## Chapter 4

## Image Measurements and Refinements

## Elements of Photogrammetry with Applications in GIS

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## 1. Introduction

- Rectangular coordinates of imaged points are the most common type of photographic measurements which are usually made on positives printed on paper, film, glass, or digital images.
- Equipment used for photographic measurements varies from simple scales to complex machines.


## 2. Coordinate Systems for Image Measurements

- The rectangular coordinate system has positive x axis in flight direction.
- The origin of the system is the intersection of fiducial lines often called the indicated principal point which is very near the true principal point for a precise mapping camera.
- Rectangular coordinates can be used to calculate the photo distances by using simple analytic geometry:

$$
a b=\sqrt{\left(x_{a}-x_{b}\right)^{2}+\left(y_{a}-y_{b}\right)^{2}}
$$



Figure 4-1 Photographic coordinate system based on side fiducials

## 3. Simple Scales for Photographic Measurements

- Engineer's scale may prove satisfactory for cases where a low accuracy is acceptable.
- Precision can be enhanced by using a magnifying glass and the finest set of graduations.
- Greater accuracy can be obtained by using a device such as the glass scale with magnifying eyepieces which can slide along the scale.


Figure 4-2 Metric (top and English (bottom) Engineer's scale
Figure 4-3 Glass scales for photographic measurements

## 4. Measuring Photo Coordinates with Simple Scales

- Marking the coordinate axes with a razor blade, pin, or sharp pencil are followed by obtaining coordinates through measuring the perpendicular distances from these axes.
- If the points on image are not clear enough for identification making a small pinprick can be helpful.
- It is important to affix the proper algebraic sign to the coordinates measured.


## 5. Comparator Measurement of Photo Coordinates

- Comparators can provide a high level of accuracy ( $2 \sim 3 \mu \mathrm{~m}$ ) in coordinate measurements while no longer in common use.
- Two types of comparator exist: monocomparators and stereocomparators with which image positions are measured by simultaneously viewing an overlapping stereo pair of photographs.
- Analytical stereoplotter is commonly used to perform the function of both type of comparators


Figure. Mann Monocomparator https://www.researchgate.net/figure/Mann-monocomparator_fig6_325425242

## 6. Photogrammetric Scanners

- Photogrammetric scanners are used to convert analog content of photograph to digital form from which coordinate measurement can take place in computer environment.
- High-quality photogrammetric scanners should be capable of producing digital images with minimum pixel sizes on the order of 5 to $15 \mu \mathrm{~m}$
- These scanners are no longer popular due to increased use of digital cameras.


## 7. Refinement of Measured Image Coordinates

- The measured photo coordinates will contain systematic errors as bellows.

1) Film distortions due to shrinkage, expansion, and lack of flatness/ CCD array distortions due to electrical signal timing issues or lack of flatness
2) Failure of photo coordinate axes to intersect at the principal point/ Failure of principal point to be aligned with center of CCD array.
3) Lens distortions
4) Atmospheric refraction distortions
5) Earth curvature distortion

## 8. Distortions of Photographic Films and Papers

- Even though film is much more stable than paper it can be distorted as much of 0 to 0.2 percent by temperature and humidity in a storage room and affected by film type and thickness.


## 9. Image Plane Distortion

- Shrinkage or expansion of a photograph can be determined by comparing the measured $(m)$ fiducial distances with their corresponding calibrated values (c):

$$
x_{a}^{\prime}=\left(\frac{x_{c}}{x_{m}}\right) x_{a} \quad y_{a}^{\prime}=\left(\frac{y_{c}}{y_{m}}\right) y_{a}
$$

- [Example 4-1] Measured fiducial distances in x and y are
233.8 \& 233.5 mm , respectively while calibrated fiducial distances are $232.604 \& 232.621 \mathrm{~mm}$.

$$
\begin{aligned}
x^{\prime} & =\left(\frac{232.604}{233.8}\right) x=0.9949 x \\
y^{\prime} & =\left(\frac{232.621}{233.5}\right) x=0.9962 y
\end{aligned}
$$

| (a) <br> Point no. | Measured Coordinates |  | Corrected Coordinates |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (b) <br> $x, \mathrm{~mm}$ | (c) <br> $y, \mathrm{~mm}$ | (d) <br> $\boldsymbol{x}^{\prime}, \mathrm{mm}$ | (e) <br> $\boldsymbol{y}^{\prime}, \mathrm{mm}$ |
| 1 | -102.6 | 95.2 | -102.1 | 94.8 |
| 2 | -98.4 | -87.8 | -97.9 | -87.5 |
| 3 | 16.3 | -36.1 | 16.2 | -36.0 |
| 4 | 65.7 | 61.8 | 65.4 | 61.6 |
| 5 | 104.9 | -73.5 | 104.4 | -73.2 |

Figure 3-19 Resolution test pattern for camera calibration.

## 10. Reduction of Coordinates to an Origin at the Principal Point

- Lens distortions are most symmetric about the principal point which photo coordinates/photogrammetric equations are based on. The principal points, however, generally does not coincide with the intersection of fiducial lines or the CCD center.
- Normally (mapping) camera manufacturers attempt to make the principal point coincide with the intersection of fiducial lines within a few micrometers so that the correction can usually be ignored in work of coarse accuracy.
- For precise analytical photogrammetric work, the principal point coordinates, $x_{p}$ and $y_{p}$ are subtracted from the measured and transformed $x$ and $y$ coordinates.


## 11. Correction for Lens Distortions

- Lens distortions typically consist of symmetric radial distortion and decentering distortion which are normally less than $5 \mu \mathrm{~m}$ in modern aerial mapping cameras.
- Symmetric radial lens distortion is unavoidable and was significantly larger than decentering distortion which is primarily caused by imperfect assembly of lens elements.
- The radial distortion value $(\Delta r)$ is the radial displacement from the ideal location to the actual image of collimator cross when the multicollimator is adopted.
- Correction procedure: coordinates relative to the principal point are calculated $(\bar{x}, \bar{y})$ and radial distortion $(\Delta r)$ is obtained from a polynomial:

$$
\Delta r=k_{1} r^{1}+k_{2} r^{3}+k_{3} r^{5}+k_{4} r^{7}
$$

, where $r=\sqrt{\bar{x}^{2}+\bar{y}^{2}}$ and $\bar{x}=x-x_{p}, \bar{y}=y-y_{p}$.

## 11. Correction for Lens Distortions



$$
\begin{aligned}
& \frac{\Delta r}{r}=\frac{\delta x}{\bar{x}}=\frac{\delta y}{\bar{y}} \\
& \delta x=\bar{x} \frac{\Delta r}{r}, \quad \delta y=\bar{y} \frac{\Delta r}{r}
\end{aligned}
$$

The corrected coordinates:

$$
\begin{aligned}
& x_{c}=\bar{x}-\delta x \\
& y_{c}=\bar{y}-\delta y
\end{aligned}
$$

Figure 4-4 Relationship between radial lens distortion and corrections to $x$ and $y$ coordinates.

## 11. Correction for Lens Distortions

[Example 4-2]

- $f=153.206 \mathrm{~mm},\left(x_{p}, y_{p}\right)=(0.008,-0.001)$
- $(x, y)=(62.579,-80.916) \rightarrow\left(x_{c}, y_{c}\right)$ ?

| Mean Radial Distortion |  |  |
| :--- | :--- | :--- |
| (a) | (b) | (c) |
| Field angle $\theta$ | $\Delta \boldsymbol{r}, \mathrm{mm}$ | $\boldsymbol{r}, \mathrm{m}$ |
| $7.5^{\circ}$ | 0.004 | 0.0202 |
| $15^{\circ}$ | 0.007 | 0.0411 |
| $22.5^{\circ}$ | 0.007 | 0.0635 |
| $30^{\circ}$ | 0.001 | 0.0885 |
| $35^{\circ}$ | -0.003 | 0.1073 |
| $40^{\circ}$ | -0.004 | 0.1286 |



From the least squares method

$$
\begin{gathered}
k_{1}=0.2296, \quad k_{2}=-35.89 \\
k_{3}=1,018, \quad k_{4}=12,100 \\
\bar{x}=x-x_{p}=0.062571 \mathrm{~m} \\
\bar{y}=y-y_{p}=-0.080915 \mathrm{~m} \\
r=\sqrt{\bar{x}^{2}+\bar{y}^{2}}=0.1023 \mathrm{~m} \\
\Delta r=k_{1} r^{1}+k_{2} r^{3}+k_{3} r^{5} \\
+k_{4} r^{7}=-0.0021 \mathrm{~mm}
\end{gathered}
$$

## 11. Correction for Lens Distortions

$$
\begin{aligned}
& \delta x=\bar{x} \frac{\Delta r}{r}=0.062571\left(\frac{-0.0021}{0.1023}\right)=-0.0013 \mathrm{~mm} \\
& \delta y=\bar{y} \frac{\Delta r}{r}=-0.080915\left(\frac{-0.0021}{0.1023}\right)=0.0017 \mathrm{~mm} \\
& x_{c}=\bar{x}-\delta x=62.571-(-0.0013)=62.572 \mathrm{~mm} \\
& y_{c}=\bar{y}-\delta y=-80.915-0.0017=-80.917 \mathrm{~mm}
\end{aligned}
$$

- USGS adopted SMAC (Simultaneous Multi-camera Analytical Calibration) for the calibration procedure which computes both symmetric radial and decentering distortion parameters directly by least squares.


## 12. Correction for Atmospheric Refraction

- Air density decreasing with increasing altitude makes light rays bent according to Snell's law, which requires corrections for the image coordinates.

- Angular distortion due to refraction, $\Delta \alpha$ increases with increasing flying height and increasing incident angle $\alpha$ :

$$
\Delta \alpha=K \tan \alpha
$$

, where $K$ is a function of $H$ (camera elevation) and $h$ (object elevation) (ex. $K=\left(7.4 \times 10^{-4}\right)(H-h)[1-$

## 12. Correction for Atmospheric Refraction

## [Example 4-4]

Elevations of Camera with 153.099 mm of focal length and an object A are $3,500 \mathrm{~m}$ and 120 m , respectively. The image point a of A has fiducial system coordinates: $x_{a}=73.287 \mathrm{~mm}$ and $y_{a}=-101.307 \mathrm{~mm}$. Compute $x^{\prime}$ and $y^{\prime}$ corrected for atmospheric refraction

$$
\begin{aligned}
& r=\sqrt{73.287^{2}+(-101.307)^{2}}=125.036 \mathrm{~mm} \\
& \tan \alpha=\frac{125.036}{153.099} \rightarrow \alpha=39.2386^{\circ} \\
& K=\left(7.4 \times 10^{-4}\right)(3.5-0.12)[1-0.02(2(3.5)-0 / 12)]=0.0022^{\circ} \\
& \Delta \alpha=K \frac{r}{f}=0.0022^{\circ}\left(\frac{125.036}{153.099}\right)=0.0018^{\circ} \\
& r^{\prime}=153.099 \tan \left(39.2386^{\circ}-0.0018^{\circ}\right)=125.029 \mathrm{~mm} \\
& \Delta r=125.036-125.029=0.008 \mathrm{~mm}
\end{aligned}
$$

## 12. Correction for Atmospheric Refraction

$$
\begin{aligned}
& \delta x=73.287\left(\frac{0.008}{125.036}\right)=0.005 \mathrm{~mm} \\
& \delta y=-101.307\left(\frac{0.008}{125.036}\right)=-0.006 \mathrm{~mm} \\
& x^{\prime}=x-\delta x=73.287-0.005=73.282 \mathrm{~mm} \\
& y^{\prime}=y-\delta y=-101.307-(-0.006)=-101.301 \mathrm{~mm}
\end{aligned}
$$

## 13. Correction for Earth Curvature

- Earth curvature corrections has been recognized not to be theoretically correct but at the same time such correction generally leads to more accurate results than ignoring it.
- Correcting photo coordinates for earth curvature may degrade the accuracy of either $x$, or $y$ coordinates depending upon the map projection used: For example, UTM (Universal Transverse Mercator coordinate system) projection with earth curvature corrections will yield more accurate elevation and $y$ values while $x$ values will be degraded because UTM projection does not curve in the $x$ direction.
- Local vertical coordinate system can avoid the need of earth curvature correction which will be described in Sec.5-5.


## 13. Correction for Earth Curvature



Figure. UTM coordinate system: 60 cells in west-east; $c \sim x$ divisions in south-north

## 14. Measurement of Feature Positions and Edges

- Under magnification or image zoom, the edges of an object will appear to be somewhat indistinct although the center can still be identified to a high precision.


Figure 4-7 Zoomed-in view of a manhole with cross-hair at center.


Figure 4-8 Zoomed-in view of a chevron target with cross-hair at the intersection of centerlines of stripes.

