Topics in Ship Structures

03 LCF S-N Curves for Notches

Reference : Fundamentals of Metal Fatigue Analysis Ch. 4 Notches

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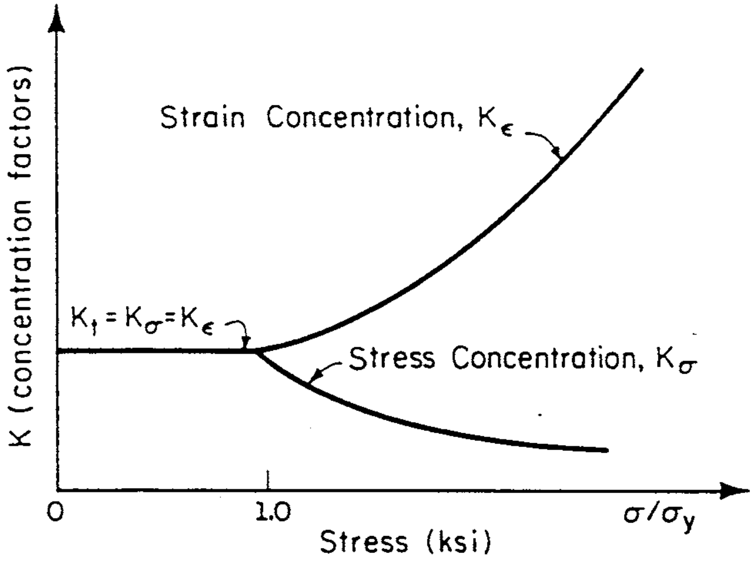
4.3.1 Notch Root Stresses and Strains

- Theoretical stress concentration factor K_t

$$K_\sigma = \frac{\sigma(\text{ notch stress})}{S(\text{nominal stress})} \qquad K_\varepsilon = \frac{\varepsilon(\text{notch strain})}{e(\text{nominal strain})}$$

- For increasing nominal stress, S, K_t remains the same, until yielding begins.



- After yielding occurs,
  - ✓ K_σ decreases with respect to K_t
  - ✓ K_ε, increases with respect to K_t

4.3.1 Notch Root Stresses and Strains

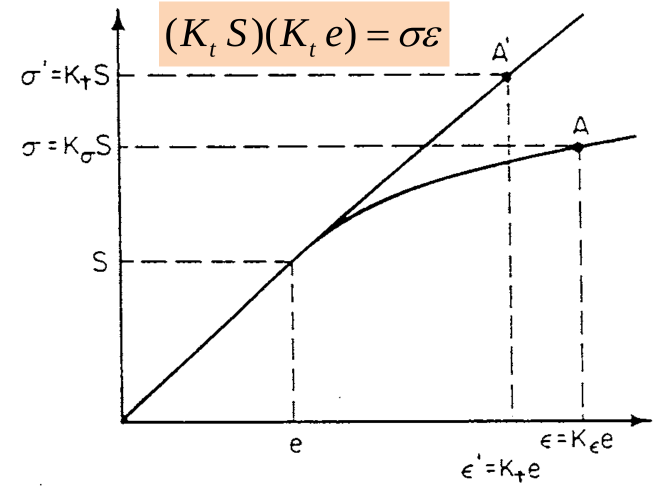
- After yielding occurs, actual local stress < predicted using K_t,

  actual local strain > predicted using K_t,

- **Neuber's rule**

$$K_t = \sqrt{K_\sigma K_\varepsilon}$$

$$K_t^2 = \frac{\sigma}{S}\frac{\varepsilon}{e}\ ,\quad K_t^2 Se = \sigma\varepsilon$$



$$K_\sigma = \frac{\sigma}{S} \qquad K_\varepsilon = \frac{\varepsilon}{e}$$

- **Nominally Elastic Behavior**

$$K_t^2 = \frac{\sigma}{S}\frac{\varepsilon}{e} = \frac{\sigma}{S}\frac{\varepsilon E}{S},\ (\because S = Ee,\ \frac{1}{e} = \frac{E}{S})$$

$$\sigma\varepsilon\ \ (notch\, repsonse) = \frac{(K_t S)^2}{E}\ \ (applied\, load)$$

- **Limited Yielding**

  when yielding occurs in the nominal stresses or strains, Hooke's law cannot no longer be used ($S \neq Ee$).

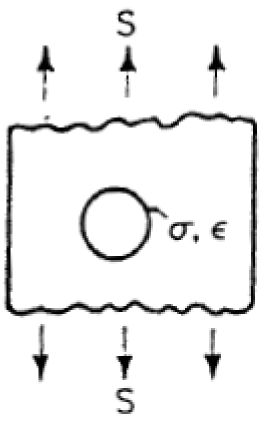
$$K_t^2 Se = \sigma\varepsilon$$

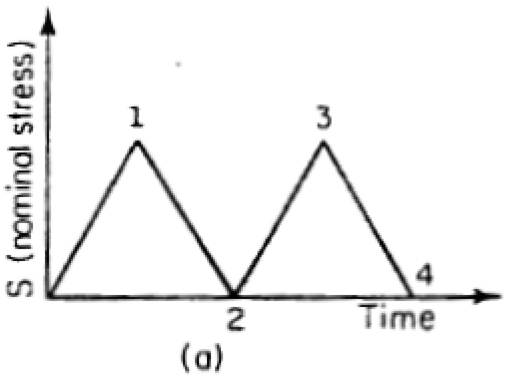
- Small notches have less effect than is indicated by K_t, Topper et al. proposed

$$\sigma\varepsilon = \frac{(K_f S)^2}{E}$$

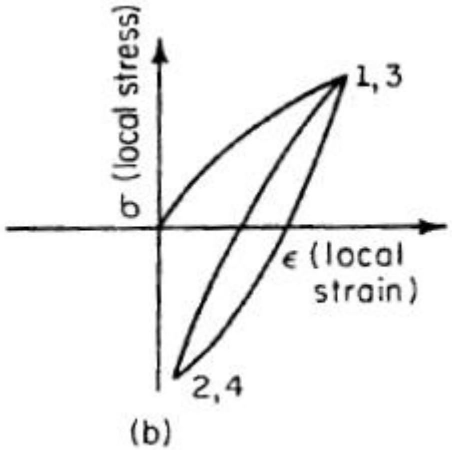
4.3.1 Notch Root Stresses and Strains

- The local stress-strain response at the notch root may vary from the nominally applied loading.
- Extreme example : Nominally applied stress is from 0 to σ_{max}, while the local stress-strain response is completely reversed.
- Due the residual stresses developed as a result of local yielding at the notch root.





Nominal stress history

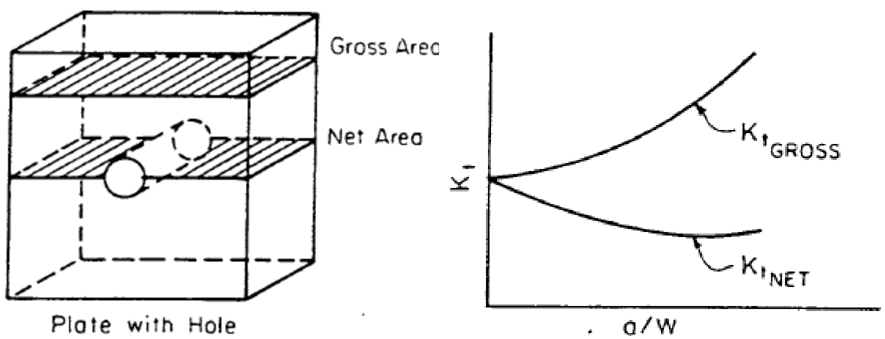


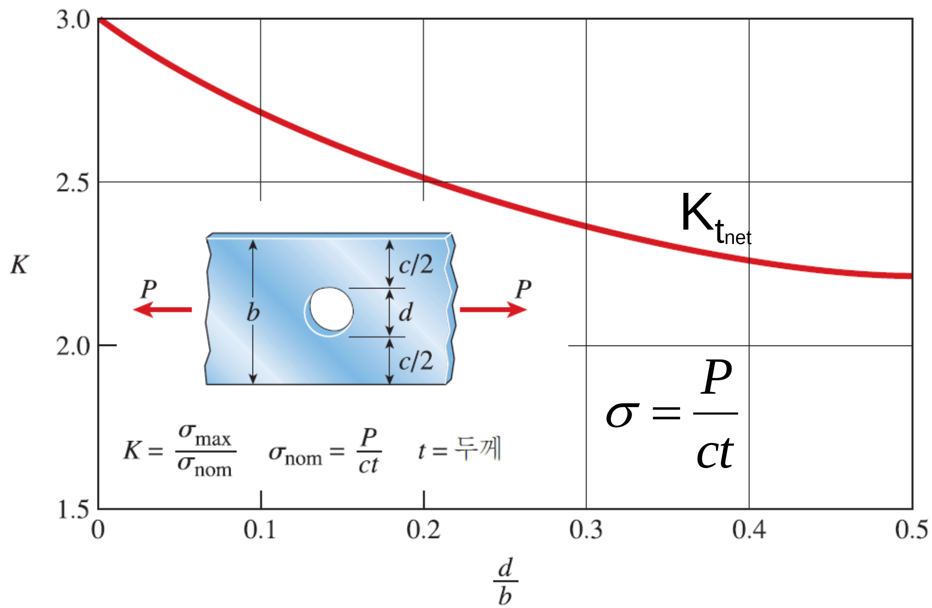
Local stress-strain response

4.3.1 Notch Root Stresses and Strains

- K_t an S must be defined using the same cross-sectional area ($K_{t_{net}}$ and S_{net} or $K_{t_{gross}}$ and S_{gross})
- $K_{t_{gross}}$ includes the nominal stress increase effect caused by area reduction and stress concentration effect in the vicinity of hole.
- $K_{t_{net}}$ includes only stress concentration effect in the vicinity of around hole

$$(K_{t_{net}})(S_{net}) \approx (K_{t_{gross}})(S_{gross})$$





Gross and net section areas (a= hole radius, W=plate width)

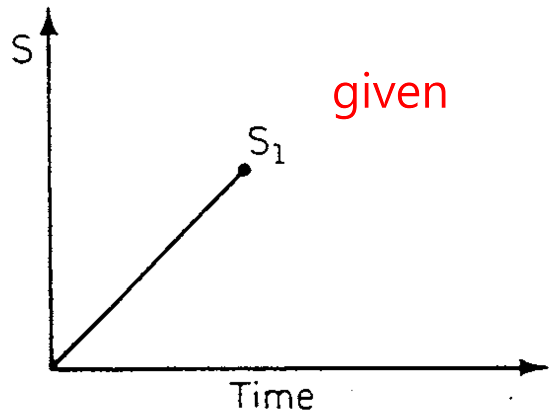
4.3.2 Example of Notch Analysis Using Neuber's Rule

- For given S_1, determine notch stress and strain.
- What to be satisfied?
  - Neuber's rule.
  - Cyclic stress-strain curve of material
- Assumptions
  1) Nominally elastic behavior
  2) K_f instead of K_t.
  3) Net section $K_{t_{net}}$ and S based on A_{net}.

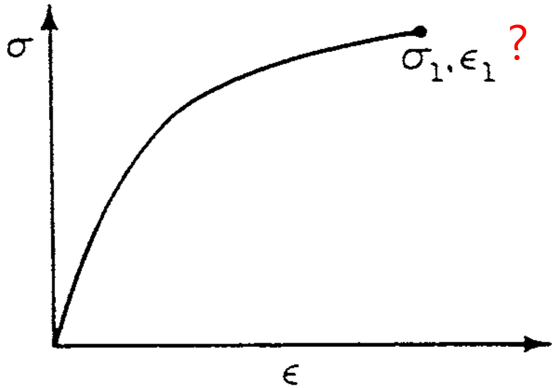
- Fatigue notch factor is dependent on material type.

$$K_f = \frac{S_e^{(unnotched)}}{S_e^{(notched)}}$$

S_e : fatigue strength



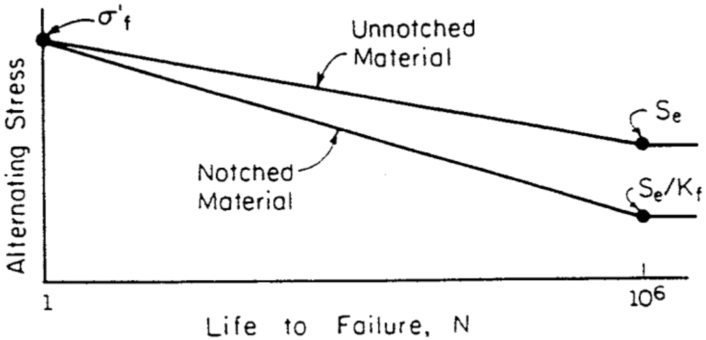
Nominal stress versus time



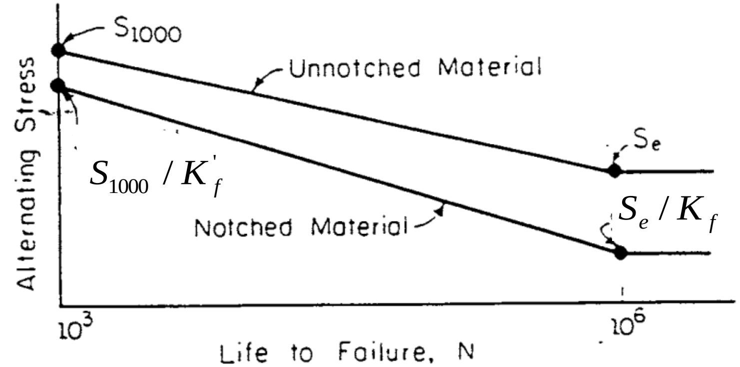
Local (notch) stress-strain

4.3.2 Example of Notch Analysis Using Neuber's Rule

- K_f is used to correct only endurance limit(S_e) of S-N Curve or entire S-N curve to be conservative ?



Modification of S-N Curve

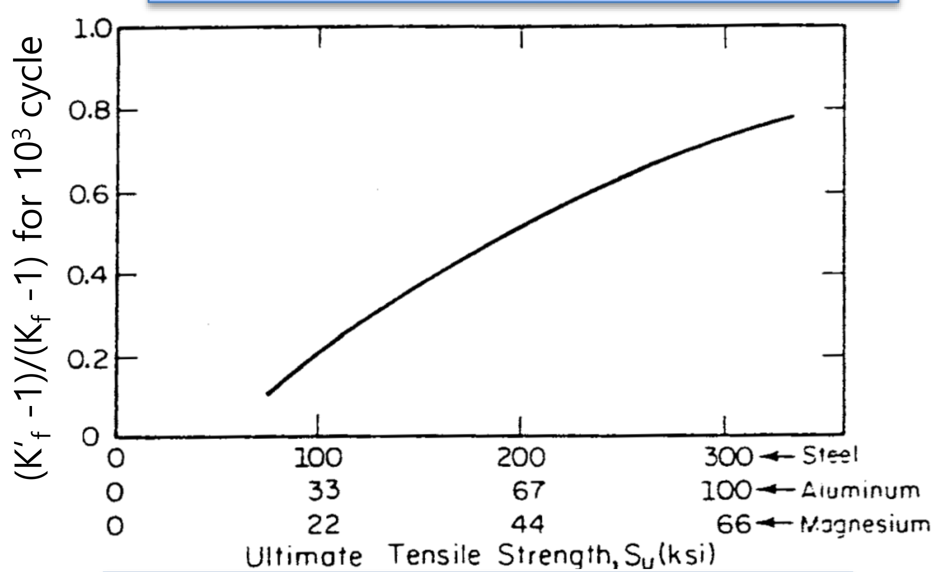


Juvinall approach

- K'_f is used to adjust S_{1000}

$$K_f = \frac{S_e^{(unnotched)}}{S_e^{(notched)}}$$

- The notch effect at short lives (10^3) is greatly reduced for low strength of soft material.
- For high strength material, the notch effect of fatigue life shortening becomes larger.



Relationship between K'_f and K_f

4.3.2 Example of Notch Analysis Using Neuber's Rule

- **Step 1 : Initial Loading**

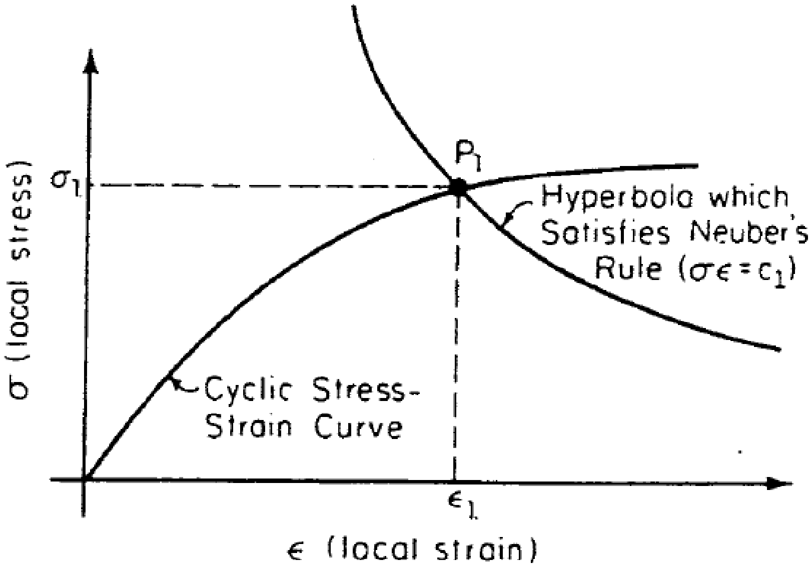
Nominally elastic version of Neuber's rule

to be determined

given

$$\sigma\varepsilon \ (notch\ repsonse) = \frac{(K_f S)^2}{E} \ (applied\ load)$$

$$\sigma\varepsilon = constant : hyperbola\ curve$$



Intersection of cyclic stress-strain curve and Neuber's hyperbola

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}$$

$$\left[\frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{1/n'}\right]\sigma = \frac{(K_f S)^2}{E}$$

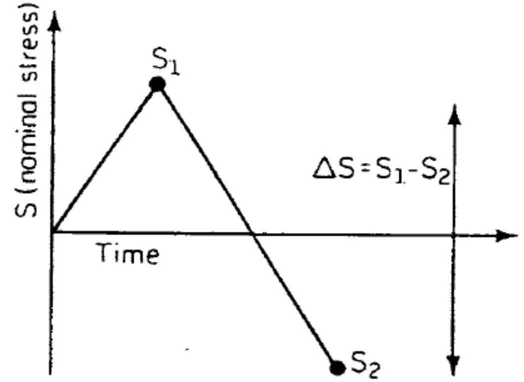
σ can be solved using numerical methods such as iteration technique

4.3.2 Example of Notch Analysis Using Neuber's Rule

- **Step 2 : Nominal Stress Reversal**

Nominal stress change ($\Delta S = S_1 - S_2$) results in $\Delta\sigma$ and $\Delta\varepsilon$.

$$\sigma\varepsilon = \frac{(K_f S)^2}{E} \Rightarrow \frac{\Delta\sigma}{2}\frac{\Delta\varepsilon}{2} = \frac{(K_f \Delta S/2)^2}{E} \Rightarrow \Delta\sigma\Delta\varepsilon = \frac{(K_f \Delta S)^2}{E}$$



Nominal stress reversal

P_2 is lies at the intersection of the hyperbola and hysteresis loop.

The origin of the axis in now at point P_1.

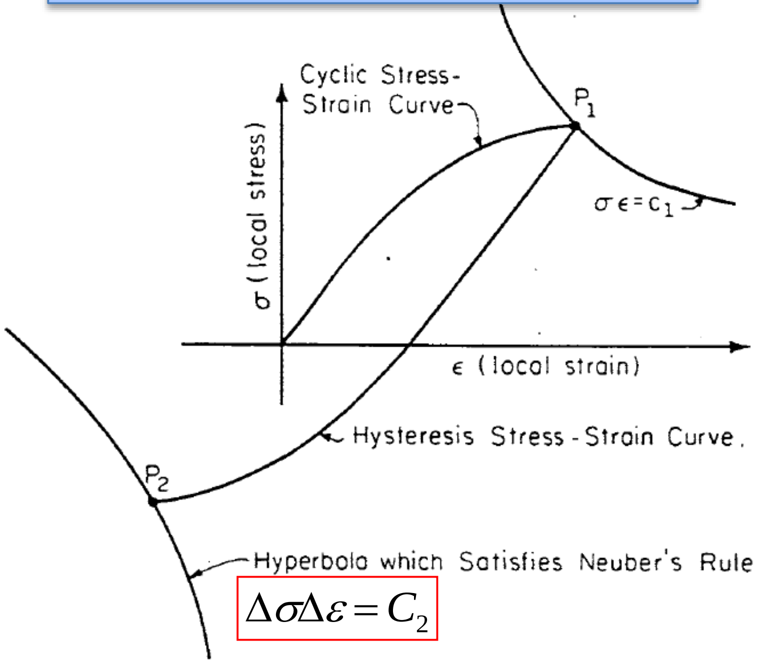
$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

$$\Downarrow$$

$$\left[\frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}\right]\Delta\sigma = \frac{(K_f \Delta S)^2}{2E}$$

$$\Downarrow$$

Δσ can be solved using iterative technique



Intersection of cyclic stress-strain curve and Neuber's hyperbola

Example 4.2

Q: A notched steel component of a bar : wide : 1.0 in, 0.25 in thick with two semi-circular edge notches with radii of 0.1 in. Determine the life of the component using a Neuber analysis.

Given 1 :

$$E = 30 \times 10^3\, ksi,\ K' = 154\, ksi, n' = 0.123$$

Hysteresis loop

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

Given 2 :

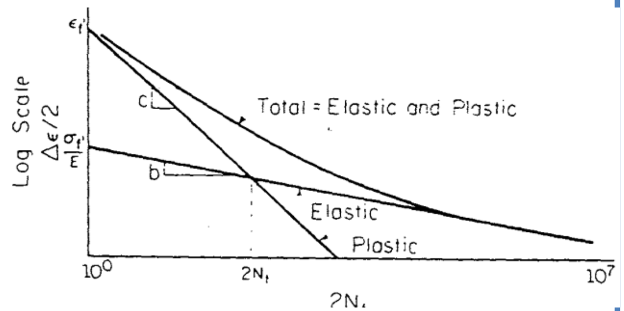
$$\sigma_f' = 169\, kis \quad b = -0.081$$

$$\varepsilon_f' = 1.14 \quad c = -0.67$$

$$K_t = 2.42$$

Strain-life equation

$$\frac{\Delta\varepsilon}{2} = \underbrace{\frac{\sigma_f}{E}(2N_f)^b}_{\text{elastic}} + \underbrace{\varepsilon_f'(2N_f)^c}_{\text{plastic}}$$



$$S = \frac{P}{A}$$

Neuber's rule & cyclic stress-strain relationship

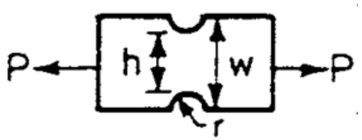
Strain-life equation

Given load P ⇒ ΔS ⇒ Δσ ⇒ Δε ⇒ N_f : fatigue life

Example 4.2

Given load P ➡ ΔS ➡ Δσ ➡ Δε ➡ N_f : fatigue life

W= 1.0 in, t=0.25, r=0.1 in, P=10 kips



- Step 3 : calculate ΔS_{net}

For most engineering applications

$$(K_{t_{net}})(S_{net}) \approx (K_{t_{gross}})(S_{gross})$$

$$\frac{\Delta S_{net}}{2} = \frac{P}{A_{net}} = \frac{10\text{kips}}{(1-0.2)(0.25)} = 50\,ksi$$

$$K_t^2 Se = \sigma\varepsilon \qquad \sigma\varepsilon = \frac{(K_f S)^2}{E}$$

- Step 2: calculate Δσ

the elastic-plastic form of Neuber's rule

$$E = 30\times10^3\,ksi,\ K' = 154\,ksi, n' = 0.123$$
$$K_t = 2.42$$

$$K_t^2 \frac{\Delta S \Delta e}{4} = \frac{\Delta\sigma\Delta\varepsilon}{4}$$

$$\frac{\Delta e}{2} = \frac{\Delta S}{2E} + \left(\frac{\Delta S}{2K'}\right)^{1/n'} \qquad \frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

$$K_t^2 \frac{\Delta S}{2}\left[\frac{\Delta S}{2E} + \left(\frac{\Delta S}{2K'}\right)^{1/n'}\right] = \frac{\Delta\sigma}{2}\left[\frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}\right]$$

Example 4.2

$$(2.42)^2(50)\left[\underbrace{\frac{50}{30\times10^3}}_{1.67e-3}+\underbrace{(\frac{50}{154})^{1/0.123}}_{1.07e-4}\right]=\frac{\Delta\sigma}{2}\left[\underbrace{\frac{\Delta\sigma}{2(30\times10^3)}}_{2.6e-3}+\underbrace{(\frac{\Delta\sigma}{2\times154})^{1/n'}}_{3.9e-3}\right] \Rightarrow \Delta\sigma=156 \text{ ksi}$$

Given load P ⇒ ΔS ⇒ Δσ ⇒ $\boxed{\Delta\varepsilon \Rightarrow N_f : \text{fatigue life}}$

- Step 3 : calculate Δε

$$\frac{\Delta\varepsilon}{2}=\frac{\Delta\sigma}{2E}+(\frac{\Delta\sigma}{2K'})^{1/n'}=0.0065$$

Fully reversed loading → mean stress =0

$$E=30\times10^3\ ksi$$
$$\sigma'_f=169\ kis \quad b=-0.081$$
$$\varepsilon'_f=1.14 \quad c=-0.67$$

- Step 4: calculate N_f.

$$\frac{\Delta\varepsilon}{2}=\frac{\sigma'_f}{E}(2N_f)^b+\varepsilon'_f(2N_f)^c \Rightarrow 2N_f=5000\ reversals$$