Interaction between X-ray and Matter Peak Position

Hammond Chapter 8, 9, 10

Pecharsky Chapter 2 Sherwood & Cooper Chap 4.13; 6.1~6.3; 8.9~8.16 Krawitz Chap 5, p119~128 Birkholz Chapter 1 Cullity Chap 3-1~3-6

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- > Incoherent (Compton) scattering λ of scattered beam increases due to partial loss of photon energy in collision with the core electrons (Compton effect).
- > Coherent scattering scattered beam has the same λ as the primary beam.





Interference

Interaction between two or more trains of waves of the same frequency emitted from coherent sources.

A series of stationary nodes and antinodes is established, known as interference.









Diffraction and information

- > When a wave interacts with an obstacle, diffraction occurs.
- > Diffracted wave contains additional information about the obstacle.
- ➤ The detailed behavior depends solely on the diffracting obstacles. → The diffracted waves can be regarded as containing information on the structure of the obstacles.



chap 6.6 & 6.7 of Sherwood & Cooper

diffraction pattern

Diffraction

Infinite plane wave with wave vector **k** and frequency w; $\psi = \psi_0 e^{i} (\mathbf{k} \cdot \mathbf{r} - wt)$

- > What happens when a wave motion interacts with an obstacle placed in its path?
- How is the wave equation modified to take account of the interaction of the wave with the obstacle?



What is X-ray Diffraction?

X-rays are an ideal probe of electromagnetic radiation for the study of crystals as the wavelength <u>λ is of the same order as the distances between</u> the atoms in crystals (Å, nm).

> Elastic scattering \rightarrow no energy transfer & no wavelength change

When the periodic array consists of crystalline matter of three dimensional (3-d) arrangement of atoms, monochromatic X-ray radiation diffracts in a number of different directions in 3-d space.

Why do we use x-rays ? read Sherwood & Cooper, Chap 6.1~6.5

X-ray diffraction

- Diffraction occurs when each object in a periodic array scatters radiation coherently, producing concerted constructive interference at specific angles.
- > The **<u>electrons</u>** in an atom **coherently** scatter light.
 - ✓ The electrons interact with oscillating electric field of light wave.
- > Atoms in a crystal form a **periodic array** of coherent scatterers.
 - ✓ The wavelength of X rays are similar to the distance between atoms.
 - Diffraction from different planes of atoms produces a diffraction pattern, which contains information about the atomic arrangement within the crystal.
- X rays are also reflected, scattered incoherently, absorbed, refracted, and transmitted when they interact with matter.

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Interaction of electromagnetic wave & matter

- > Electric field (E) & magnetic field (H) propagate through the matter.
- Effect of E >>> effect of H
- > When a particle of charge q is placed in **E**, F = qE = ma $a = \frac{q}{m}E$
- ➤ E oscillates w/ very high v. → make the charged particle oscillate. → particle radiates E-M wave (scattering).

intensity of scattered wave $I \propto |a|^2 \propto (\frac{q}{m})^2 \mathbf{E}^2$

> Neutron (no charge), proton, electron vs. E

$$m_{p} = 1800 m_{e} \qquad I_{p} \propto \left(\frac{e}{m_{p}}\right)^{2}$$

$$I \text{ of radiation scattered by e'} \qquad I_{e} \propto \left(\frac{e}{m_{e}}\right)^{2} \qquad \qquad \frac{I_{e}}{I_{p}} = \left(\frac{e}{m_{e}}\right)^{2} \left(\frac{m_{p}}{e}\right)^{2} = 1800^{2}$$

> XRD looks at the <u>electron distribution</u> in a crystal.



Interaction of X-ray & crystal

- > XRD looks at the **<u>electron distribution</u>** in a crystal.
- > XRD does not directly look at the positions of the nuclei of atoms.
- > Atom of atomic number Z \rightarrow intensity of scattered wave $\propto (\frac{Ze}{m_e})^2$
- ➢ High Z atoms (heavy atoms) are much more effective scattering centers. →
 XRD cannot give much info on light elements.
- ➤ Intensities of scattered X-ray → locations of electrons in one unit cell → electron density distribution (e' density map)

Diffraction pattern

- > Each reflection (diffraction peak) has an (hkl) index and a measured intensity.
- > Each reflection index defines a set of parallel planes that slice thru the crystal.





> Given all F(hkl) (amplitude & phase for each reflection), $\rho(xyz)$ can be calculated.

> Given $\rho(xyz)$ (i.e., the structure), F(hkl) can be calculated.

ightarrow F, $\varphi \leftarrow$ FT ightarrow $\rho(xyz)$

Kinematical vs. Dynamical theories of diffraction

> Kinematical theory

- ✓ <u>A beam scattered once is not scattered again.</u>
- ✓ Interaction of diffracted beam with crystal is negligibly small.
- Crystal consists of individual mosaic blocks.
- Size of the crystallites is small.
- Misalignment of crystallites is large enough, so that interaction of X-ray with matter at length scale larger than the size of the mosaics is negligible.

Dynamical theory

- ✓ Accounts for <u>scattering of diffracted beam & other interactions of</u> <u>waves</u> inside the crystal.
- ✓ Needed when crystals are nearly perfect or when there is a strong interaction of the radiation with the material (electron diffraction).
- Many dynamical effects (primary & 2ndary extinction, simultaneous diffraction, thermal diffuse scattering, etc.) are accounted for as corrections to the kinematical diffraction model.
 Rigaku Journal, 25(2), 2009, X-ray thin film measurement techniques

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Range of Applications of X-Ray Analytical Methods

- > Qualitative and quantitative element analysis (XRF; X-ray fluorescence)
- > Qualitative and quantitative phase analysis (XRD)
- ➢ % crystallinity
- > Micro-strain and crystallite size determination
- Residual stress and texture analysis
- Grazing incidence diffraction (GID)
- > X-ray reflectometry (XRR; X-ray reflection)
- High Resolution X-ray Diffraction (HRXRD)
- Structure solution and refinement
- > Micro-diffraction (phase identification, texture, stress...)
- > Nano-structure investigations by small angle X-ray scattering (SAXS)

- > Qualitative phase analysis (Identification of unknown phases)
- Quantitative phase analysis
- Accurate lattice parameter measurement
- ➢ % crystallinity
- Measurement of crystal size
- Measurement of internal elastic strains
- Preferred orientation measurement
- Cation site disorder
- Micro-diffraction (phase identification, texture, stress...)
- Structure refinement (vs. single crystal)















XRPD pattern



Camera vs. Diffractometer

Diffraction camera

- \checkmark I is measured thru amount of blackening it produces on a <u>film.</u>
- ✓ All diffraction lines recorded simultaneously. Variation in I of incident beam during exposure has no effect on the relative I.
- ✓ Quantitative measurements of line position & intensity need at least two steps (recording pattern on the film + microphotometer record of the film).

Diffractometer

- ✓ I is measured directly by an electronic X-ray detector.
- ✓ Diffraction lines recorded one after another → incident beam intensity must be kept constant → voltage & current needs to be stabilized.
- ✓ Quantitative measurement of line position & intensity is made in one operation.

► Laue

- ✓ Crystals consist of <u>3-D network of rows of atoms.</u>
- ✓ Crystal behaves as a <u>3D diffraction grating</u>.
- ✓ Laue equations

➢ Bragg

- ✓ Crystals consist of <u>planes of atoms</u> which behaves as reflecting planes.
- Strong reflected beam is produced when the path difference between reflections from successive planes in a family is equal to whole number of wavelengths.

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✓ Bragg's law



Bragg's Law

- > Braggs W.H. and W.L. Bragg (father & son)
- ▶ 1913 1914



- > Realized that X-ray scattering could be imagined as reflections from planes.
- > Showed how it could be used to determine atomic structure of NaCl.
- ➢ Nobel prize 1915

$n\lambda = 2d \sin\theta$

d = distance of lattice planes

 λ = x-ray wavelength









 $(AB + BC) = (d_{hkl} \sin \theta + d_{hkl} \sin \theta) = 2d_{hkl} \sin \theta.$

Hence for constructive interference:

 $n\lambda = 2d_{hkl}\sin\theta,$

where *n* is an integer (the order of reflection or diffraction).

$$\lambda = 2\left(\frac{d_{hkl}}{n}\right)\sin\theta = 2d_{nh\,nk\,nl}\sin\theta$$
 Bragg's law



Laue Index; see Hammond p141





$\lambda = 2d \sin \theta$

- Condition for diffraction
 - ✓ Incident beam
 - \checkmark Diffracted beam > co-planar
 - ✓ Plane normal

 > sinθ = λ/2d (e.g. d = 4 Å)
 ✓ If λ = 1000 Å, sinθ = 125 → The crystal could not possibly diffract. (No θ can meet the diffraction condition.)
 ✓ If λ = 0.01 Å, sinθ = 0.00125 → Diffraction angle too small to be measured
 ✓ If λ = 2 Å, sinθ = 0.25, θ = 14.5° → Diffraction angle can be easily measured

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- Bragg's law = reciprocal lattice point for reflecting plane (hkl) should intersect the sphere.
- ➤ If the reciprocal lattice point does not intersect the sphere, then the Bragg's law is not satisfied. → no diffracted beam











If λ can change continuously \rightarrow other planes can reflect as their reciprocal lattice points successively intersect the sphere. \rightarrow Laue's original X-ray experiment using white radiation



- All the planes in the shaded region satisfy Bragg's law for the particular sphere on which they lie (for that particular λ).
- ➤ Monochromatic radiation → crystal and the sphere should move to have more intersection (to have diffracted beams from more planes).



- > Simulated and indexed powder XRD pattern for bulk (1 µm) wurtzite CdS.
- The inset shows the crystal structure of wurtzite CdS with the (100), (002), and (101) planes highlighted.





- Graphical representation of preferred orientation for nanoparticles having different shapes: (a) spheres, (b) cubes, and (c) rods.
- Simulated XRD patterns for varying degrees of alignment (*i.e.*, preferred orientation) of wurtzite CdS particles along specific crystallographic directions:
 (d) [100], (e) [001], and (f) [110].



 $\lambda = 2d \sin \Theta$

The value of d can be obtained from the peak position (2θ) of the XRD pattern.

SSIV

The change of d can be obtained from XRD @ many different angles. \rightarrow info on strain \rightarrow info on stress





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✓ Size & shape of the unit cell (← $\lambda = 2$	2d sinθ)
Intensity	
✓ Atomic scattering factor	
\checkmark Structure factor (atomic position, occu	upancy, etc.)
✓ Polarization	
✓ Multiplicity	
✓ Temperature	4
✓ Microabsorption	
✓ Crystallite size	H
✓ Residual stress	
✓ Preferred orientation (texture)	1
✓ Degree of crystallinity	20
✓ Anomalous scattering	
✓ Source intensity, voltage drift, take-of detector dead time, etc.	f angle, slit width, axial divergence,

D-spacing accuracy > 2theta Diffractometer misalignment ✓ Geometry (crystal system, lattice > Specimen displacement error parameter) (shape & size) Specimen transparency error > Intensity > Problems in establishing true peak ✓ Atom type position ✓ Arrangement Background ✓ Orientation Κα2 Shape of diffraction lines ✓ Instrument broadening ✓ Particle dimension ✓ Strain

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Factor affecting the Relative Intensity of Bragg Reflections

	Instrument sensitive
 Structure sensitive 	✓ Absolute intensities
\checkmark Atomic scattering factor	Source intensity
✓ Structure factor	 Diffractometer efficiency
✓ Polarization	 Take-off angle of tube
✓ Multiplicity	 Receiving slit width
✓ Temperature	 Axial divergence allowed
	✓ Relative intensities
 Sample sensitive Absorption 	 Divergence slit aperture
	 Detector dead-time
✓ Degree of crystallinity	
✓ Particle orientation	Measurement sensitive
	✓ Method of peak area measurement
	✓ Method of background subtraction
	✓ K α 2 stripping or not
	✓ Degree of data smoothing employed
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Crystal structure determination

➤ Two step process

(1) Determination of the size & shape of the unit cell \leftarrow peak position

(2) Determination of lattice type & distribution of the atoms in the structure ← intensities of the diffraction spots

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