Innovative Ship and Offshore Plant Design

Part I. Ship Design

Ch. 2 Design Equations

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Contents

Ch. 2 Design Equations

1. Owner's Requirements

Owner's Requirements

- ☑ Ship's Type
  - Cargo Capacity: Cargo Hold Volume / Containers in Hold & on Deck / Car Deck Area
  - Water Ballast Capacity
  - Service Speed at Design Draft with Sea Margin, MCR/NCR Engine Power & RPM
- ☑ Dimensional Limitations: Panama canal, Suez canal, Strait of Malacca, St. Lawrence Seaway, Port limitations
- ☑ Maximum Draft (T_{max})
- ☑ Daily Fuel Oil Consumption (DFOC): Related with ship's economics
- ☑ Special Requirements
  - Ice Class, Air Draft, Bow/Stern Thruster, Special Rudder, Twin Skeg
  - Delivery day, with ( )$ penalty per delayed day
  - Abt. 21 months from contract
  - Material & Equipment Cost + Construction Cost + Additional Cost + Margin

Principal Particulars of a Basis Ship

- ☑ At early design stage, there are few data available to determine the principal particulars of the design ship.
- ☑ Therefore, initial values of the principal particulars can be estimated from (called also as ' ' or ' '), whose main dimensional ratios and hull form coefficients are similar with the ship being designed.
- ☑ The include main dimensions, hull form coefficients, speed and engine power, DFOC, capacity, cruising range, crew, class, etc.

Example) VLCC (Very Large Crude oil Carrier)

308.000DWT

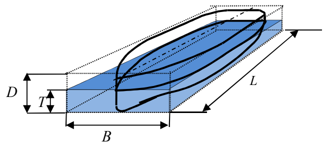
318.000DWT

Principal Dimensions and Hull Form Coefficients

☑ The principal dimensions and hull form coefficients decide many characteristics of a ship, e.g. stability, cargo hold capacity, resistance, propulsion, power requirements, and economic efficiency.

☑ Therefore, the determination of the principal dimensions and hull form coefficients is most important in the ship design.

☑ The length L, breadth B, depth D, immersed depth (draft) T, and block coefficient C_B should be determined first.



2. Design Constraints

Design Constraints

In the ship design, the principal dimensions cannot be determined arbitrarily; rather, they have to satisfy following :

1) constraint

- : Hydrostatic equilibrium ➡

2) constraints

- Owner's requirements
  - Ship's type, Deadweight (DWT)[*ton*],
  - Cargo hold capacity (V_{CH})[m^3], ➡
  - Service speed (V_S)[*knots*], ➡ **Daily fuel oil consumption(DFOC)**[*ton/day*]
  - Maximum draft (T_{max})[*m*],
  - Limitations of main dimensions (Canals, Sea way, Strait, Port limitations : e.g. Panama canal, Suez canal, St. Lawrence Seaway, Strait of Malacca,
  - Endurance[*n.m*[1)]],

1) n.m = nautical mile
1 n.m = 1.852 km

3) constraints

International Maritime Organization [**IMO**] regulations,
International Convention for the Safety Of Life At Sea [**SOLAS**],
International Convention for the Prevention of Marin Pollution from Ships [**MARPOL**],
International Convention on Load Lines [**ICLL**],
Rules and Regulations of **Classification** Societies

(1) Physical Constraint

Physical Constraint

- **Physical constraint**
  - **Floatability**

For a ship to float in sea water, the total weight of the ship (W) must be equal to the buoyant force (F_B) on the immersed body

➡ :

$$F_B \stackrel{!}{=} W \quad ...(1)$$

$$W = LWT + DWT$$

***Lightweight**(LWT) reflects the weight of vessel being ready to go to sea without cargo and loads. And lightweight can be composed of:
LWT = Structural weight + Outfit weight + Machinery weight

***Deadweight**(DWT) is the weight that a ship can load till the maximum allowable immersion(at the scantling draft(T_s)).
And deadweight can be composed of:
DWT= Payload+ Fuel oil + Diesel oil+ Fresh water +Ballast water + etc.

Physical Constraint

- Physical constraint: hydrostatic equilibrium

$$F_B = W \quad ...(1)$$
$$W = LWT + DWT$$

∇ : the immersed volume of the ship.
ρ : density of sea water = 1.025 Mg/m³

(L.H.S) **What is the (F_B)?**
According to the ,
the buoyant force on an immersed body has the same magnitude as the weight of the fluid displaced by the body.

$$F_B = g \cdot \rho \cdot V$$

Volume
Mass

In shipbuilding and shipping society, those are called as follows :
- **Displacement volume** ∇
- **Displacement mass** Δ_m
- **Displacement** Δ

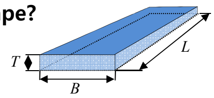
Buoyant Force is the weight of the displaced fluid.

In shipbuilding and shipping society, **buoyant force** is called in another word, **displacement (Δ)**.

Block Coefficient (C_B)

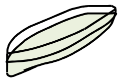
V : immersed volume
V_{box} : volume of box
L : length, B : breadth
T : draft

Does a ship or an airplane usually have box shape?



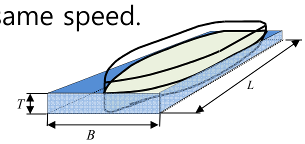
No!

They have a streamlined shape.



Why does a ship or an airplane has a streamlined shape?

They have a streamlined shape **to minimize the drag force** experienced when they travel, so that the propulsion engine needs a smaller power output to achieve the same speed.



Block coefficient(C_B) is **the ratio of the immersed volume to the box bounded by L, B, and T.**

$$C_B \equiv \frac{V}{V_{box}} = \frac{V}{L \cdot B \cdot T}$$

Shell Appendage Allowance

$C_B = \frac{V}{L \cdot B \cdot T}$

V : immersed volume
V_{box} : volume of box
L : length, B : breadth
T : draft
C_B : block coefficient

The immersed volume of the ship can be expressed by block coefficient.

$$V_{molded} = L \cdot B \cdot T \cdot C_B$$

In general, we have to consider the **displacement of shell plating and appendages such as propeller, rudder, shaft, etc.** additionally.
Thus, The total immersed volume of the ship can be expressed as following:

$$V_{total} = L \cdot B \cdot T \cdot C_B \cdot (1+\alpha)$$

Where the hull dimensions length L, beam B, and draft T are the dimensions of the immerged hull to the inside of the shell plating,

thus α is which adapts the **molded volume to the actual volume** by accounting for the volume of the shell plating and appendages (typically about 0.002~0.0025 for large vessels).

$$\boxed{F_B = g \cdot \rho \cdot V_{total} = \rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha)}$$

Weight Equation

- **Physical constraint: hydrostatic equilibrium**

$$F_B = W \quad ...(1)$$

(R.H.S) $W = LWT + DWT$

(L.H.S) $F_B = \rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha)$

ρ : density of sea water = 1.025 Mg/m³
α : displacement of shell, stern and appendages
C_B : block coefficient
g : gravitational acceleration

$$\boxed{\rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = LWT + DWT \; ...(2)}$$

The equation (2) describes the physical constraint to be satisfied in ship design,

Unit of the Lightweight and Deadweight

- **Physical constraint**: hydrostatic equilibrium

$$F_B = W \quad ...(1)$$

$$\boxed{\rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = LWT + DWT \quad ...(2)}$$

What is the unit of the lightweight and deadweight?

Weight vs. Mass

Question: Are the "weight" and "mass" the same? 

Answer: No!

is a measure of the amount of matter in an object.

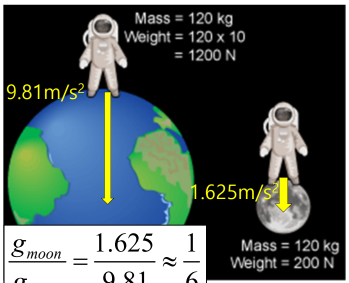
is a measure of the force on the object caused by the universal gravitational force.

<u>Gravity causes weight.</u>

Mass of an object does not change, but its weight can change.

For example, an astronaut's weight on the moon is one-sixth of that on the Earth.

But the astronaut's mass does not change.



$$\frac{g_{moon}}{g_{earth}} = \frac{1.625}{9.81} \approx \frac{1}{6}$$

g_{moon} : gravitational acceleration on the moon
g_{earth} : gravitational acceleration on the earth

Mass Equation

• **<u>Physical constraint</u>**: hydrostatic equilibrium

$$F_B = W \qquad ...(1)$$

In shipping and shipbuilding world, is used instead of for the unit of the lightweight and deadweight in practice.

Actually, however, the weight equation is "mass equation".

$$\boxed{\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = LWT + DWT \qquad ...(3)}$$

where, ρ = 1.025 Mg/m³

(2) Economical Constraints

Volume Equation

Economical Constraints: Required Cargo Hold Capacity ➡ Volume Equation

- **Economical constraints**

  - **Owner's requirements (Cargo hold capacity**[m^3]**)**
  - **The main dimensions have to satisfy the required cargo hold capacity (V_{CH}).**

$$\boxed{V_{CH} = f(L, B, D)}$$

:

- **It is checked whether the depth will allow the <u>required cargo hold capacity</u>.**

Service Speed & DFOC (Daily Fuel Oil Consumption)

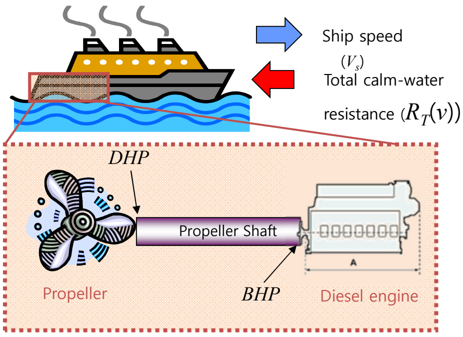
Economical Constraints : Required DFOC (Daily Fuel Oil Consumption) ➡ Hull Form Design and Hydrodynamic Performance Equation

☑ **Goal: Meet the Required DFOC.**

At first, we have to estimate <u>total calm-water resistance of a ship</u>

$$EHP = \boxed{R_T(v)} \cdot V_s$$

Then, the can be predicted by estimating propeller efficiency, hull efficiency, relative rotative efficiency, shaft transmission efficiency, sea margin, and engine margin.

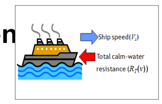


Economical Constraints : Required DFOC (Daily Fuel Oil Consumption) ➡ Propeller and Engine Selection

① EHP (Effective Horse Power)

$$EHP = \boxed{R_T(v)} \cdot V_s \quad \text{(in calm water)}$$

⇦ **Resistance Estimation**



② DHP (Delivered Horse Power)

$$DHP = \frac{EHP}{\boxed{\eta_D}}$$

(η_D : Propulsive efficiency)

$\eta_D = \eta_O \cdot \eta_H \cdot \eta_R$

η_O : Open water efficiency

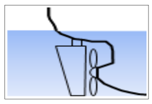
η_H : Hull efficiency

η_R : Relative rotative efficiency

⇦ **Propeller Efficiency**

Thrust deduction and wake (due to additional resistance by propeller)

Hull-propeller interaction



③ BHP (Brake Horse Power)

$$BHP = \frac{DHP}{\eta_T}$$

(η_T : Transmission efficiency)

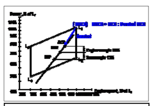
④ NCR (Normal Continuous Rating)

$$NCR = BHP\left(1 + \frac{\text{Sea Margin}}{100}\right)$$

⑤ DMCR (Derated Maximum Continuous Rating)

$$\boxed{DMCR = \frac{NCR}{\text{Engine Margin}}}$$

⇨ **Engine Selection**



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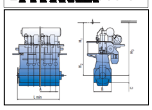
⑥ NMCR (Nominal Maximum Continuous Rating)

$$NMCR = \frac{DMCR}{\text{Derating rate}}$$

⇦ **Engine Data**

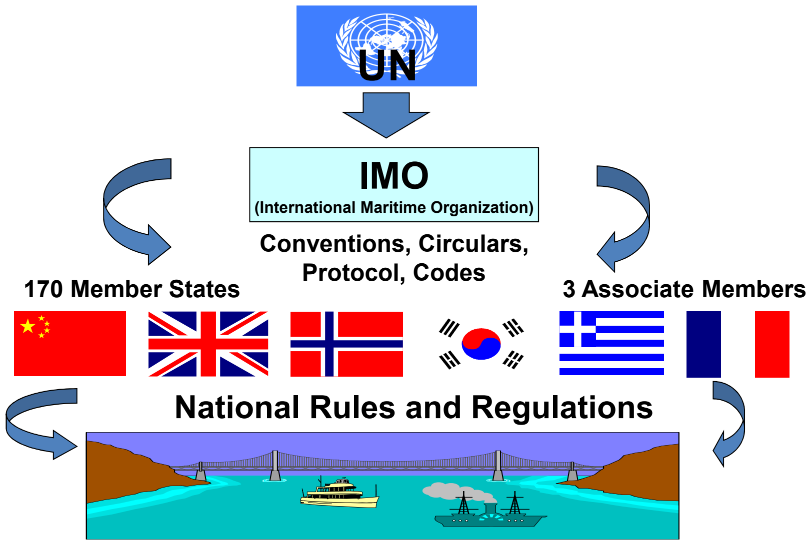


(3) Regulatory Constraints

Regulatory Constraints - Rules by Organizations

☑ International Maritime Organizations (IMO)
☑ International Labor Organizations (ILO)
☑ Regional Organizations (EU, ...)
☑ Administrations (Flag, Port)
☑ Classification Societies
☑ International Standard Organizations (ISO)

IMO (International Maritime Organization)

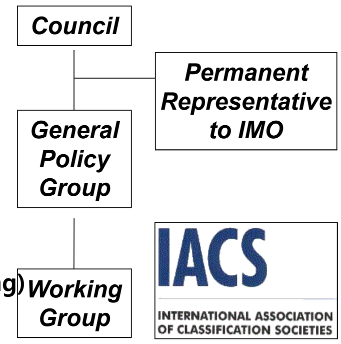


IMO Instruments

- ☑ Conventions
  - / / / COLREG / ITC / AFS / BWM ......
- ☑ Protocols
  - ■ MARPOL Protocol 1997 / ICLL Protocol 1988
- ☑ Codes
  - ■ ISM / LSA / IBC / IMDG / IGC / BCH / BC / GC ......
- ☑ Resolutions
  - ■ Assembly / MSC / MEPC
- ☑ Circulars
  - ■ MSC / MEPC / Sub-committees ......

Regulatory Constraints
- Rules by Classification Societies

☑ 10 Members
- ABS (American Bureau of Shipping)
- DNV (Det Norske Veritas)
- LR (Lloyd's Register)
- BV (Bureau Veritas)
- GL (Germanischer Lloyd)
- KR (Korean Register of Shipping)
- RINA (Registro Italiano Navale)
- NK (Nippon Kaiji Kyokai)
- RRS (Russian Maritime Register of Shipping)
- CCS (China Classification Society)

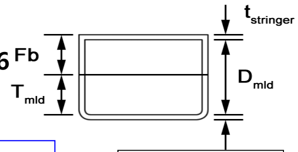


☑ 2 Associate Members
- CRS (Croatian Register of Shipping)
- IRS (Indian Register of Shipping)

Required Freeboard

Required Freeboard of ICLL 1966

- **Regulatory constraints**
  - **International Convention on Load Lines (ICLL)1966**



$D_{Fb} = D_{mld} + t_{stringer}$

$$D_{Fb} - T \geq Fb_{ICLL}(L, B, D_{mld}, C_B)$$

:

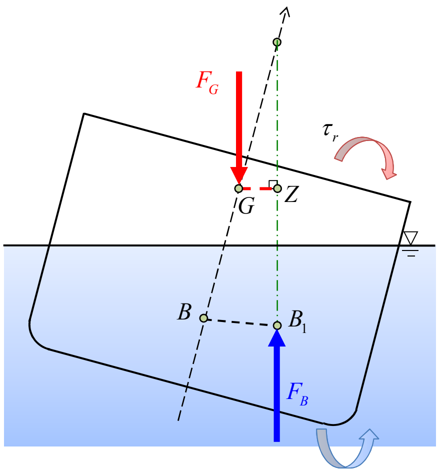
✓ **Check : Actual freeboard ($D_{Fb} - T$) of a ship should not be less than the freeboard required by the ICLL 1966 regulation (Fb_{ICLL}).**

Freeboard (Fb) means the distance between the water surface and the top of the deck at the side (at the deck line). It includes the thickness of freeboard deck plating.

- The freeboard is closely related to the draught.
  A 'freeboard calculation' in accordance with the regulation determines whether the desired depth is permissible.

Required Stability

Definition of GZ (Righting Arm)

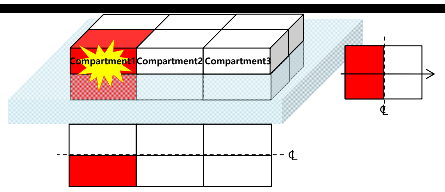


$$\tau_r = GZ \cdot F_B$$

τ_r :

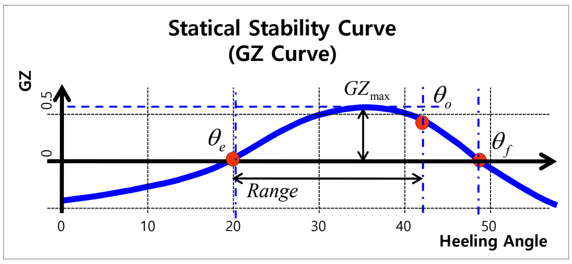
GZ :

Damage of a Box-Shaped Ship (GZ Curve)



Immersion + Trim + Heel

✓ To measure the damage stability, we should find the a statical stability curve(GZ curve) of this damage case by finding the new center of buoyancy(B) and center of mass(G).



θ_e: Equilibrium heel angle

θ_v: minimum(θ_f, θ_o)

(in this case, θ_v equals to θ_o)

GZ_{max}: Maximum value of *GZ*

Range: Range of positive righting arm

Flooding stage: Discrete step during the flooding process

θ_f: Angle of flooding (righting arm becomes negative)

θ_o: Angle at which an "opening" incapable of being closed weathertight becomes submerged

Structural Design in accordance with the Rule of the Classification Society

Regulatory Constraint: Ship Structural Design in accordance with Rule of the Classification Society

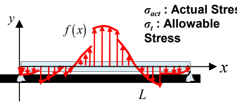
$Load: f(x)$

cause

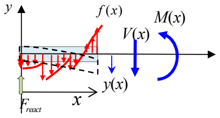
$Shear\ Force: V(x)$
$Bending\ Moment: M(x)$
$Deflection: y(x)$

'relations' of load, S.F., B.M., and deflection

$\frac{dV(x)}{dx} = -f(x)$, $\frac{dM(x)}{dx} = V(x)$, $EI\frac{d^2y(x)}{dx^2} = M(x)$



σ_{act} : Actual Stress
σ_l : Allowable Stress



- **Ship Structural Design**

What is designer's **major** interest?

- Safety:
Won't "It" fail under the load?

a ship	global
a stiffener	local
a plate	

What is our interest?

- Safety: *Won't it fail under the load?*

Stress should meet : $\boxed{\sigma_{act} \le \sigma_l}$, where $\sigma_{act} = \frac{M}{I_{\bar{y}}/\bar{y}_i} = \frac{M}{Z}$

- Geometry: *How much it would be bent under the load?*

Differential equations of the defection curve $\boxed{EI\frac{d^4y(x)}{dx^4} = -f(x)}$

what kinds of load f cause hull girder moment?

$\boxed{\sigma_{act.} \le \sigma_l}$, $\sigma_{act.} = \frac{M_S + M_W}{Z_{mid}}$, M_S = Still water bending moment, M_W = Vertical wave bending moment

$f(x) = f_S(x) + f_W(x)$
$V(x) = V_S(x) + V_W(x)$
$M(x) = M_S(x) + M_W(x)$

Hydrostatics

$f_S(x)$: load in still water = weight + buoyancy

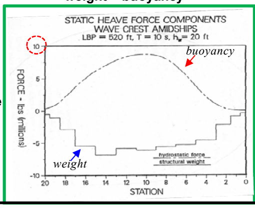
$f_S(x)$: load in still water

$V_S(x) = \int_0^x f_S(x)dx$

$V_S(x)$: still water shear force

$M_S(x) = \int_0^x V_S(x)dx$

$M_S(x)$: still water bending moment



Hydrodynamics

$f_W(x)$: load in wave = added mass + diffraction + damping + Froude-Krylov + mass inertia

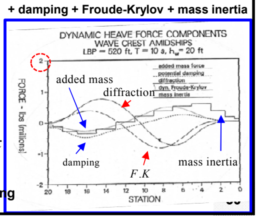
$f_W(x)$: load in wave

$V_W(x) = \int_0^x f_W(x)dx$

$V_W(x)$: wave shear force

$M_W(x) = \int_0^x V_W(x)dx$

$M_W(x)$: vertical wave bending moment



Regulatory Constraint: Ship Structural Design in accordance with Rule of the Classification Society

- **Ship Structural Design**

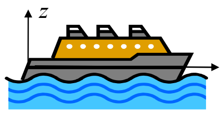
What is designer's **major** interest?

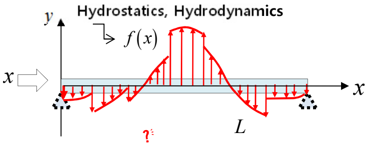
- Safety:

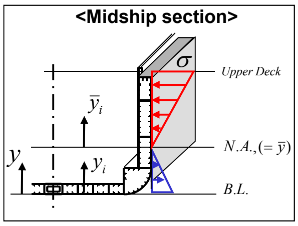
Won't 'It' fail under the load?

a ship } global
a stiffener
a plate } local

a ship







Actual stress on midship section should be less than allowable stress.

$\boxed{\sigma_{act.} \le \sigma_{allow}}$, $\sigma_{act.} = \dfrac{M_{mid}}{Z_{mid}} = \dfrac{M_s + M_w}{I_{ship,N.A}/\bar{y}_i}$

Allowable stress by Rule : (for example) , $\sigma_{allow} = 175 f_1 \ [N/mm^2]$

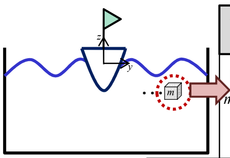
How we can meet the rule?

'Midship design' is to arrange the structural members and fix the thickness of them to secure enough section modulus to the rule.

, M_w : vertical wave bending moment
, M_s : still water bending moment
, $I_{ship,N.A.}$: moment of inertia from N.A. of Midship section

Hydrostatic and Hydrodynamic Forces acting on a Ship in Waves

Equations of Motion of a Fluid Element
- From Cauchy Eq. to Bernoulli Eq.



✓Equations of motion of a Fluid Element

Lagrangian & Eulerian Description

Shear force

Curl & Rotation

Newton's 2nd Law $m\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body force} + \text{Surface force})$ ⇒ Cauchy Equation ⇒ ①② Navier-Stokes Equation ⇒ ③ Euler Equation ⇒ ④⑤ Bernoulli Equation

$$\rho\frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho\left|\nabla\Phi\right|^2 + \rho g z = 0$$

Microscopic/ Macroscopic Derivation (RTT[1])

Mass Conservation Law ⇒ ④⑤ Laplace Equation ⇒ $\nabla^2\Phi = 0$ $(\mathbf{V} = \nabla\Phi)$

① Newtonian fluid: Fluid whose stress versus strain rate curve is linear.

② Stokes assumption: Definition of viscosity coefficient (μ, λ) due to linear deformation and isometric expansion

③ Inviscid fluid

④ Irrotational flow

⑤ Incompressible flow

1) RTT: Reynolds Transport Theorem

$\mathbf{r}$: displacement of a fluid particle with respect to the time

$\mathbf{V} = \frac{d\mathbf{r}}{dt}, \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$

* Lagrangian specification of the flow field: a way of looking at fluid motion where the observer follows an individual fluid parcel as it moves through space and time.
* Eulerian specification of the flow field: a way of looking at fluid motion that focuses on specific locations in the space through which the fluid flows as time passes.

Equations of Motion of a Fluid Element and Continuity Equation

1) Kundu, P.K., Cohen, I.M., Fluid Mechanics 5th, Academic Press, 2012
* A **Newtonian fluid**: fluid whose stress versus strain rate curve is linear.
** Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

Cauchy Equation: $\rho\frac{d\mathbf{V}}{dt} = \rho\mathbf{g} + \nabla\bullet\sigma \quad ,\left(\mathbf{V} = [u, v, w]^T\right)$

⇓ ① Newtonian fluid* ② Stokes assumption**

Navier-Stokes Equation: (in general form) $\rho\frac{d\mathbf{V}}{dt} = \rho\mathbf{g} - \nabla P + \mu\left(\frac{1}{3}\nabla(\nabla\cdot\mathbf{V}) + \nabla^2\mathbf{V}\right)$

$(\mu = 0)$ ⇓ ③ Inviscid fluid

Euler Equation: $\rho\frac{d\mathbf{V}}{dt} = \rho\mathbf{g} - \nabla P$

$\rho = \rho(P)$ ⇓ ④ Barotropic flow(A fluid whose density is a function of only pressure.)

Euler Equation: (another form) $\frac{\partial\mathbf{V}}{\partial t} + \nabla B = \mathbf{V}\times\boldsymbol{\omega} \quad ,\left(B = \frac{1}{2}q^2 + gz + \int\frac{dP}{\rho} \quad , \quad q^2 = u^2 + v^2 + w^2\right)$

$\boldsymbol{\omega} = \nabla\times\mathbf{V}$
If $\boldsymbol{\omega} = 0$, (irrotational flow)
then $\nabla\times\mathbf{V} = 0$
If $\nabla\times\mathbf{V} = 0$, then $\mathbf{V} = \nabla\Phi$.

⑤ Steady flow $\left(\frac{\partial\mathbf{V}}{\partial t} = 0\right)$ ⇓ along the streamlines and vortex lines

Bernoulli Equation (Case 1) $B = \text{Constant}$ $\left(\frac{1}{2}q^2 + gz + \int\frac{dP}{\rho} = C\right)$

$\left(q^2 = \left|\nabla\Phi\right|^2\right)$ ⇓ ⑥ unsteady, irrotational flow

Bernoulli Equation (Case 2) $\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left|\nabla\Phi\right|^2 + gz + \int\frac{dP}{\rho} = F(t)$

$(\rho = constant)$ ⇓ ⑦ incompressible flow

Bernoulli Equation (Case 3) $\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left|\nabla\Phi\right|^2 + gz + \frac{P}{\rho} = F(t)$

Continuity Equation $\frac{\partial\rho}{\partial t} + \nabla\bullet\rho\mathbf{V} = 0$

⇓ ⑦ Incompressible flow $\rho = constant, \left(\frac{\partial\rho}{\partial t} = 0\right)$

$\nabla\bullet\mathbf{V} = 0$

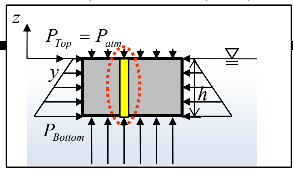
⇓ ⑥ Irrotational flow $(\mathbf{V} = \nabla\Phi)$

Laplace Equation $\nabla^2\Phi = 0$

Newtonian fluid,
Stokes assumption,
Inviscid fluid,
Unsteady flow,
Irrotational flow,
Incompressible flow

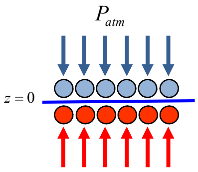
Meaning of F(t) in Bernoulli Equation and Gauge Pressure

1) Gauge pressure: The net pressure of the difference of the total pressure and atmospheric pressure



Bernoulli Equation

$$\frac{\partial \Phi}{\partial t}+\frac{P}{\rho}+\frac{1}{2}|\nabla\Phi|^2+gz=F(t)$$



If a fluid element is in static equilibrium state on the free surface (z=0), then

$$\frac{\partial \Phi}{\partial t}=0,\ \nabla\Phi=0,\quad P=P_{atm}$$

$$\cancel{\frac{\partial \Phi}{\partial t}}+\frac{P}{\rho}+\frac{1}{2}\cancel{|\nabla\Phi|^2}+\cancel{gz}=F(t)\longrightarrow \frac{P_{atm}}{\rho}=F(t)$$

(Atmospheric pressure (P_{atm})) = (Pressure at z=0)

$$\therefore \frac{\partial \Phi}{\partial t}+\frac{P}{\rho}+\frac{1}{2}|\nabla\Phi|^2+gz=\frac{P_{atm}}{\rho}$$

✓What is the pressure on the bottom of an object ?

$$\frac{\partial \Phi}{\partial t}+\frac{P_{Bottom}}{\rho}+\frac{1}{2}|\nabla\Phi|^2+gz=\frac{P_{atm}}{\rho}$$

$$\frac{\partial \Phi}{\partial t}+\frac{\cancel{P_{atm}}+P_{Fluid}}{\rho}+\frac{1}{2}|\nabla\Phi|^2+gz=\frac{\cancel{P_{atm}}}{\rho}$$

$$\therefore \frac{\partial \Phi}{\partial t}+\frac{\boxed{P_{Fluid}}}{\rho}+\frac{1}{2}|\nabla\Phi|^2+gz=0$$

'gauge pressure'

※ In case that R.H.S of Bernoulli equation is expressed by zero, pressure P means the pressure due to the fluid which excludes the atmospheric pressure.

If the motion of fluid is small, square term could be neglected.

$$\frac{\partial \Phi}{\partial t}+\frac{P_{Fluid}}{\rho}+\cancel{\frac{1}{2}|\nabla\Phi|^2}+gz=0$$

$$P_{Fluid}=\underbrace{-\rho\frac{\partial \Phi}{\partial t}}_{P_{dynamic}}\underbrace{-\rho gz}_{P_{static}}=0$$

'Linearized Bernoulli Equation'

Pressure and Force acting on a Fluid Element

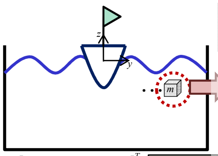
$\mathbf{r}$: displacement of particle with respect to time

$\mathbf{V}=\frac{d\mathbf{r}}{dt},\ \mathbf{a}=\frac{d^2\mathbf{r}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude-Krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

1) RTT: Reynolds Transport Theorem
2) SWBM: Still Water Bending Moment
3) VWBM: Vertical Wave Bending Moment

✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
④ Irrotational flow
⑤ Incompressible flow



$\mathbf{r}=[\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6]^T$

ξ_1 : surge, ξ_4 : roll
ξ_2 : sway, ξ_5 : pitch
ξ_3 : heave, ξ_6 : yaw

✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Shear force

Curl & Rotation

Newton's 2nd Law
$m\ddot{\mathbf{r}}=\sum\mathbf{F}=$ (Body force + Surface force)

Cauchy equation → Navier-Stokes equation ①② → Euler equation ③ → Bernoulli equation ④⑤

Microscopic/ Macroscopic Derivation(RTT[1])

Mass Conservation Law ④⑤ → Laplace Equation

$$\nabla^2\Phi=0\quad(\mathbf{V}=\nabla\Phi)$$

$$\rho\frac{\partial \Phi}{\partial t}+P+\frac{1}{2}\rho|\nabla\Phi|^2+\rho gz=0$$

Velocity potential Φ

$\Phi=\Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

✓Calculation of Fluid Force

Linearization
$\left(\frac{1}{2}\rho|\nabla\Phi|^2=0\right)$

$$P=-\rho gz-\rho\frac{\partial \Phi}{\partial t}$$

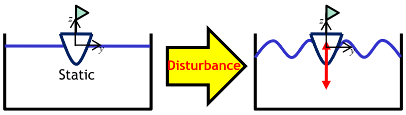
$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})=\iint_{S_B}P\mathbf{n}dS=\mathbf{F}_{Buoyancy}(\mathbf{r})+\mathbf{F}_{F.K}(\mathbf{r})+\mathbf{F}_D(\mathbf{r})+\mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

(S_B : wetted surface area)

* A **Newtonian fluid** : fluid whose stress versus strain rate curve is linear.
** Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

Forces acting on a Ship in Waves (1/2)

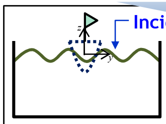
$$\rho\frac{\partial\Phi}{\partial t}+P+\frac{1}{2}\rho|\nabla\Phi|^2+\rho g\,z=0$$



✓ **Pressure due to the fluid elements around the ship in wave**

: Velocity, acceleration, pressure of the fluid elements are changed due to the motion of fluid, then the pressure of fluid elements acting on the ship is changed.

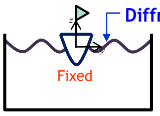
Linearization



Incident wave velocity potential (Φ_I)

✓ **Velocity potential of incoming waves that are independent of the body motion**

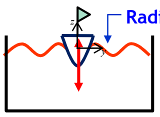
+



Diffraction wave velocity potential (Φ_D)

✓ **Velocity potential of the disturbance of the incident waves by the body that is fixed in position**[1)]

+



Radiation wave velocity potential (Φ_R)

✓ **Velocity potential of the waves that are induced due to the body motions, in the absence of the incident waves.**[1)]

✓ **Total Velocity Potential**

$$\Phi_T=\Phi_I+\Phi_D+\Phi_R$$

Superposition theorem

For homogeneous linear PDE, the superposed solution is also a solution of the linear PDE[2)].

$$P=-\rho gz-\rho\frac{\partial\Phi_T}{\partial t}$$

$$\mathbf{F}_{Fluid}=\iint_{S_B}P\mathbf{n}dS$$

$$=\mathbf{F}_{static}+\mathbf{F}_{F.K}+\mathbf{F}_D+\mathbf{F}_R$$

1) Newman, J.N., Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp.287
2) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch.12.1, pp.535

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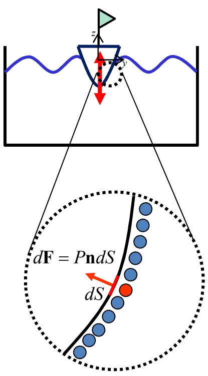
Forces acting on a Ship in Waves (2/2)

Φ_I : Incident wave velocity potential
Φ_D: Diffraction potential
Φ_R: Radiation potential



$d\mathbf{F}$: Infinitesimal force of the fluid elements acting on the ship

dS : Infinitesimal Area

$\mathbf{n}$: Normal vector of the infinitesimal Area

$\mathbf{r}=[\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6]^T$
ξ_1 : surge , ξ_4 : roll
ξ_2 : sway , ξ_5 : pitch
ξ_3 : heave , ξ_6 : yaw

✓ Bernoulli Equation

$$\rho\frac{\partial\Phi}{\partial t}+P+\cancel{\frac{1}{2}\rho|\nabla\Phi|^2}+\rho gz=0$$

Linearization

✓ Laplace Equation

$$\nabla^2\Phi=0$$

$$\Phi=\Phi_I+\Phi_D+\Phi_R$$

Linear combination of the basis solutions

Basis solutions

$$P_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})'=-\rho gz-\rho\frac{\partial\Phi}{\partial t}=\boxed{-\rho gz}-\rho\left(\frac{\partial\Phi_I}{\partial t}+\frac{\partial\Phi_D}{\partial t}+\frac{\partial\Phi_R}{\partial t}\right)$$

Pressure of the fluid elements acting on the ship

$$=\boxed{P_{Buoyancy}(\mathbf{r})}+\underbrace{P_{F.K}(\mathbf{r})+P_D(\mathbf{r})+P_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})}_{P_{dynamic}}$$

$$P_{Fluid}\quad=\mathbf{F}_{Buoyancy}(\mathbf{r})+\mathbf{F}_{F.K}(\mathbf{r})+\mathbf{F}_D(\mathbf{r})+\mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

(S_B : wetted surface)

Integration over the wetted surface area of the ship
(**Forces and moments acting on the ship due to the fluid elements)**

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})=\iint_{S_B}P\;\mathbf{n}dS$$

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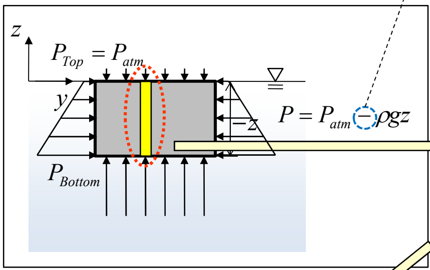
Hydrostatic Pressure and Buoyant Force acting on a Ship

※ Pressure: Force per unit area applied in a direction perpendicular to the surface of an object.
To calculate force, we should multiply pressure by area and normal vector of the area.

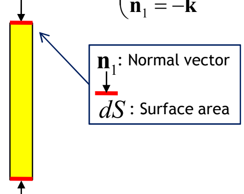
According to the reference frame, (-) sign is added because the value of z is (-).

✓ What is the force acting on the bottom of an object?



: Force acting on the upper differential area

$$d\mathbf{F}_{Top} = P_{Top}\cdot\mathbf{n}_1 dS \quad \begin{pmatrix} P_{Top} = P_{atm} - \rho g\cdot 0 \\ \mathbf{n}_1 = -\mathbf{k} \end{pmatrix}$$



$$d\mathbf{F}_{Bottom} = P_{Bottom}\cdot\mathbf{n}_2 dS \quad \begin{pmatrix} P_{Bottom} = P_{atm} - \rho g z \\ \mathbf{n}_2 = \mathbf{k} \end{pmatrix}$$

: Force acting on the lower differential area

$$\begin{aligned} d\mathbf{F} &= d\mathbf{F}_{Top} + d\mathbf{F}_{Bottom} \\ &= P_{Top}\cdot\mathbf{n}_1 dS + P_{Bottom}\cdot\mathbf{n}_2 dS \\ &= \cancel{P_{atm}}(-\mathbf{k})dS + (\cancel{P_{atm}} - \rho g z)\mathbf{k}dS \\ &= -\rho g z\mathbf{k}dS = \mathbf{k}(-\rho g z\cdot dS) \end{aligned}$$

: Force due to the atmospheric pressure is vanished.

$$\mathbf{F} = \int d\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \,, (P = P_{static} = -\rho g z)$$

Static fluid pressure excluding the atmospheric pressure.

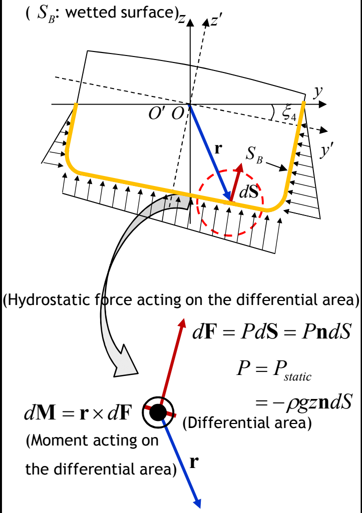
$$= -\rho g \iint_{S_B} \mathbf{n}z dS$$

Cf) Linearized Bernoulli Eq.

$$P = \underbrace{-\rho g z}_{P_{static}} \underbrace{- \rho\frac{\partial\Phi}{\partial t}}_{P_{dynamic}}$$

Hydrostatic Force and Moment acting on a Ship

In case that ship is inclined about x- axis
(Front view)
(S_B: wetted surface)



▪ Hydrostatic force (Surface force) is calculated by integrating the differential force over the wetted surface area.

✓ Hydrostatic force acting on the differential area

$$d\mathbf{F} = P\cdot d\mathbf{S} = P\cdot\mathbf{n}dS$$

P is hydrostatic pressure, P_{static}.

$$P = P_{static} = -\rho g z$$

$$d\mathbf{F} = P_{static}\cdot\mathbf{n}dS = -\rho g z\cdot\mathbf{n}dS$$

✓ Total force (S_B: wetted surface area)

$$\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \;\Rightarrow\; \boxed{\mathbf{F} = -\rho g\iint_{S_B} z\mathbf{n}dS}$$

▪ Hydrostatic Moment : (Moment) = (Position vector) X (Force)

✓ Moment acting on the differential area

$$d\mathbf{M} = \mathbf{r}\times d\mathbf{F} = \mathbf{r}\times P\mathbf{n}dS = P(\mathbf{r}\times\mathbf{n})dS$$

✓ Total moment

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r}\times\mathbf{n})dS \;\Rightarrow\; \boxed{\mathbf{M} = -\rho g\iint_{S_B} z(\mathbf{r}\times\mathbf{n})dS}$$

Buoyant Force

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th, Wiley, Ch. 10.7, p.458-463
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th, Wiley, Ch. 9.9, p.414-417

✓ Hydrostatic force

$$\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS \qquad (S\text{: wetted surface area})$$

By divergence theorem[1],

$$\left(\iint_S f \cdot \mathbf{n} dA = \iiint_V \nabla f dV \right)$$

$$\mathbf{F} = \rho g \iiint_V \nabla z dV \qquad \left(\nabla z \overset{2)}{=} \frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} + \frac{\partial z}{\partial z}\mathbf{k} = \mathbf{k} \right)$$

$$= \mathbf{k} \rho g \iiint_V dV$$

$$= \mathbf{k} \rho g V(t)$$

When ship moves, the displacement volume (*V*) of the ship is changed with time.
That means *V* is the function of time, *V*(*t*).

: The buoyant force on an immersed body has the same magnitude as the weight of the fluid displaced by the body[1]. And the direction of the buoyant force is opposite to the gravity (≒Archimedes' Principle)

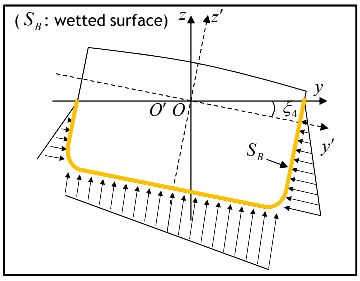
※ The reason why (-) sign is disappeared.
: Divergence theorem is based on the outer unit vector of the surface.
Normal vector for the calculation of the buoyant force is based on the inner unit vector of the surface, so (-) sign is added, and then divergence theorem is applied.

Hydrostatic Moment

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th, Wiley, Ch. 10.7, p.458-463
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th, Wiley, Ch. 9.9, p.414-417

✓ Hydrostatic moment

$$\mathbf{M} = -\rho g \iint_{S_B} (\mathbf{r} \times \mathbf{n}) z dS = \rho g \iint_{S_B} (\mathbf{n} \times \mathbf{r}) z dS$$



By divergence theorem[1] ,

$$\left(\iiint_V \nabla \times \mathbf{F} dV = \iint_S \mathbf{n} \times \mathbf{F} dA \right)$$

$$\mathbf{M} = -\rho g \iiint_V (\nabla \times \mathbf{r}) z dV$$

Because direction of normal vector is opposite, (-) sign is added

$$\left(\nabla \times \mathbf{r}\, z \overset{2)}{=} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & yz & z^2 \end{vmatrix} = \mathbf{i}\left(\frac{\partial}{\partial y} z^2 - \frac{\partial}{\partial z} yz \right) + \mathbf{j}\left(\frac{\partial}{\partial z} xz - \frac{\partial}{\partial x} z^2 \right) + \mathbf{k}\left(\frac{\partial}{\partial x} yz - \frac{\partial}{\partial y} xz \right) = -\mathbf{i}y + \mathbf{j}x \right)$$

$$\therefore \mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV$$

Hydrodynamic Forces
Calculated from 6 DOF (Degree of Freedom) Equations of Ship Motions (1/2)

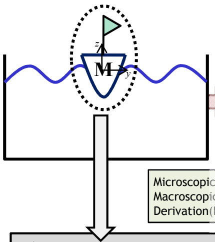
$\mathbf{r}$: displacement of particle with respect to time

$\mathbf{V} = \frac{d\mathbf{r}}{dt}, \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude-Krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

1) RTT: Reynolds Transport Theorem
2) SWBM: Still Water Bending Moment
3) VWBM: Vertical Wave Bending Moment

✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
④ Irrotational flow
⑤ Incompressible flow



✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Newton's 2nd Law $m\ddot{r} = \sum F = (\text{Body force} + \text{Surface force})$

Cauchy Equation → (①②) Navier-Stokes Equation (Shear force) → (③) Euler Equation → (④⑤) Bernoulli Equation (Curl & Rotation)

Microscopic/ Macroscopic Derivation(RTT[1])

Mass Conservation Law → (④⑤) Laplace Equation

$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$

$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$

✓6 D.O.F equations of motions

① Coordinate system (Reference frame)
(Water surface-fixed & Body-fixed frame)
② Newton's 2nd Law

$$\mathbf{M\ddot{r}} = \sum \mathbf{F} = (Body\ \ Force) + (Surface\ \ Force)$$

$$= \mathbf{F}_{gravity}(\mathbf{r}) \quad + \quad \mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}})$$

Nonlinear terms ➡ Nonlinear equation
➡ Difficulty of getting analytic solution

Numerical Method

Velocity potential Φ

$\Phi = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

✓Calculation of Fluid Force

Linearization

$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$$

$$\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

(S_B: wetted surface)

(displacement: $\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$)

$\xi_1 : surge \quad \xi_4 : roll$
$\xi_2 : sway \quad \xi_5 : pitch$
$\xi_3 : heave \quad \xi_6 : yaw$

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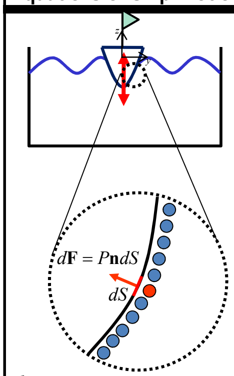
Hydrodynamic Forces
Calculated from 6 DOF (Degree of Freedom) Equations of Ship Motions (2/2)

$\mathbf{F}_{F.K}$: Froude-Krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

Φ_I : Incident wave velocity potential
Φ_D: Diffraction potential
Φ_R: Radiation potential



$d\mathbf{F}$: Force of fluid elements acting on the infinitesimal surface of a ship

dS: Infinitesimal surface area

$\mathbf{n}$: Normal vector of the infinitesimal surface area

$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$

$\xi_1 : surge \quad \xi_4 : roll$
$\xi_2 : sway \quad \xi_5 : pitch$
$\xi_3 : heave \quad \xi_6 : yaw$

$\mathbf{M}_A$: 6×6 added mass matrix
$\mathbf{B}$: 6×6 damping coeff. matrix
$\mathbf{C}$: 6×6 restoring coeff. matrix

✓ Surface forces: Fluid forces acting on a ship

$$\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) + \mathbf{F}_{F.K}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) + \mathbf{F}_D(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) + \mathbf{F}_R(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

: Fluid forces are obtained by integrating the fluid hydrostatic and hydrodynamic pressure over the wetted surface of a ship.

✓ 6 D.O.F equations of motion
Newton's 2nd Law

$$\mathbf{M\ddot{r}} = \sum \mathbf{F} = (\text{Body Force}) + (\text{Surface Force})$$

$$= \mathbf{F}_{gravity} \quad + \quad \mathbf{F}_{Fluid} \quad + \mathbf{F}_{external}$$

Body force Surface force

$$\mathbf{M\ddot{r}} = \mathbf{F}_{gravity}(\mathbf{r}) + \mathbf{F}_{Buoyancy}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) + \mathbf{F}_{Hydrodynamic}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) + \mathbf{F}_{external,\ dynamic} + \mathbf{F}_{external,\ static}$$

Assume that forces are constant or proportional to the displacement, velocity and acceleration of the ship.

$$\mathbf{M\ddot{r}} = \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}}) + \mathbf{F}_{ext,\ dynamic} + \mathbf{F}_{ext,\ static}$$

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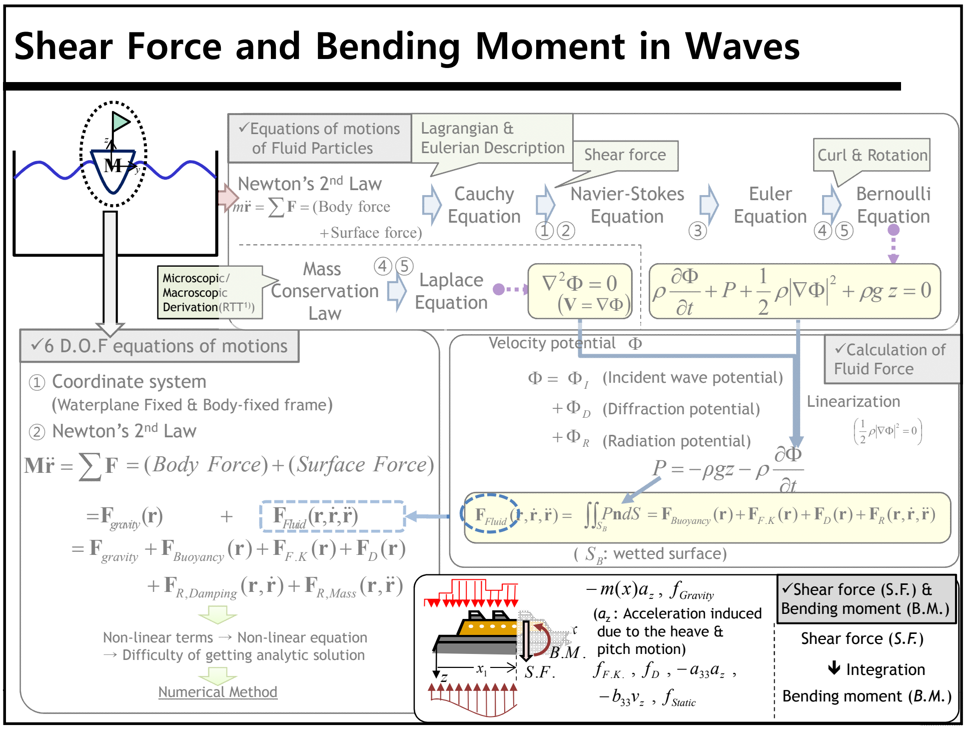
Shear Force and Bending Moment in Waves
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Shear force
Curl & Rotation
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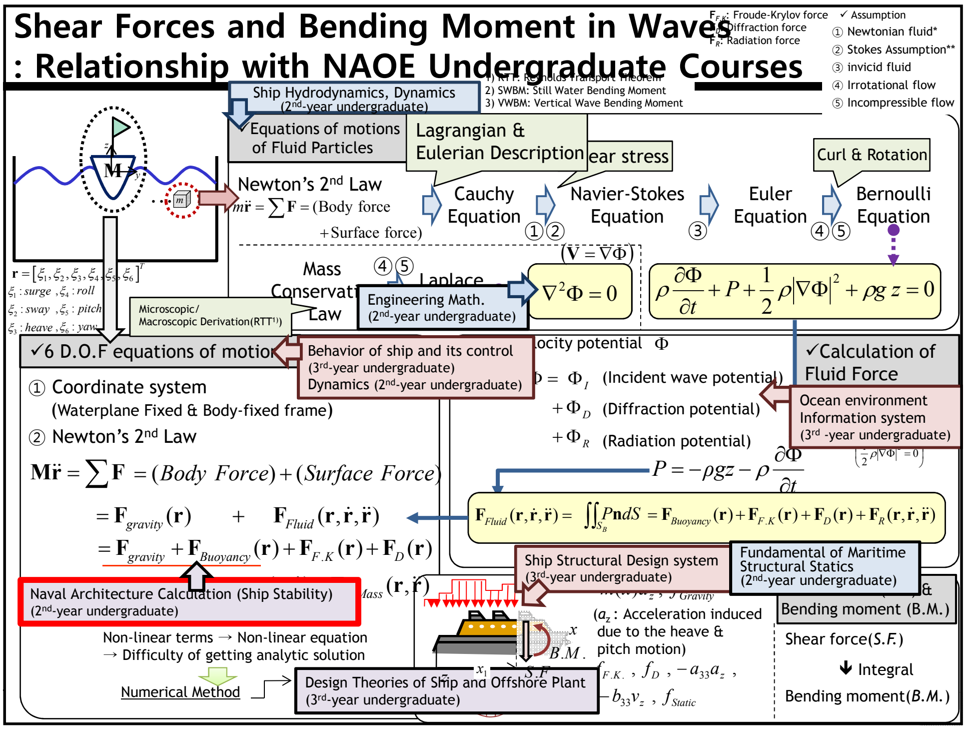
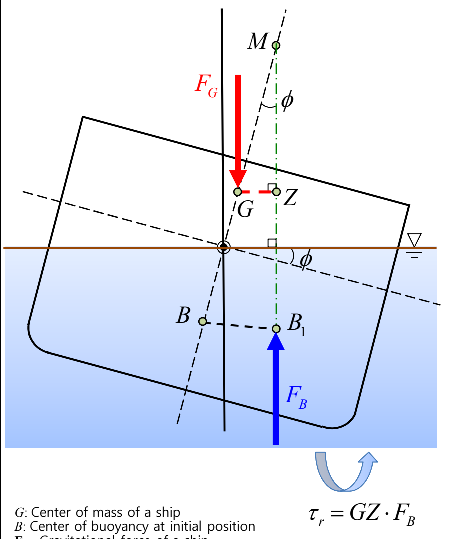
Mass Conservation Law
Laplace Equation
$\nabla^2\Phi = 0$ $(V = \nabla\Phi)$
$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$
✓6 D.O.F equations of motions
① Coordinate system
(Waterplane Fixed & Body-fixed frame)
② Newton's 2nd Law
$M\ddot{r} = \sum F = (Body\ Force) + (Surface\ Force)$
$= F_{gravity}(r) + F_{Fluid}(r, \dot{r}, \ddot{r})$
$= F_{gravity} + F_{Buoyancy}(r) + F_{F.K}(r) + F_D(r)$
$+ F_{R,Damping}(r, \dot{r}) + F_{R,Mass}(r, \ddot{r})$
Non-linear terms → Non-linear equation
→ Difficulty of getting analytic solution
Numerical Method
Velocity potential Φ
$\Phi = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)
✓Calculation of Fluid Force
Linearization
$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$
$F_{Fluid}(r, \dot{r}, \ddot{r}) = \iint_{S_B} P n dS = F_{Buoyancy}(r) + F_{F.K}(r) + F_D(r) + F_R(r, \dot{r}, \ddot{r})$
(S_B: wetted surface)
$-m(x)a_z$, $f_{Gravity}$
(a_z: Acceleration induced due to the heave & pitch motion)
$f_{F.K.}$, f_D, $-a_{33}a_z$,
$-b_{33}v_z$, f_{Static}
B.M.
S.F.
✓Shear force (S.F.) & Bending moment (B.M.)
Shear force (S.F.)
Integration
Bending moment (B.M.)


Shear Forces and Bending Moment in Waves
: Relationship with NAOE Undergraduate Courses
$F_{F.K}$: Froude-Krylov force
F_D: Diffraction force
F_R: Radiation force
1) RTT: Reynolds Transport Theorem
2) SWBM: Still Water Bending Moment
3) VWBM: Vertical Wave Bending Moment
✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
④ Irrotational flow
⑤ Incompressible flow
Ship Hydrodynamics, Dynamics (2nd-year undergraduate)
✓Equations of motions of Fluid Particles
Lagrangian & Eulerian Description
Curl & Rotation
Newton's 2nd Law
Cauchy Equation
Navier-Stokes Equation
Euler Equation
Bernoulli Equation
Engineering Math. (2nd-year undergraduate)
Behavior of ship and its control (3rd-year undergraduate)
Dynamics (2nd-year undergraduate)
✓Calculation of Fluid Force
Ocean environment Information system (3rd -year undergraduate)
① Coordinate system
(Waterplane Fixed & Body-fixed frame)
② Newton's 2nd Law
$M\ddot{r} = \sum F = (Body\ Force) + (Surface\ Force)$
Naval Architecture Calculation (Ship Stability) (2nd-year undergraduate)
Ship Structural Design system (3rd-year undergraduate)
Fundamental of Maritime Structural Statics (2nd-year undergraduate)
Design Theories of Ship and Offshore Plant (3rd-year undergraduate)
Non-linear terms → Non-linear equation
→ Difficulty of getting analytic solution
Numerical Method
Bending moment (B.M.)
Shear force (S.F.)
Integral
Bending moment (B.M.)

Roll Period

Required Minimum Roll Period
- For example, min. 12 sec

I_a : added moment of inertia
b : damping moment coefficient



G: Center of mass of a ship
B: Center of buoyancy at initial position
$\mathbf{F}_G$: Gravitational force of a ship
$\mathbf{F}_B$: Buoyant force acting on a ship
M : Metacenter

Derivation of the equation of roll motion of a ship:

$I\ddot{\phi} = \sum \tau$ **(Euler equation)**

$$= \tau_{body} + \tau_{surface}$$

$$= \tau_{gravity} + \tau_{fluid}$$

$$= \tau_{gravity} + \tau_{hydrostatic} + \tau_{F.K} + \tau_{diffraction} + \tau_{radiation}$$

$$= \tau_r \qquad + \qquad \tau_{exciting} \qquad -I_a\ddot{\phi} - b\dot{\phi}$$

(add mass)

$$\tau_r = GZ \cdot F_B$$
$$\approx GM \cdot \sin\phi \cdot F_B$$
$$= GM \cdot \sin\phi \cdot \rho g V$$
$$\approx GM \cdot \phi \cdot \rho g V$$

For small ϕ
$\sin\phi \cong \phi$

$$= -\rho g V \cdot GM \cdot \phi + \tau_{exciting} - I_a\ddot{\phi} - b\dot{\phi}$$

Equation of roll motion of a ship

$$\therefore (I + I_a) \cdot \ddot{\phi} + b\dot{\phi} + (\rho g V \cdot GM) \cdot \phi = \tau_{exciting}$$

Calculation of Natural Roll Period (1/2)

$$(I+I_a)\cdot\ddot{\phi}+b\dot{\phi}+(\rho gV\cdot GM)\cdot\phi=\tau_{exciting}$$ ← Second Order Linear Ordinary Differential Equation

- Objectives: Find the natural frequency of roll motion
  : No exciting moment $(\tau_{exciting}=0)$
- Assumption: No damping moment $(b\dot{\phi}=0)$

$$(I+I_a)\cdot\ddot{\phi}+(\rho gV\cdot GM)\cdot\phi=0$$

Try: $\phi=e^{\lambda t}$

$$(I+I_a)\cdot\lambda^2\cdot e^{\lambda t}+(\rho gV\cdot GM)\cdot e^{\lambda t}=0$$

$$\{(I+I_a)\cdot\lambda^2+(\rho gV\cdot GM)\}\cdot e^{\lambda t}=0$$

$$(I+I_a)\cdot\lambda^2+(\rho gV\cdot GM)=0,\ (e^{\lambda t}\neq 0)$$

$$\lambda_{1,2}=\pm\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i$$

$$\therefore\phi=C_1e^{\lambda_1\cdot t}+C_2e^{\lambda_2\cdot t}=C_1e^{\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i\cdot t}+C_2e^{-\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i\cdot t}$$

Calculation of Natural Roll Period (2/2)

$$\phi=C_1e^{\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i\cdot t}+C_2e^{-\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i\cdot t}$$

Euler's formula $(e^{i\phi}=\cos\phi+i\sin\phi)$

$$\boxed{\phi=C_1\cos\left(\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot t\right)+C_2\sin\left(\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot t\right)}$$

Angular frequency (ω)

Because $\omega=\dfrac{2\pi}{T_\phi}$, the natural roll period is as follows:

$$T_\phi=\frac{2\pi}{\omega}$$

$$=2\pi\sqrt{\frac{I+I_a}{\rho gV\cdot GM}}$$

Effect of GM on the Natural Roll Period

Roll period

$$T_\phi = 2\pi\sqrt{\frac{I+I_a}{\rho g V \cdot GM}}$$

(Assumption) $I + I_a \cong (k \cdot B)^2 \cdot \rho \cdot V$

- ✓ 2k: 0.32~0.39 for full load condition
- ✓ 2k: 0.37~0.40 for ballast condition

$$T_\phi = 2\pi\sqrt{\frac{(k\cdot B)^2 \cdot \rho \cdot V}{\rho \cdot g \cdot V \cdot GM}}$$

$$\approx \frac{2\pi \cdot k \cdot B}{\sqrt{g \cdot GM}}$$

$$\boxed{= \frac{2 \cdot k \cdot B}{\sqrt{GM}}} \quad ,(\sqrt{g} \approx \pi)$$

Does a ship in a light condition roll quickly or slowly? What does this indicate?

Approximate Roll period of ship

That is, a stiff ship or crank ship (heavy), one with a large metacentric height will roll quickly whereas a tender ship (light), one with a small metacentric height, will roll slowly.

Effect of GM on the Tangential Inertial Force due to the Roll Motion

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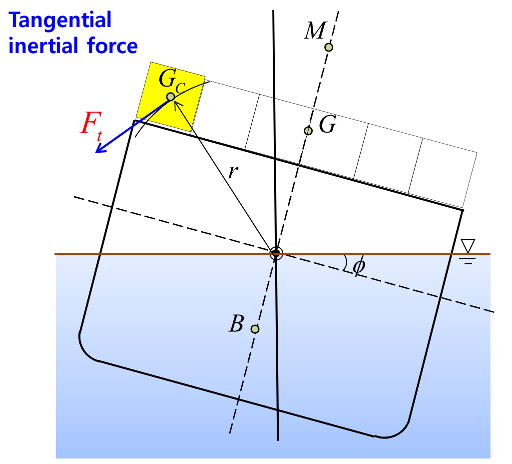

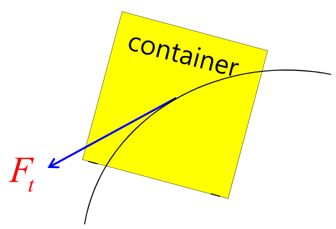

Effect of GM on the Angular Acceleration of a Ship

$\omega = \sqrt{\dfrac{\rho g V \cdot GM}{I + I_A}}$

Roll motion of a ship:

$$\phi = C_1 \cos(\omega \cdot t) + C_2 \sin(\omega \cdot t)$$

$$\Downarrow$$

$$\phi = \sqrt{C_1^{\,2} + C_2^{\,2}} \cos(\omega \cdot t + \beta), \beta : \text{phase}$$

Angular acceleration of a ship:

$$\begin{aligned} \ddot{\phi} &= \sqrt{C_1^{\,2} + C_2^{\,2}}\, \omega^2 \cos(\omega \cdot t + \beta) \\ &= A\omega^2 \cos(\omega \cdot t + \beta) \quad ,(A = \sqrt{C_1^{\,2} + C_2^{\,2}}) \end{aligned}$$