⊙ Random fields

- One-point statistics

  for example, covariance of velocity $\langle u_i(\underline{x},t)\, u_j(\underline{x},t) \rangle$

  Reynolds stresses $\langle u_i u_j \rangle$

- N-point statistics

  2-pt $\langle u_i(\underline{x},t)\, u_j(\underline{x}',t) \rangle$

  3-pt $\langle u_i(\underline{x})\, u_j(\underline{x}')\, u_k(\underline{x}'') \cdots \rangle$

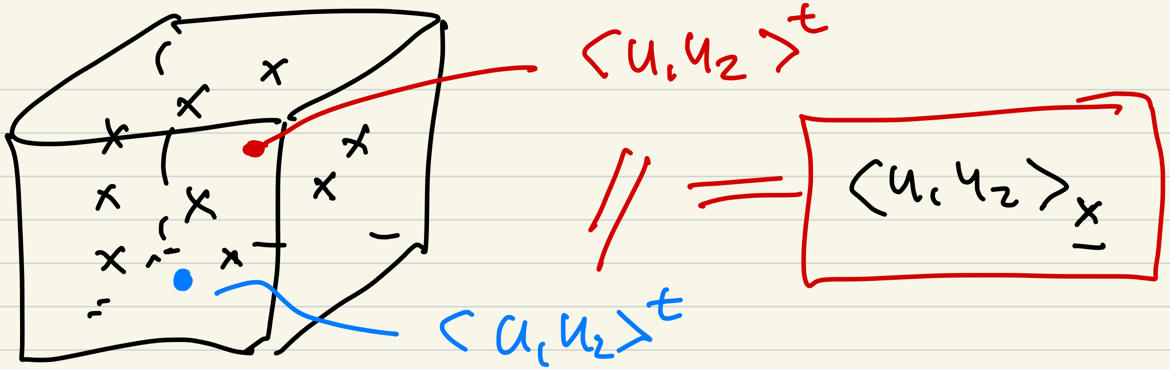
- statistical stationarity and homogeneity

  $U(\underline{x},t)$ is statistically stationary

  if all statistics are invariant under a shift in time

$U(\underline{x},t)$ is statistically homogeneous

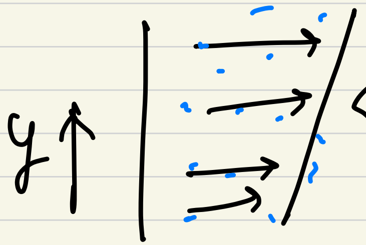
if all statistics are invariant under a shift in position.



If $U(\underline{x}, t)$ is stat. homo., $\langle U \rangle$ is uniform in space.

Homogeneous $\boxed{\text{turbulence}}$

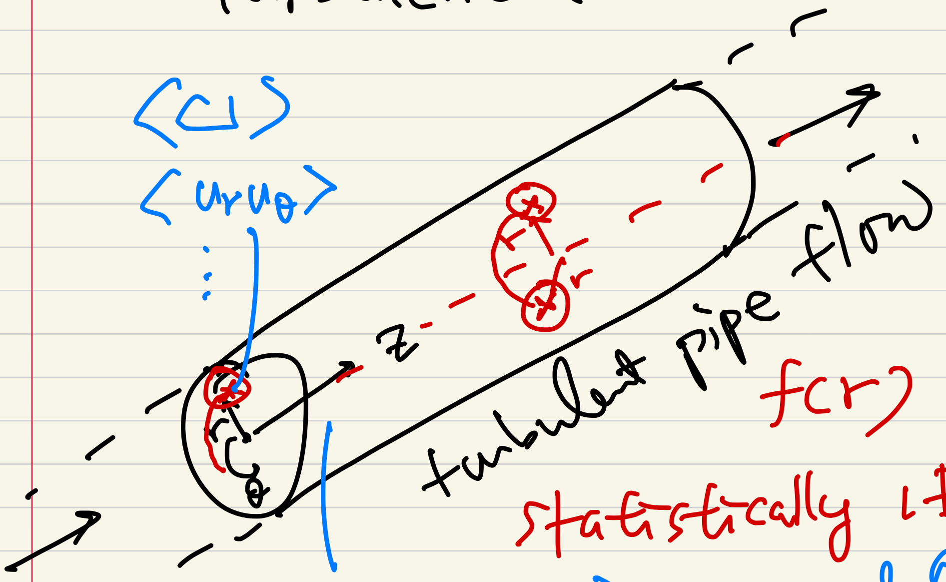
= fluctuating velocity $\underline{u}(\underline{x}, t)$ is stat. homo.
but $\langle U \rangle$ does not have to be uniform.



$\langle U \rangle \rightarrow \dfrac{\partial \langle U \rangle}{\partial y} = \text{const}$: const. mean shear
unif.

$\searrow$ homo. turbulence

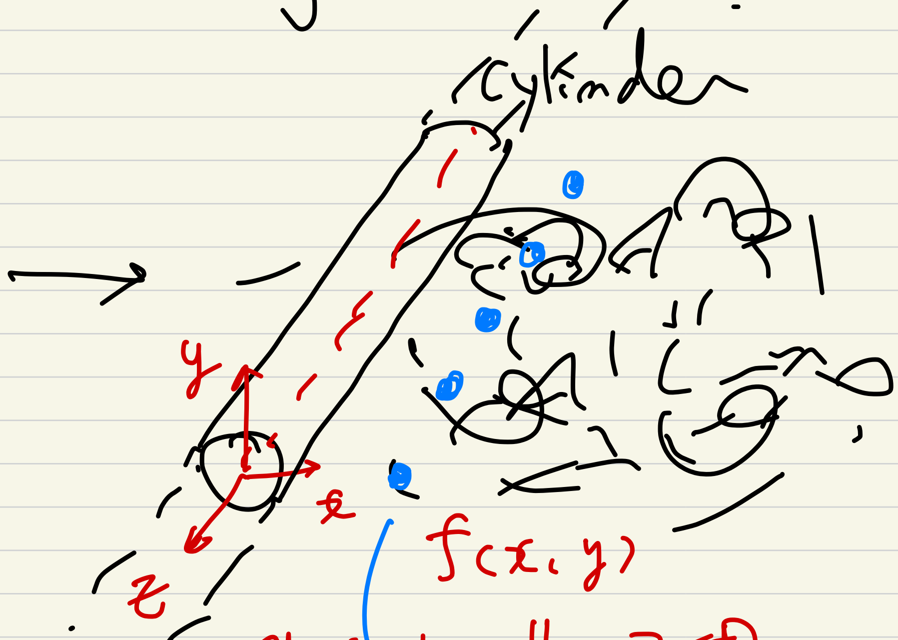
Turbulent flows can be statistically 2D or 1D



$f(r)$

statistically 1-D

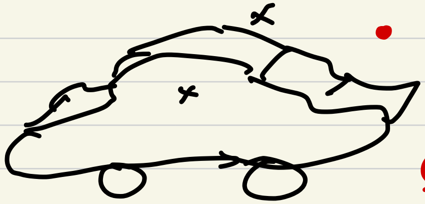
homogeneous in z and θ



$f(x, y)$

statistically 2-D

homogeneous in z.



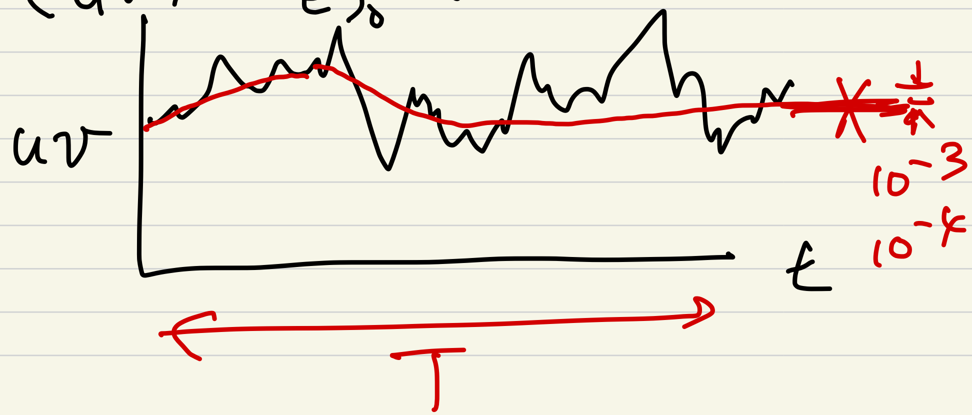
$f(x, y, z)$

statistically 3-D

<u>convergence of statistics</u>

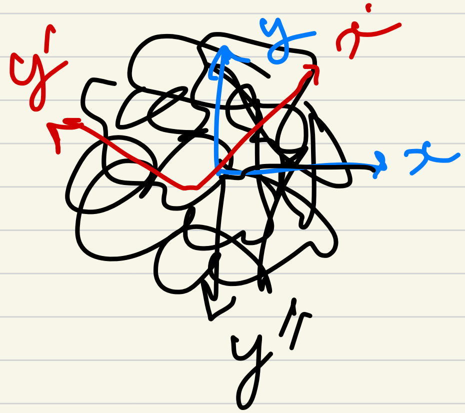
$\langle \cdot \rangle$

$$\langle uv \rangle = \frac{1}{t}\int_0^t uv\,dt$$



Isotropic turbulence

turbulent flow is statistically invariant under rotations and reflections of the coord. system.



homogeneous and isotropic turbulence

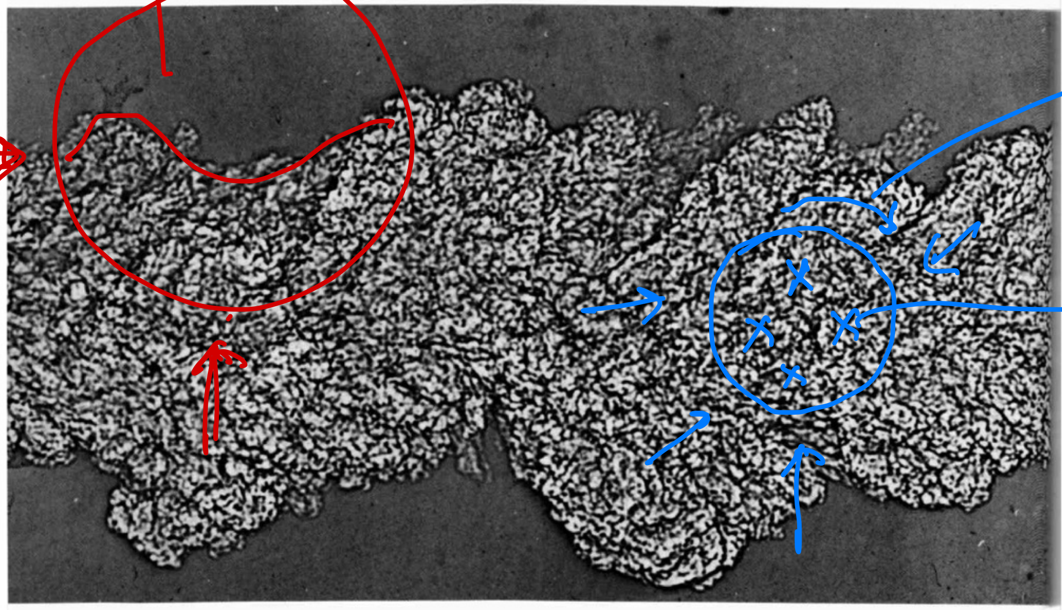
homogeneity + isotropy

6. Turbulence

inhomogeneous

isotropic

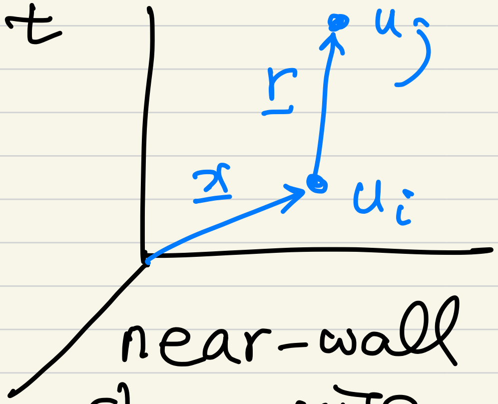
homo.



151. Turbulent wake far behind a projectile. A bullet has ~~been shot through the atmosphere at~~ supersonic speed, and is now several hundred wake diameters to the left. This short-duration shadowgraph shows the remarkable sharpness of the irregular boundary between the highly turbulent wake produced by the bullet and the almost quiescent air in irrotational motion outside. *Photograph made at Ballistic Research Laboratories, Aberdeen Proving Ground, in Corrsin & Kistler 1954*

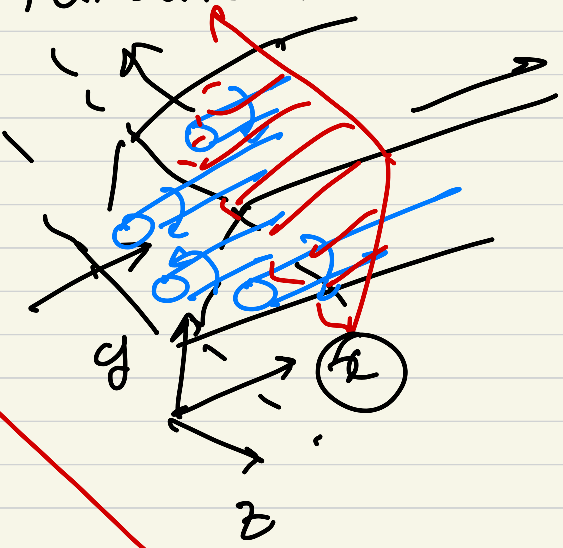
- two-point correlation : information on spatial structure of flow field.

$$R_{ij}(\underline{r}, \underline{x}) = \langle u_i(\underline{x}, t)\, u_j(\underline{x} + \underline{r}, t) \rangle_t$$

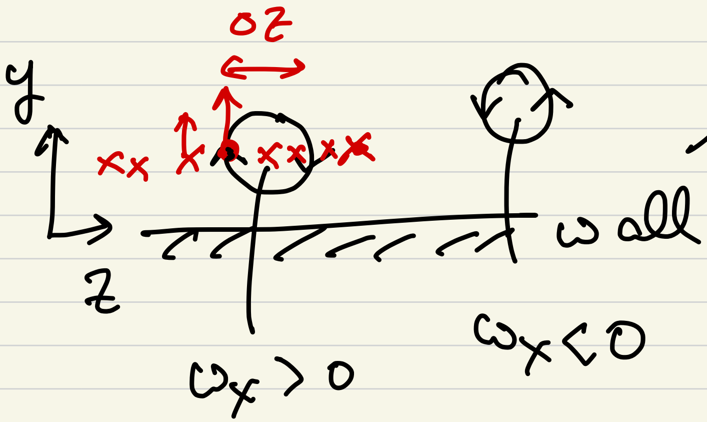


near-wall streamwise vortex

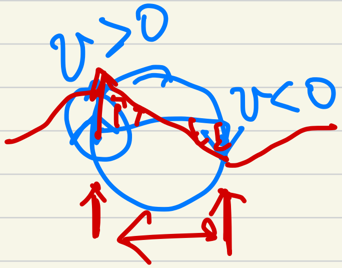
turbulent channel flow

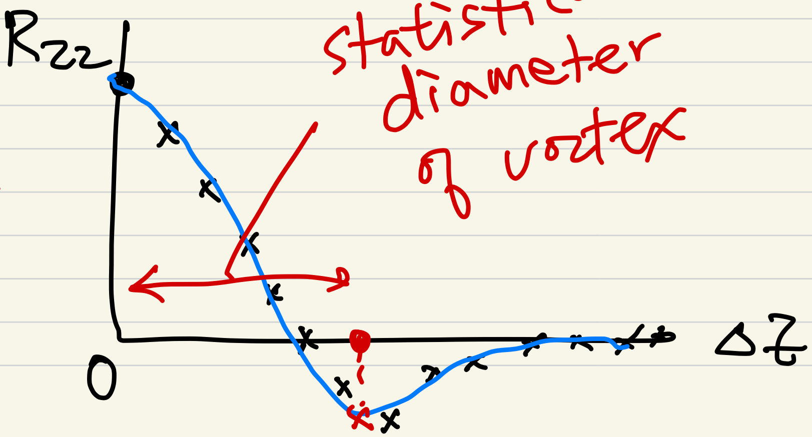


homo. direction

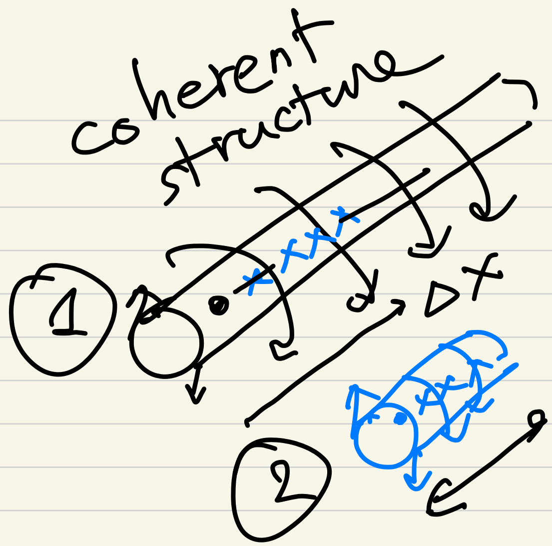


$$\langle v(\cdot, t)\, v(\cdot + \Delta z, t) \rangle = R_{zz}$$

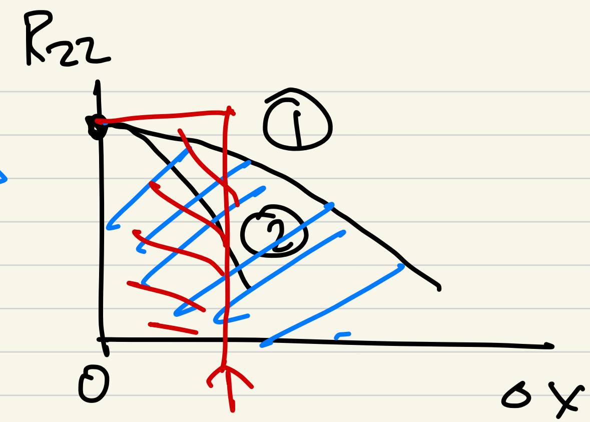




statistical diameter of vortex



$\langle v(\cdot, t)\, v(\cdot + \Delta x, t) \rangle$



Integral length scale

$$L_{11}(\underline{x}) = \frac{1}{R_{11}(0, \underline{x})} \int_0^\infty R_{11}(\underline{e}_1 r, \underline{x})\, dr$$

r: separation distance Δx, Δz

Wavenumber spectra

For homo. turbulence, $R_{ij}(\underline{r}, \underline{x})$ is indep. of $\underline{x}$.

$\rightarrow R_{ij}(\underline{r})$

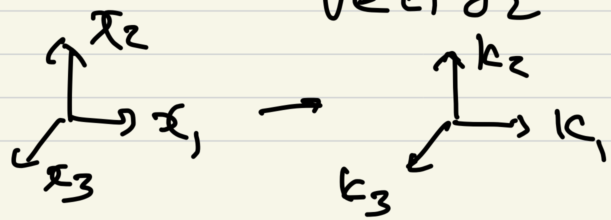
$\rightarrow$ wavenumber spectra

$$\Phi_{ij}(\underline{k}) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} e^{-i\underline{k}\cdot\underline{r}} R_{ij}(\underline{r})\, d\underline{r}$$

$\underline{k}$: wavenumber vector

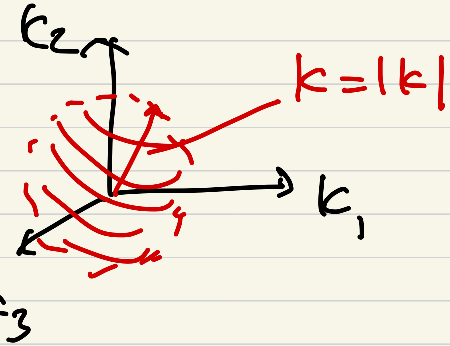
$$R_{ij}(\underline{r}) = \iiint_{-\infty}^{\infty} e^{i\underline{k}\cdot\underline{r}} \Phi_{ij}(\underline{k})\, d\underline{k}$$

$R_{ii}(0) = \int \Phi_{ii}\, d\vec{k}$



$$\boxed{\langle u_i u_j \rangle = R_{ij}(0) = \iiint_{-\infty}^{\infty} \Phi_{ij}(\underline{k})\, d\underline{k}}$$: Φ_{ij} is the contribution to $\langle u_i u_j \rangle$ with $\underline{k}$.

· One-dimensional energy spectrum ft.



$$E(k) = \iiint_{-\infty}^{\infty} \frac{1}{2} \Phi_{ii}(\underline{k})\, \delta(|\underline{k}| - k)\, d\underline{k}$$

$$\boxed{\int_0^{\infty} E(k)\, dk = \frac{1}{2} R_{ii}(0) = \frac{1}{2}\langle u_i u_i \rangle} = \frac{1}{2}\langle u_1 u_1 + u_2 u_2 + u_3 u_3 \rangle$$

turbulent kinetic energy

↳ $E(k)$ is the contribution to turbulent kinetic energy, $\frac{1}{2}\langle u_i u_i \rangle$, from all modes with $|\underline{k}|$ in the range of $k \le |\underline{k}| \le k + dk$.

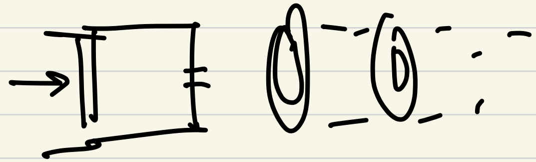
· Probability & averaging

· time average for stat. stationary flows

$$\langle U(t) \rangle_T = \frac{1}{T} \int_t^{t+T} U(t')\, dt'$$

- ensemble average for repeatable flows

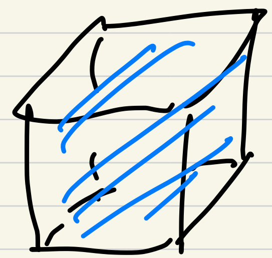
$$\langle U(t)\rangle_N = \frac{1}{N}\sum_{n=}^{N} U^{(n)}(t)$$



- spatial average for homogeneous turbulence

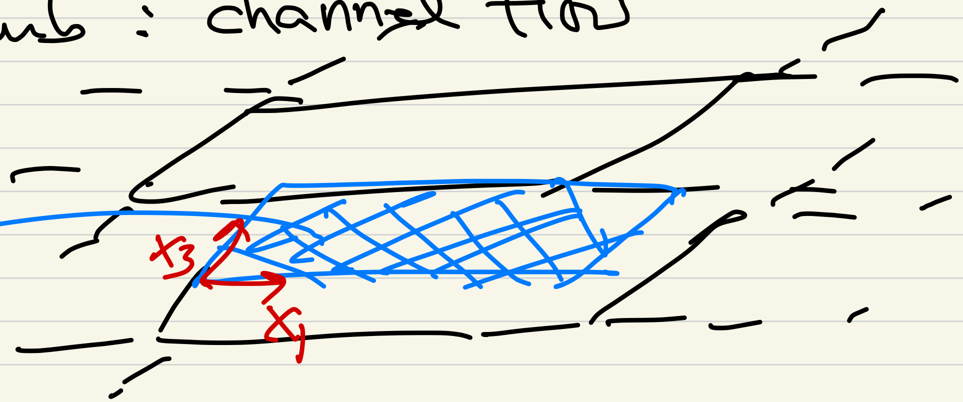
$$\langle U(t)\rangle_L = \frac{1}{L^3}\iiint_0^L U(x,t)\,dx_1\,dx_2\,dx_3$$

3-D homo. turb. →



2-D homo. turb : channel flow

2D - homo.



$$\frac{1}{L^2}\iint \cdot \, dx_1\, dx_3$$

Ch. 4