Ship Stability

Ch. 3 Transverse Stability Due to Cargo Movement

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Contents

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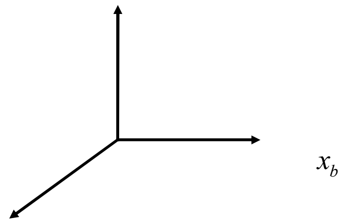
1. Rotational Transformation of a Position Vector to a Body in Fluid

System of Coordinates



n-frame: Inertial frame $x_n\, y_n\, z_n$ or $x\, y\, z$

Point E: Origin of the inertial frame(n-frame)

b-frame: Body fixed frame $x_b\, y_b\, z_b$ or $x'\, y'\, z'$

Point O: Origin of the body fixed frame(b-frame)

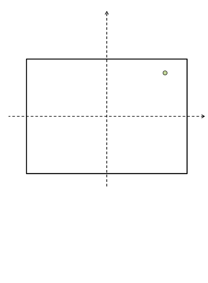
1) *coordinate system*

The right handed coordinate system with the axis called x_b(or x'), y_b(or y'), and z_b(or z') is **fixed to the object**. This coordinate system is called ***body fixed coordinate system*** or ***body fixed reference frame (b-frame)***.

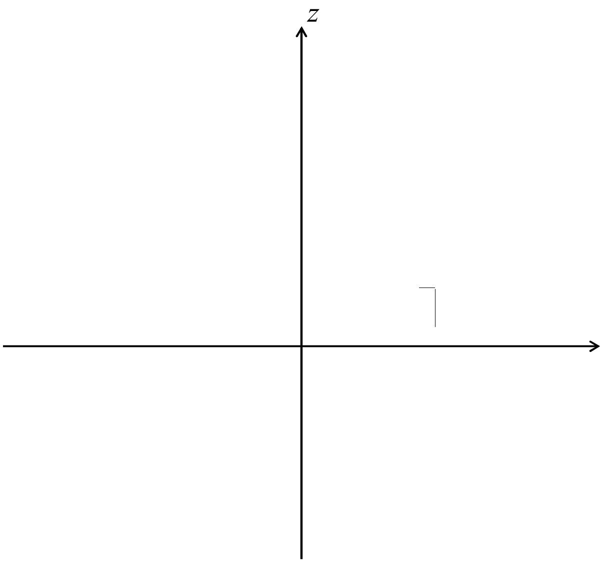
2) *coordinate system*

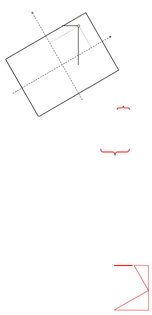
The right handed coordinate system with the axis called x_n(or x), y_n(or y) and z_n(or z) is **fixed to the space**. This coordinate system is called ***space fixed coordinate system*** or ***space fixed reference frame*** or ***inertial frame (n-frame)***.

In general, a change in the position and orientation of the object is described with respect to the inertial frame. Moreover Newton's 2nd law is only valid for the inertial frame.

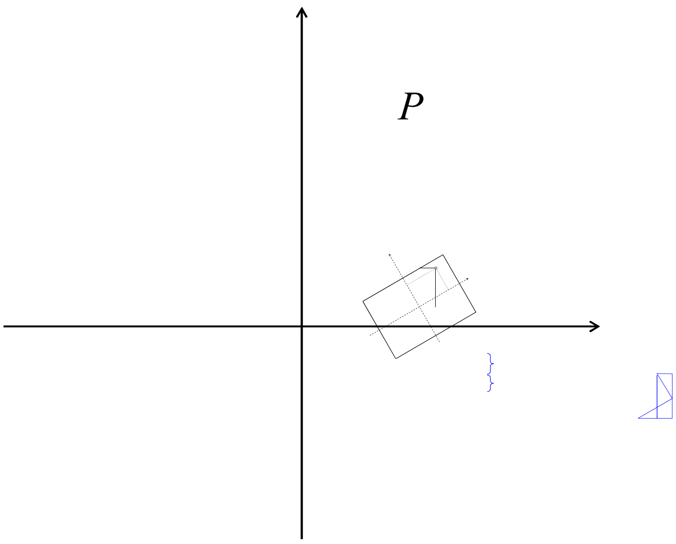


Rotation of the Object with an Angle of ϕ and then Representation of the Point "P" on the Object with Respect to the Inertial Frame





Coordinate Transformation of a Position Vector



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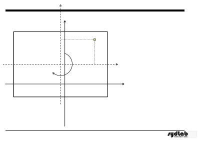
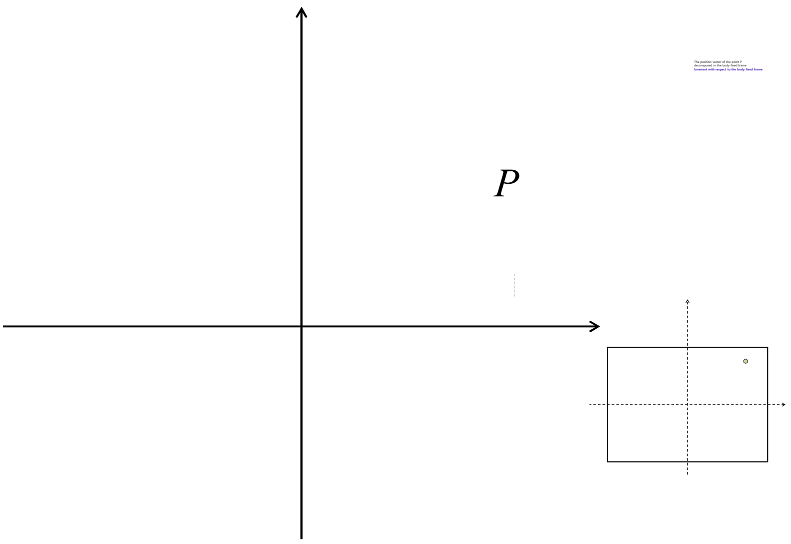
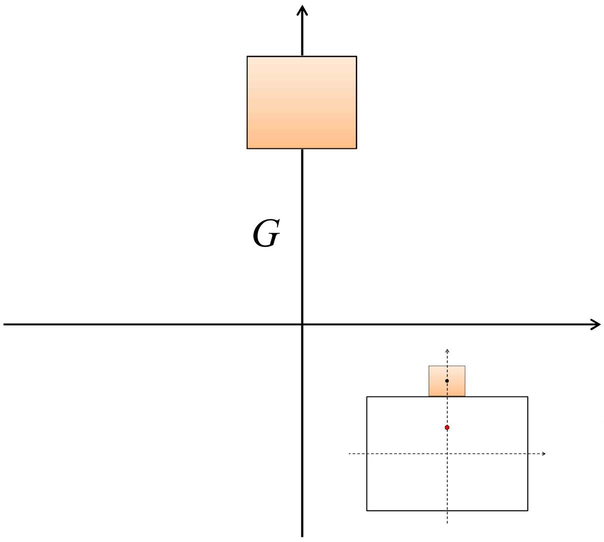
The position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame
The position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame



Representation of a Point "P" on the Object with Respect to the Body Fixed Frame (Decomposed in the Body Fixed Frame)



Change of the Total Center of Mass Caused by Moving a Load of Weight "w" with Distance "d" from "g" to "g_1"



"Change of
the center of mass"

The position vector of the changed total center of mass G_1 decomposed in the body fixed frame

, where

is the weight of the moving load

is total weight of the object.

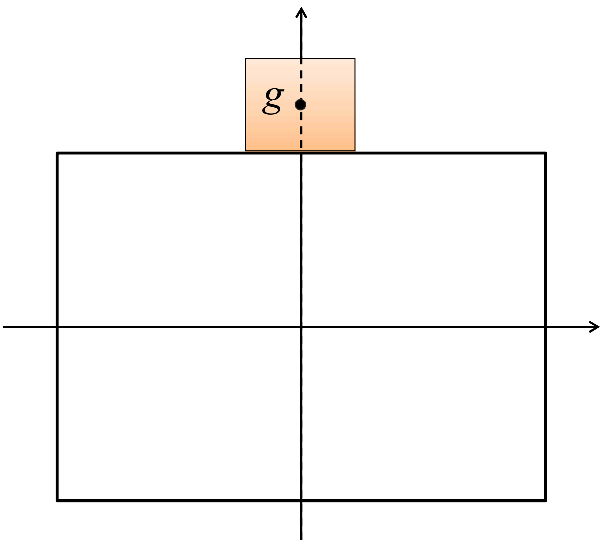
[Reference] Kinds of 2nd Moment

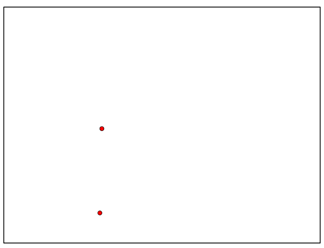
☑ 2nd Moment of Area = Area Moment of Inertia
- Area × Arm^2

☑ 2nd Moment of Volume = Volume Moment of Inertia
- Volume × Arm^2

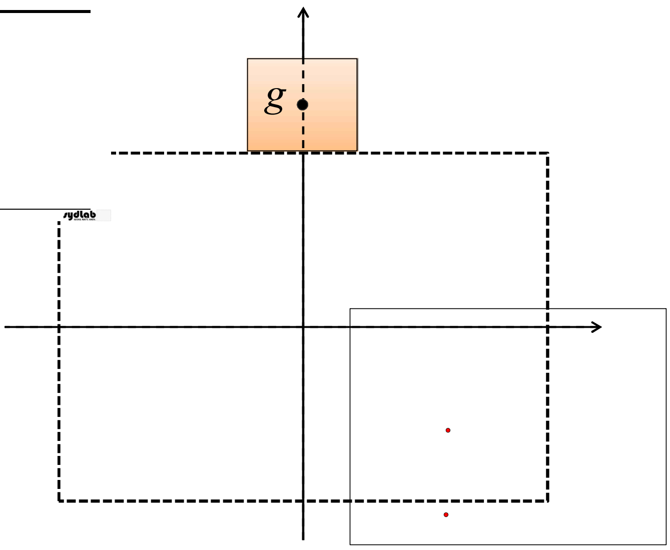
☑ 2nd Moment of Mass = Mass Moment of Inertia
- Mass × Arm^2

Change of the Total Center of Mass Caused by Moving a Load - Initial State

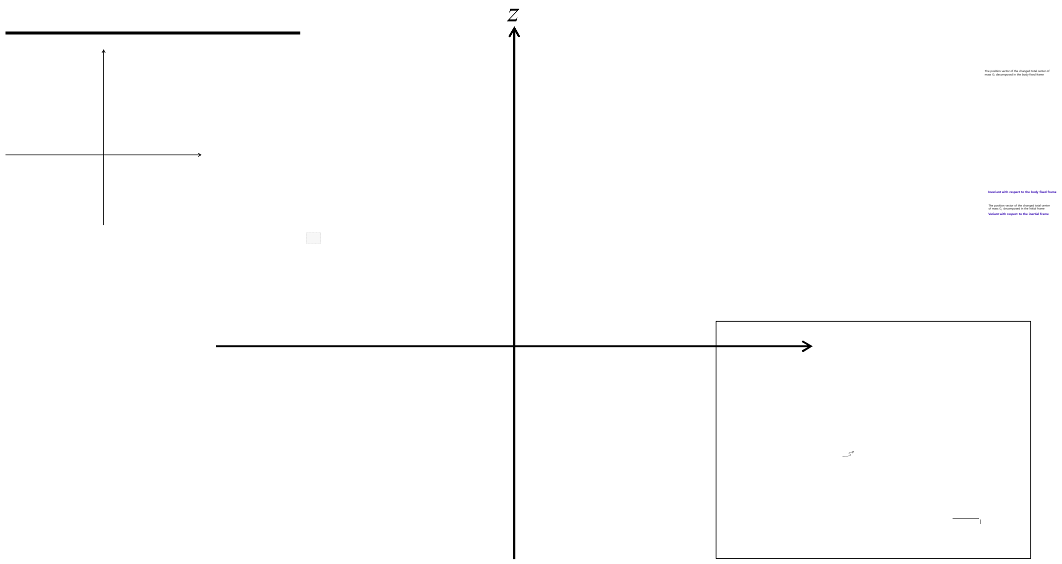


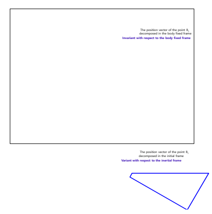


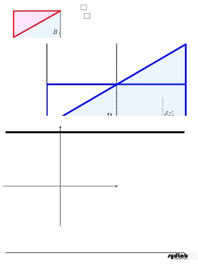
Change of the Total Center of Mass Caused by Moving a Load - Moving a Load



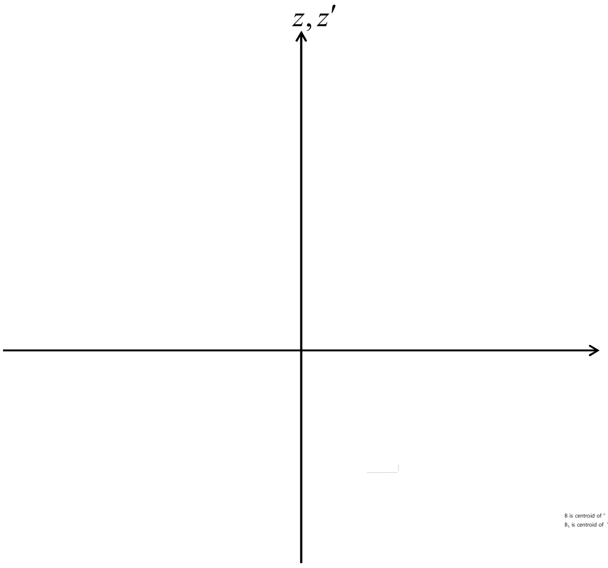
Rotation of the Object with an Angle of "-ϕ" and then Representation of the Total Center of Mass with Respect to the Inertial Frame

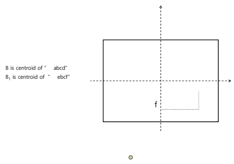






(1) Calculate the initial centroid "B" of the rectangle for z'<0 with respect to the body fixed frame.
(2) Then calculate new centroid "B_1" caused by moving a partial triangular area with respect to the body fixed frame.





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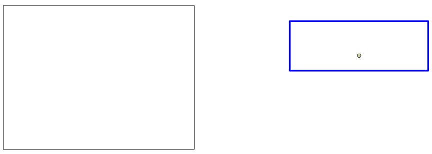
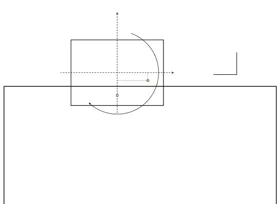
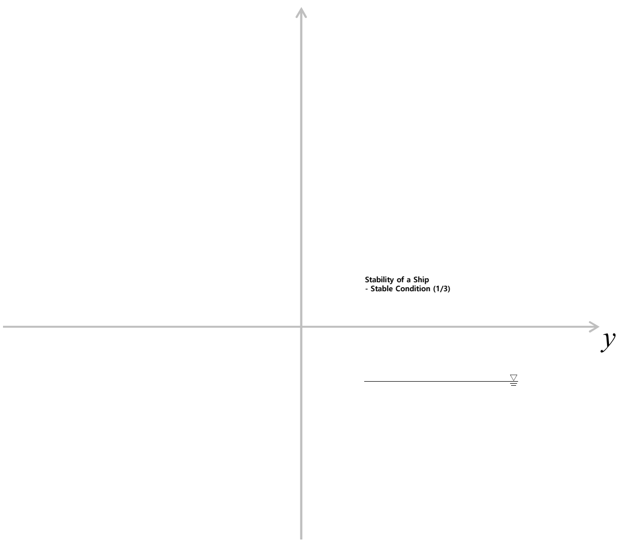




2. Application of Rotational Transformation of a Position Vector to a Body in Fluid

Stability of a Ship
- Stable Condition (1/3)

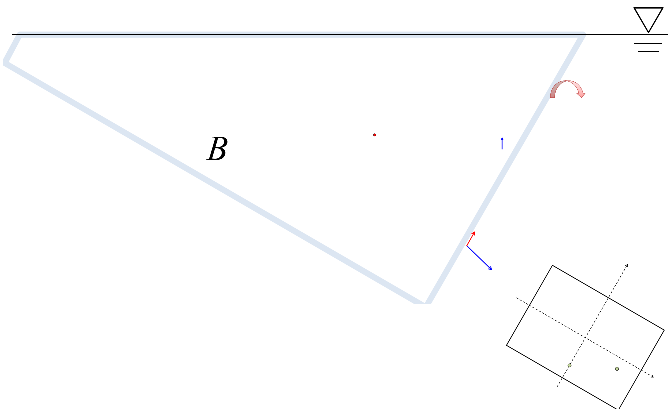


Heeling moment

Stability of a Ship
- Stable Condition (2/3)

Resultant moment abo
x-axis through point O

Heeling
moment



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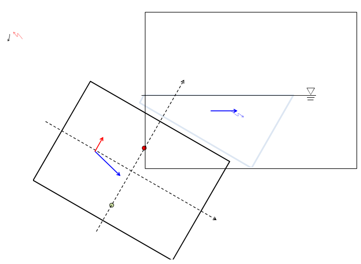
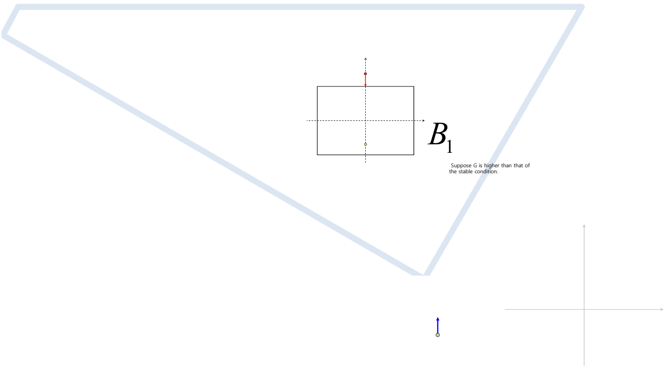
Resultant moment abo
x-axis through point O

Stability of a Ship
- Stable Condition (3/3)



Stability of a Ship
- Neutral Condition (1/3)

Heeling moment



Suppose G is higher than that of the stable condition.

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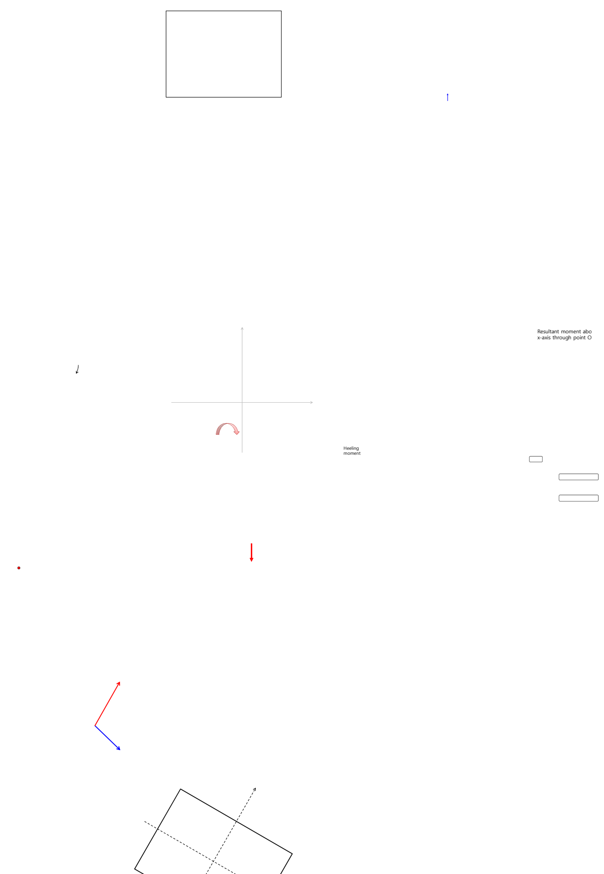
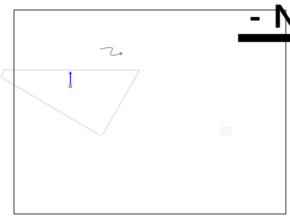


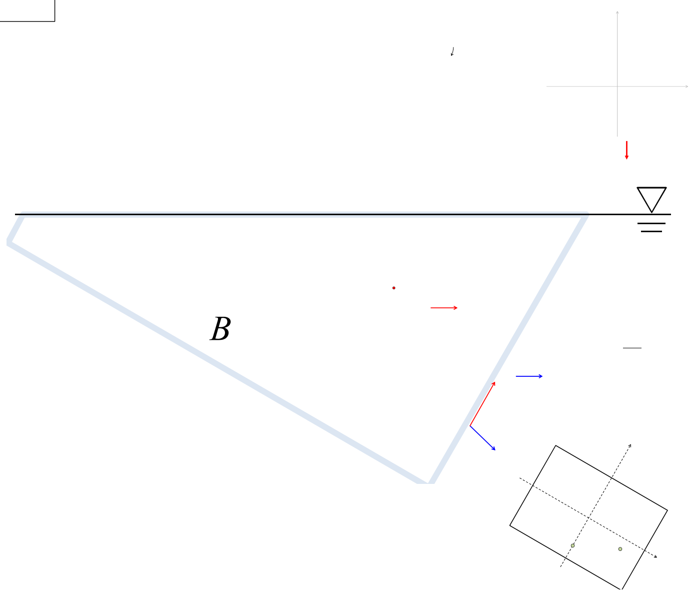
Resultant moment abo
x-axis through point O

Heeling moment

Stability of a Ship
- Neutral Condition (3/3)



If G and B_1 are on on
calculate resultant mo
x-axis through point O



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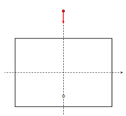
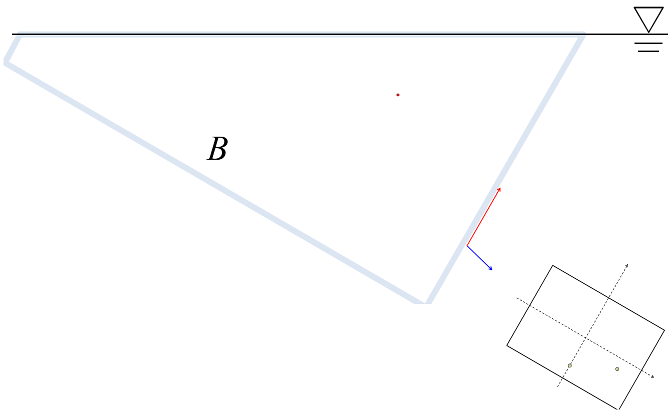
Stability of a Ship
- Unstable Condition (1/3)

Heeling moment



Stability of a Ship
- Unstable Condition (2/3)



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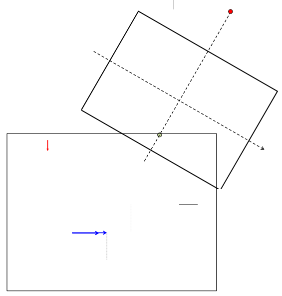
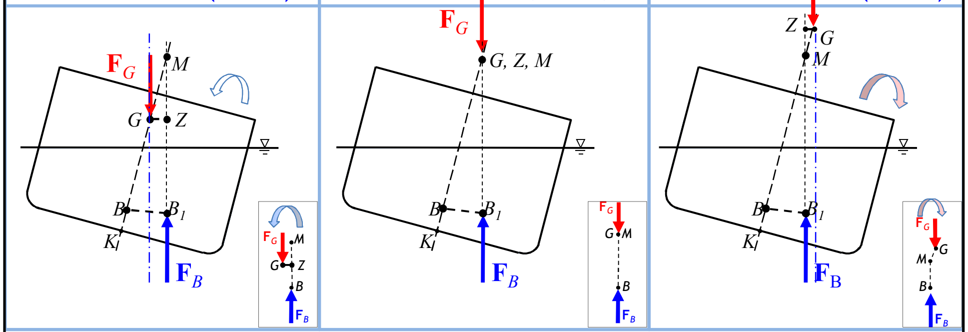


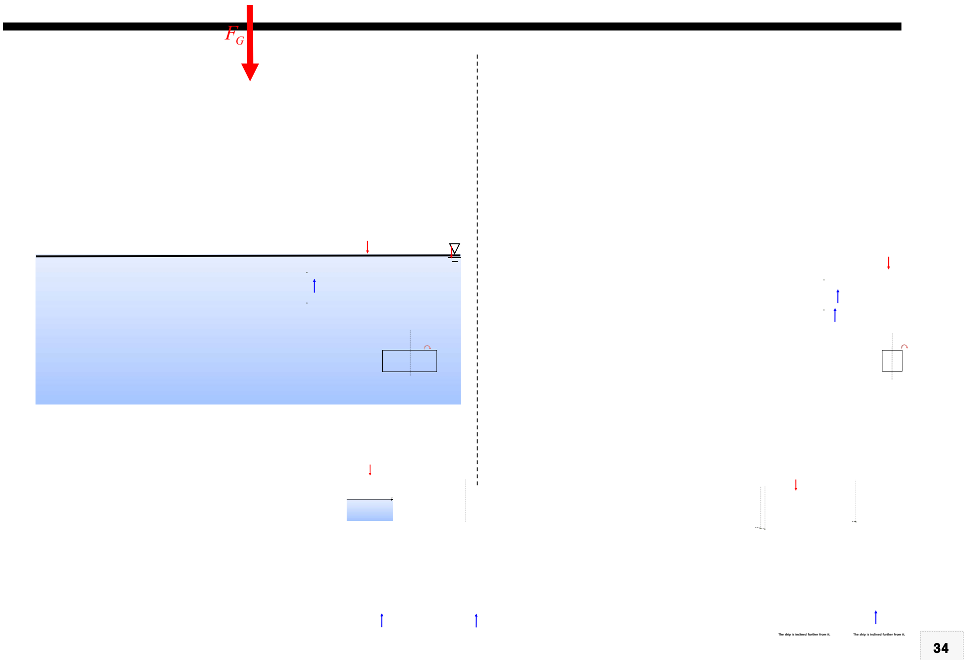
Stability of a Ship According to Relative Position between "G", "B", and "M" at Small Angle of Inclination

- **Righting (Restoring) Moment:** Moment to return the ship to the upright floating position
- **Stable / Neutral / Unstable Condition:** Relative height of G with respect to M is one measure of stability.

• Stable Condition ( ) • Neutral Condition ( ) • Unstable Condition ( )



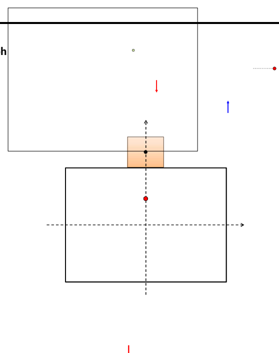
G: Center of mass *K*: Keel
B: Center of buoyancy at upright position B_1: Changed center of buoyancy
F_G : Weight of ship F_B: Buoyant force acting on ship
Z: The intersection of the line of buoyant force through B_1 with the transverse line through G
M: The intersection of the line of buoyant force through B_1 with the centerline of the ship

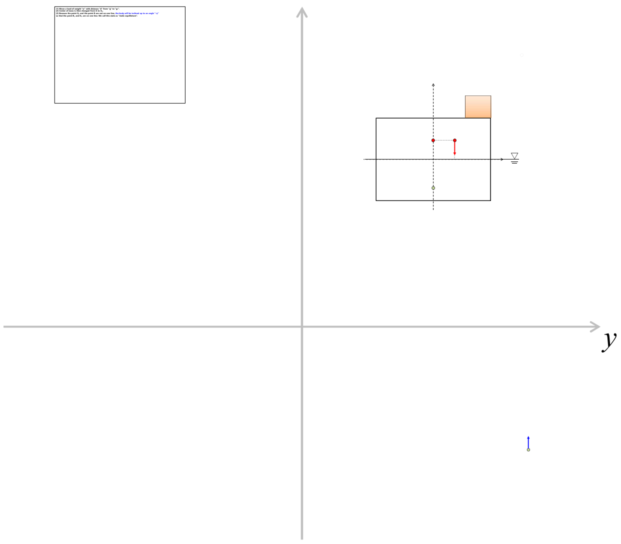


3. Calculation of the Inclination Angle Caused by Moving a Load

(1) Move a load of weight "w" with distance "d" from "g" to "g_1".
(2) Center of mass is then changed from G to G_1.
(3) Because the point G_1 and the point B are not on one line, the body will be inclined up to an angle "$-\phi$" so that the point B_1 and G_1 are on one line. We call this state as "static equilibrium".

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$





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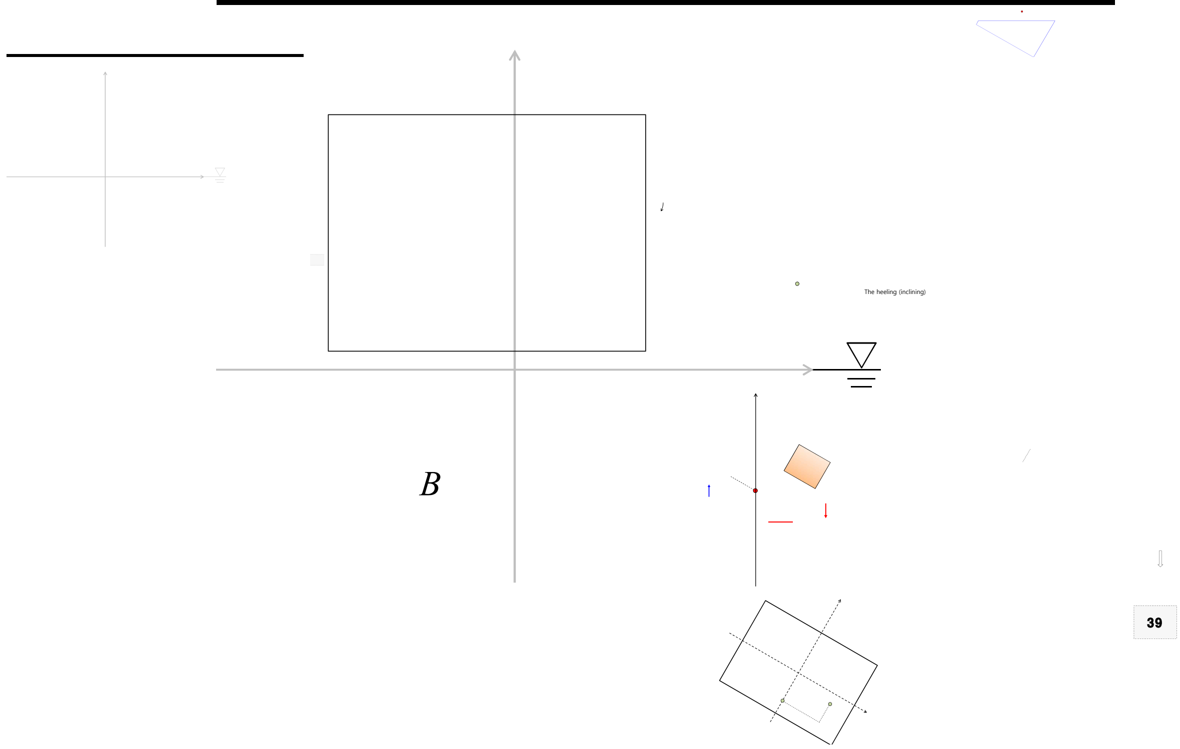
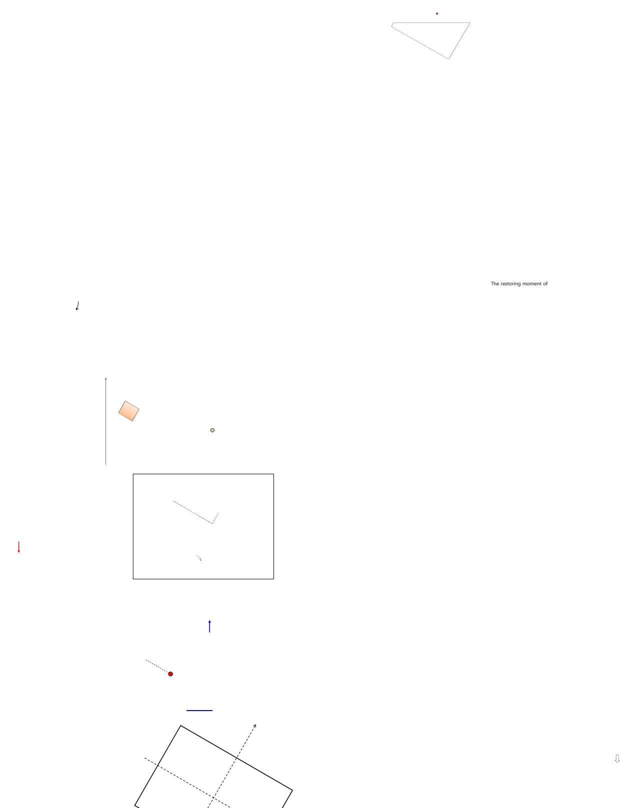
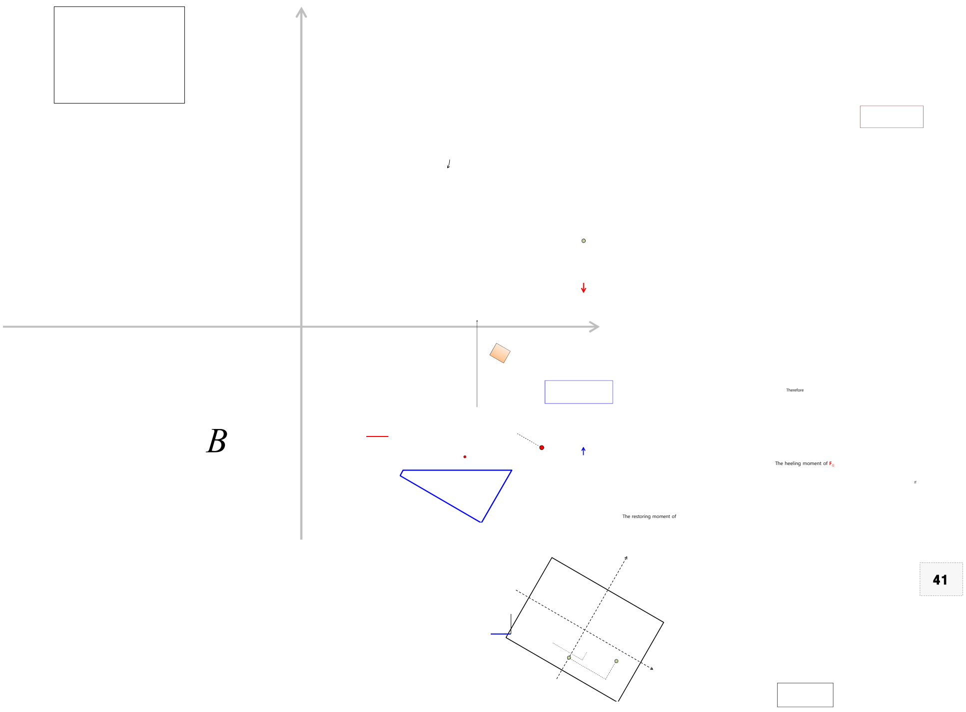
Calculation of the Heeling(Inclining) Angle
- Heeling (Inclining) Moment



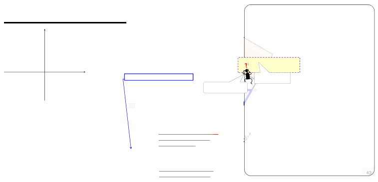


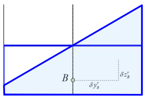
The Heeling Angle can be Calculated by Equating the Heeling Moment and the Restoring Moment in "Static Equilibrium". (1/5)



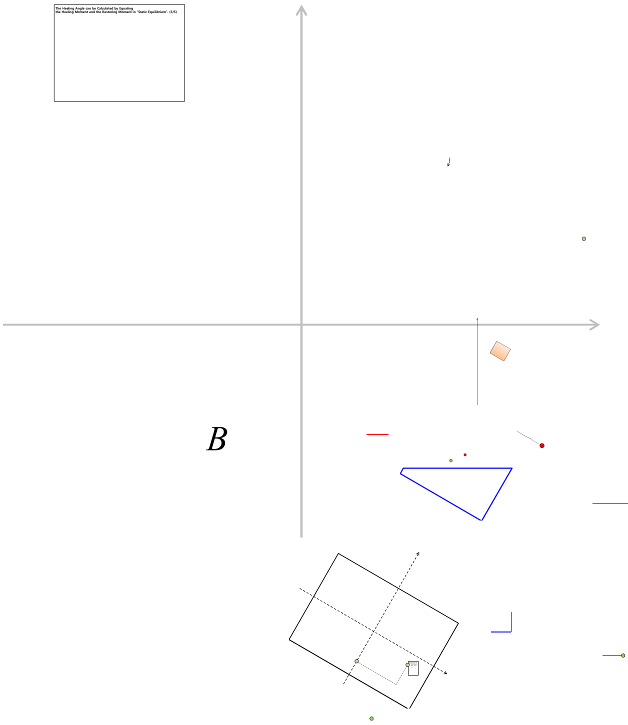
1.

To calculate GZ, and should be known.





The Heeling Angle can be Calculated by Equating
the Heeling Moment and the Restoring Moment in "Static Equilibrium". (3/5)



1. M does not change at small ar
inclination. (about 7° to 10°)
2. For wall sided ship, the subme
volume and emerged volume
3. Main deck is not submerged a
bottom is not emerged.

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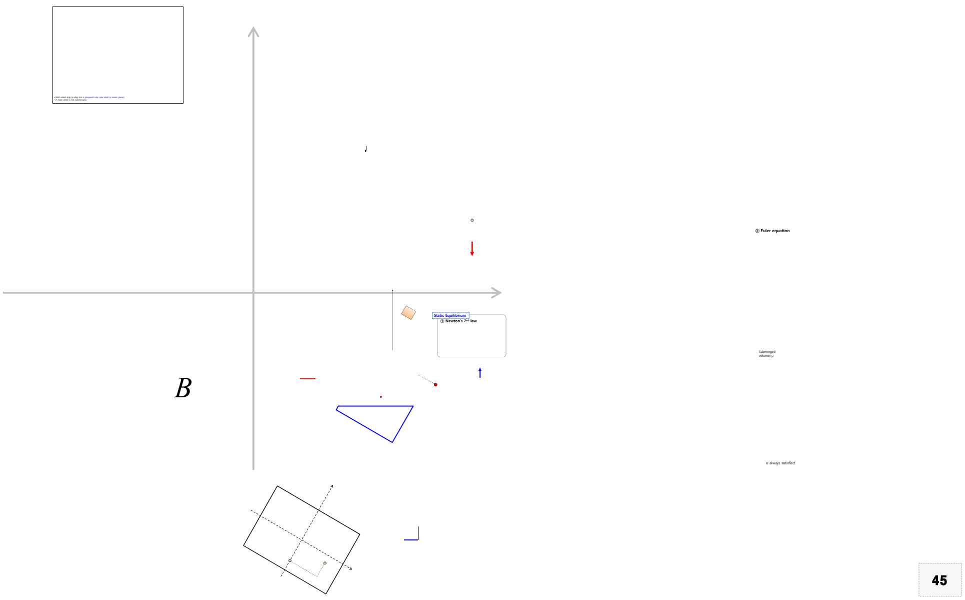
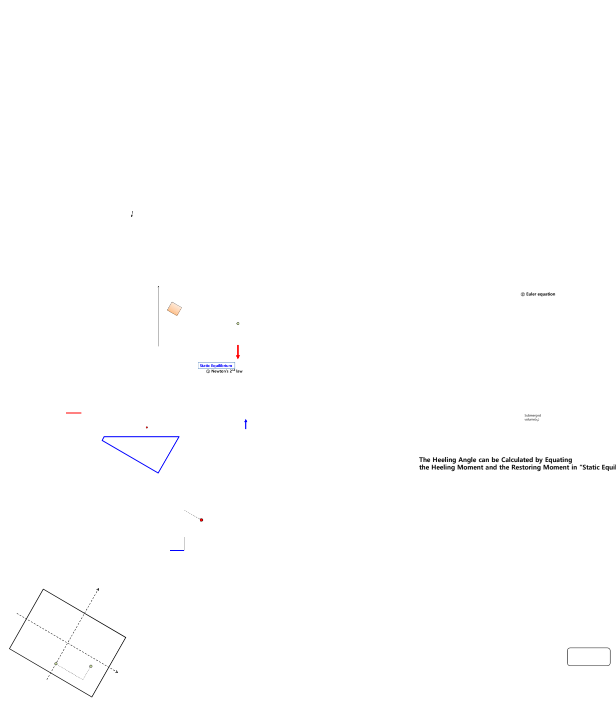
The heeling moment of W





The Heeling Angle can be Calculated by Equating the Heeling Moment and the Restoring Moment in "Static Equilibrium". (5/5)

Numerical Method for Solving Nonlinear Equation (1/11)

$$f_1(x) = 0$$

: Nonlinear function of one variable

Assumption

Find:

, where

Given:

Taylor series expansion of at

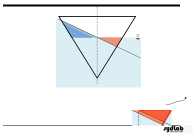
Find:

, where

Numerical Method for Solving Nonlinear Equation (2/11)

: Nonlinear function of one variable

L.H.S



$$f_1(x)=0$$

Given:

Find:

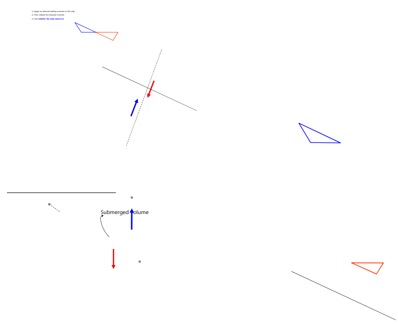
, where

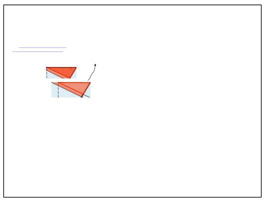
Numerical Method for Solving Nonlinea

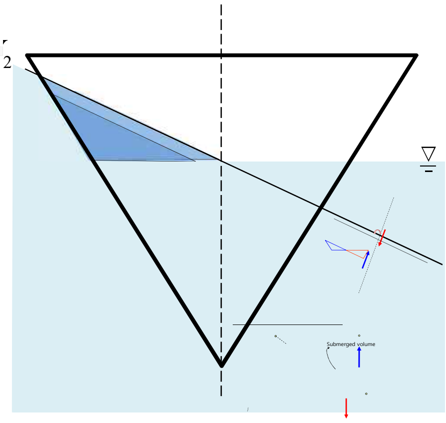
: Nonlinear function of one variable

Known

Check







Static Equilibrium

① Newton's 2nd law

② Euler equation

In this case, the submerged volume and immerged volume are not same at the heeling angle "Φ". And the draft "z" should be adjusted to maintain same displacement.

It means that the following equations should be satisfied.

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: Nonlinear functions of two variables

Assumption

Find:

, where

Numerical Method for Solving Nonlinear Equation (4/11)

Numerical Method for Solving Nonlinear Equation (6/11)

$$f_1(x_1, x_2) = 0, \quad f_2(x_1, x_2) = 0$$

Given:

: Nonlinear functions of two variables

Find:

, where

Assumption:

Taylor series expansion of at

Linearization

Given:

Find:

Numerical Method for Solving Nonlinear Equation (6/11)

Assumption:

, where

: Nonlinear functions of two variables

Given:

$$f_1(x_1, x_2) = 0, \quad f_2(x_1, x_2) = 0$$

Find:

: Nonlinear functions of two variables

Numerical Method for Solving Nonlinear Equation (7/11)

, where

Assumption:

Matrix form

Numerical Method for Solving Nonlinear Equation (8/11)

Given:

Find:

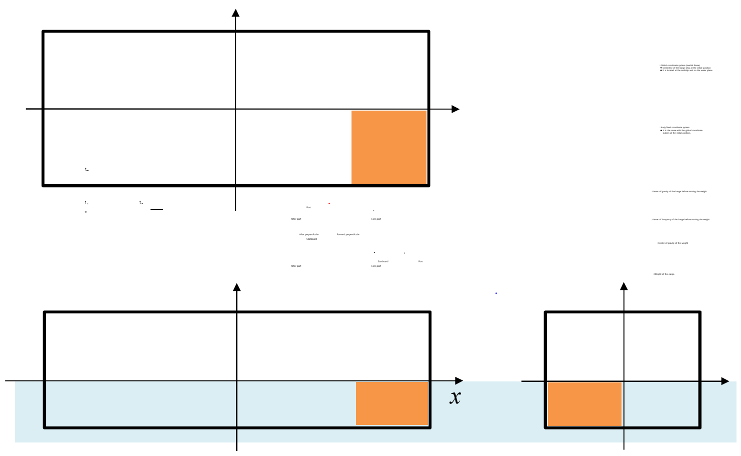
Assumption:

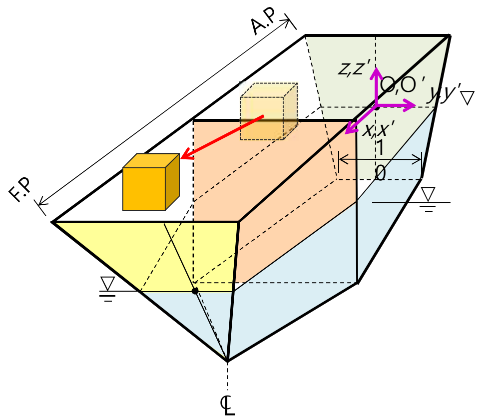
, where

: Nonlinear functions of two variables

Known

Calculation of Position and Orientation of a Barge When Cargo is Moved (1/2)





x

Numerical Method for Solving Nonlinear Equation (9/11)

Find:

Assumption $z^* = z^{(0)} + \delta z^{(0)},\ \phi^* = \phi^{(0)} + \delta\phi^{(0)},\ \theta^* = \theta^{(0)} + \delta\theta^{(0)}$

, where

Given:

✓ Governing equation of hydrostatic equilibrium

Find:

Known

where

Assumption

✓ Governing equation of hydrostatic equilibrium

Given:

Find:

$$\begin{bmatrix} -F\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\ -M_T\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\ -M_L\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial \phi} & \frac{\partial F}{\partial \theta} \\ \frac{\partial M_T}{\partial z} & \frac{\partial M_T}{\partial \phi} & \frac{\partial M_T}{\partial \theta} \\ \frac{\partial M_L}{\partial z} & \frac{\partial M_L}{\partial \phi} & \frac{\partial M_L}{\partial \theta} \end{bmatrix}_{z^{(0)},\phi^{(0)},\theta^{(0)}} \begin{bmatrix} \delta z^{(0)} \\ \delta \phi^{(0)} \\ \delta \theta^{(0)} \end{bmatrix}$$

Numerical Method for Solving Nonlinear Equation (11/11)

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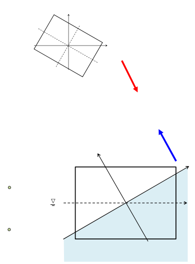
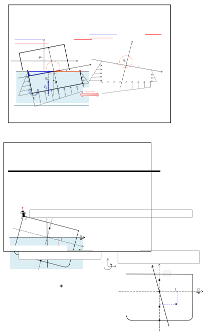


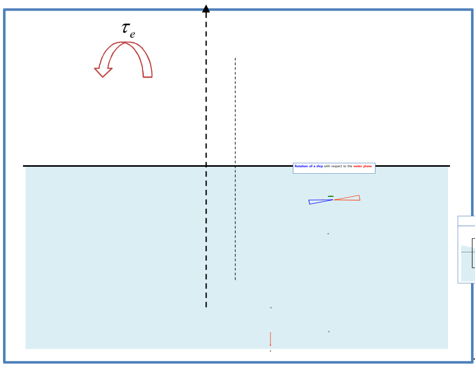
(a)



Orientation of a Ship with Respect to the Different Reference Frame (2/2)

Inclination of a ship can be represented either with respect to the **water plane fixed frame ("inertial reference frame")** or the **body fixed reference frame**.

Are these two phenomena with respect to the different reference frames the same?



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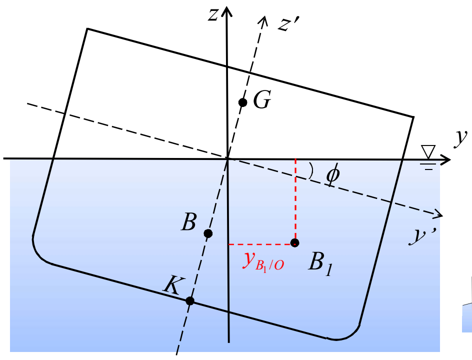

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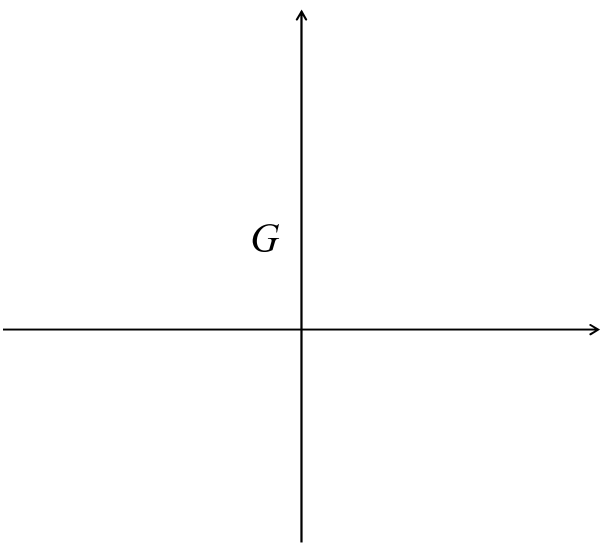
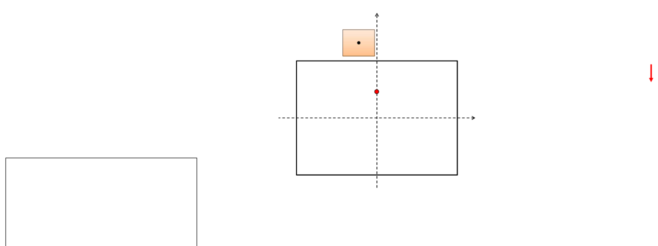
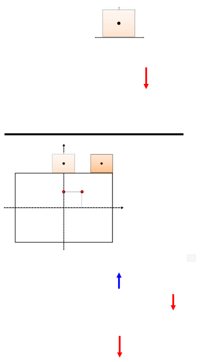
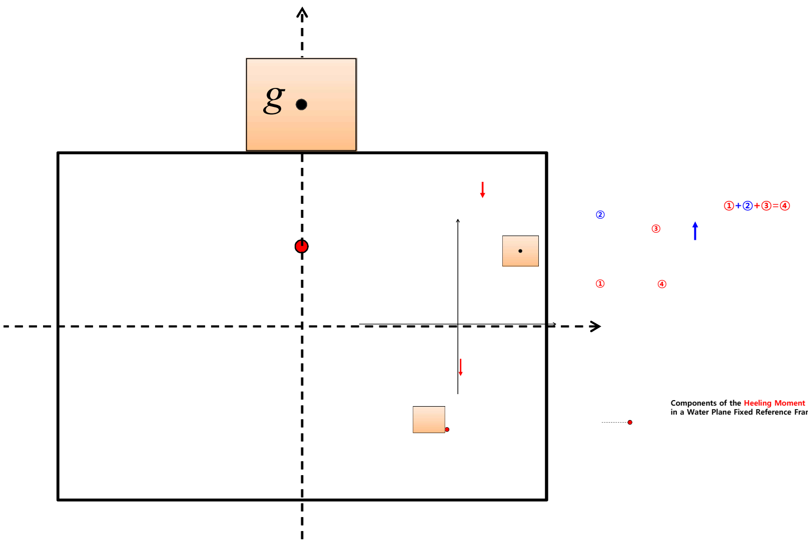
4. Components of the Heeling and Restoring Moment Described in the Water Plane Fixed Frame and the Body Fixed Frame

Components of the Heeling Moment Described in a Water Plane Fixed Reference Frame (1/3)









Components of the Heeling Moment Described in a Water Plane Fixed Reference Frame (2/3)

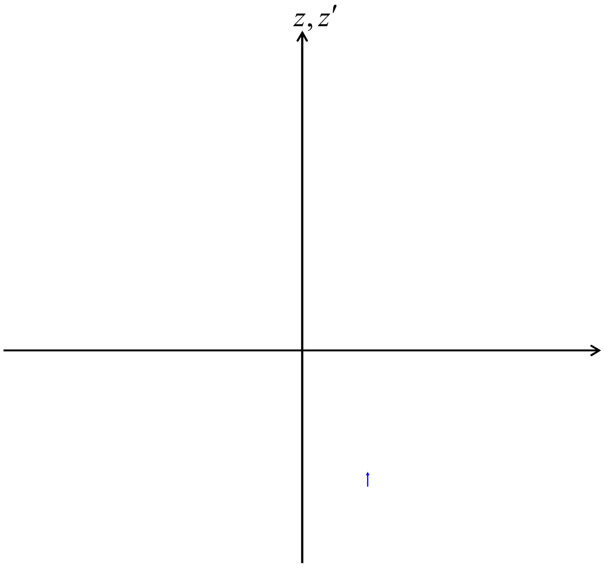
71

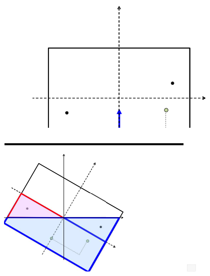
②

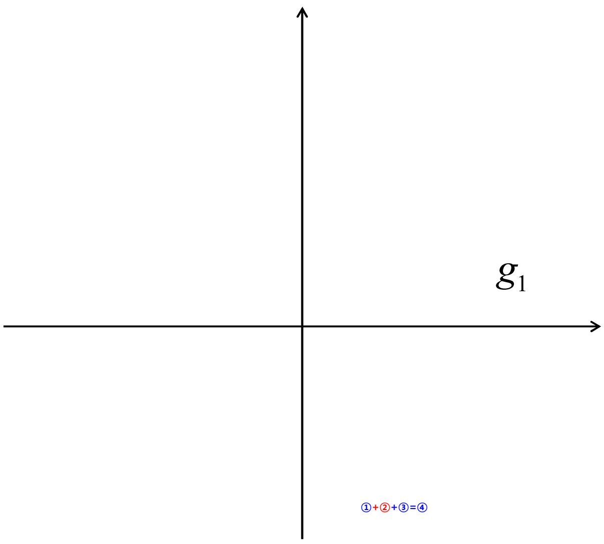
③

①+②+③=④

Components of the Restoring Moment Described in the Water Plane Fixed Reference Frame (1/4)







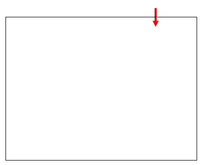
③

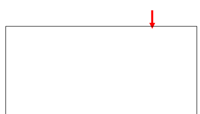
①+②+③=④

④

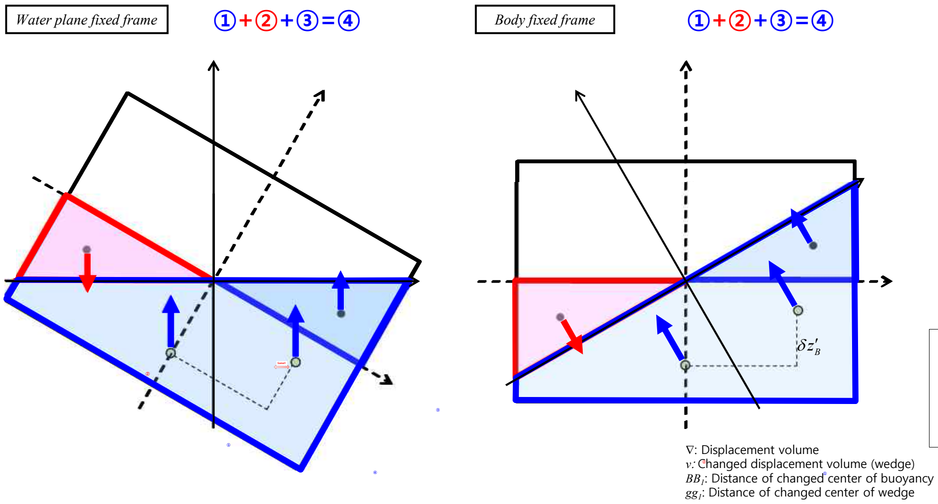
Components of the Restoring Moment Described in the Water Plane Fixed Reference Frame (3/4)

75



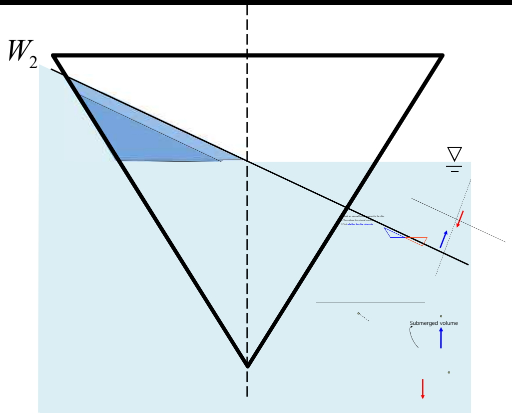


Moment Described in the Water Plane Fixed Reference Frame and the Body Fixed Reference Frame



The moments described in the water plane fixed reference frame can be described in the body fixed reference frame by decomposing the forces and moment arms in the body fixed reference frame.

If the Submerged Volume and the Emerged Volume are not Same, the Draft should be Adjusted (1/3)

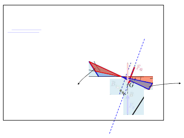


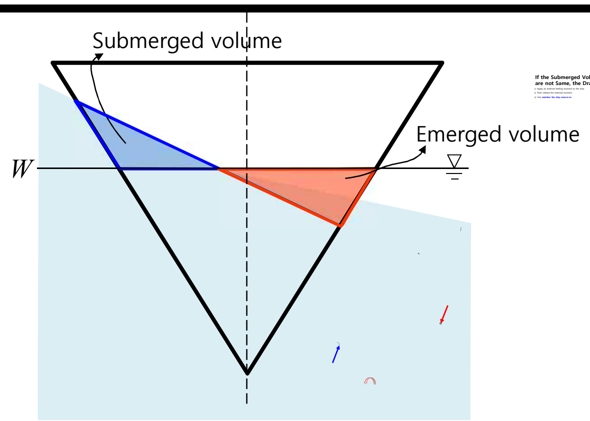
For the ship to be in static equilibrium, the magnitude of the buoyant force has to be equal to the magnitude of the weight: .

① A ship with triangular section shape is inclined with an angle of .

② In this case, the and the are not same.

③ Thus, the draft should be adjusted to maintain same displacement (volume).





If the Submerged Volume and the Emerged Volume are not Same, the Draft should be Adjusted (3/3)

For the ship to be in static equilibrium, the buoyant force and gravitational force have to be on one line, so that the total moment about the transverse axis through any point becomes 0:

⑤ The submerged volume and the emerged volume are not same.

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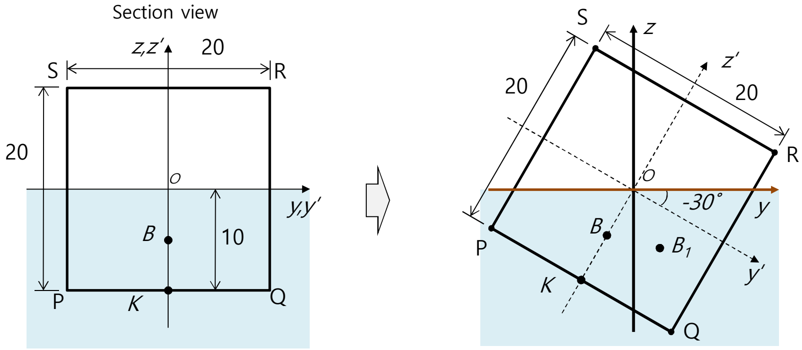
Method ① Direct Calculation of Center of Buoyancy with Respect to the Water Plane Fixed Frame (1/6)

Example) A ship is inclined about x-axis through origin O with an angle of -30°. Calculate center of buoyancy with respect to the water plane fixed frame.

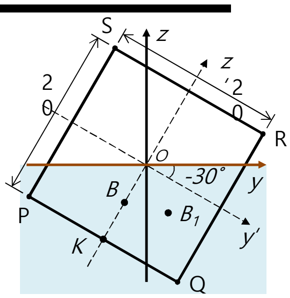
- Given: Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30˚
- Find: Center of buoyancy(y_{B1}, z_{B1})

G: Center of mass K: Keel
B: Center of buoyancy B_1: Changed center of buoyancy



Example) A ship is inclined about x-axis through origin O with an angle of -30°. Calculate center of buoyancy with respect to the water plane fixed frame.

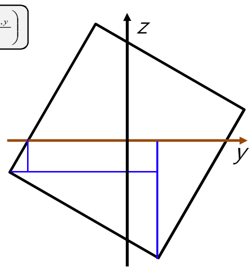
Sol.) P, Q, R, and S with respect to the water plane fixed frame



$P'(y'_P, z'_P)$

Sol.) Area

$$(y_{B_1}, z_{B_1}) = \left(\frac{M_{A,z}}{A}, \frac{M_{A,y}}{A} \right)$$



Method ① Direct Calculation of Center of Buoyancy with Respect to the Water Plane Fixed Frame (4/6)

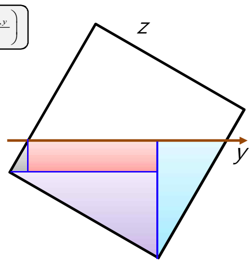
Example) A ship is inclined about x-axis through origin O with an angle of -30°. Calculate center of buoyancy with respect to the water plane fixed frame.

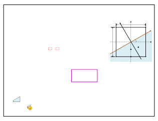
- Given: Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30°
- Find: Center of buoyancy(y_{B1}, z_{B1})

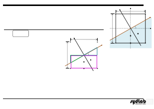
G: Center of mass K: Keel
B: Center of buoyancy B_1: Changed center of buoyancy

Sol.) Centroid

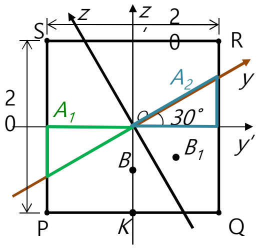
$$(y_{B_1}, z_{B_1}) = \left(\frac{M_{A,z}}{A}, \frac{M_{A,y}}{A}\right)$$

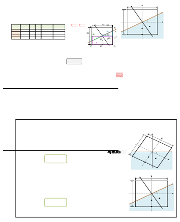






Sol.) Area $(y'_{B_1}, z'_{B_1}) = \left(\frac{M'_{A,z'}}{A}, \frac{M'_{A,y'}}{A}\right)$





Sol.) 1st moment of area $(y'_{B_1}, z'_{B_1}) = \left(\frac{M'_{A,z'}}{A}, \frac{M'_{A,y'}}{A} \right)$

The 1st moments and centroid of the total area are calculated as follows.

Centroid of total area with respect to the body fixed frame

Calculation of Center of Buoyancy of Ship with Constant Section
- Comparison of the Results of Method ① and Method ②

Calculation Result

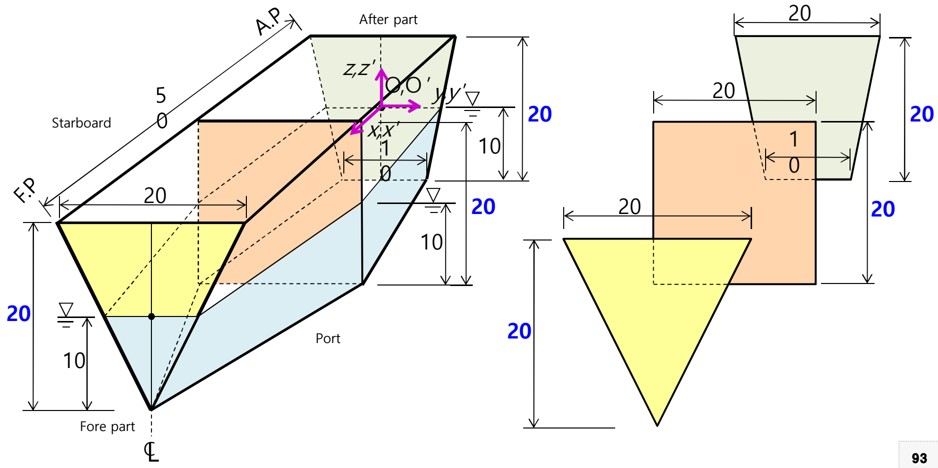
Same

Calculation Result

[Example] Calculation of Center of Buoyancy (1/17)

A ship with three varied section shape is given. When this ship is inclined about x axis with an angle of -30° at an intermediate state, calculate y and z coordinates of the center of buoyancy (with respect to the water plane fixed frame).

- Given: Length(L) 50m, Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30˚
- Find: Center of buoyancy($y_{\nabla,\phi}$, $z_{\nabla,\phi}$) after heeling



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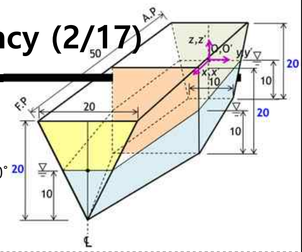
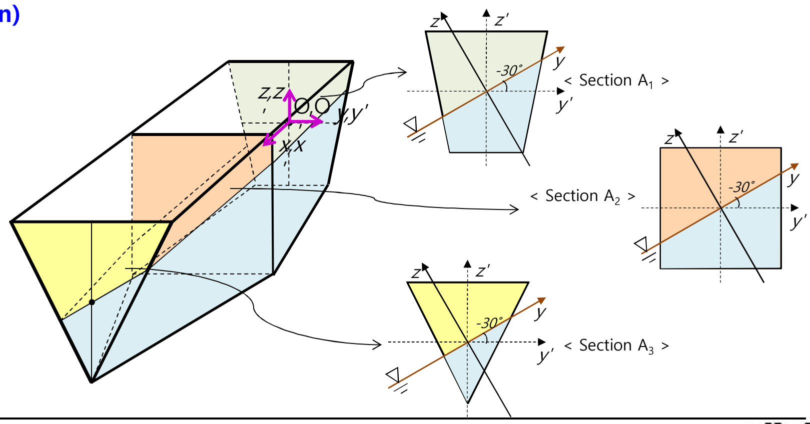
[Example] Calculation of Center of Buoyancy (2/17)



- Given: Length(L) 50m, Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30˚
- Find: Center of buoyancy($y_{\nabla,\phi}$, $z_{\nabla,\phi}$) after heeling

Solution)



Naval Architectural Calculation, Spring 2018, Myung-Il Roh

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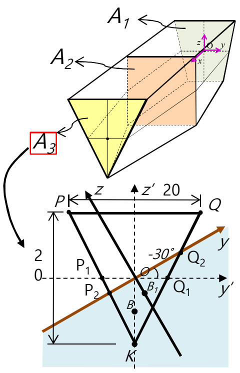


< Section A_3 >

In the same way of previous example, calculate center of buoyancy with respect to the body fixed frame at first, then transform the center of buoyancy to that with respect to the water plane fixed frame. (Method ②)

Coordinates of P_1, P_2, Q_1, and Q_2 of section A_3

- Coordinates of P_1, Q_1 are given (-5,0), (5,0) by geometric shape.

To find P_2 and Q_2, calculate equations of straight lines PK and KQ.

The equation of straight line PK $z' = -2y' - 10$

Solution)

②-A_3: 1st moment of area of section A_3

• Calculate 1st moment of area to obtain the centroid of section A_3 with respect to the body fixed frame.

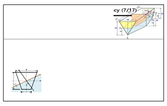
	Area			Area×$y'_{c\_3\_i}$	Area×$z'_{c\_3\_i}$
① A_{3_0}	50.00	0.00	-3.33	0.00	-166.67
② A_{3_1}	-5.60	-2.96	-0.75	16.57	4.18
③ A_{3_2}	10.15	4.01	1.35	40.69	13.73
①-②+③	54.55			57.26	-148.76

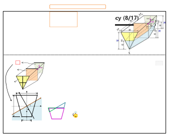
• Centroid of section A_3 with respect to the body fixed frame is calculated as follows.

$$(y'_{c\_3}, z'_{c\_3}) = \left(\frac{M_{A_3,z'}}{Area_{A_3}}, \frac{M_{A_3,y'}}{Area_{A_3}} \right)$$

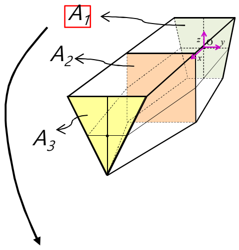
$$= \left(\frac{57.26}{54.55}, \frac{-148.76}{54.55} \right) = (1.05, -2.73)$$

[Example] Calculation of Center of Buoyancy (6/17)





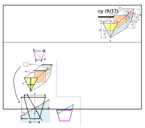
Solution)

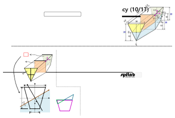


Coordinates of R_1, R_2, S_1, and S_2 of section A_1

- Coordinates of R_1 and S_1 are given as (-7.5, 0), (7.5 ,0) by geometrical shape.
- Calculate equations of straight lines RR_3 and SS_3 to find R_2 and S_2.

The equation of straight line RR_3 $\quad z' = -4y' - 30$





Centroid of section A_1
(We divide the trapezoid into two triangles)

[Example] Calculation of Center of Buoyancy (9/17)

$$\text{Cetroid of } A_{1\_0} = \left(\frac{-2.5\times 50 + 1.67\times 75}{125}, \frac{-6.67\times 50 - 3.33\times 75}{125}\right)$$
$$= (0, -4.67)$$

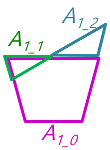
②- A_1: 1st moment of area of section A_1

- Calculate the 1st moment of area in order to know centroid of section A_1 with respect to the body fixed frame.

	Area			$M_{A_1,z'}$ Area×$y'_{c\_1\_i}$	Area×$z'_{c\_1\_i}$
① A_{1_0}	125.00	0.00	-4.67	0.00	-583.34
② A_{1_1}	-14.19	-4.68	-1.26	66.48	17.90
③ A_{1_2}	18.98	5.42	1.69	102.90	32.02
①-②+③	129.79			169.38	-533.42

: 1st moment of area of the section A_1 about the z' axis in y' direction

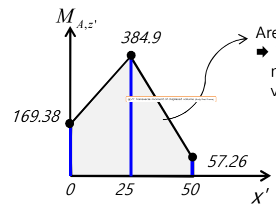
: 1st moment of area of the section A_1 about the y' axis in z' direction



- Centroid of section A_1 with respect to the body fixed frame is calculated as follows.

Solution)

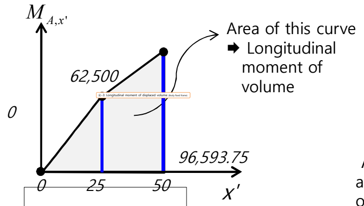
[Example] Calculation of Center of Buoyancy (13/17)



After finding each transverse moment of the sectional area about the z′ axis($M_{A,y}$), the transverse moment of the displaced volume can be obtained by integration of the transverse moment of the sectional area over the length of ship.

[Example] Calculation of Center of Buoyancy (14/17)

[Example] Calculation of Center of Buoyancy (15/17)



After finding each longitudinal moment of the sectional area about the x′ axis($M_{A,x'}$), the longitudinal moment of the displaced volume can be obtained by integration of the transverse moment of the sectional area over the length of ship.

[Example] Calculation of Center of Buoyancy (16/17)

Solution)

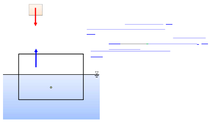
· Center of buoyancy with respect to the water plane fixed frame, then, can be obtained by rotational transformation.

⑥ Center of buoyancy (Water plane fixed frame)

[Example] Calculation of Center of Buoyancy (17/17)

$$\begin{pmatrix} y_{\nabla,c} \\ z_{\nabla,c} \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} 1.71 \\ -4.21 \end{pmatrix}$$

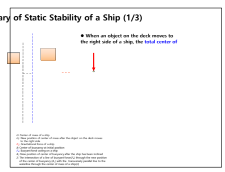
$$= \begin{pmatrix} \cos(-30) & -\sin(-30) \\ \sin(-30) & \cos(-30) \end{pmatrix} \begin{pmatrix} 1.71 \\ -4.21 \end{pmatrix} = \begin{pmatrix} -0.63 \\ -4.50 \end{pmatrix}$$

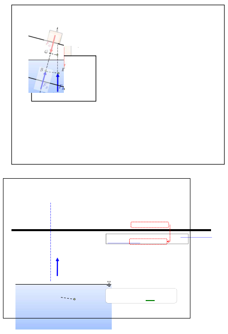


7. Summary

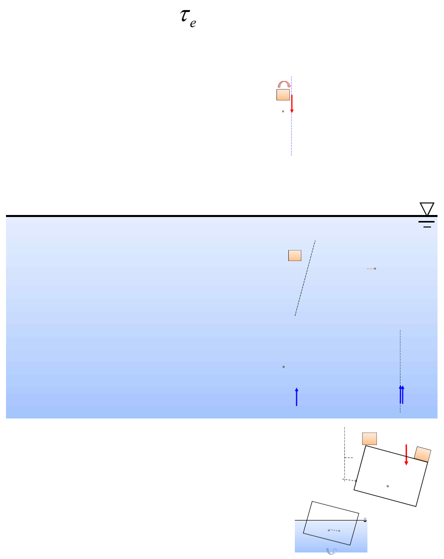
G

● Because the buoyant force and the gravitational force are not on one line, the





Summary of Static Stability of a Ship (2/3)



● The total moment will only be zero when the buoyant force and the gravitational force are on one line. If the moment becomes zero, the ship is in **static equilibrium state**.

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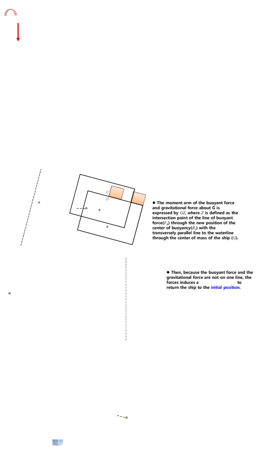
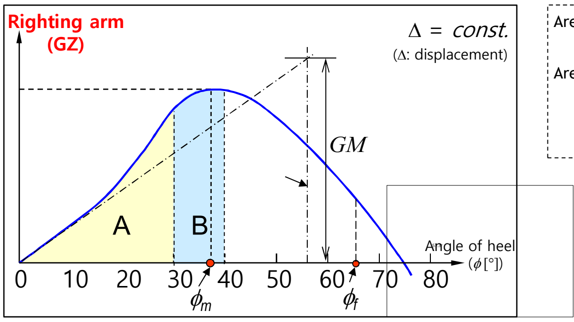


Evaluation of Stability
: Merchant Ship Stability Criteria – IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.3.1)

☑ **IMO recommendation on intact stability for passenger and cargo ships.**



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f: Heel angle at which openings in the hull

ϕ_m: Heel angle of maximum righting arm

※ After receiving approval of calculation of IMO regulation from Owner and Classification Society, ship construction can proceed.

IMO Regulations for Intact Stability

(a) Area A ≥ 0.055 m-rad

(b) Area A + B ≥ 0.09 m-rad

(c) Area B ≥ 0.030 m-rad

→ **The work and energy considerations (dynamic stability)**

(d) GZ ≥ 0.20 m at an angle of heel equal to or greater than 30°

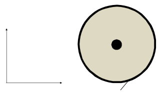
(e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25°.

(f) The initial metacentric height GM_o should not be less than 0.15 m.

→ **Static considerations**

<1st moment of area>

Area$_a$



Movement of Centroid Caused by Movement of Area (1/3)

Let us consider 1st moment of area about z axis through origin g.

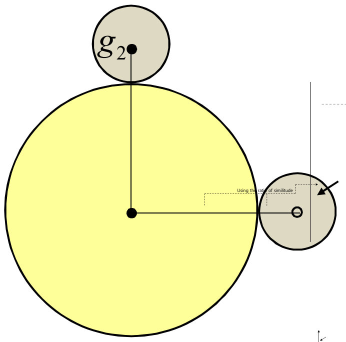
… ①

G_1 : Centroid of total area, $Area_g$: Total area
g : Centroid of the large circle, $Area_{A\text{-}a}$: Area of the large circle
g_1 : Centroid of the small circle, $Area_a$: Area of the small circle

Movement of Centroid Caused by Movement of Area (2/3)

… ②

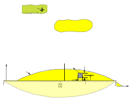
… ③



From ①, ②, ③,

Triangle $\triangle G_1 g G_2$ and $\triangle g_1 g g_2$ are similar.
(by SAS(Side-Angle-Side) similarity theorem)

G_1 : Centroid of total area, Area_A : Total area
g : Centroid of the large circle, Area_{A-a} : Area of the large circle
g_1 : Centroid of the small circle, Area_a : Area of the small circle



[Reference] Area, Moments, Centroid, and Moments of Inertia

- Transverse Moment of Inertia (I_T)

$$x_1 \to 0,\ x_2 \to L,\ f_1(x) \to 0,\ f_2(x) \to b(x)$$

On the other side :

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