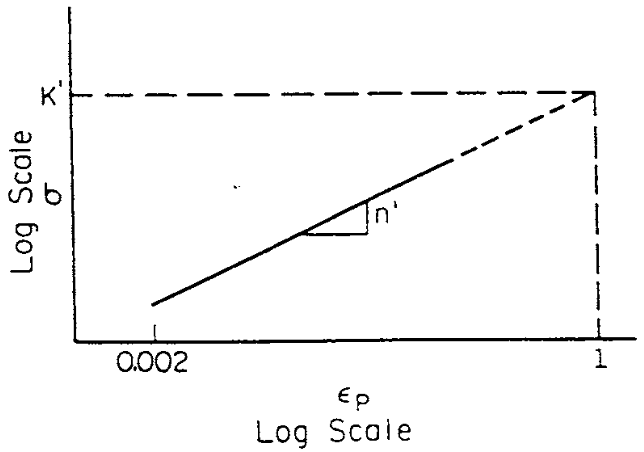
ε = *cyclically stable plastic strain amplitude*

K' = *cyclic strength coefficient*

n' = *cyclic strain hardening exponent (0.10 ~0.25, average 0.15)*

- Total strain is

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$$

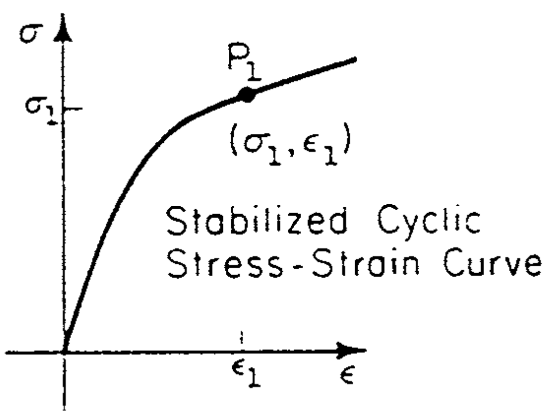


Log-log plot of true cyclic stress versus true cyclic plastic strain

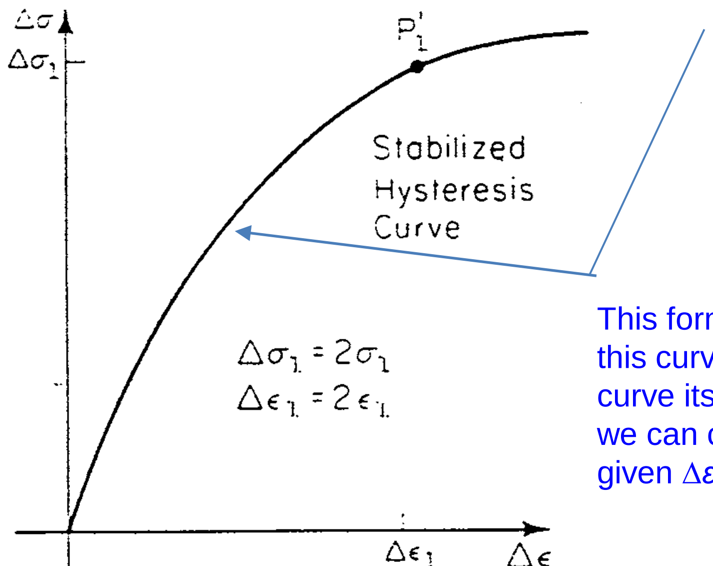
Hysteresis loop by Massing's hypothesis

- Total strain is $$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$$
- An arbitrary point $P_1(\sigma_1, \varepsilon_1)$ on Cyclic Stress-Strain Curve, $$\varepsilon_1 = \frac{\sigma_1}{E} + \left(\frac{\sigma_1}{K'}\right)^{1/n'}$$
- From Massing's hypothesis, P_1 can be located on hysteresis curve , $P'_1(\Delta\sigma_1, \Delta\varepsilon_1)$.

$$\Delta\sigma_1 = 2\sigma_1,\ \Delta\varepsilon_1 = 2\varepsilon_1 \Rightarrow \frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'} \Rightarrow \Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$



Cyclic stress-strain Curve

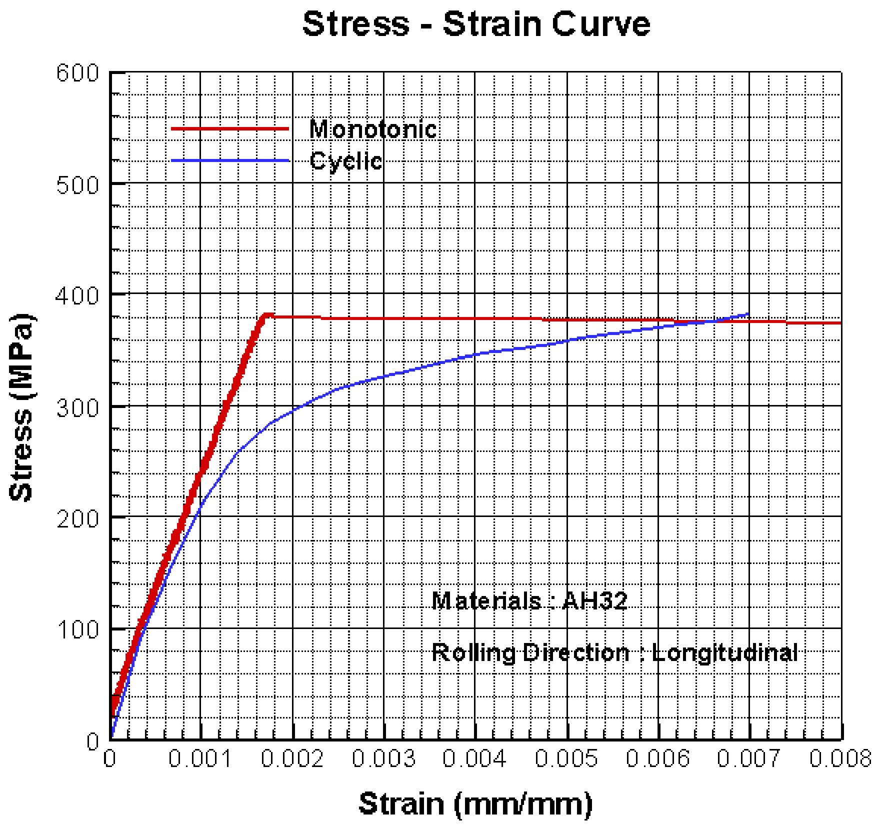


Hysteresis Curve

This formula represents this curve not Hysterias curve itself. Using this, we can calculate $\Delta\sigma$ for given $\Delta\varepsilon$

Example of Experiment data

- Example of actual Monotonic and Cyclic Stress Strain Curve.



Example 2.1

Q : Consider a test specimen with the following material properties :

E = modulus of elasticity = 30 X 10³ ksi

n′ = cyclic strain hardening exponent =0.202

K′ = cyclic strength coefficient = 174.6 ksi

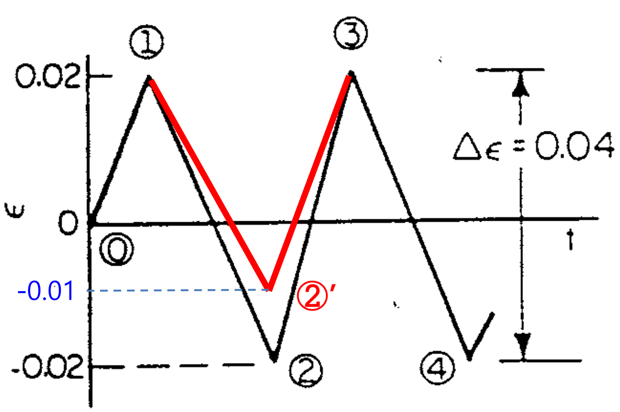
Fully reversed cyclic strain with a strain range, Δε, of 0.04. Determine the stress-strain response of the material.

Initial application of strain follows stress- strain curve

① $\varepsilon_1 = \frac{\sigma_1}{E} + (\frac{\sigma_1}{K'})^{1/n'}$ $\quad 0.02 = \frac{\sigma_1}{30\times10^3 ksi} + (\frac{\sigma_1}{176.4ksi})^{1/0.202}$ $\quad \sigma_1 = 77.1ksi$

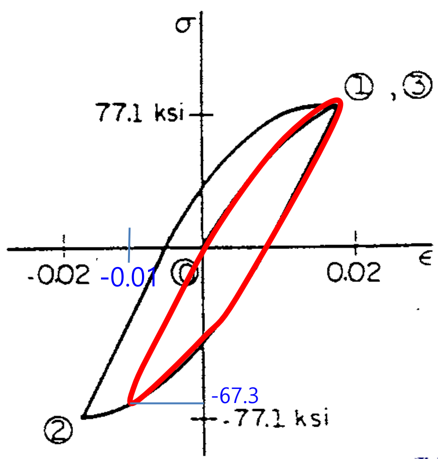
All successive strains follows hysteresiss curve.

② $\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2(\frac{\Delta\sigma}{2K'})^{1/n'}$ ⇨ $\Delta\sigma = 154.2ksi$ ⇨ $\varepsilon_2 = \varepsilon_1 - \Delta\varepsilon = -0.02ksi$, $\sigma_2 = \sigma_1 - \Delta\sigma = -77.1ksi$

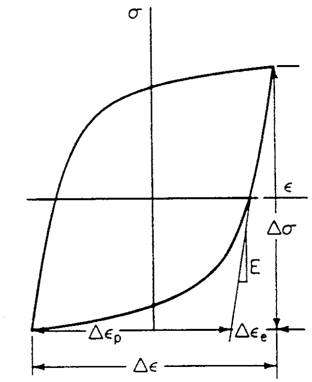


If ① → ②′

$\Delta\sigma = 144.4ksi$
$\varepsilon_2 = \varepsilon_1 - \Delta\varepsilon = -0.01ksi$
$\sigma_2 = \sigma_1 - \Delta\sigma = -67.3ksi$



Low Cycle Fatigue Calculation Procedure



Low Cycle Fatigue

↑

True Strain Amplitude

↑

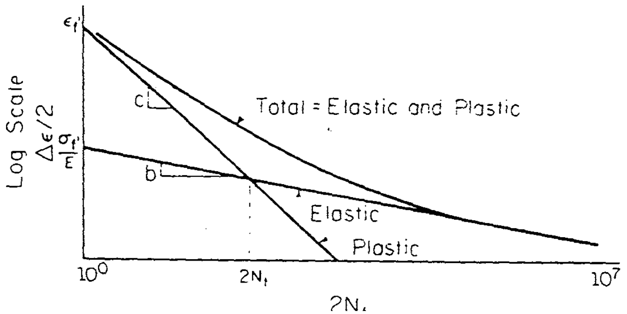
True Strain-Stress Relations under inelastic loading : **Hysteresis Curve**

↑

Stabilized Cyclic Strain-Stress Curve

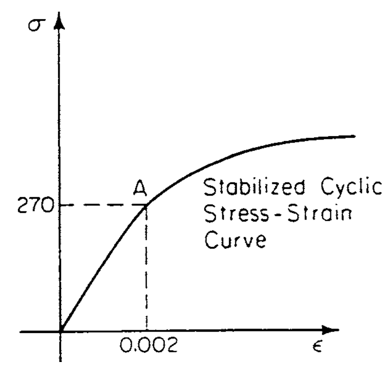
↑

1) Companion Sample
2) Incremental Step Test



Strain-life curve

Massing's hypothesis



Strain Life Curve

- Stress life (S-N) data on a log-log scale.

$$\frac{\Delta\sigma}{2} = \sigma'_f (2N_f)^b$$

$\Delta\sigma/2$ = *true stress amplitude*
$2N_f$ = *reversals to failure (1 rev =1/2 cycle)*
σ'_f = *fatigue strength coefficient*
b = *fatigue strength exponent*

- Plastic strain –life (ε_1-N) data on log-log coordinates by Coffin and Manson

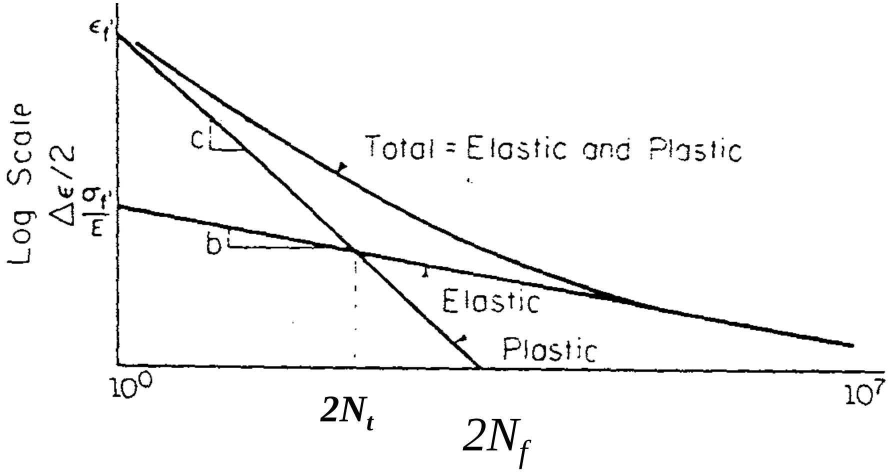
$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2N_f)^c$$

$\Delta\varepsilon_p/2$ = *plastic strain amplitude*
$2N_f$ = *reversals to failure*
ε'_f = *cyclic strength coefficient (≈true facture ductility,* ε_f*)*
c = *fatigue ductility exponent (-0.5~-0.7)*

- Total strain and the elastic term

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2} \qquad \frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E}$$

$$\Rightarrow \frac{\Delta\varepsilon}{2} = \underbrace{\frac{\sigma'_f}{E}(2N_f)^b}_{\text{elastic}} + \underbrace{\varepsilon'_f (2N_f)^c}_{\text{plastic}}$$



Strain-life curve

Strain Life Curve

Low cycle S-N curve

$$\frac{\Delta\sigma}{2} = \sigma'_f\,(2N_f)^b$$

$$\frac{\Delta\sigma}{2} = \sigma'_f\,(N)^b$$

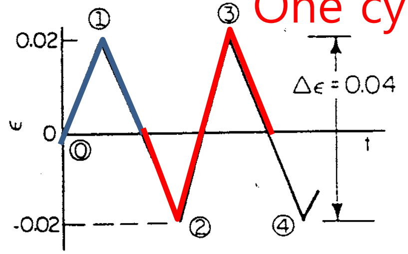
$$\boxed{N = \left(\frac{1}{\sigma'_f}\right)^{-b}\left(\frac{\Delta\sigma}{2}\right)^{-b} = c(\Delta\sigma)^{-b}}$$

High cycle S-N curve

$$\boxed{N = \bar{a}\ \Delta\sigma^{-m}}$$

One reversal

One cycle



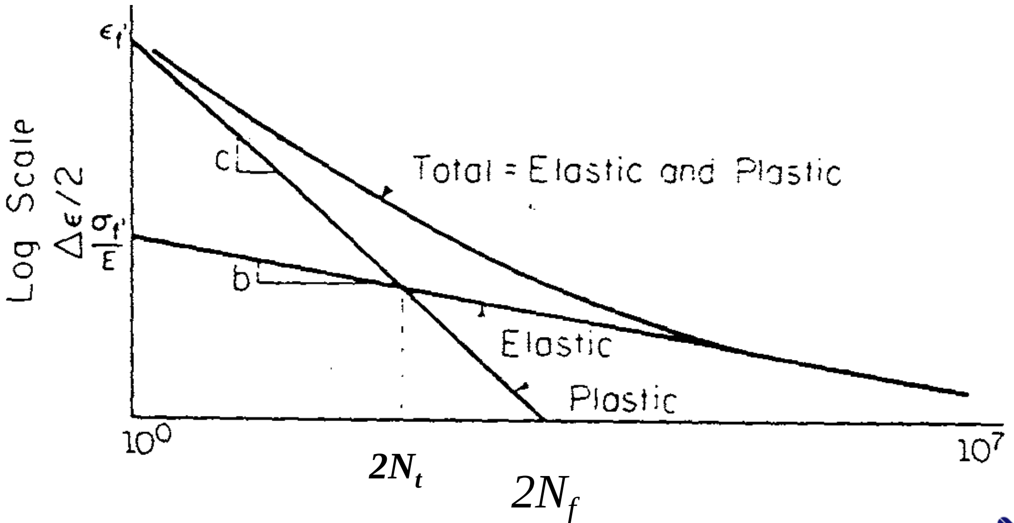
Low cycle S-N curve in strain-life relationship

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b$$

$$2N_f = N \qquad \frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(N)^b$$

$$N = \left(\frac{E}{\sigma'_f}\right)^{-b}\left(\frac{\Delta\varepsilon}{2}\right)^{-b}$$

When N=1 $\quad \dfrac{\Delta\varepsilon}{2} = \dfrac{\sigma'_f}{E}$



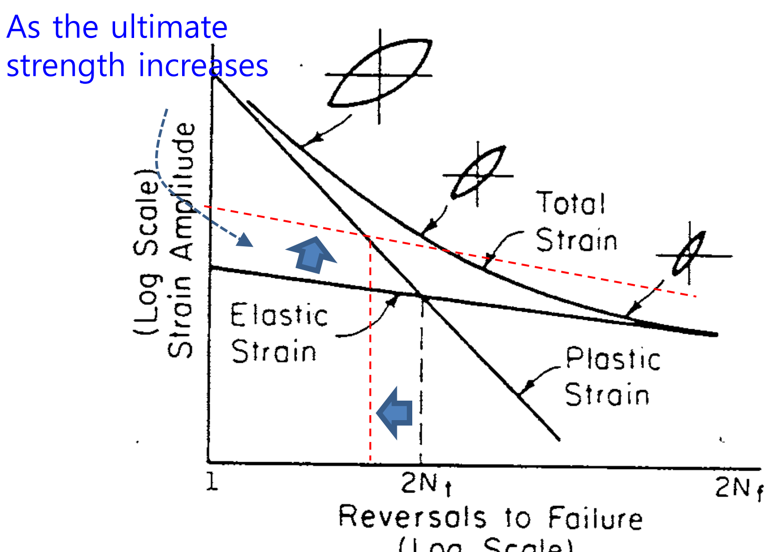
Strain Life Curve

- Transition fatigue life, $2N_t$

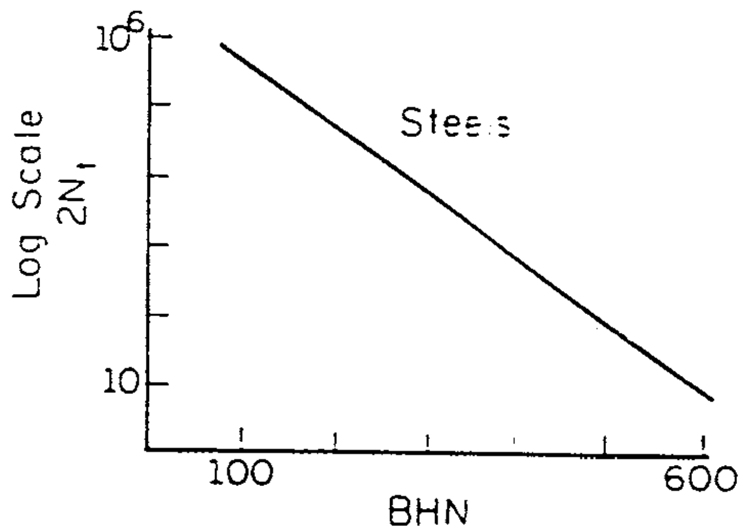
$$\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\varepsilon_p}{2} \Rightarrow \frac{\sigma'_f}{E}(2N_f)^b = \varepsilon'_f(2N_f)^c \quad at \quad N_f = N_t \Rightarrow 2N_t = \left(\frac{\varepsilon'_f E}{\sigma'_f}\right)^{1/(b-c)}$$

- Short lives : more plastic strain, wider loop.

  Long lives : less plastic loop, narrower loop.
- As the ultimate strength increases, the transition life decreases and elastic strains dominate for a greater portion of the life range.



Shape of the hysteresis curve in relation to the strain-life curve



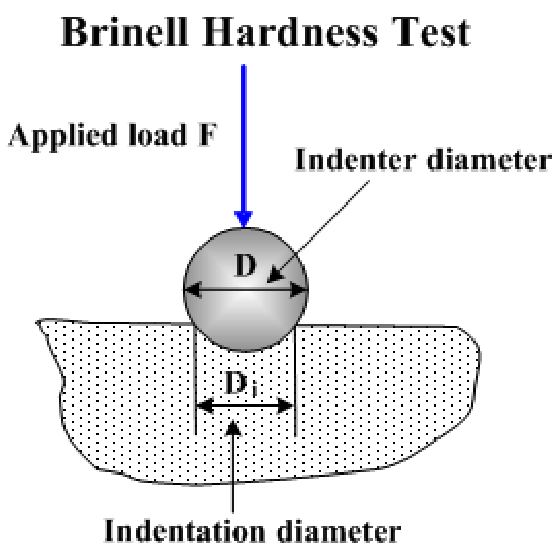
Relationship between transition life and hardness for steels

Brinell Hardness number

- A measure of the hardness of a material obtained by pressing a hard steel ball into its surface.
- the ratio of the load on the ball in kilograms to the area of the depression made by the ball in square millimeters
- Endurance limit (Fatigue Limit) of S-N Curve of base material is related to hardness

S_e(ksi) ≈ 0.25 x BHN for BHN <400

100 ksi (=689MPa) for BHN > 400



$$BHN = \frac{2F}{\pi D(D - \sqrt{D^2 - D_i^2})}$$

Strain-Life Curve

- Definition of failure
  - ✓ Separation of specimen : common for uniaxial loading
  - ✓ Development of given crack length (often 1.0mm)
  - ✓ Loss of specified load carrying capability (often 10 or 50% load drop)

  → Not a large difference in life between these criteria

- Factor of 2
  - ✓ Strain-life approach measure life in terms of reversals ($2N$), the stress-life method Cycles (N)
  - ✓ Strain-life approach uses both strain range ($\Delta\varepsilon$) and amplitude (ε_a).
  - ✓ hysteresis curve can be modeled as twice the Cyclic σ-ε curve versus

Methods to determine Properties

- The strain-life equation requires four empirical constants ($b, c, \varepsilon'_f, \sigma'_f$). These can be obtained from fatigue data.

$$\boxed{\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f\,(2N_f)^c} \Rightarrow 2N_f = \left(\frac{\Delta\varepsilon_p}{2\varepsilon'_f}\right)^{1/c}$$

$$\boxed{\frac{\Delta\sigma}{2} = \sigma'_f\,(2N_f)^b} \Rightarrow \frac{\Delta\sigma}{2} = \sigma'_f \left(\frac{\Delta\varepsilon_p}{2\varepsilon'_f}\right)^{b/c}$$

$$\frac{\Delta\sigma}{2} = \sigma \qquad \frac{\Delta\varepsilon}{2} = \varepsilon$$

$$\boxed{\sigma = K'(\varepsilon_p)^{n'}}$$

$$\sigma'_f \left(\frac{\varepsilon_p}{\varepsilon'_f}\right)^{b/c} = K'(\varepsilon_p)^{n'} \Rightarrow \boxed{\sigma'_f \left(\frac{1}{\varepsilon'_f}\right)^{b/c} (\varepsilon_p)^{b/c} = K'(\varepsilon_p)^{n'}}$$

$$\boxed{n' = \frac{b}{c}} \qquad \boxed{K' = \frac{\sigma'_f}{(\varepsilon'_f)^{n'}}}$$

- Although these, relationships may be useful, K' and n' are usually obtained from a curve fit of the cyclic stress –strain data using $\sigma = K(\varepsilon_p)^n$

Methods to determine Properties

- Approximate methods
  - ✓ Fatigue strength coefficient σ'_f

    $\sigma'_f \approx \sigma_f$ (corrected for necking)

    $\sigma_f \approx S_u + 50\text{ksi}$ (steels with hardness below 500BHN)

  - ✓ Fatigue strength exponent, b : -0.05~-0.12 for most metals, average of -0.085

  - ✓ Fatigue ductility coefficient, ε'_f : $\varepsilon'_f \approx \varepsilon_f$ where $\varepsilon_f = \ln \dfrac{1}{1-RA}$

    RA : the reduction in area

  - ✓ Fatigue ductility exponent c : not well defined, -0.5~-0.7

Example 2.2

Q : From the monotonic and cyclic strain-life data for smooth steel specimens. Determine the cyclic stress-strain and strain-life constants

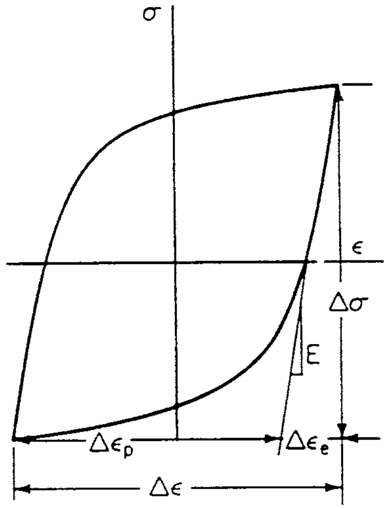
Monotonic data $S_y = 158$ ksi, E=2.84 X103 ksi

$S_u = 168$ ksi, $\sigma_f = 228$ ksi

%RA = 52 $\varepsilon_f = 0.734$

Smooth Specimen-Cyclic Data

Total Strain Amplitude, $\Delta\epsilon/2$	Stress Amplitude, $\Delta\sigma/2$ (ksi)	Plastic Strain Amplitude, $\Delta\epsilon_p/2$[a]	Reversals to Failure, $2N_f$
0.0393	162.5	0.0336	50
0.0393	162	0.0336	68
0.02925	155	0.0238	122
0.01975	143.5	0.0147	256
0.0196	143.5	0.0145	350
0.01375	136.5	0.00894	488
0.00980	130.5	0.00521	1,364
0.00980	126.5	0.00534	1,386
0.00655	121	0.00229	3,540
0.00630	119	0.00211	3,590
0.00460	114	0.00059	9,100
0.00360	106	0.00000	35,200
0.00295	84.5	0.00000	140,000



$$\frac{\Delta\varepsilon_p}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_e}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\sigma}{2E}$$

Example 2.2

Smooth Specimen-Cyclic Data

Total Strain Amplitude, $\Delta\epsilon/2$	Stress Amplitude, $\Delta\sigma/2$ (ksi)	Plastic Strain Amplitude, $\Delta\epsilon_p/2^a$	Reversals to Failure, $2N_f$
0.0393	162.5	0.0336	50
0.0393	162	0.0336	68
0.02925	155	0.0238	122
0.01975	143.5	0.0147	256
0.0196	143.5	0.0145	350
0.01375	136.5	0.00894	488
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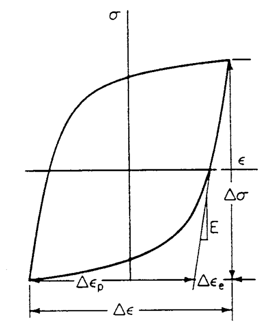
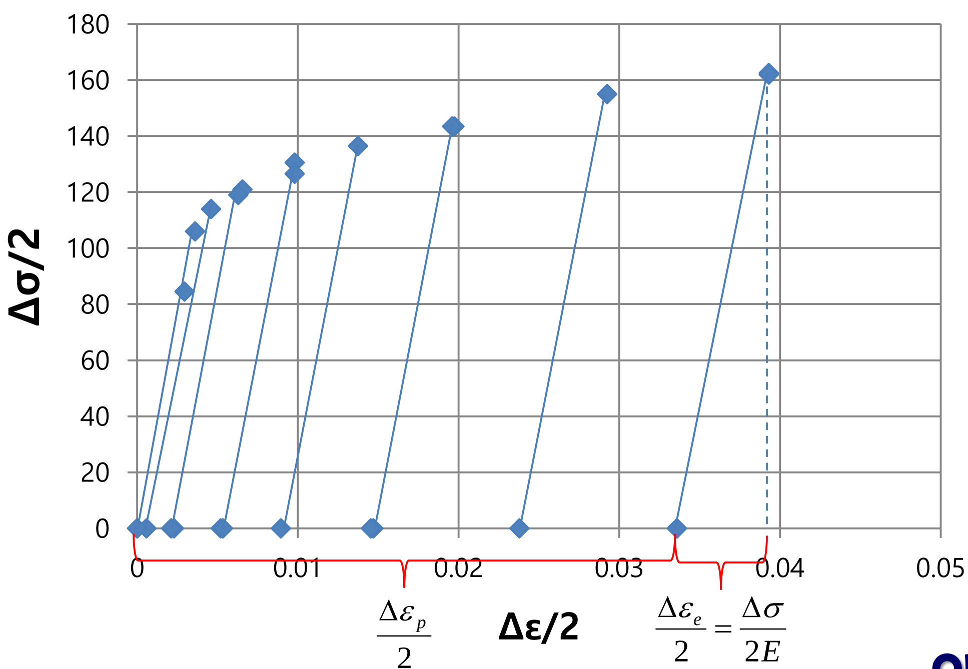
0.00460	114	0.00059	9,100
0.00360	106	0.00000	35,200
0.00295	84.5	0.00000	140,000



$$\frac{\Delta\varepsilon_p}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\varepsilon_e}{2} = \frac{\Delta\varepsilon}{2} - \frac{\Delta\sigma}{2E}$$



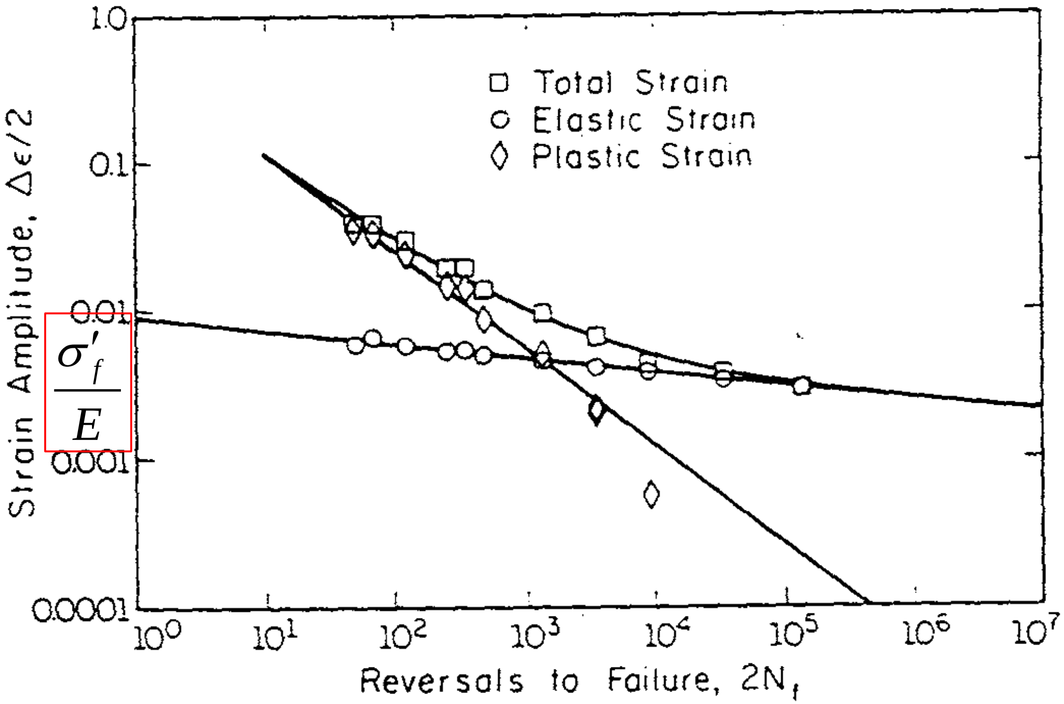
Example 2.2

- Fatigue strength coefficients σ'_f and b by fitting a power law relationship between $\Delta\sigma/2$ and $2N_f$

$$\frac{\Delta\sigma}{2} = \sigma'_f (2N_f)^b \Rightarrow \frac{\Delta\sigma}{2E} = \frac{\sigma'_f}{E}(2N_f)^b$$

- Fatigue ductility coefficients ε'_f and c by fitting a power law relationship between $\Delta\varepsilon_p/2$ and $2N_f$

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f (2N_f)^c$$



Strain-life Curve

Example 2.2

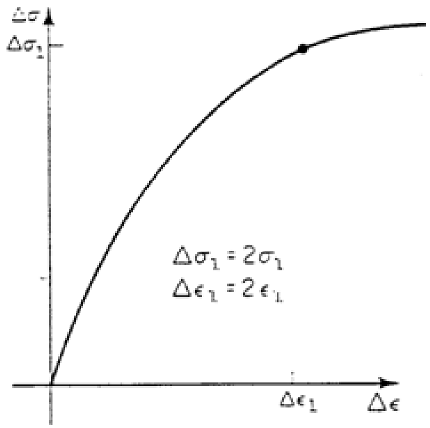
- The cyclic strength coefficient, K′ and the cyclic strain hardening exponent, n′.

1) by fitting a power law relationship to stress amplitude $\Delta\sigma/2$ versus plastic strain amplitude $\Delta\,\varepsilon_p/2$. → Preferred

$$\sigma = K'(\varepsilon_p)^{n'} \qquad K' = 216\,ksi,\ \ n' = 0.094$$

2) From the relationship

$$K' = \frac{\sigma'_f}{(\varepsilon'_f)^{n'}} \qquad n' = \frac{b}{c} \quad \Rightarrow \quad K' = 227\,ksi,\ \ n' = 0.104$$



From strain-life data v.s. from approximations

Value	Determined from Strain–Life Data	Determined Using Approximations
σ'_f	222	228
b	−0.076	−0.085
ϵ'_f	0.811	0.734
c	−0.732	−0.6

$\sigma'_f \approx \sigma_f$

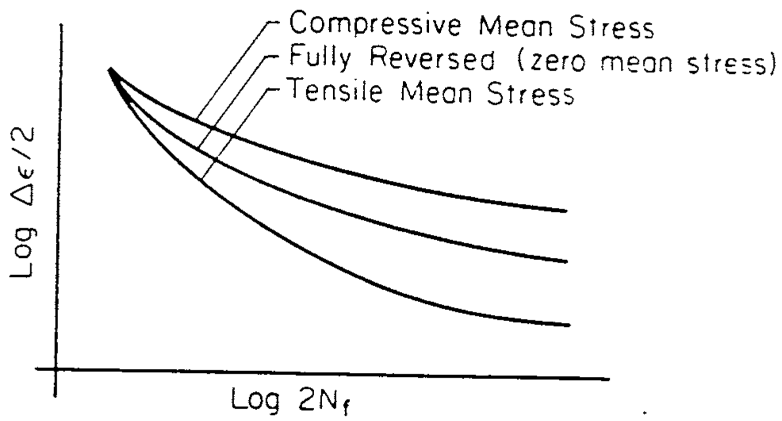
b : average of -0.085

$\varepsilon'_f \approx \varepsilon_f$

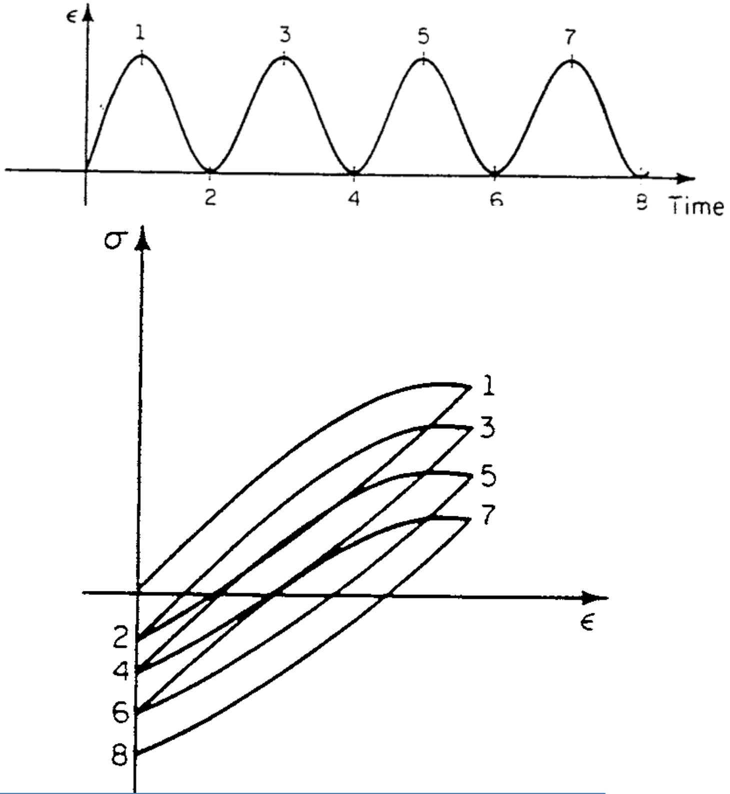
c : -0.5~-0.7

Mean stress effect

- Mean strain is negligible but mean stress has a significant effect on the fatigue life.
- At longer lives, mean compressive stress effect is valid.
- At high strain amplitudes (0.5% to 1% or above), mean stress tends toward zero.



Effect of mean stress on strain-life curve



Mean stress relaxation

Modification to strain-life equation I

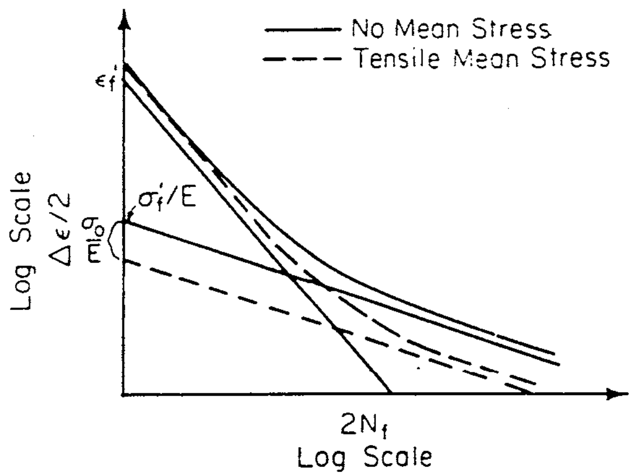
- Morrow suggested. $\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E} = \frac{\sigma'_f - \sigma_0}{E}(2N_f)^b \Rightarrow 2N_f = \left(\frac{2(\sigma'_f - \sigma_0)}{\Delta\varepsilon_e E}\right)^{-b}$, here $b < 0$

"Fatigue life decreases as mean tensile stress σ_0 increase."

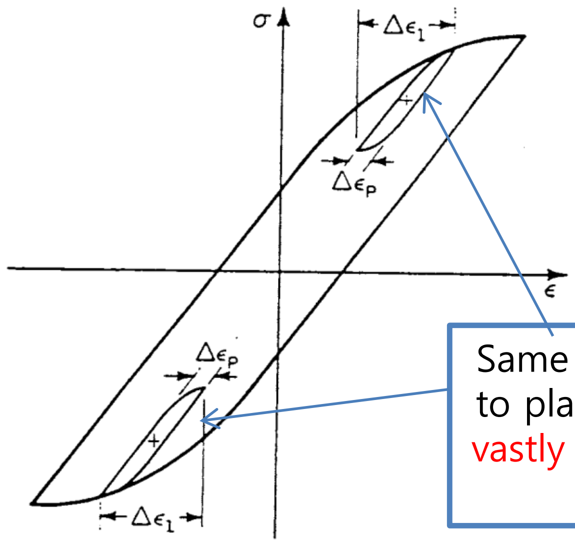
- The strain-life equation,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_0}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

Ratio of elastic to plastic strain is dependant on mean stress?



Morrow's mean stress correction



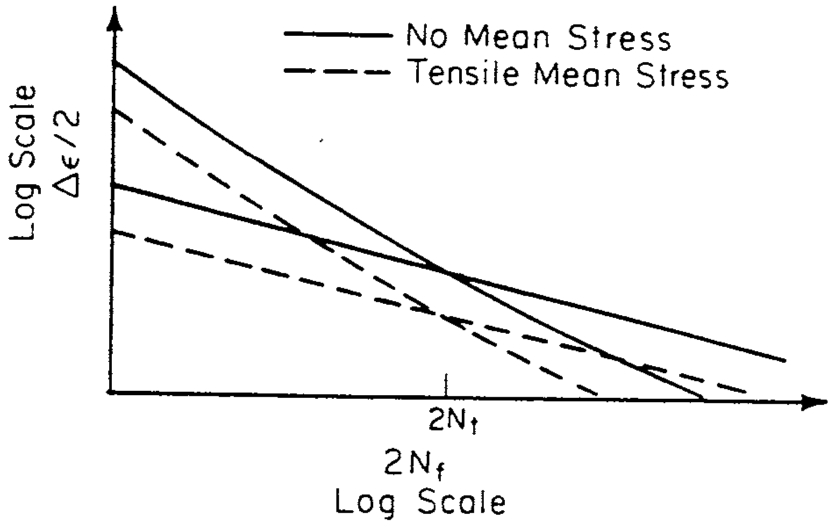
Same ratio of elastic to plastic strain, but, vastly different mean stress

Independence of elastic/plastic strain ratio from mean stress

Modification to strain-life equation I

- Manson and Halford's modification

$$\frac{\Delta\varepsilon}{2}=\frac{\sigma'_f-\sigma_0}{E}(2N_f)^b+\varepsilon'_f\left(\frac{\sigma'_f-\sigma_0}{\sigma'_f}\right)^{c/b}(2N_f)^c$$



- Too much mean stress effect at short lives.

- Smith, Watson, and Topper (SWT)'s modification. For completely reversed loading

$$\sigma_{\max}=\frac{\Delta\sigma}{2}=\sigma'_f(2N_f)^b$$

- Multiplying the strain-life equation by this term, results in

$$\sigma_{\max}\frac{\Delta\varepsilon}{2}=\frac{(\sigma'_f)^2}{E}(2N_f)^{2b}+\sigma'_f\,\varepsilon'_f\,(2N_f)^{b-c}$$

- The term σ_{max}

$$\sigma_{\max}=\frac{\Delta\sigma}{2}+\sigma_0 \qquad \sqrt{\sigma_{\max}\Delta\varepsilon}\propto N_f$$

It becomes undefined when σ_{max} is negative.
No fatigue damage occurs when σ_{max} <0 ?