Ship Stability

Ch. 2 Review of Fluid Mechanics

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1. Hydromechanics and Hydrostatics

Introduction to Hydromechanics

- Today, the branch of physics, which encompasses the theories and laws of the behavior of water and other liquids, is known as hydromechanics.

- Hydromechanics itself is subdivided into three fields:

(1) Hydrostatics, which deals with liquids at rest.
(2) Hydrodynamics, which studies liquids in motion.
(3) Hydraulics, dealing with the practical and engineering applications of hydrostatics and hydrodynamics.

Meaning of Hydrostatics

- What is Hydrostatics?

Hydrostatics (from Greek *hydro*, meaning water, and *statics* meaning rest, or calm) describes the behavior of water in a state of rest.

This science also studies the forces that apply to immersed and floating bodies, and the forces exerted by a fluid.

Definition of Pressure

- **Pressure***

Let a small pressure-sensing device be suspended inside a fluid-filled vessel.

We define the pressure on the piston from the fluid as the force divided by area, and it has units Newton per square meter called 'Pascal'.

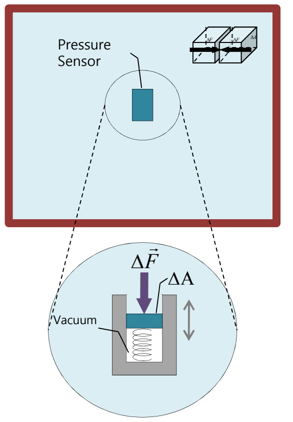
$$P = \frac{\Delta F}{\Delta A} \qquad (1\ \text{Pa} = 1N/m^2)$$

One newton per square meter is one Pascal.

We can find by experiment that at a given point in a fluid at rest, the pressure have the same value no matter how the pressure sensor is oriented.

Pressure is a scalar, having no directional properties, and force is a vector quantity.

But ΔF is only the magnitude of the force.



ΔF : Magnitude of normal force on area ΔA
ΔA : Surface area of the piston

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.361, 2004

Two Principles of Hydrostatics

Hydrostatics mainly consists of two principles.

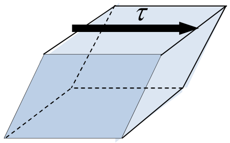
1. **Pascal's principle (also Pascal's law) says that the pressure applied to an enclosed fluid is transmitted undiminished.**

2. **Archimedes' principle states that the buoyant force on an immersed body has the same magnitude as the weight of the fluid which is displaced by the body.**

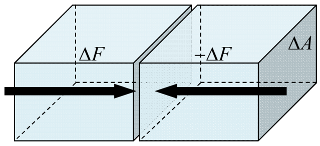
Definition of Fluid

- **What is a Fluid?***

A fluid, in contrast to a solid, is a substance that can flow, because it cannot withstand a shearing stress.



It can, however, exert a force in the direction perpendicular to its surface.



ΔF : Magnitude of perpendicular force between the two cubes
ΔA : Area of one face of one of the cubes

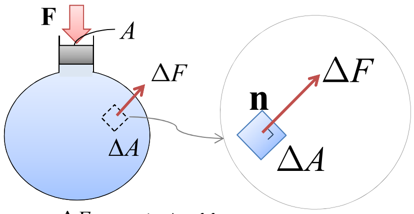
* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.360, 2004

2. Pascal's Principle

Pascal's Principle

- We will now consider a fluid element in static equilibrium in a closed container filled with a fluid which is either a gas or a liquid. The velocity of flow is everywhere zero.

- At first, we will neglect gravity. If a force F is applied on the cap of the container with an area A in this direction, then a pressure of F/A is applied.



ΔF : Magnitude of force

$$\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = P$$

Pascal's Principle

Pascal's Principle

In the absence of gravity, the pressure is the same everywhere in this container.
That is what's called Pascal's principle.

A change in the pressure applied an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of its container*.

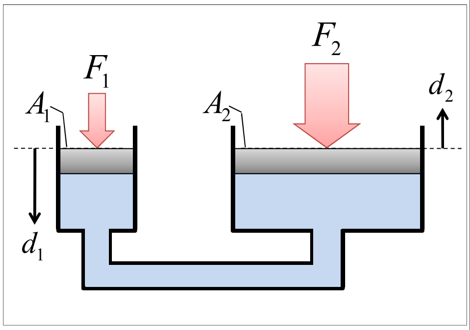
* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.366, 2004

Application of the Pascal's Principle

Pascal's Principle:
A change in the pressure applied an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of its container*.

• The idea of a Hydraulic jack

Pascal's Principle: $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



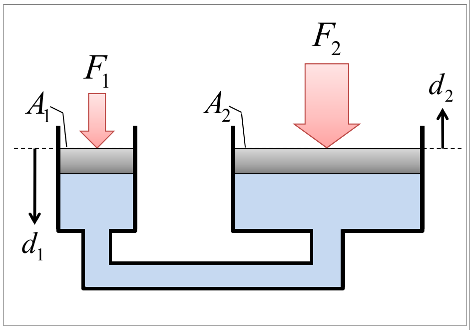
Consider a vessel with two pistons having area A_1 and area A_2. The vessel is filled with liquid everywhere.

Now a force F_1 on A_1 and a force F_2 on A_2 are applied. So the pressure on the left piston is F_1/A_1.

According to the Pascal's principle, everywhere in the fluid, the pressure must be the same. The pressure on the right piston, F_2/A_2 must be the same as the pressure F_1/A_1, if the liquid is not moving. The effect of gravity does not change the situation very significantly.

Example of Design of Hydraulic Jack (1/5)

Pascal's Principle: $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Example) If $\frac{A_2}{A_1} = 100$, then $\frac{F_2}{F_1} = 100$.

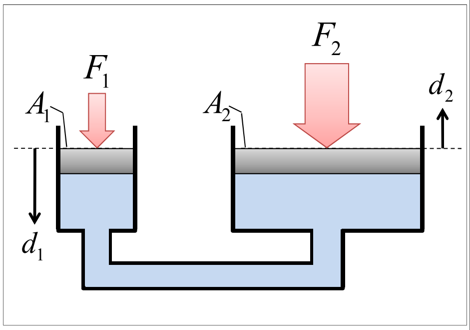
(Pascal's Principle)

Example of Design of Hydraulic Jack (2/5)

Pascal's Principle: $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

Displaced Volume: $A_1 d_1 = A_2 d_2$

(Incompressible fluid)

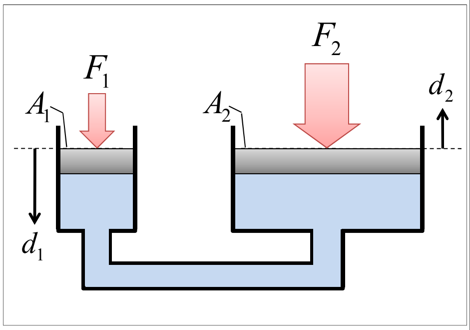


Example of Design of Hydraulic Jack (3/5)

Pascal's Principle: $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

Displaced Volume: $A_1 d_1 = A_2 d_2$

(Incompressible fluid)



Conservation of Energy:

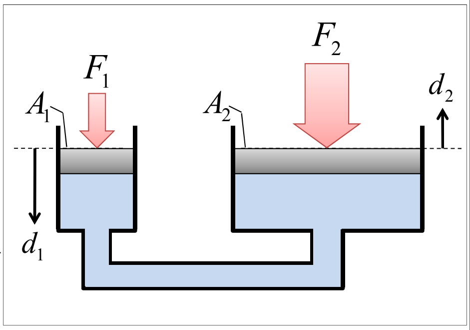
$$F_1 d_1 = \underbrace{\left(\frac{A_1}{A_2} F_2\right)}_{F_1 \text{ (Pascal's Principle)}} \underbrace{\left(\frac{A_2}{A_1} d_2\right)}_{d_1 \text{ (Displaced Volume)}} = F_2 d_2$$

Example of Design of Hydraulic Jack (4/5)

Pascal's Principle: $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

Displaced Volume: $A_1 d_1 = A_2 d_2$
(Incompressible fluid)

Conservation of Energy: $F_1 d_1 = F_2 d_2$



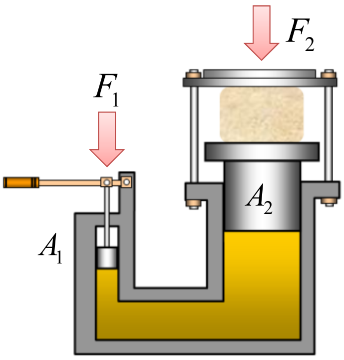
Example) If $\dfrac{A_2}{A_1} = 100$ → $100F_1 = F_2$
(Pascal's Principle)

→ $d_1 = 100 d_2$
(Displaced Volume)

$$\overset{\times 100}{F_1} d_1 = F_2 \underset{\times 100}{d_2}$$

∴ **Conservation of Energy is satisfied.**

Example of Design of Hydraulic Jack (5/5)



Change of a Flat Tire by using hydraulic jack

[Example]
A force of 30 N is applied to the smaller cylinder of a hydraulic jack.
If the area of this cylinder is 10 cm^2 and the area of the large cylinder is 100 cm^2 what is the force exerted by the large cylinder?

[Solution]
Force (F_2) = $F_1 \times A_2 / A_1$ = 30×100/10 = 300 N

3. Hydrostatic Pressure

Hydrostatic Pressure (1/9)

- Hydrostatic Pressure

As every diver knows, the pressure increases with depth below the water.

As every mountaineer knows, the pressure decreases with altitude as one ascends into the atmosphere.

The pressure encountered by the diver and the mountaineer are usually called hydrostatic pressures, because they are due to fluids that are static (at rest).

Here we want to find an expression for hydrostatic pressure as a function of depth or altitude.

Hydrostatic Pressure (2/9)

Now, **gravity**, of course, **has an effect on the pressure in the fluid.**

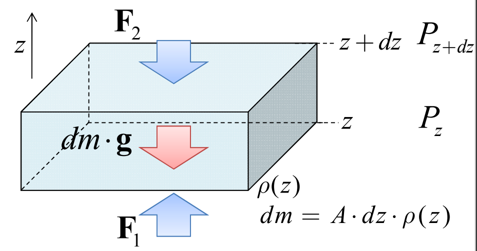
Hydrostatic pressure is due to fluids that are **static** (at rest).

Thus, there has to be **static equilibrium**.

Consider a fluid element in the fluid itself and assume the upward vertical direction as the positive z-coordinate.

The mass of the fluid element is the volume times the density, and the volume is face area times delta z, and then times the density, which may be a function of z.

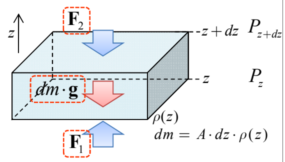
Fluid Element



$\rho(z)$ = Density of the fluid element
dm = Mass of the fluid element
A = Horizontal base (or face) area
F_1 = Force that acts at the bottom surface (due to the water below the rectangular solid)
F_2 = Force that acts at the top surface (due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Hydrostatic Pressure (3/9)

Fluid Element



$\rho(z)$ = Density of the fluid element
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F_1 = Force that acts at the bottom surface (due to the water below the rectangular solid)
F_2 = Force that acts at the top surface (due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Newton's 2nd Law : $\sum \mathbf{F} = m\ddot{\mathbf{z}}$

(Static Equilibrium : $\ddot{\mathbf{z}} = 0$**)**

$$\sum \mathbf{F} = 0$$

$$\mathbf{F}_1 - \mathbf{F}_2 - dm \cdot \mathbf{g} = 0$$

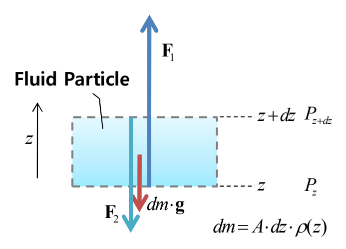
To describe the behavior of the fluid element, we apply the Newton's 2nd law to the free body diagram for the fluid element, as shown in the figure.

The gravitational force acting on the fluid element is delta m times g in the downward direction. **The pressure force, which is always perpendicular to the surfaces**, acting on the bottom surface is F_1 in the upward direction, whereas the pressure force acting on the top surface is F_2 in the downward direction.

We only **consider forces in the vertical direction**, because **all forces in the horizontal direction will cancel**, for obvious reasons. **The fluid element is not going anywhere. It is just sitting still in the fluid.** Thus, the fluid element is in static equilibrium.

For the fluid element to be in static equilibrium, the upward force F_1 minus downward force F_2 minus delta mg **must be zero.**

Free-body diagram for the fluid element

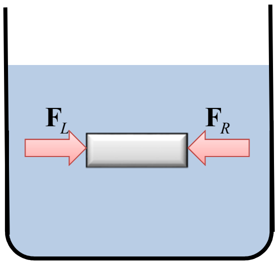


Hydrostatic Pressure (4/9)

Reference) Static Equilibrium

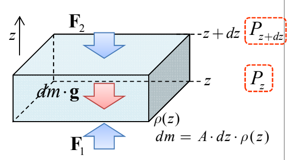
If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium (at rest with respect to the observer).
Furthermore, all points at the same depth must be at the same pressure.
If this was not the case, a given portion of the fluid would not be in equilibrium.

For example, consider the small block of fluid. If the pressure were greater on the left side of the block than on the right, $\mathbf{F}_L$ would be greater than $\mathbf{F}_R$, and the block would accelerate and thus would not be in equilibrium.



Hydrostatic Pressure (5/9)

Fluid Element



$\rho(z)$ = Density of the fluid element
dm = Mass of the fluid element
A = horizontal base(or face) area
F_1 = Force that acts at the bottom surface(due to the water below the rectangular solid)
F_2 = Force that acts at the top surface(due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Newton's 2nd Law : $\sum \mathbf{F} = m\ddot{\mathbf{z}}$

$$\mathbf{F}_1 - \mathbf{F}_2 - dm\cdot\mathbf{g} = 0$$

Three forces act on vertically.
Thus we can consider magnitude of vectors only.

$$\Rightarrow P_z\cancel{A} - P_{z+dz}\cancel{A} - \cancel{A}\cdot dz\cdot\rho(z)\cdot g = 0$$

$$P_z - P_{z+dz} - dz\cdot\rho(z)\cdot g = 0$$

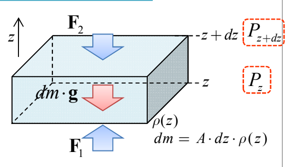
$$P_z - P_{z+dz} = dz\cdot\rho(z)\cdot g$$

$\times(-1)$

$$P_{z+dz} - P_z = -dz\cdot\rho(z)\cdot g$$

Hydrostatic Pressure (6/9)

Fluid Element



$\rho(z)$ = Density of the fluid element
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A = horizontal base(or face) area
F_1 = Force that acts at the bottom surface(due to the water below the rectangular solid)
F_2 = Force that acts at the top surface(due to the water above the rectangular solid)
P_{z+dz} = Pressure at $z+dz$
P_z = Pressure at z

Newton's 2nd Law : $\sum \mathbf{F} = m\ddot{z}$

$$\mathbf{F}_1 - \mathbf{F}_2 - dm\cdot\mathbf{g} = 0$$

$$P_{z+dz} - P_z = -dz\cdot\rho(z)\cdot g$$

$\div(dz)$

$$\frac{P_{z+dz} - P_z}{dz} = -\rho(z)\cdot g$$

$$\lim_{dz\to 0}\frac{P_{z+dz} - P_z}{dz} = -\rho(z)\cdot g = \frac{dP}{dz}$$

$$\therefore \frac{dP}{dz} = -\rho(z)\cdot g$$

: Change of Hydrostatic Pressure
(Due to gravity)

Hydrostatic Pressure (7/9)

$\frac{dP}{dz} = -\rho(z)\cdot g$: Change in Hydrostatic Pressure (**Due to gravity**)

Calculate the pressure difference between z_1 and z_2.

$$dP = -\rho(z)\cdot g\cdot dz$$

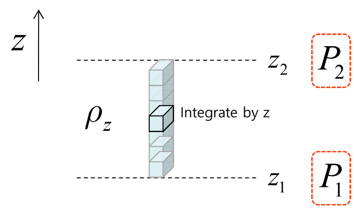
Integrate from z_1 to z_2.

$$\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2}\rho(z)\cdot g\cdot dz$$

Most liquids are practically incompressible. In other words, the density of the liquid cannot really change. And so therefore, we could always use the constant density, ρ, instead of the varying density ρ(z). We will assume from now on that fluids are completely incompressible. We can, then, do a very simple integration.

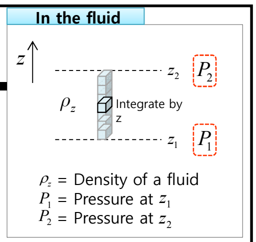
We have now dP in the L.H.S, which we can integrate from some value P_1 to P_2. And that equals now minus rho g dz in the R.H.S, integrated from z_1 to z_2.

In the fluid



ρ_z = Density of a fluid
P_1 = Pressure at z_1
P_2 = Pressure at z_2

Hydrostatic Pressure (8/9)



Calculate the pressure difference between z_1 and z_2.

$$\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_2} \rho(z)\cdot g\cdot dz$$

L.H.S: $\int_{P_1}^{P_2} dP = P_2 - P_1$

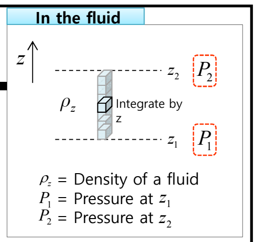
R.H.S: $-\int_{z_1}^{z_2} \rho(z)\cdot g\cdot dz = -\rho g\int_{z_1}^{z_2} dz = -\rho g(z_2 - z_1)$

Assume : Incompressible Fluid (ρ= constant)

L.H.S=R.H.S

$\therefore \boxed{P_2 - P_1 = -\rho g(z_2 - z_1)}$ **: Pascal's Principle (also Pascal's Law)**

Hydrostatic Pressure (9/9)



$$P_2 - P_1 = -\rho g(z_2 - z_1)$$

$$P_1 - P_2 = \rho g(z_2 - z_1)$$

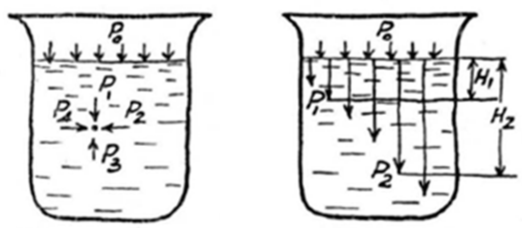
We multiply a minus sign here, so we switch these around: ρ g times z_2 minus z_1.
What it means is we see immediately that if z_2 minus z_1 is positive, i.e. z_2 is higher than z_1, the pressure at P_1 is larger than the pressure at P_2.
This is the hydrostatic pressure.

Hydrostatic Pressure (Incompressible fluid due to gravity)

The pressure at a point in a fluid in static equilibrium depends on the depth of that point, but not on any horizontal dimension of the fluid or its container.*

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.363, 2004

Three Basic Characteristics of Pressure in a Body of Fluid



$P_1 = P_2 = P_3 = P_4$	$P_2 > P_1$	$P \to \angle 90^\circ$
Hydrostatic pressure at any point in a body of water is equal in all directions.	Pressure in a body of water increases with depth of water.	Hydrostatic pressure is always applied perpendicular to any submerged body.

<Graphic presentation of the concept of hydrostatic pressure>

* Polevoy, S. L., Water Science and Engineering, Blackie Academic and Professional, pp.78, 1996

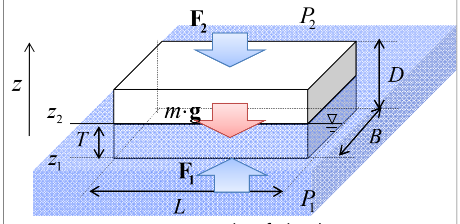
4. Archimedes' Principle and Buoyant Force

Archimedes' Principle and Buoyant Force (1/4)

- **Static equilibrium of a rigid body in a fluid**

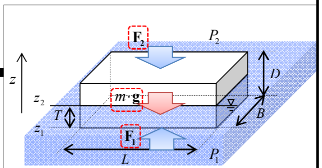
Consider a simple box shaped barge that floats in a fluid. That means the barge is in static equilibrium.
Thus, the gravitational force on the barge in the downward direction must be equal to a net upward force on it from the surrounding fluid, so called 'buoyant force'.

The length of the barge is L, the breadth is B, the depth is D, the immersed depth is T, its density is ρ_{obj}, and the density of the fluid is ρ_{sw}.
Let be the upward vertical direction as the positive z-coordinate. We define, then, the level of the bottom surface as z_1 and the level of the immersed depth as z_2.
On the top surface of the barge, there is the atmospheric pressure P_2, which is the same as it is on the fluid. And on the bottom surface we have a pressure P_1 in the fluid.



L = Length of the barge
B = Breadth of the barge
D = Depth of the barge
T = Draft of the barge(=z_2-z_1)
ρ_{sw} = Density of the fluid
ρ_{obj} = Density of the barge

Archimedes' Principle and Buoyant Force (2/4)



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge(=z_2-z_1)
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

- **Static equilibrium of a barge in a fluid**

Newton's 2nd Law: $\sum \mathbf{F} = m \cdot \ddot{\mathbf{z}}$

(Static Equilibrium: $\ddot{\mathbf{z}} = 0$ **)**

$$\rightarrow \sum \mathbf{F} = 0$$

Assumption: Buoyant force of air is neglected.

$P_1 - P_2 = \rho_{sw} g T$ (Pascal's Law)

$\boxed{\mathbf{F}_1 - \mathbf{F}_2} - m\mathbf{g} = 0$

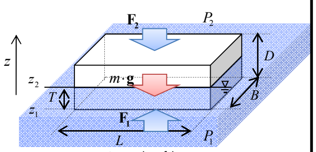
$\mathbf{F}_1 - \mathbf{F}_2$: **Buoyant Force** $\mathbf{F}_B$

$\mathbf{F}_1$: Force which contains the hydrostatic pressure
$\mathbf{F}_2$: Force which contains the atmospheric pressure

To describe the behavior of the barge in the fluid, we apply the Newton's 2nd law to the barge as shown in the figure.
The gravitational force acting on the barge is mass, m, times g in the downward direction. The hydrostatic pressure force, which is always perpendicular to the surfaces, acting on the bottom surface is F_1 in the upward direction, whereas the atmospheric pressure force acting on the top surface is F_2 in the downward direction.
We only consider forces in the vertical direction, because all forces in the horizontal direction will cancel. If there were any net tangential component force, then the barge would start to move. The barge, however, is static, that means the barge is not moving anywhere. It is just sitting still in the fluid. Thus, the barge is in static equilibrium. For the barge to be in static equilibrium, the upward force F_1 minus downward force F_2 minus delta mg must be zero. Here the net upward hydrostatic pressure force, F_1-F_2, is so called the 'Buoyant force'.

Archimedes' Principle and Buoyant Force (3/4)

- **Buoyant force:** $F_B = F_1 - F_2$



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge($=z_2-z_1$)
ρ_{sw} = Density of sea water
ρ_{B} = Density of a barge

$\rightarrow F_B = (L \cdot B) \cdot P_1 - (L \cdot B) \cdot P_2$
$\quad = (L \cdot B) \cdot (P_1 - P_2)$

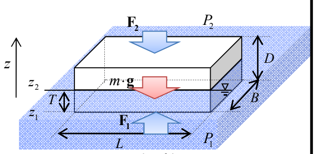
Assumption: Buoyant force of air is neglected.

Substitution: $P_1 - P_2 = \rho_{sw} g T$ (Pascal's Law)

$\rightarrow F_B = (L \cdot B) \cdot \rho_{sw} g T$

$F_B = (L \cdot B \cdot T) \cdot \rho_{sw} \cdot g$

Archimedes' Principle and Buoyant Force (4/4)



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge($=z_2-z_1$)
ρ_{sw} = Density of sea water
ρ_{B} = Density of a barge

$$\boxed{\therefore F_B = \underbrace{\underbrace{(L \cdot B \cdot T)}_{\text{Volume}} \cdot \rho_{sw}}_{\text{Mass}} g}$$

Buoyant force is the weight of the displaced fluid.

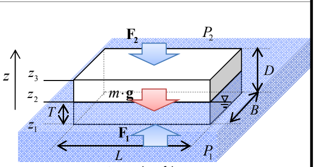
This is a very special case of a general principle which is called Archimedes' Principle.

Archimedes' Principle*

When a body is fully or partially submerged in a fluid, a buoyant force F_B from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight of the fluid which is displaced by the body.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.368, 2004

[Reference] Buoyant Force of Air



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge(=z_2-z_1)
ρ_{sw} = Density of sea water
ρ_{B} = Density of a barge

- **Static equilibrium of a barge**

$$F_B = (L \cdot B) \cdot (P_1 - P_2)$$

Apply Pascal's Law: $\int_{P_1}^{P_2} dP = -\int_{z_1}^{z_3} \rho(z) \cdot g \cdot dz$

(Due to gravity)

(z_1~z_2: fluid, z_2~z_3: air)

L.H.S: $\int_{P_1}^{P_2} dP = P_2 - P_1$

R.H.S: $-\int_{z_1}^{z_3} \rho(z) \cdot g \cdot dz = -\int_{z_1}^{z_2} \rho_{sw} g dz - \int_{z_2}^{z_3} \rho_{air} g dz$

(Air, sea water: incompressible)

$$= -\rho_{sw} g \int_{z_1}^{z_2} dz - \rho_{air} g \int_{z_2}^{z_3} dz$$

$$= -\rho_{sw} g (z_2 - z_1) - \rho_{air} g (z_3 - z_2)$$

L.H.S=R.H.S

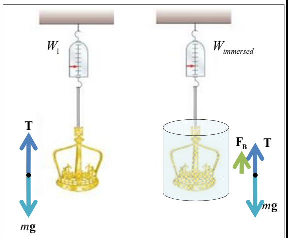
$$\rightarrow P_1 - P_2 = \rho_{sw} g T + \rho_{air} g (D - T) \quad \Rightarrow \quad P_1 - P_2 = \rho_{sw} g T$$

$\rho_{air} \simeq 1.2 kg/m^3, \rho_{sw} \simeq 1025 kg/m^3$

Ratio of ρ_{sw} to ρ_{air} is $\frac{1025}{1.2} \simeq 854$, ($\rho_{air} \ll \rho_{sw}$)

So buoyant force of air is negligible.

Archimedes' Principle and Buoyant Force - Example: Archimedes and Crown Problem (1/2)



W_1 =Weight of the crown
V =Volume of the crown
ρ_{crown} =Density of the crown
$W_{immersed}$ =Weight immersed in water
ρ_w =Density of the water

- **Apparent weight of a body in a fluid**

If we place a crown on a scale that is calibrated to measure weight then the reading on the scale is the crown's weight. However, if we do this underwater, the upward buoyant force on the crown from the water decreases the reading.

That reading is then an apparent weight. In general, an apparent weight is the actual weight of a body minus the buoyant force on the body.

$$\begin{pmatrix}\text{apparent}\\ \text{weight}\end{pmatrix} = \begin{pmatrix}\text{actual}\\ \text{weight}\end{pmatrix} - \begin{pmatrix}\text{magnitude of}\\ \text{buoyant force}\end{pmatrix}$$

Weight Loss

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.369, 2004

Archimedes' Principle and Buoyant Force
- Example: Archimedes and Crown Problem (2/2)

Question)

Is the crown made of pure gold?

Answer)

$$W_1 = V\rho_{crown}g$$

$$\rightarrow W_{immersed} = V\rho_{crown}g - V\rho_w g$$

(Apparent Weight)

W_{Loss}: Weight Loss(Buoyant Force)

$$\frac{W_1}{W_{Loss}} = \frac{V\rho_{crown}g}{V\rho_w g} = \frac{\rho_{crown}}{\rho_w}$$

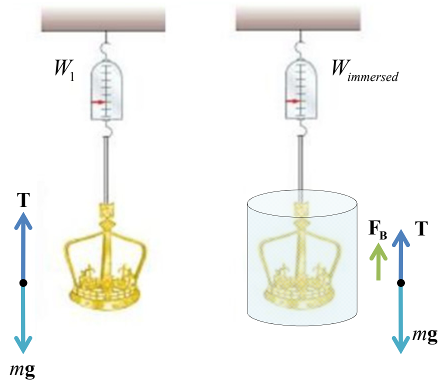
W_1 (Measure), W_{Loss} (Measure), ρ_{crown} (Find & Compare), ρ_w (Known)

Archimedes lived in the third century B.C. Archimedes had been given the task to determine whether a crown was pure gold. He had the great vision to do the following: He takes the crown and he weighs it in a normal way. So the weight of the crown - we call it W_1 - is the volume of the crown times the density of which it is made. If it is gold, it should be 19.3 gram per centimeter cube (19.3 ton/m^3), and so volume of the crown x rho crown is the mass of the crown and multiplying mass by g is the weight of the crown. Cf. Silver: 10.49 ton/m^3

Now he takes the crown and he immerses it in the water. And he has a spring balance, and he weighs it again. And he finds that the weight is less and so now he has the weight immersed in the water.

So what he gets is the weight of the crown minus the buoyant force, which is the weight of the displaced water. And the weight of the displaced water is the volume of the crown times the density of water times g. And so V x rho w x g is 'weight loss'.

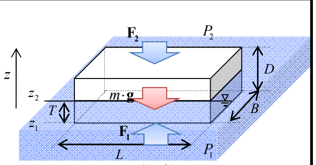
And he takes W_1 and divides by the weight loss and it gives him rho of the crown divided by rho of the water. And he knows rho of the water, so he can find rho of the crown. It's an amazing idea; he was a genius.



W_1 =Weight of the crown
V =Volume of the crown
ρ_{crown} =Density of the crown
$W_{immersed}$ =Weight immersed in water
ρ_w =Density of the water

* Serway, R. A., College Physics, 8th Ed., Brooks/Cole, pp.287, 2009.

Archimedes' Principle and Buoyant Force
- Condition for Floating



L = Length of barge
B = Breadth of barge
D = Depth of barge
T = Draft of barge
ρ_{sw} = Density of sea water
ρ_{obj} = Density of a barge

- **Condition for floating**

$$F_B = mg \quad (T < D)$$

- **For this barge to float, the buoyant force must be equal to gravitational force.**

$$(\cancel{L} \cdot \cancel{B} \cdot T) \cdot \rho_{sw} \cancel{g} = (\cancel{L} \cdot \cancel{B} \cdot D) \cdot \rho_{obj} \cancel{g}$$

$\rightarrow$ $\rho_{sw} > \rho_{obj}$: Float

Necessary condition for floating

$\rho_{sw} < \rho_{obj}$: Sink

Archimedes' Principle and Buoyant Force
- Example: Floating Iceberg

Question)

What percentage of the volume of ice will be under the level of the water?

$$\rho_{ice} = 0.92\text{g/cm}^3, \rho_w = 1.0\text{g/cm}^3$$

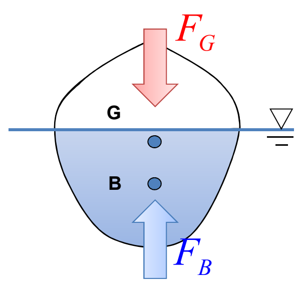
Answer)

$$Mg = V_{tot}\rho_{ice}g = V_{uw}\rho_w g$$

$$\longrightarrow \frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_w}$$

$$\frac{\text{Underwater Volume}}{\text{Total Volume}} = \frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_w} = 0.92$$

$\therefore 92\%$ **of the iceberg is in underwater.**



M : Mass of iceberg
V_{tot} : Total volume of iceberg
V_{uw} : Volume underwater
ρ_{ice} : Density of the ice,
ρ_w : Density of underwater
g : Acceleration of gravity

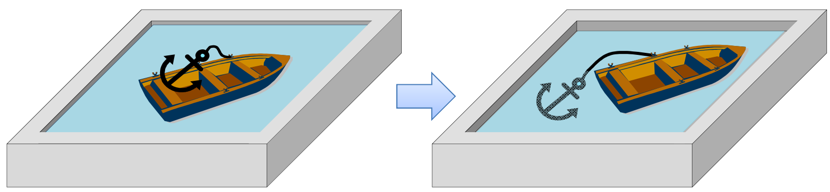
* Ohanian, H. C., Physics, 2nd Ed., W. W. Norton & Company, pp.478, 1989

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (1/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is

(a) Dropped into the water or
(b) Thrown onto the surrounding ground?
(c) Does the water level in the pool move upward, move downward, or remain the same if, instead, a cork (or buoy) is dropped from the boat into the water, where it floats?

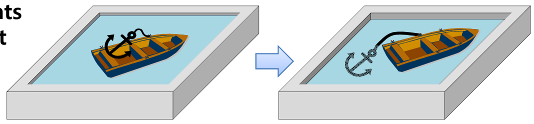


* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (2/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



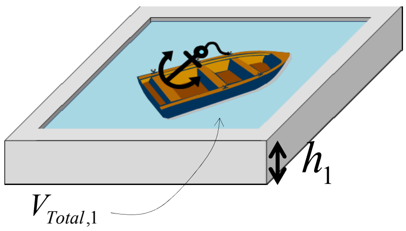
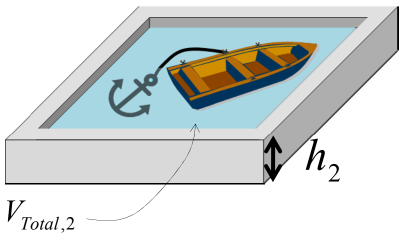
(a) Dropped into the water

Answer)

The volume under the water level is composed of the water and the volume displaced by the boat and anchor. After the anchor is dropped into the water, the buoyant force exerted on the anchor cannot compensate the weight of the anchor.
Thus the water level moves down.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (3/6)

Stage1	**Stage2**
	

If the shape of water tanks are same, the waterline will be proportional to total volume (volume of water + volume displaced by the boat and the anchor).

$$h_1 = \frac{V_{Total,1}}{A} \qquad h_2 = \frac{V_{Total,2}}{A}$$

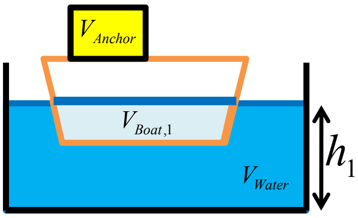
h : Waterline
A : Bottom area
V_{Total} : Total volume

Example: Waterline will change? (4/6)
(a) Dropped into the water (1/2)

$h = \frac{V_{Total}}{A}$

$V = \frac{M}{\rho} = \frac{Mg}{\rho g} = \frac{W}{\rho g}$

Stage1

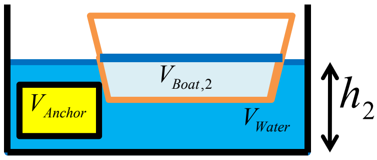


$$V_{Total,1} = V_{Boat,1} + V_{Water}$$

$$V_{Boat,1} = \frac{W_{Baot} + W_{Anchor}}{\rho_{Water} g}$$ (floating condition)

$$= \frac{W_{Boat} + W_{Anchor}}{\rho_{Water} g} + V_{Water}$$

Stage2



$$V_{Total,2} = V_{Boat,2} + V_{Anchor} + V_{Water}$$

$$V_{Boat,2} = \frac{W_{Boat}}{\rho_{Water} g}, \quad V_{Anchor} = \frac{W_{Anchor}}{\rho_{Anchor} g}$$

$$= \frac{W_{Boat}}{\rho_{Water} g} + \frac{W_{Anchor}}{\rho_{Anchor} g} + V_{Water}$$

h_1 : Height of the waterline in stage 1
h_2 : Height of the waterline in stage 2
W_{Baot} : Weight of the boat
W_{Anchor} : Weight of the anchor
ρ_{Water} : Density of the water

$V_{Boat,1}$: Displaced volume by the ship with the anchor
$V_{Boat,2}$: Displaced volume by the ship without the anchor
V_{Anchor} : Displaced volume by the anchor
V_{Water} : Volume of the water which is invariant
ρ_{Anchor} : Density of the anchor

$\rho_{Anchor} > \rho_{Water}$

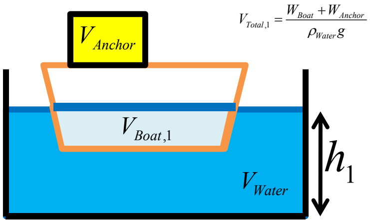
Example: Waterline will change? (4/6)
(a) Dropped into the water (2/2)

$h = \frac{V_{Total}}{A}$

$V = \frac{M}{\rho} = \frac{Mg}{\rho g} = \frac{W}{\rho g}$

Stage1

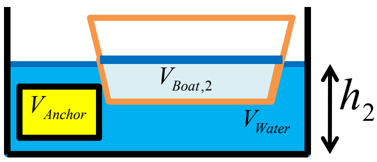
$V_{Total,1} = \frac{W_{Boat} + W_{Anchor}}{\rho_{Water} g} + V_{Water}$



Stage2

$\rho_{Luggage} > \rho_{Water}$

$V_{Total,2} = \frac{W_{Boat}}{\rho_{Water} g} + \frac{W_{Anchor}}{\rho_{Anchor} g} + V_{Water}$



$$V_{Total,1} - V_{Total,2} = \left(\cancel{\frac{W_{Boat}}{\rho_{Water} g}} + \frac{W_{Anchor}}{\rho_{Water} g} + \cancel{V_{Water}}\right) - \left(\cancel{\frac{W_{Boat}}{\rho_{Water} g}} + \frac{W_{Anchor}}{\rho_{Anchor} g} + \cancel{V_{Water}}\right)$$

$$= \left(\frac{W_{Anchor}}{\rho_{Water} g}\right) - \left(\frac{W_{Anchor}}{\rho_{Anchor} g}\right)$$

$$= \frac{W_{Anchor}}{g}\left(\frac{1}{\rho_{Water}} - \frac{1}{\rho_{Anchor}}\right) > 0 \quad (\because \rho_{Anchor} > \rho_{Water}, \frac{1}{\rho_{Anchor}} < \frac{1}{\rho_{Water}})$$

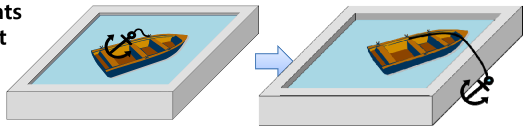
$$V_{Total,1} > V_{Total,2}, \quad \boxed{\therefore h_1 > h_2}$$

The waterline will go down!

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (5/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(b) Thrown onto the surrounding ground

Answer)

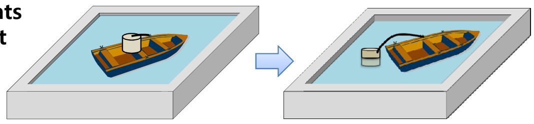
After the anchor is thrown onto the surrounding ground, the ground supports the weight of the anchor. So buoyant force exerted on the anchor is zero.
Thus the water level moves down.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Waterline will change? (6/6)

Question)

A boat with an anchor on board floats in a swimming pool that is somewhat wider than the boat. Does the pool water level move up, move down, or remain the same if the anchor is



(c) If, instead, a cork is dropped from the boat into the water, where it floats, does the water level in the pool move upward, move downward, or remain the same?

Answer)

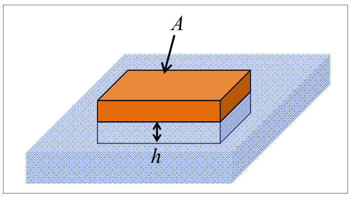
After the cork is dropped from the boat into the water, the cork floats in the water. So the buoyant force exerted on the cork has the same magnitude as that of the weight of the cork. Thus the volume displaced by the cork remains the same.
And the water level also remains the same.

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.377, 2004

Archimedes' Principle and Buoyant Force
- Example: Floating Down the River (1/2)

Question)*

A raft is constructed of wood having a density of 600 kg/m^3. Its water plane area is 5.7 m^2, and its volume is 0.60 m^3. When the raft is placed in fresh water of density 1,000 kg/m^3, as in the figure, to what depth does the raft sink in the water?



<A raft partially submerged in water>

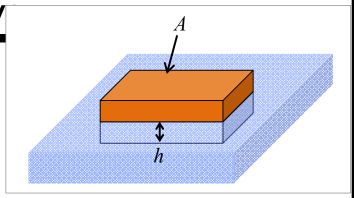
Hint)

The magnitude of the upward buoyant force acting on the raft must equal the weight of the raft if the raft is to float. In addition, from Archimedes' Principle the magnitude of the buoyant force is equal to the weight of the displaced water.

* Serway, R. A., College Physics, 8th Ed., Brooks/Cole, pp.273, 2009.

Archimedes' Principle and Buoyant Force
- Example: Floating Down the River (2/2)

Question)*



<A raft partially submerged in water>

Answer)

The magnitude of the upward buoyant force (B) acting on the raft equals the weight of the displaced water, which in turn must equal the weight of the raft:

$$B = \rho_{water} g V_{water} = \rho_{water} g A h$$

Because the area A and density ρ_{water} are known, we can find the depth h to which the raft sinks in the water:

$$h = \frac{w_{raft}}{\rho_{water} g A} \quad \text{......... (1)}$$

The weight of the raft is

$$w_{raft} = \rho_{wood} g V_{raft} = (600 kg/m^3)(9.8 m/s^2)(0.60 m^3) = 3.5 \times 10^3 N$$

Therefore, substitution into (1) gives

$$h = \frac{3.5 \times 10^5 \mathrm{N}}{(1000 kg/m^3)(9.8 m/s^2)(5.7 m^2)} = 0.060\mathrm{m}$$

* Serway, R. A., College Physics, 8th Ed., Brooks/Cole, pp.273, 2009.

Archimedes' Principle and Buoyant Force
- Example: 302,000DWT VLCC

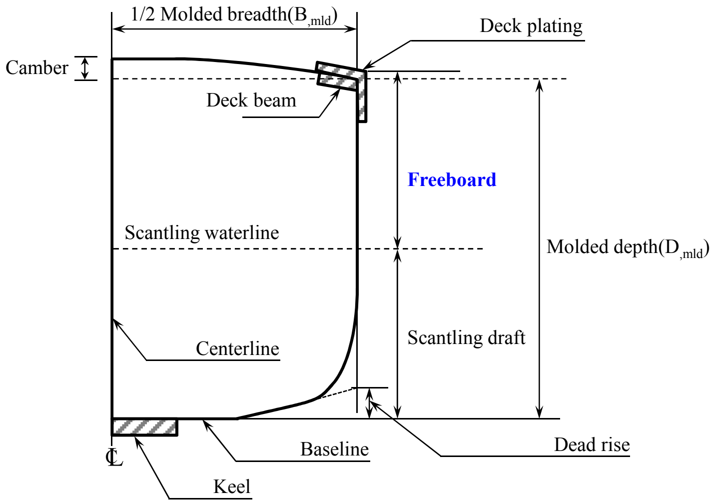
Question)

A 302,000DWT VLCC has a mass of 41,000 metric tons when empty and it can carry up to 302,000 metric tons of oil when fully loaded. Assume that the shape of its hull is approximately that of a rectangular parallelepiped of 300m long, 60m wide, and 30m high.

(a) What is the draft of the empty tanker, that is, how deep is the hull submerged in the water? Assume that the density of the sea water is $1.025 \mathrm{Mg/m^3}$.

(b) What is the draft of the fully loaded tanker?

Archimedes' Principle and Buoyant Force
- Freeboard (1/2)

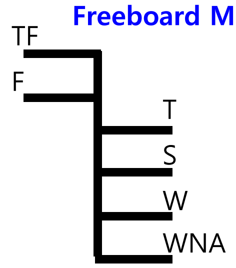


$$Freeboard = Depth(D_{mld}) - Draft(T) + t_{deckplating}$$

Archimedes' Principle and Buoyant Force - Freeboard (2/2)

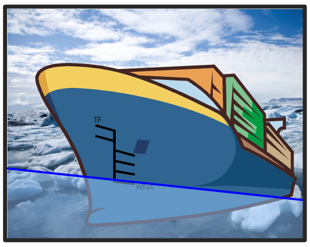
$$W = F_B = (L \cdot B \cdot T) \cdot \rho_{sw} g$$

Freeboard Mark



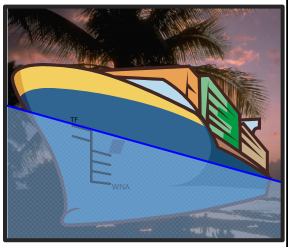
TF – Tropical Fresh Water
F – Fresh Water
T – Tropical Sea Water
S – Summer Sea Water
W – Winter Sea Water
WNA – Winter North Atlantic

The heaviest water is in the North Atlantic in winter time. Ships there displace much less water than in other areas of the world ocean.



The density of water in the world ocean is 1.026 g/cm^3. The density of water in the North Atlantic is 1.028 g/cm^3.

Tropical fresh water is lightest. It occurs in tropical rivers (Amazon, Congo, and others). Some of these rivers are navigable by ocean steamers.



The density of water in navigable tropical rivers is 0.997 g/cm^3.

* Polevoy, S. L., Water Science and Engineering, Blackie Academic and Professional, p.93-97, 1996

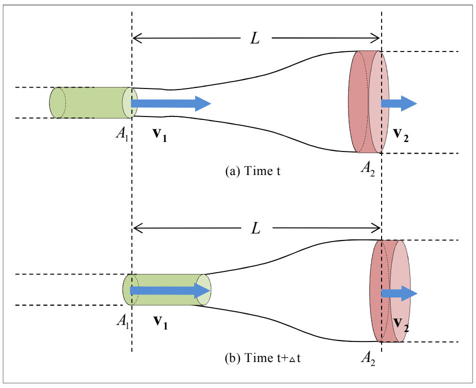
5. The Equation of Continuity and Bernoulli Equation

The Equation of Continuity* (1/3)

● **The Equation of Continuity**

The equation of continuity of flow is a mathematical expression of the law of conservation of mass for flow.

Here we wish to derive an expression that relates v and A for the steady flow of an ideal fluid through a tube with varying cross section.



The Equation of Continuity* (2/3)

● **The Equation of Continuity**

The volume ΔV of fluid that has passed through the dashed line in that time interval Δt is

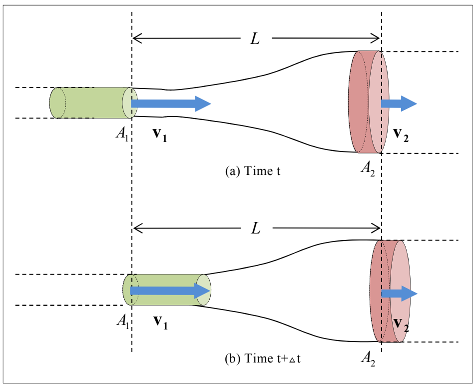
$$\Delta V = A \cdot \Delta x = A \cdot v \cdot \Delta t$$

Apply to both the left and right ends of the tube segment, we have

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$\rightarrow \therefore \underline{A_1 v_1 = A_2 v_2}$$

: **Equation of Continuity**
for the flow of an ideal fluid



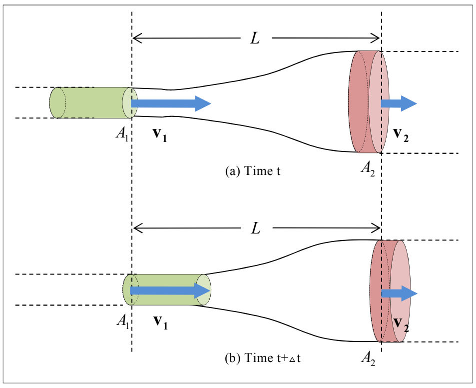
The Equation of Continuity* (3/3)

- The Equation of Continuity

$$A_1 v_1 = A_2 v_2$$: **Equation of Continuity** for the flow of an ideal fluid

This relation between speed and cross-sectional area is called the equation of continuity for the flow of an ideal fluid.

The flow speed increases when we decrease the cross-sectional area through which the fluid flows.



* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.371, 2004

Bernoulli's Equation (1/9)

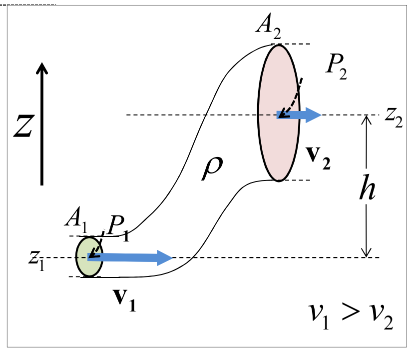
- **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

Assumption: incompressible fluid (density is constant.)

(1) If this fluid is completely static, it seems that it is not moving.

$$P_1 - P_2 = \rho g(z_2 - z_1) = \rho g h$$ **: Pascal's Law**



Bernoulli's Equation (2/9)

- **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

Assumption: incompressible fluid

(1) If this fluid is completely static, it seems that it is not moving.

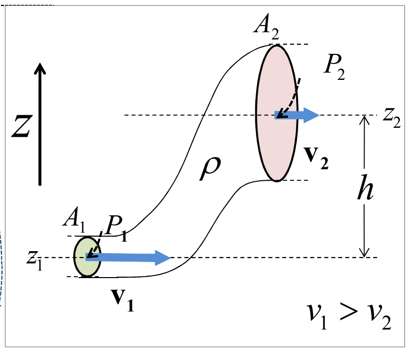
$$P_1 - P_2 = \rho g(z_2 - z_1) = \rho gh$$ **: Pascal's Law**

Have the same dimension of $\dfrac{\text{Energy}}{\text{Volume}}$

mgh : Gravitational Potential Energy

$\dfrac{\text{Mass}}{\text{Volume}} = \text{Density}$

$\rho gh = \dfrac{\text{Gravitational Potential Energy}}{\text{Volume}}$



Bernoulli's Equation (3/9)

- **Bernoulli's Equation**

We can apply the principle of conservation of energy to the fluid.

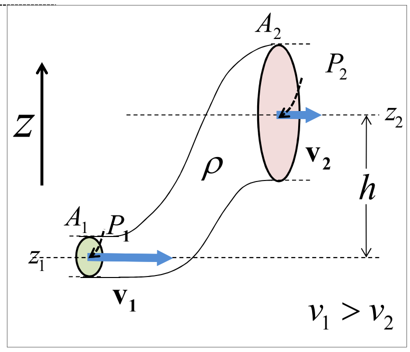
Assumption: incompressible fluid

(2) If we now set this whole machine in motion, there are three players.

$$\boxed{\frac{\text{Kinetic Energy}}{\text{Volume}}} + \boxed{\frac{\text{Gravitational Potential Rnergy}}{\text{Volume}}} + \boxed{\text{P}}$$

Apply the Conservation of Energy

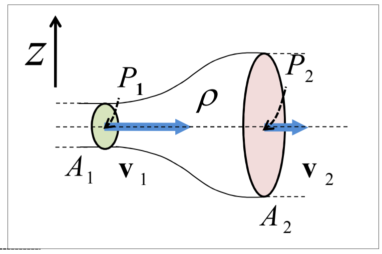
$$\boxed{\frac{1}{2}\rho v^2 + \rho gz + P_z = \text{Constant}}$$ **: Bernoulli's Equation**



Bernoulli's Equation (4/9)

- **Example: Eliminate 'z'**

If we take z to be a constant, so that the fluid does not change elevation as it flows,



If we assume that $A_1 < A_2$,

By the **Equation of Continuity** (ideal fluid)

$$A_1 v_1 = A_2 v_2$$

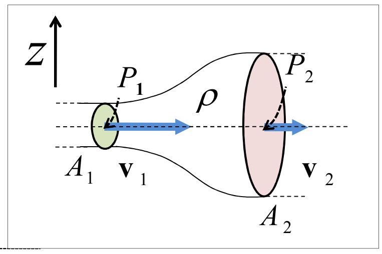
$$\rightarrow \quad A_1 < A_2 \rightarrow v_1 > v_2$$

Bernoulli's Equation (5/9)

Bernoulli's Equation : $\frac{1}{2}\rho v^2 + \rho g z + P_z = \text{Constant}$

- **Example: Eliminate 'z'**

If we take z to be a constant, so that the fluid does not change elevation as it flows,



Bernoulli' Equation becomes

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

$$\rightarrow \quad v_1 > v_2 \rightarrow P_1 < P_2$$

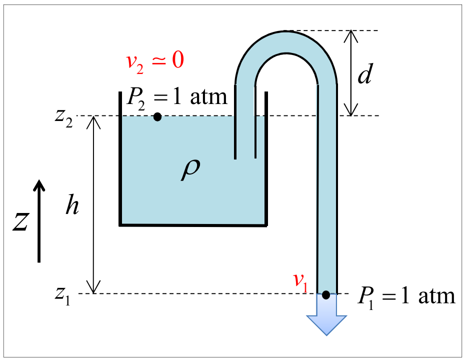
Which tell us that:

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.*

Bernoulli's Equation (6/9)
- Example: Siphon* (: Eliminate 'P') (1/3)

Figure on the right side shows a siphon, which is a device for removing liquid from a container, and a kind of tube.

A tube must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is the same level with the tube opening at z_1. The liquid has density ρ and negligible viscosity.

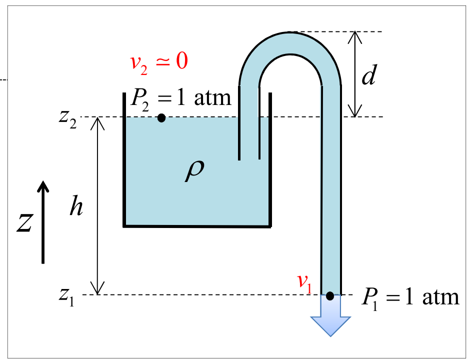


(a) With what speed does the liquid emerge from the tube at z_1?
(b) Theoretically, what is the greatest possible height d that a siphon can lift water?

* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.383-384, 2004

Bernoulli's Equation (6/9)
- Example: Siphon* (: Eliminate 'P') (2/3)

(a) With what speed does the liquid emerge from the tube at z_1?



Bernoulli's Equation:

$$\frac{1}{2}\rho v^2 + \rho g z + P_z = \text{Constant}$$

$P_1 = P_2 \rightarrow$ **P term is eliminated.**

$$\rightarrow \frac{1}{2}\cancel{\rho} v_1^2 + \cancel{\rho} g z_1 = \cancel{\rho} g z_2$$

$$\frac{1}{2}v_1^2 + g z_1 = g z_2 \quad \frac{1}{2}v_1^2 = g(z_2 - z_1)$$

$$\rightarrow \frac{1}{2}v_1^2 = g(h)$$

$$\therefore v_1 = \sqrt{2gh}$$ **Conversion of gravitational potential energy to kinetic energy**

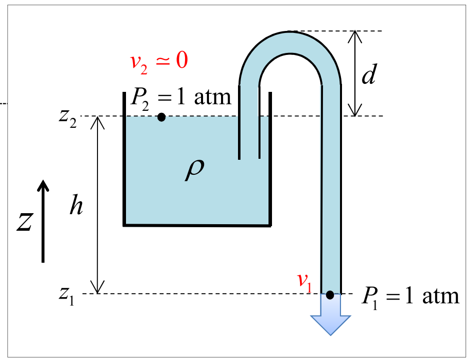
Bernoulli's Equation (6/9)
- Example: Siphon* (: Eliminate 'P') (3/3)

(b) Theoretically, what is the greatest possible height d that a siphon can lift water?

Barometric Pressure:

$$1\ \text{atm} = 1.01\times10^{5}\text{Pa}$$
$$= 760\text{torr}$$
$$\simeq 10\text{m (Water)}$$

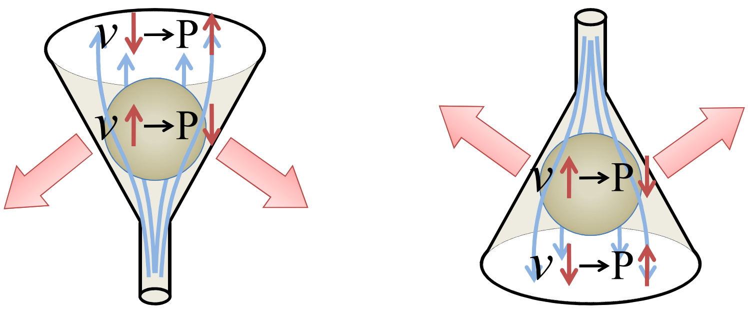
Therefore, This siphon would only work if d is less than 10m.



* 1 atm = 101,325 Pascal = 101,325 N/m²

Bernoulli's Equation (7/9)
- Example: Funnel with a Ping-Pong Ball

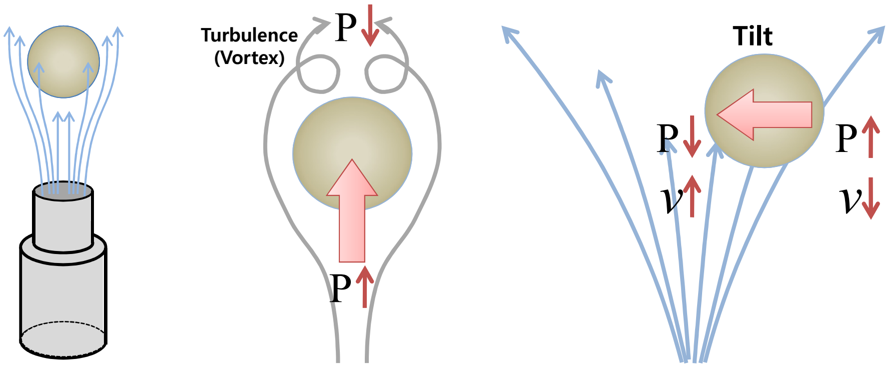
How is the position of a ping-pong ball?



Movie Clip

Bernoulli's Equation (8/9)
- Example: Ping-Pong Ball in the Jet of Air*

If you place a ping-pong ball in the jet of air from a vacuum cleaner hose aimed vertically upward, the ping-pong ball will be held in stable equilibrium with this jet. Explain this by means of Bernoulli's equation. (Hint: The speed of air is maximum at the center of the jet.)

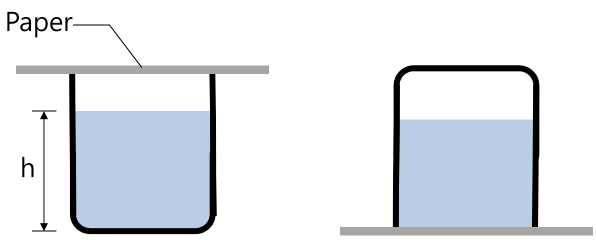


* Ohanian, H. C., Physics, 2nd Ed., W W. Norton & Company, p.354-355, pp.486, 1989

Bernoulli's Equation (9/9)
- Example: A Glass Filled with Water* (1/2)

Partially fill a tall drinking glass with water to depth h. Cut a square of sturdy paper somewhat wider than the mouth of the glass. Place the paper over the mouth. Spread the fingers of your left hand over the paper, pressing it against the mouth of the glass.

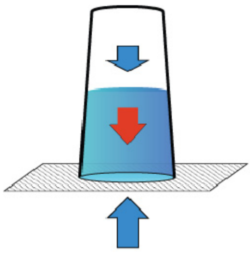
Grab the glass with your right hand and then as rapidly as you can, invert it with your left hand and then as rapidly as you can, invert it with your left hand still pressing the paper against the rim. Chances are you can then remove your left hand without the water pouring out. If h=11.0cm, what is the gauge pressure of the air now trapped in the above the water?



* Halliday, D., Fundamentals of Physics, 7th Ed., Wiley, pp.385, 2004

Bernoulli's Equation (9/9)
- Example: A Glass Filled with Water* (2/2)

- ☑ Any object in air is subject to pressure from air molecules colliding with it. At sea level, the mean air pressure is one "atmosphere" (=101,325 Pascals in standard metric units).
- ☑ The blue arrows indicate the forces due to air pressure above and below the water. The red arrow indicates the force of gravity. Together, the three forces balance out to cancel each other.



- ☑ The pressure of the outside air acts against the paper, and forces it against the glass, because it is stronger than the pressure of the water (weight of the water).