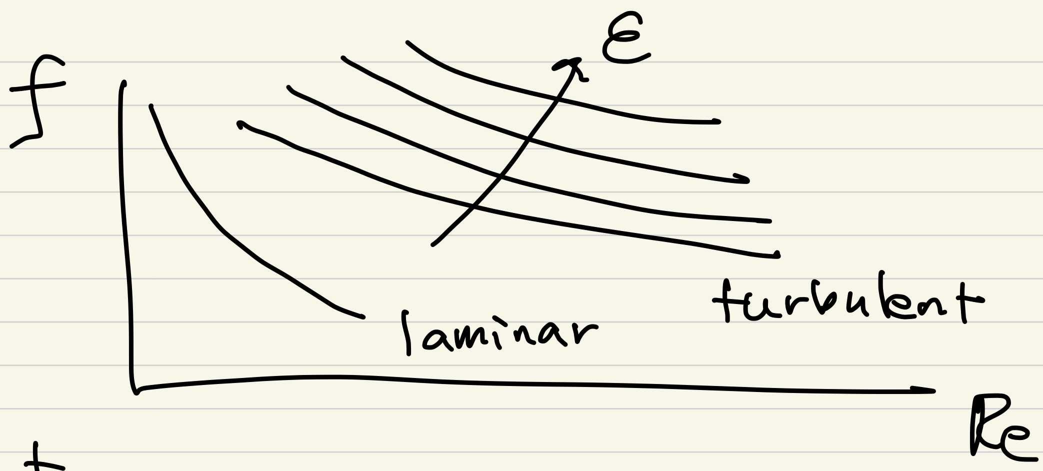
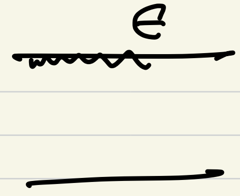
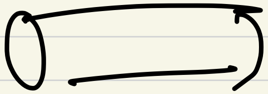
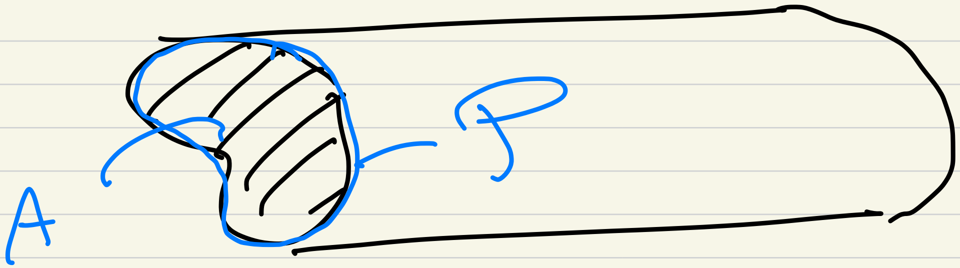
Moody chart







Non-circular duct

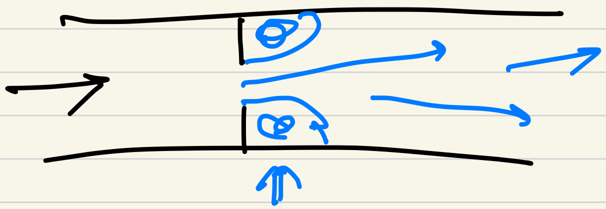


$$D_h \equiv 4A/P$$

hydraulic diameter $\quad 4\cdot\frac{\pi}{4}d^2/2\pi r = d$

$$Re_{D_h} = V D_h/\nu$$

Minor losses in pipe system

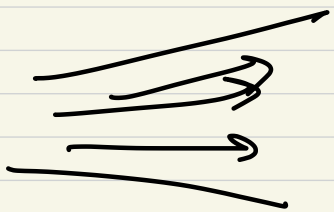


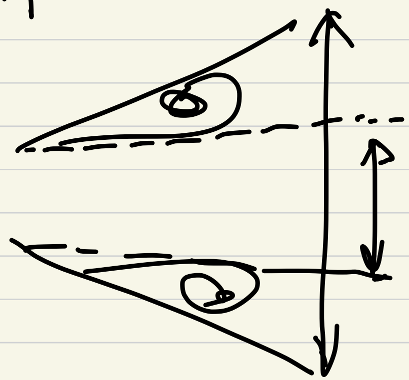
Diffuser

$\rightarrow V_2 < V_1 \Rightarrow P_2 > P_1$

$A_2 > A_1$

$\cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 V_2$

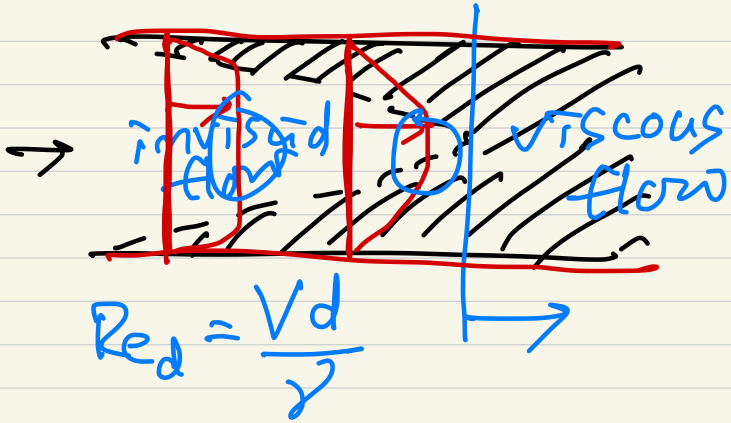




Fluid meters
- orifice
- nozzle
- venturi

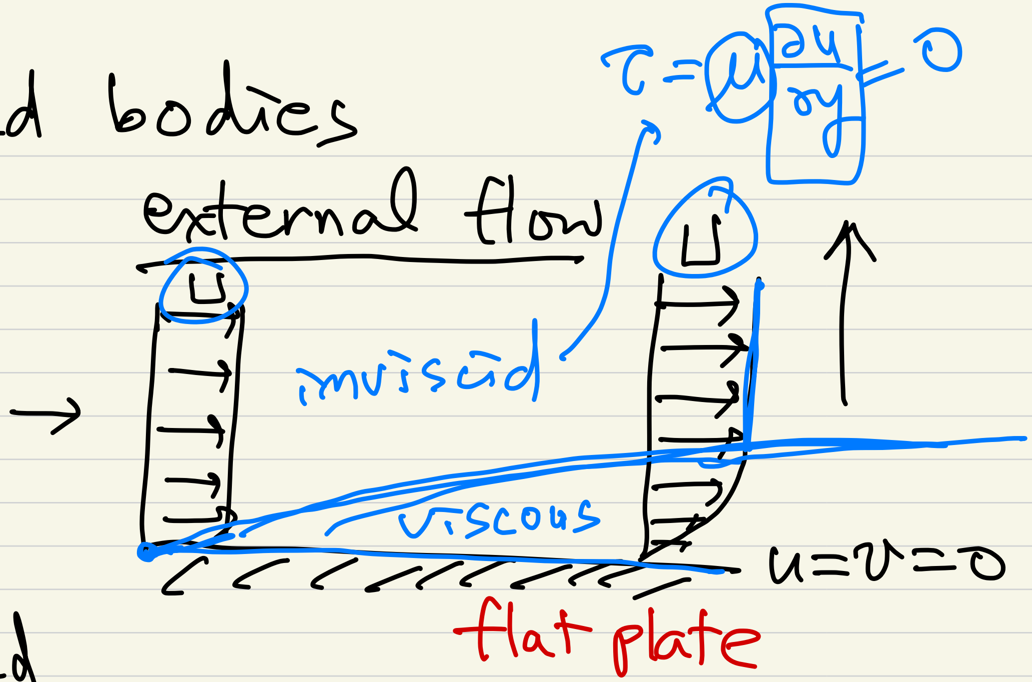
Ch. 7 Flow past immersed bodies

internal flow V



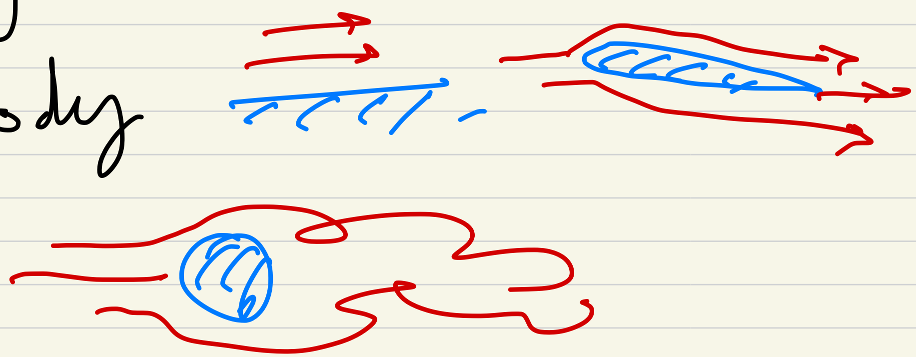
vs.

external flow



7.1 Reynolds number and geometry effects

two types : { streamlined body
bluff body



⊙ $\boxed{\text{Flat plate}}$

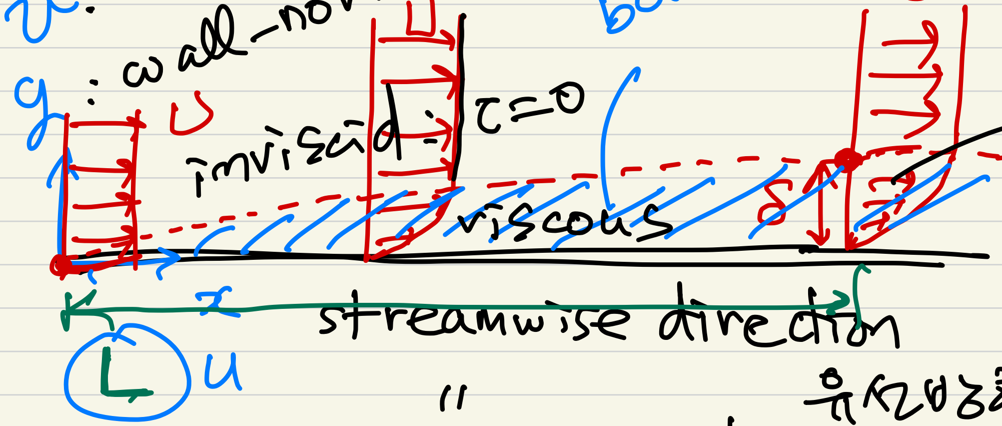
U → free-stream velocity 자유유동속도

v : wall-normal velocity 벽에 수직한 방향 속도

y : wall-normal direction 벽에 수직한 방향

boundary layer (경계층)

δ : boundary layer thickness $\delta = \delta(x)$ 경계층 두께



$u(x, y)$

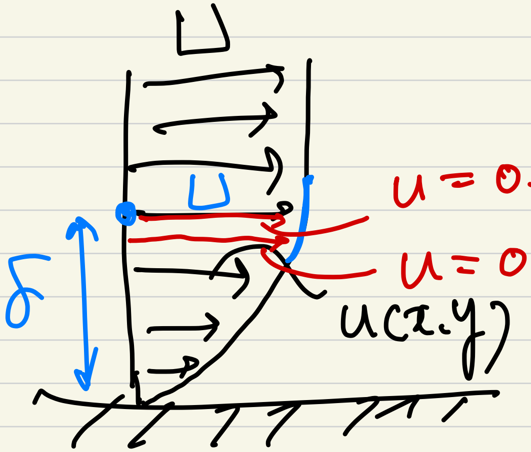
$\delta \ll L$

← no-slip b.c. $u = v = 0$

x : streamwise direction 유선방향

u : streamwise velocity 속도

$$Re_x = \frac{Ux}{\nu}, \quad Re_\delta = \frac{U\delta}{\nu} \quad \text{vs.} \quad Re_d = \frac{Ud}{\nu}$$

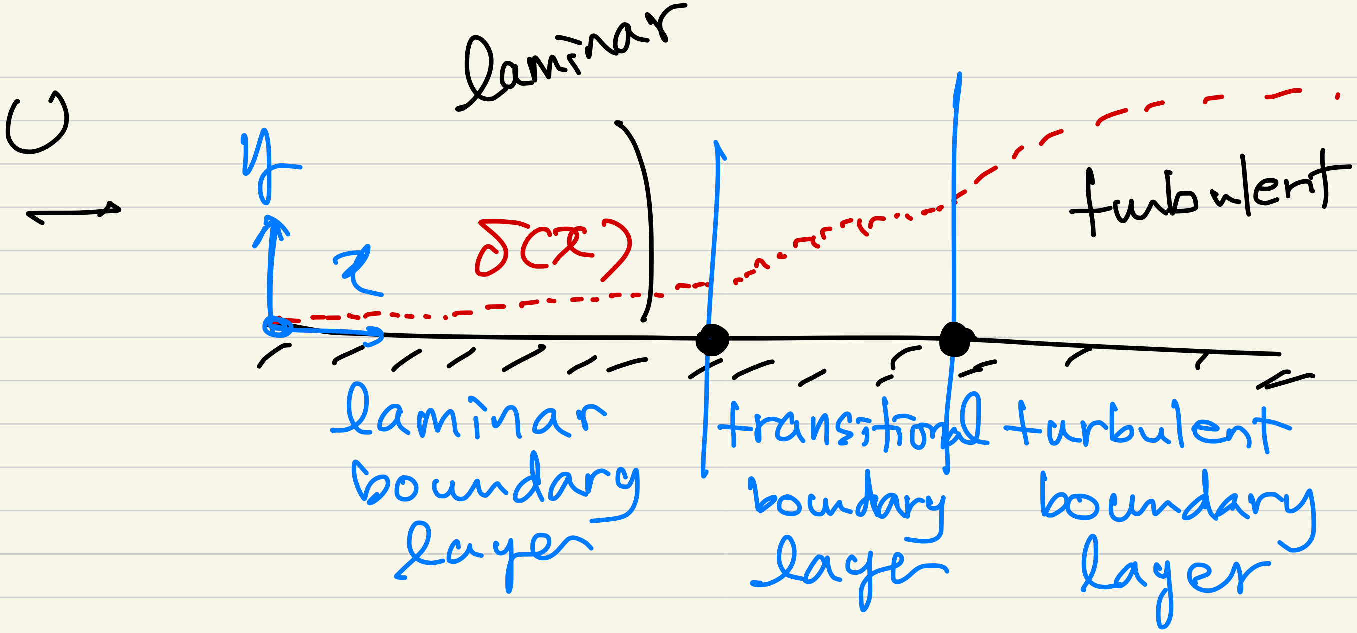


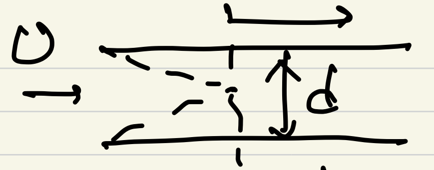
$\delta_{0.999U}$

$u = 0.99U \rightarrow \delta_{0.99U}$

$u = 0.95U \rightarrow \delta_{0.95U}$

] δ





$Re_d = Ud/\nu$

임계

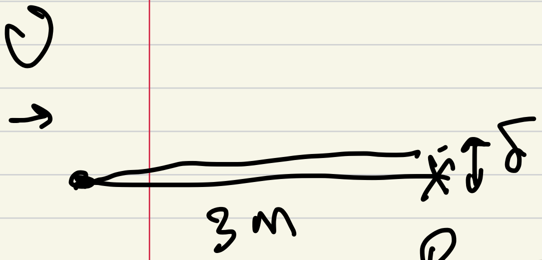
> 2500 turb

$<$ 〃 lam.

$$\frac{\delta}{x} = \begin{cases} \dfrac{5.0}{\sqrt{Re_x}} & \text{laminar} \quad \text{Blasius (1908)} \\[2ex] \dfrac{0.16}{Re_x^{1/7}} & \text{turbulent} \end{cases}$$

$\rightarrow \delta_{lam} \sim x^{\frac{1}{2}}$

$\rightarrow \delta_{turb} \sim x^{\frac{6}{7}}$



air

$x = 3m$

$U = 10m/s$

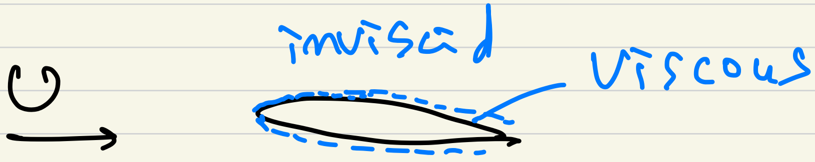
$Re_x = \frac{Ux}{\nu} = 2\times10^6$:

$\delta_{lam.} = \frac{5.0\times3}{\sqrt{2\times10^6}} \doteqdot 1cm$

$\delta_{turb} = \frac{0.16\times3}{(2\times10^6)^{1/7}} = 6cm$

thin boundary layer

- flow around slender bodies



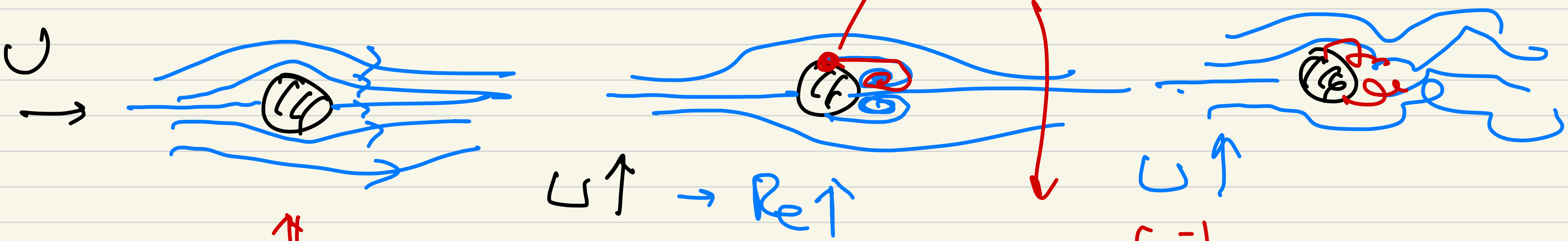
plates
airfoils] thin boundary layer

↑

theory

- flow around bluff bodies

flow separation
유동박리

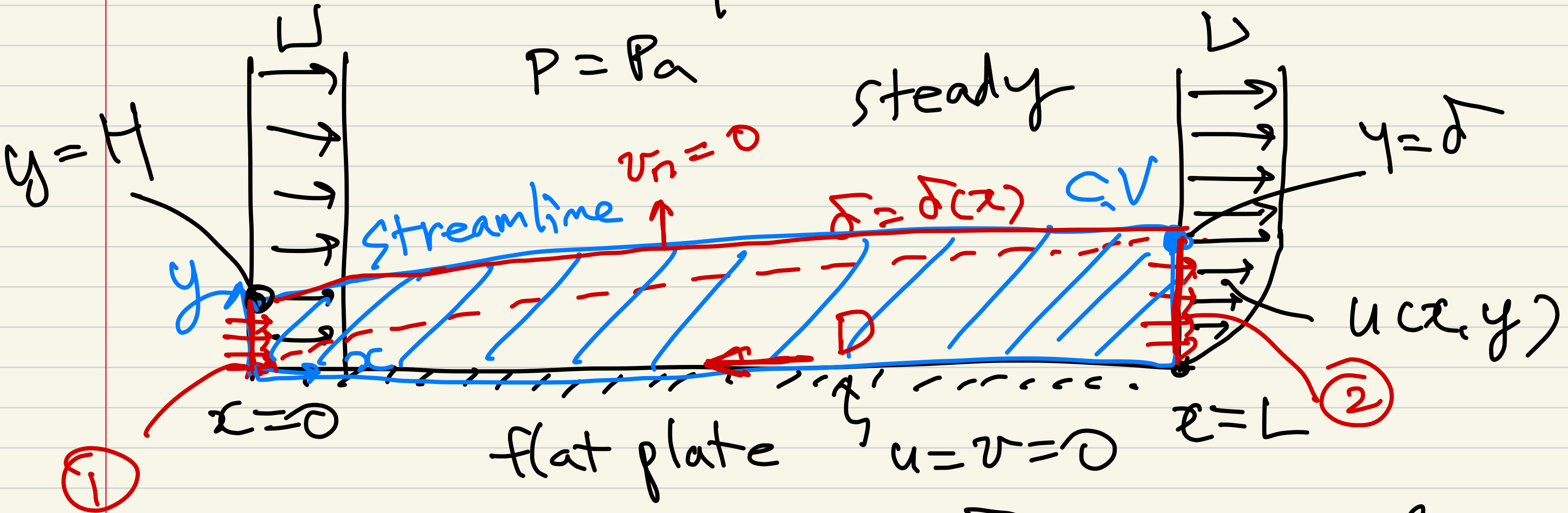


↑

theory
stokes flow

theory fails
↳ experiment
CFD
computational
fluid dynamics

7.2 Momentum-integral estimates



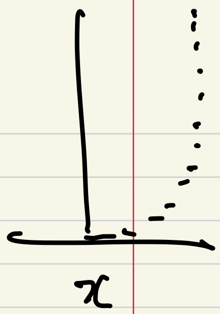
b: width

continuity : $UH = \int_0^\delta u\,dy \rightarrow U^2 H = \int_0^\delta u U\,dy$

x-mtm : $\sum F_x = -D = \int_{cs} \rho u (\underline{V}\cdot\underline{n})\,dA = \underbrace{\int_0^\delta \rho u^2 dy \cdot b}_{②} - \underbrace{\rho U^2 H b}_{①}$

$= \rho b \left(\int_0^\delta u^2 dy - \int_0^\delta u U\,dy \right)$

$\therefore\ D = \rho b \int_0^\delta (uU - u^2)\,dy = \rho b U^2 \underbrace{\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}_{\equiv\, \theta(x)}$



$$\boxed{\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy}$$

: momentum thickness
운동량 두께

$$\rightarrow \boxed{D(x) = \rho b U^2 \theta(x)} \rightarrow \frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

$$D(x) \equiv \int_0^x \tau_w(x)\,dx \cdot b \rightarrow \frac{dD}{dx} = \tau_w b$$

$$\boxed{\tau_w = \rho U^2 \frac{d\theta}{dx}}$$

wall shear stress
벽전단응력

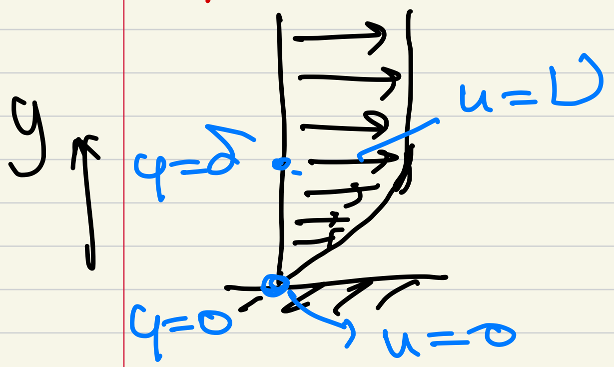
$$c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U^2} = 2\frac{d\theta}{dx} \rightarrow \boxed{\frac{c_f}{2} = \frac{d\theta}{dx}}$$

skin-friction coefficient
마찰계수

valid for laminar & turbulent flows

momentum-integral relation for flat-plate boundary layer flow

- <u>von Karman's assumption on velocity profile</u>



$$\left.\begin{aligned} u &= 0 \ @\ y = 0 \\ u &= U \ @\ y = \delta \\ \frac{\partial u}{\partial y} &= 0 \ @\ y = \delta \end{aligned}\right\} \quad u = a + by + cy^2$$

$$\downarrow$$

$$\boxed{u(x,y) = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)} \quad 0 \le y \le \delta$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \frac{2}{15}\delta$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu \cdot \frac{2U}{\delta} = \rho U^2 \frac{d\theta}{dx} = \frac{2}{15}\rho U^2 \frac{d\delta}{dx}$$

$$\rightarrow \frac{\delta^2}{2} = 15\frac{\nu x}{U}$$

$$\mu \frac{\cancel{2}U}{\delta} = \frac{\cancel{2}}{15}\rho U^{\cancel{2}} \frac{d\delta}{dx}$$

$$\frac{15}{\cancel{2}}\frac{\mu}{\rho}\frac{1}{U}dx = \delta d\delta$$

$$15\frac{\nu}{U}dx = \delta d\delta$$

$$\frac{\delta^2}{2}$$

$$\rightarrow \boxed{\frac{\delta}{x} = 5.5\sqrt{\frac{\nu}{Ux}} = \frac{5.5}{\sqrt{Re_x}}}$$

10% error

$$\rightarrow \delta \sim \sqrt{x}$$

cf. $\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$ Blasius

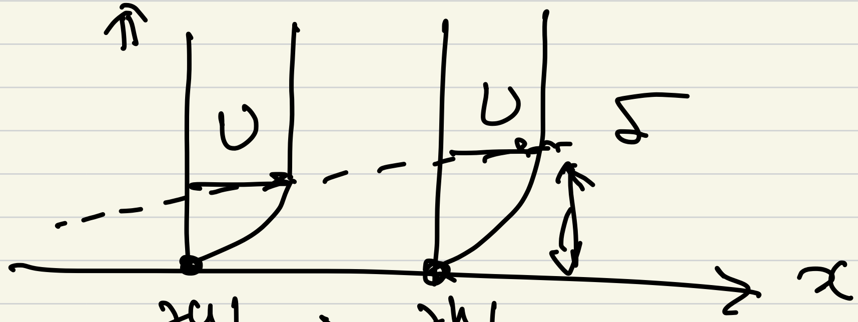
$$\frac{\theta}{x} = \frac{\theta}{\delta}\cdot\frac{\delta}{x} = \frac{2}{15}\,\frac{5.5}{\sqrt{Re_x}} = \frac{0.733}{\sqrt{Re_x}} \rightarrow \theta \sim \sqrt{x}$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\mu U/\delta}{\frac{1}{2}\rho U^2} = \frac{4\mu}{\rho U \delta} = \frac{x}{\delta}\cdot\frac{4\mu}{\rho U x} = \frac{0.73}{\sqrt{Re_x}}$$

$$\rightarrow c_f \sim x^{-\frac{1}{2}}$$

$$c_f = \frac{0.664}{\sqrt{Re_x}} \text{ Blasius}$$

10% error



$$c_f \sim x^{-\frac{1}{2}}$$

$$\left.\frac{\partial u}{\partial y}\right|_w > \left.\frac{\partial u}{\partial y}\right|_w$$

$$\tau_w > \tau_w$$