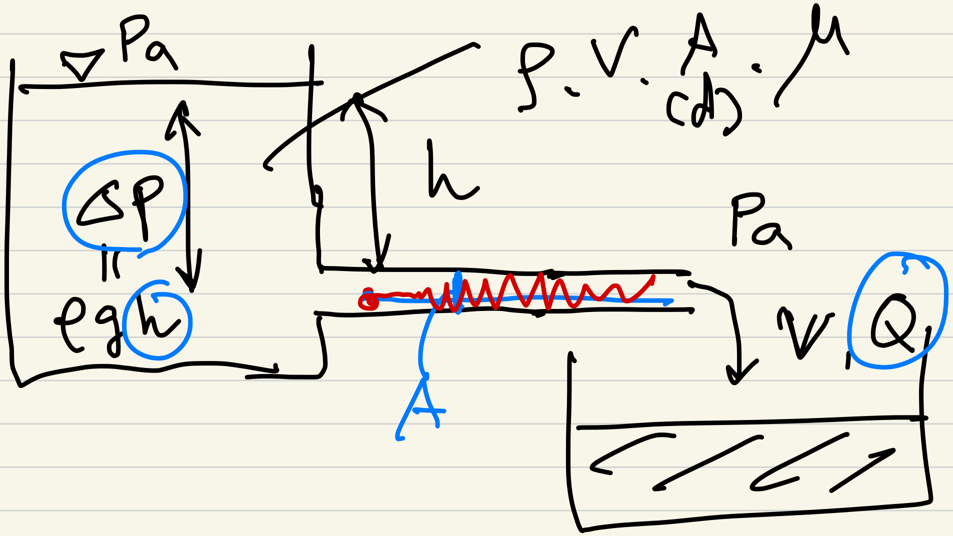
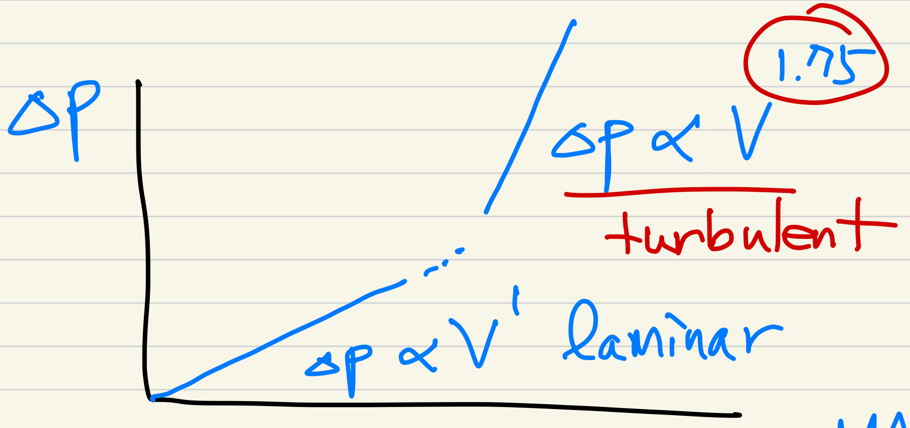
Ch.6 Viscous flow in ducts

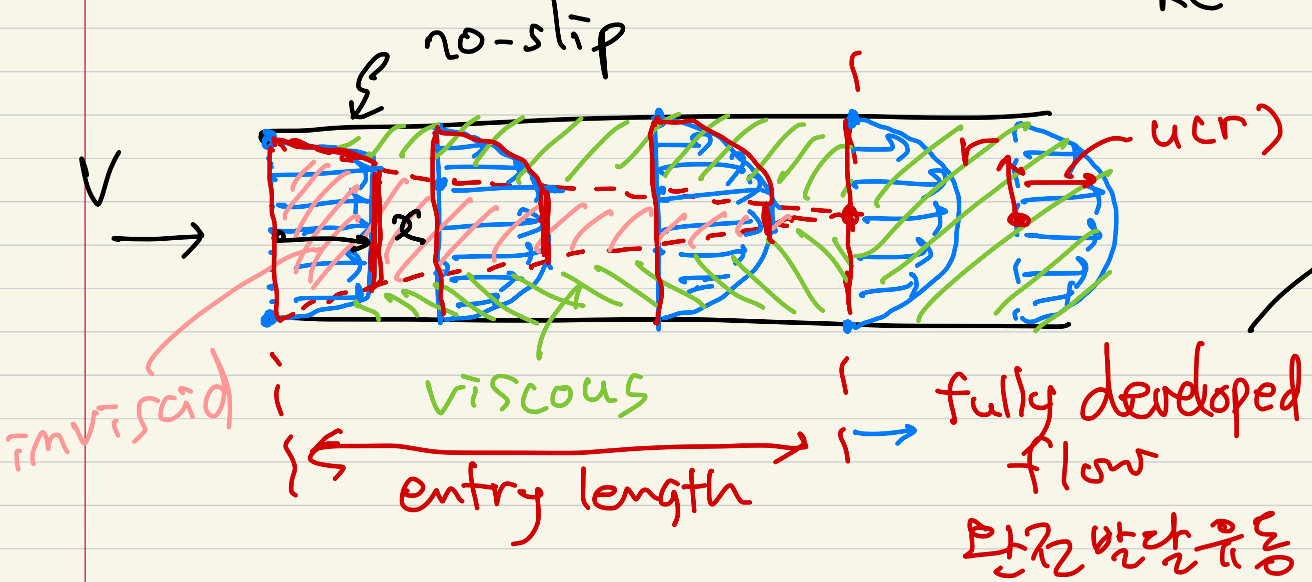
internal flow ↔ external flow
Ch.7

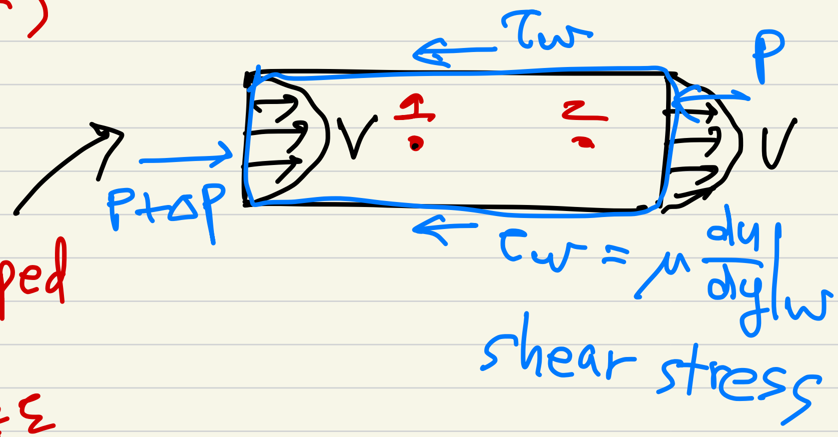




$Q = VA$

$Re = \dfrac{\rho V d}{\mu}$: Reynolds number





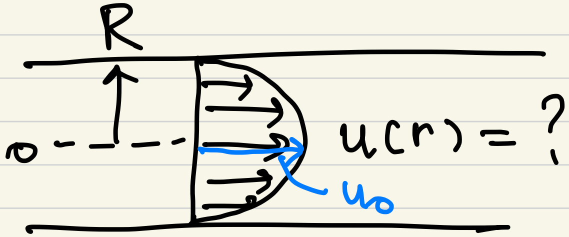
head loss

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2 + h_f \qquad \frac{\partial u}{\partial x} = 0$$

$$h_f = f\frac{L}{d}\frac{V^2}{2g}$$

f : Darcy friction factor

$$f \equiv \frac{8\tau_w}{\rho V^2}, \quad V = \int_0^R u(r) \cdot 2\pi r\, dr / A \quad \text{bulk velocity}$$

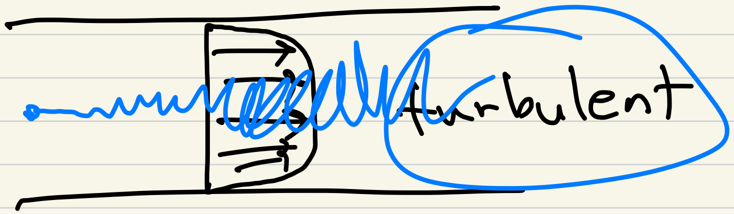


$u(r) = ?$

$\therefore\ u = u_0\left[1 - \left(\frac{r}{R}\right)^2\right]$ laminar flow

Poiseuille flow

$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_w = -\mu \frac{\partial u}{\partial r}\Big|_R$$

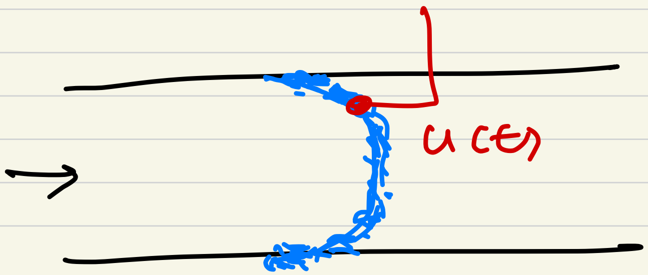


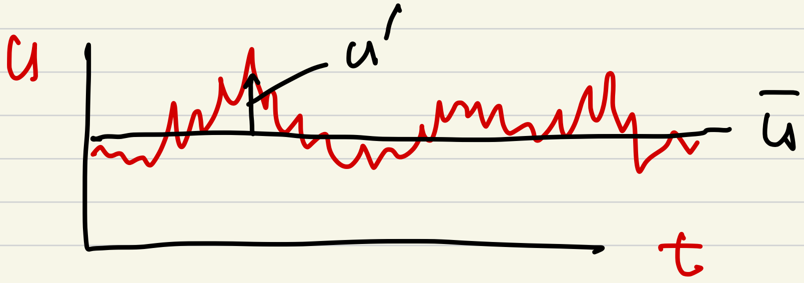
$$Re_d = \frac{Vd}{\nu} \simeq 2500 \quad \begin{array}{l} \uparrow \text{ turb.} \\ \downarrow \text{ lam.} \end{array}$$

water. $\nu = 10^{-6}\ m^2/s$

$d = 5cm$

$$V = \frac{2500 \times \nu}{d} = 0.05\ m/s$$





- Reynolds time-averaging concept

  " velocity decomposition

$$\boxed{u(t) = \bar{u} + u'(t)} \rightarrow \bar{u} = \overline{\bar{u}} + \overline{u'}$$

instantaneous velocity 순간속도 | mean velocity 평균속도 | fluctuating velocity 속도섭동

$$\overline{\bar{u}} = \bar{u} \Rightarrow \overline{u'} = 0$$

- Navier-Stokes eq.

$$\overline{\rho\frac{\partial u}{\partial t}}^{t} + \overline{\rho\frac{\partial}{\partial x}(uu)}^{t} + \overline{\rho\frac{\partial}{\partial y}(uv)}^{t} + \overline{\rho\frac{\partial}{\partial z}(uw)}^{t} = \overline{-\frac{\partial p}{\partial x}}^{t} + \overline{\mu\nabla^2 u}^{t}$$

$$\rho\frac{\partial u}{\partial x}(uu) + \rho\frac{\partial}{\partial y}(uv) + \rho\frac{\partial}{\partial z}(uw) = \left(\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z}\right)$$

$$-\frac{\partial p}{\partial x} = -\frac{\partial \bar{p}}{\partial x}, \qquad \mu\nabla^2 u = \mu\nabla^2\bar{u}$$

" $\cancel{\rho\frac{\partial \bar{u}}{\partial t}} = 0$

$$\overline{\frac{\partial}{\partial x}(uu)} = \overline{\frac{\partial}{\partial x}(\bar{u}+u')(\bar{u}+u')}^{t} = \frac{\partial}{\partial x}\left(\overline{\bar{u}\bar{u}} + \overline{2\bar{u}u'} + \overline{u'u'}\right)$$

$$\overline{\bar{u}\bar{u}} = \bar{u}\bar{u}, \qquad \overline{2\bar{u}u'} = 2\bar{u}\overline{u'} = 0, \qquad \overline{u'u'} \neq 0$$

$$\overline{\frac{\partial}{\partial y}(uv)} = \frac{\partial}{\partial y}\left(\bar{u}\bar{v} + \overline{u'v'}\right)$$

$$\frac{\partial}{\partial z}\overline{(uw)} = \frac{\partial}{\partial z}\left(\bar{u}\,\bar{w} + \overline{u'w'}\right)$$

$$\mu\nabla^2 u = \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right)$$

$$\Rightarrow \rho\frac{\partial}{\partial x}(\bar{u}\bar{u}) + \rho\frac{\partial}{\partial y}(\bar{u}\bar{v}) + \rho\frac{\partial}{\partial z}(\bar{u}\bar{w}) = -\frac{\partial\bar{p}}{\partial x}$$

$$+ \frac{\partial}{\partial x}\left(\mu\frac{\partial\bar{u}}{\partial x} - \rho\overline{u'u'}\right) + \frac{\partial}{\partial y}\boxed{\left(\mu\frac{\partial\bar{u}}{\partial y} - \rho\overline{u'v'}\right)} + \frac{\partial}{\partial z}\left(\mu\frac{\partial\bar{u}}{\partial z} - \rho\overline{u'w'}\right)$$

viscous stress

turbulent stress
or Reynolds stress ???

$$\frac{\partial}{\partial t}(\overline{u'v'}) + \cdots = \frac{\partial}{\partial x}(\overline{u'u'v'}) + \cdots$$

??

closure problem.

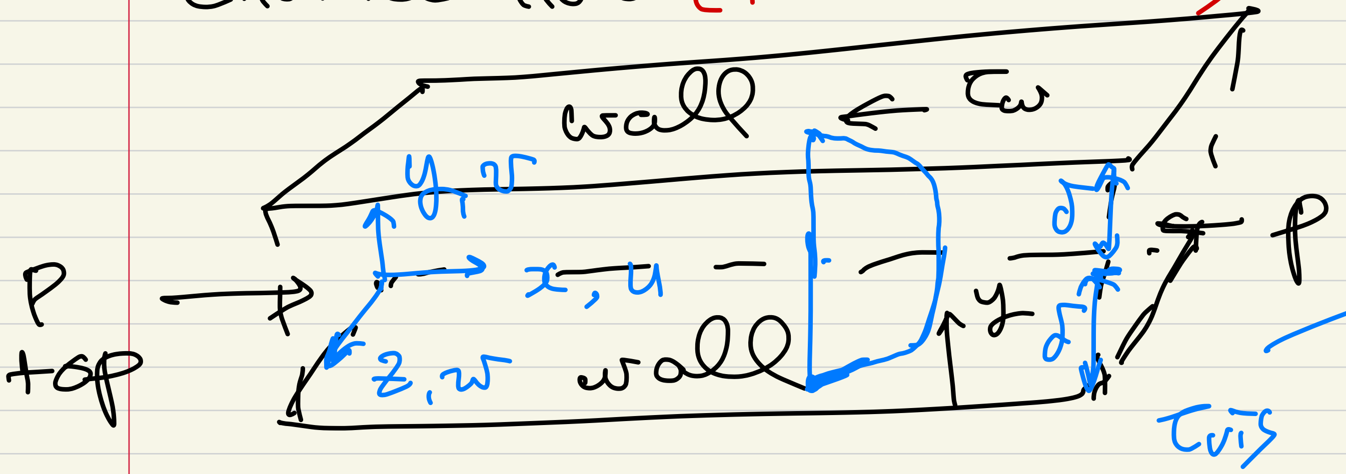
⇙

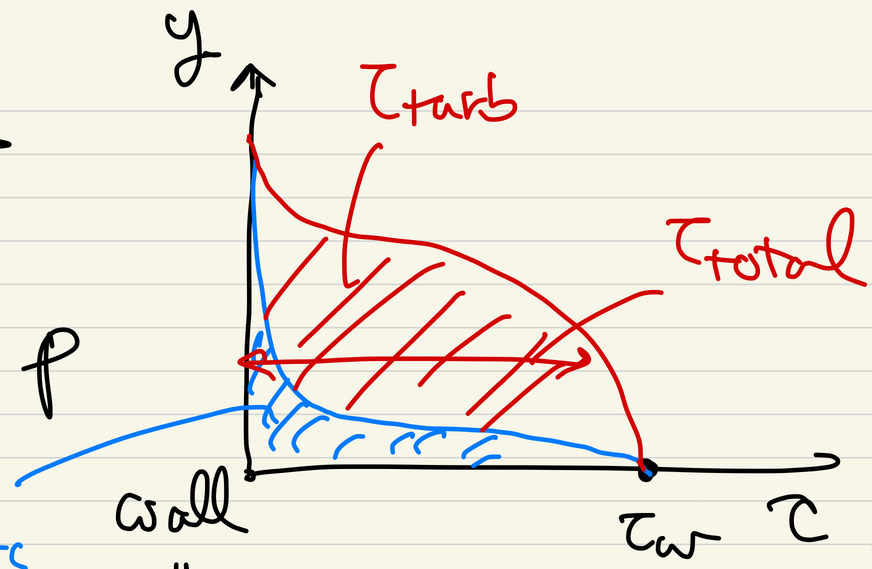
turbulence modeling

$$\tau = \mu\frac{\partial\bar{u}}{\partial y} - \rho\overline{u'v'}$$

$$\tau_{total} \qquad \tau_{vis} \qquad \tau_{turb}$$

Channel flow (turbulent flow)

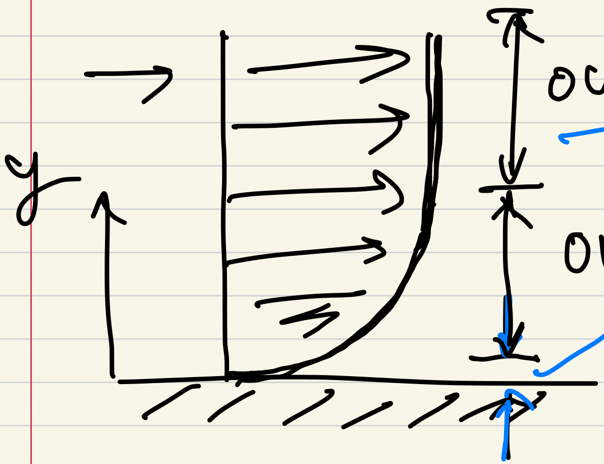




wall
$\parallel$
$u' = v' = w' = 0$

$\tau_w = \mu \frac{\partial \bar{u}}{\partial y}\Big|_w - \rho \overline{u'v'}$

vel.

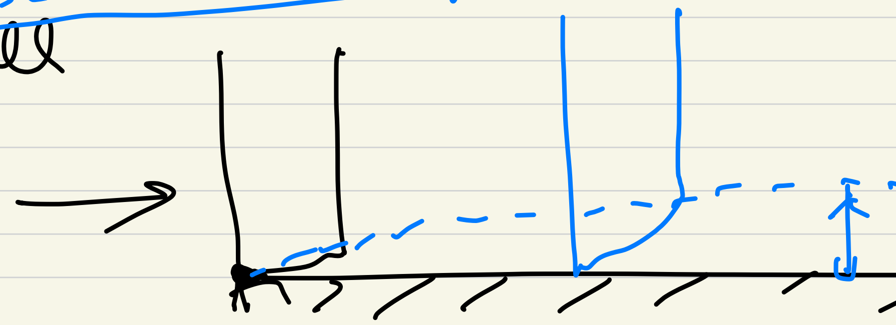


outer layer : $\tau_{turb} > \tau_{vis}$

overlap layer ✓

viscous wall layer : $\tau_{turb} < \tau_{vis}$

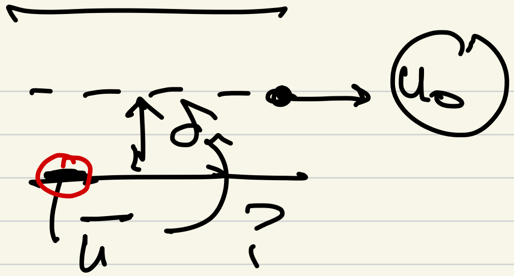
Ch. 7



δ : boundary layer thickness

- wall layer : Prandtl (1930)

$$\bar{u} = f(\mu, \rho, y, \tau_w, \cancel{\delta}) \neq f(\delta)$$



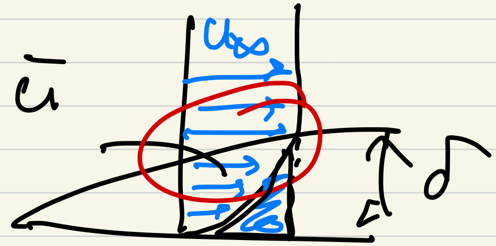
dimensional analysis

$$\rightarrow \frac{\bar{u}}{u^*} = F\left(\frac{yu^*}{\nu}\right), \quad u^* = \sqrt{\frac{\tau_w}{\rho}}$$: wall-shear vel.

벽전단속도

$$u^+ \equiv \frac{\bar{u}}{u^*}, \quad y^+ \equiv \frac{yu^*}{\nu}$$

$$\rightarrow \boxed{u^+ = F(y^+)}$$: law of the wall 벽법칙



- Outer layer : von Karman (1933)

$$u_\infty - \bar{u} = g(\delta, \rho, y, \tau_w, \cancel{\mu}) \neq g(\mu)$$

$u_\infty - \bar{u}$: velocity defect

$$\rightarrow \boxed{\frac{u_\infty - \bar{u}}{u^*} = G\left(\frac{y}{\delta}\right)}$$: velocity defect law

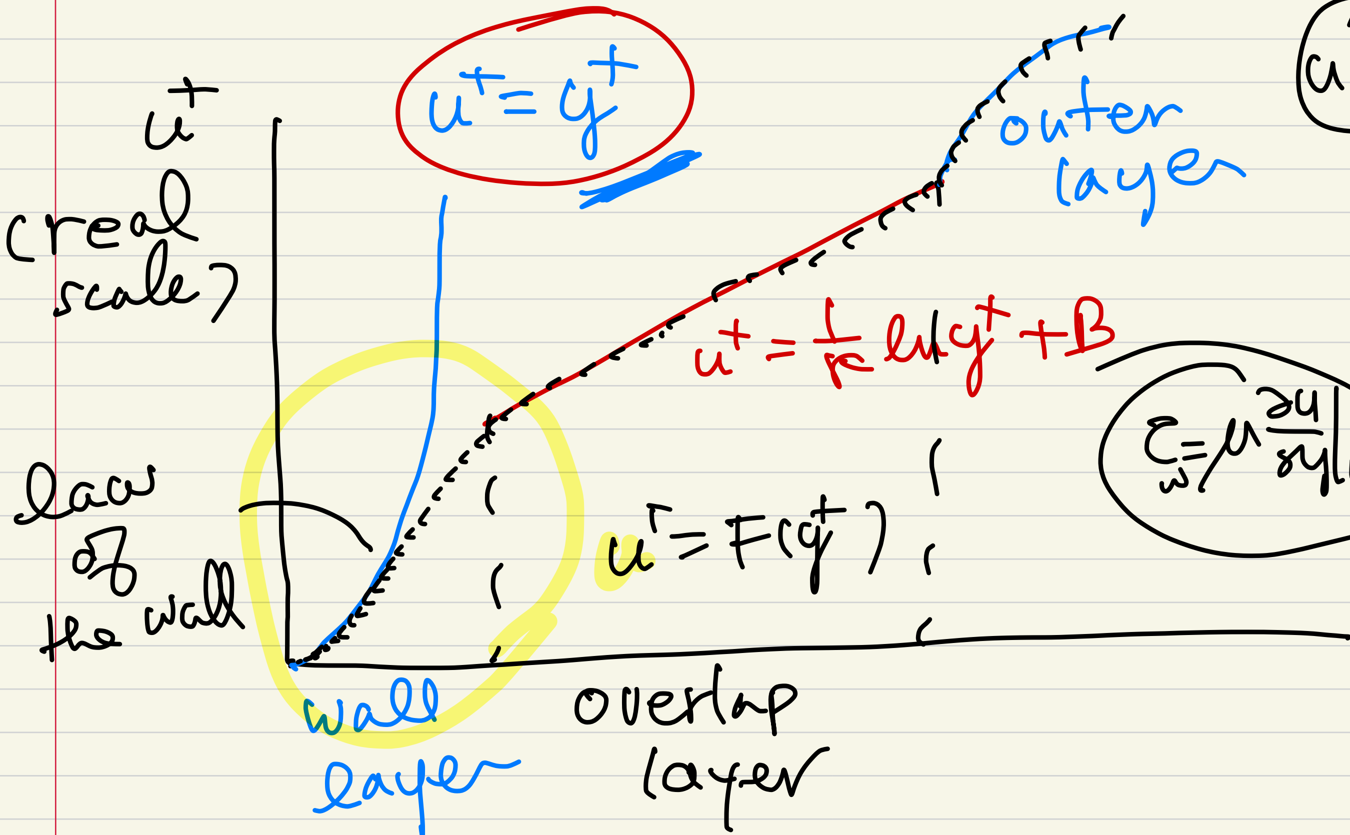
· Overlap layer (log layer) : Millikan (1937)

$$\rightarrow \quad \frac{\bar{u}}{u^*} = \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B \quad : \text{log law}$$

$$\rightarrow \quad \boxed{u^+ = \frac{1}{\kappa} \ln y^+ + B}$$

$\kappa = 0.41$: von Karman const.

$B = 5.0$



$$u(y) = \cancel{u(0)} + y \left.\frac{\partial u}{\partial y}\right|_w + \frac{1}{2} y^2 \left.\frac{\partial^2 u}{\partial y^2}\right|_0 + \cdots$$

$$= y \frac{\tau_\omega}{\mu} + O(y^2)$$

$$= y \cdot \frac{\rho u^{*2}}{\mu} \qquad u^* = \sqrt{\frac{\tau_\omega}{\rho}}$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_w$$

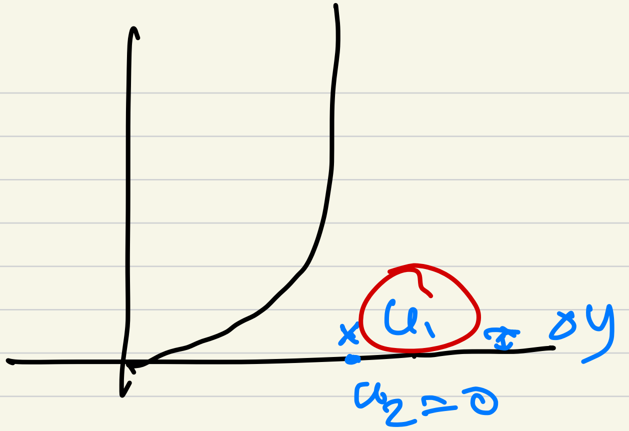
$$\rightarrow \tau_\omega = \rho u^{*2}$$

$$\frac{u}{u^*} = \frac{y u^*}{\nu} \rightarrow u^+ = y^+$$

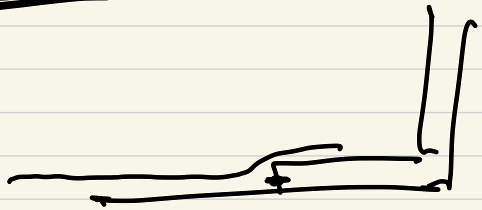
$\tau_w \rightarrow$ drag

$\Uparrow$ $\uparrow$ measure?

$\mu \left.\frac{\partial u}{\partial y}\right|_w$



$$\left.\frac{\partial u}{\partial y}\right|_w = \frac{u_1 - u_2}{\delta y} = \frac{\tau_w}{\mu} \; !$$



u ✗ $\rightarrow$ τ_w ✗

$$\boxed{\frac{\bar{u}}{u^*} = \frac{1}{\kappa} \ln \frac{y u^*}{\nu} + B}$$

least square method $\rightarrow u^* \rightarrow \tau_w$