Lecture Note of Naval Architectural Calculation

Ship Stability

Ch. 5 Initial Longitudinal Stability

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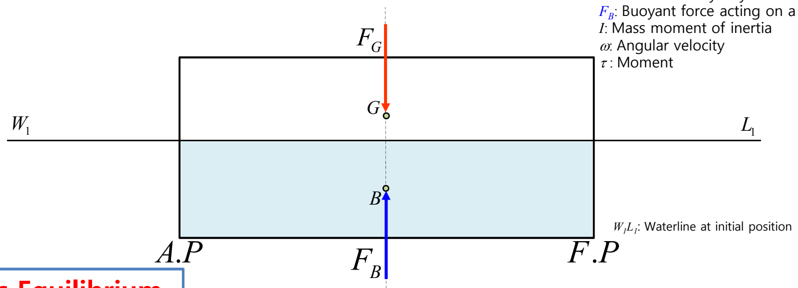
Contents

Ch. 5 Initial Longitudinal Stability

1. Longitudinal Stability

Static Equilibrium

m: Mass of a ship
a: Acceleration of a ship
G: Center of mass of a ship
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
I: Mass moment of inertia
ω: Angular velocity
τ: Moment



Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$

$$\boxed{\therefore F_G = F_B}$$

② **Euler equation**

$$I\dot{\omega} = \sum \tau$$

for the ship to be in static equilibrium

$$0 = \sum \tau \quad ,(\because \dot{\omega} = 0)$$

When the buoyant force (F_B) and the gravitational force (F_G) are on one line, the total moment about the transverse axis through any point becomes 0.

It does not matter where the axis of rotation is selected if the two forces are on one line.

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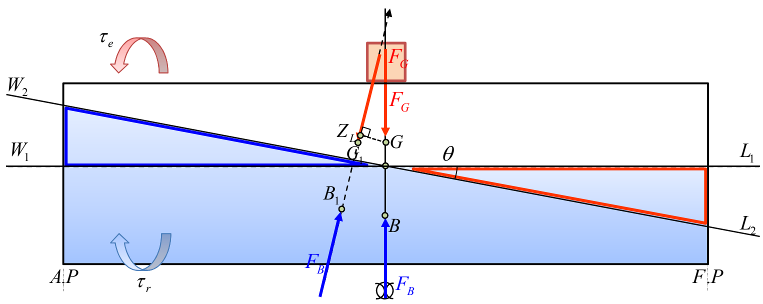
Longitudinal Stability - Stable Equilibrium

B_1: Changed position of the center of buoyancy after the ship has been trimmed
W_1L_1: Waterline at initial position
W_2L_2: Waterline after trim
$A.P$: after perpendicular, $F.P$: forward perpendicular



① Produce an external trim moment by moving a weight in the longitudinal direction (τ_e).

② Then, release the external moment by moving the weight to its original position.

③ Test **whether it returns to its initial equilibrium position**.

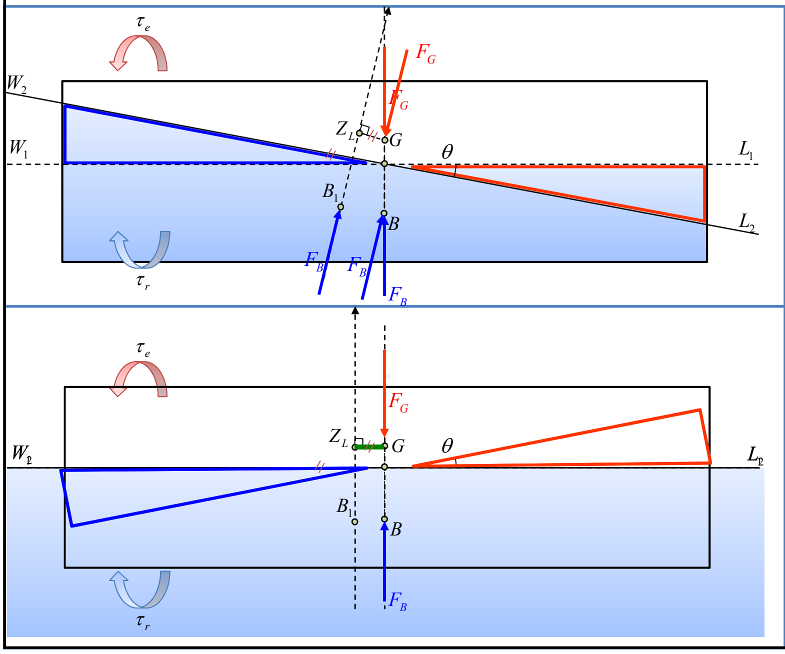
Z_L: Intersection point of the vertical line to the waterline W_2L_2 through the changed position of the center of buoyancy (B_1) with the horizontal line parallel to the waterline W_2L_2 through the center of mass of the ship (G)

Return to its initial equilibrium position

Stable

Longitudinal righting moment (τ_r) ⇨ **GZ_L ➡ Longitudinal righting arm**

Reference Frames

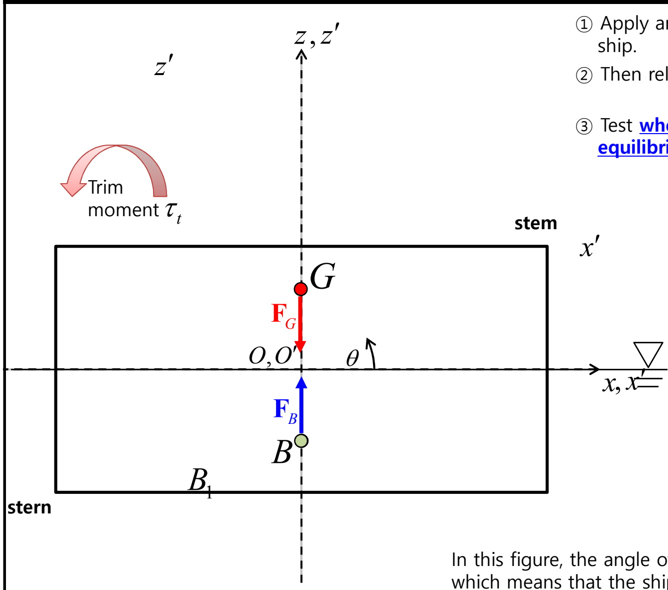


Rotation of the water plane fixed frame while the body (ship) fixed frame is fixed

Same in view of the Mechanics!!

Rotation of the body (ship) fixed frame with respect to the water plane fixed frame

Longitudinal Stability of a Ship - Stable Condition (1/3)



① Apply an external trim moment to the ship.

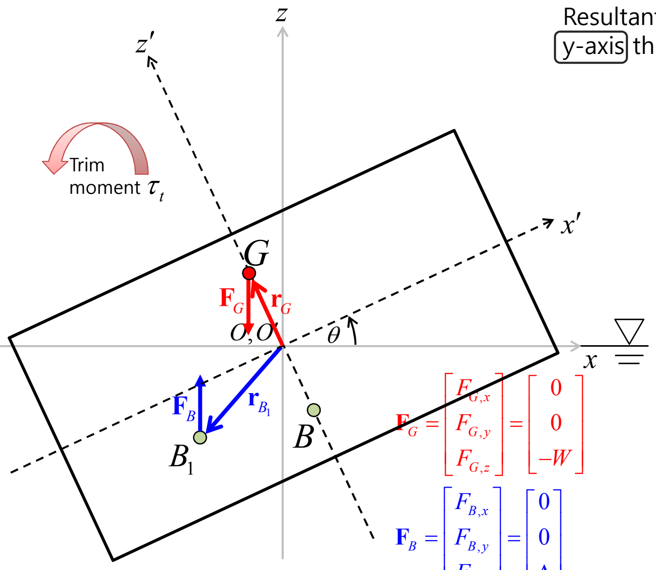
② Then release the external moment.

③ Test **whether it returns to its initial equilibrium position**.

In this figure, the angle of inclination, θ, is negative, which means that the ship is trimmed by the stern.

Longitudinal Stability of a Ship - Stable Condition (2/3)

$$\mathbf{r}_G \times \mathbf{F}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & y_G & z_G \\ F_{G,x} & F_{G,y} & F_{G,z} \end{vmatrix} = \begin{array}{l} \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ +\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ +\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \end{array}$$



In this figure, the angle of inclination, θ, is negative, which means that the ship is trimmed by the stern.

$$\mathbf{F}_G = \begin{bmatrix} F_{G,x} \\ F_{G,y} \\ F_{G,z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -W \end{bmatrix} \qquad \mathbf{F}_B = \begin{bmatrix} F_{B,x} \\ F_{B,y} \\ F_{B,z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Delta \end{bmatrix}$$

Resultant moment about $\boxed{\text{y-axis}}$ through point O ($\boldsymbol{\tau}^e$):

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$\begin{aligned} &= \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ &\quad \boxed{+\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x})} \\ &\quad +\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \\ &\quad +\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot F_{B,y}) \\ &\quad \boxed{+\mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot F_{B,x})} \\ &\quad +\mathbf{k}(x_{B_1} \cdot F_{B,y} - y_{B_1} \cdot F_{B,x}) \end{aligned}$$

$$\begin{aligned} &= \mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot \cancel{F_{G,x}}) \\ &\quad +\mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot \cancel{F_{B,x}}) \end{aligned}$$

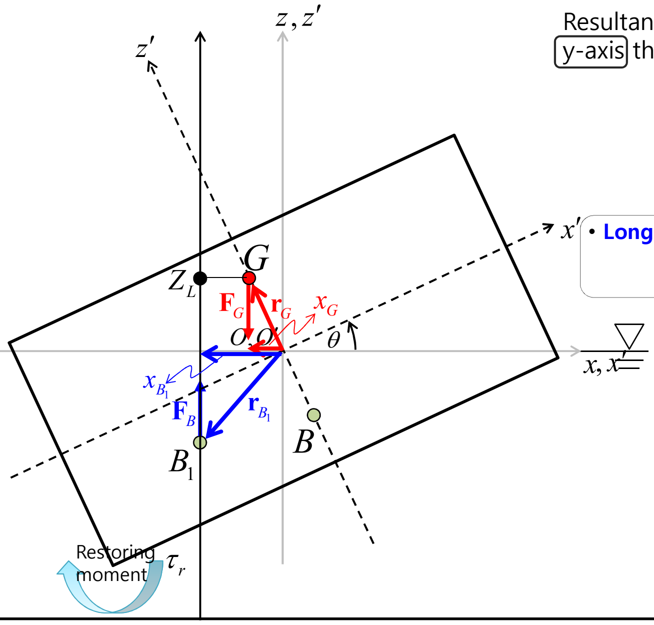
$$= \mathbf{j}(-x_G \cdot (-W) - x_{B_1} \cdot \Delta)$$

If $W = \Delta$

$$= \mathbf{j}(-x_G \cdot (-\Delta) - x_{B_1} \cdot \Delta)$$

$$= \mathbf{j} \cdot \Delta(x_G - x_{B_1})$$

Longitudinal Stability of a Ship - Stable Condition (3/3)



Resultant moment about $\boxed{\text{y-axis}}$ through point O ($\boldsymbol{\tau}^e$):

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \mathbf{j} \cdot \Delta(x_G - x_{B_1})$$

$$= \mathbf{j} \cdot \Delta \cdot GZ_L$$

- **Longitudinal Righting Moment**

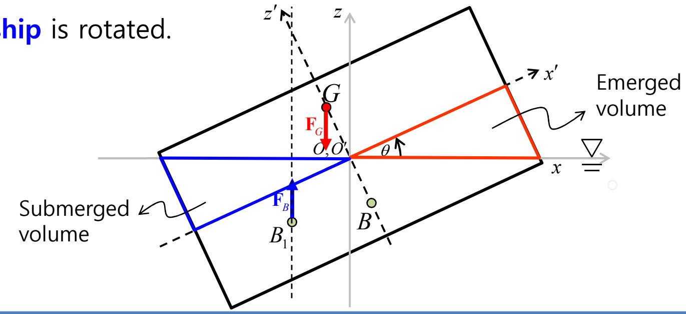
$$\tau_r = \Delta \cdot GZ_L$$

The moment arm induced by the buoyant force and gravitational force is expressed by GZ_L, where Z_L is the intersection point of the line of buoyant force(Δ) through the new position of the center of buoyancy(B_1) with a transversely parallel line to a waterline through the center of the ship's mass(G).

Stable!!

Position and Orientation of a Ship with Respect to the Water Plane Fixed and Body(Ship) Fixed Frame

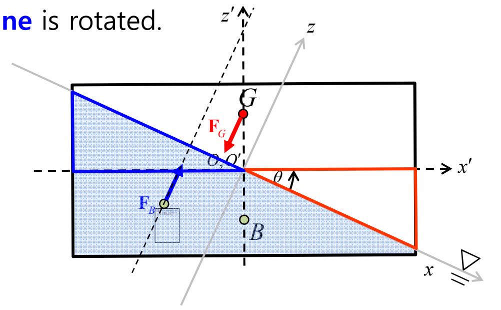
The ship is rotated.



Rotation of the body (ship) fixed frame while the water plane fixed frame is fixed

Same in view of the Mechanics!!

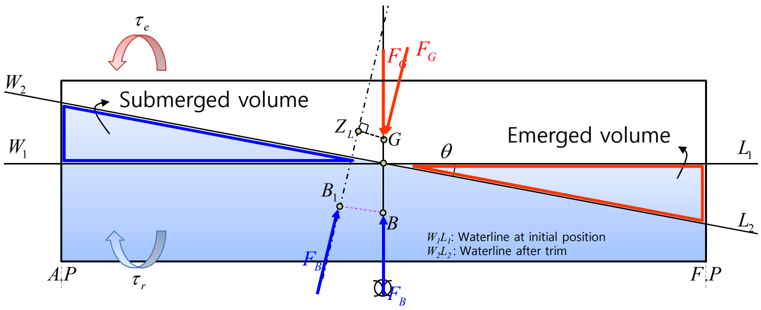
The water plane is rotated.



Rotation of the water plane fixed frame while the body (ship) fixed frame is fixed

2. Longitudinal Stability in Case of Small Angle of Trim

Assumptions for Small Angle of Trim

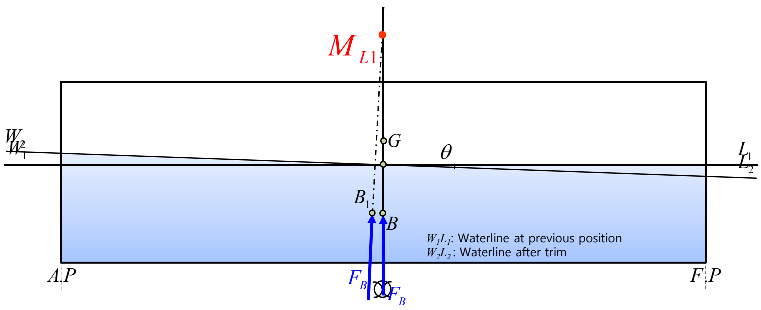


Assumptions

① **Small angle of inclination** (3~5° for trim)

② **The submerged volume** and **the emerged volume** are to be the **same**.

Longitudinal Metacenter (M_L) (1/3)



Longitudinal Metacenter (M_L) ※ Metacenter "M" is valid for small angle of inclination.

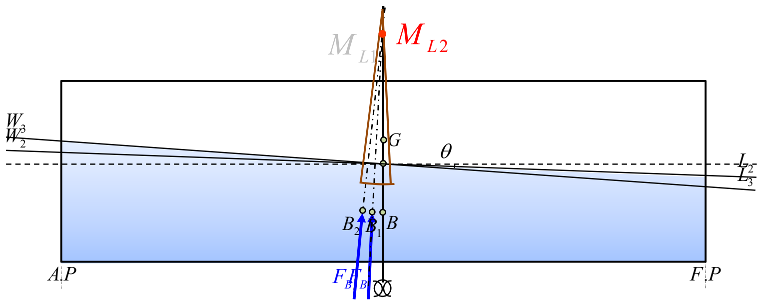
The **intersection point** of

a vertical line through the center of buoyancy **at a previous position** (**B**)

with a vertical line through the center of buoyancy **at the present position** (**B_1**)

after the ship has been trimmed.

Longitudinal Metacenter (M_L) (2/3)



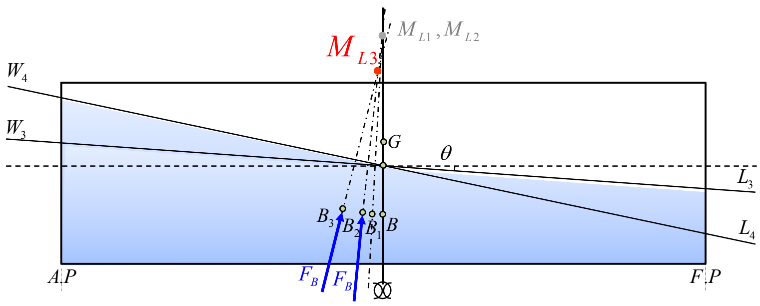
M_L **remains at the same position** for small angle of trim, up to **about 3~5 degrees**.

As the ship is inclined with a small trim angle, **B** moves on the arc of circle whose center is at **M_L**.

The BM_L is **longitudinal metacentric radius**.

The GM_L is **longitudinal metacentric height**.

Longitudinal Metacenter (M_L) (3/3)



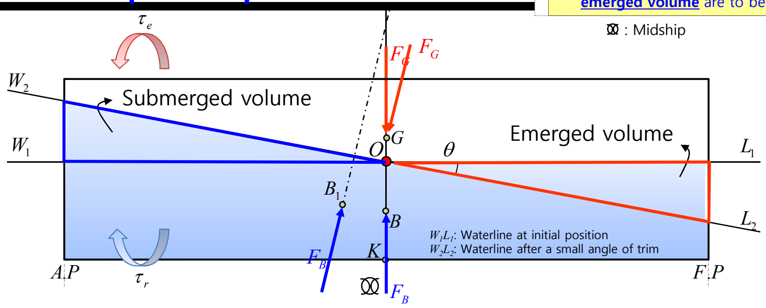
M_L **does not remain in the same position** for large trim angles **over 5 degrees**.

Thus, the longitudinal metacenter, M_L, is only **valid for a small trim angle.**

Longitudinal Stability for a **Box-Shaped Ship**

Assumption
① **A small trim angle** (3~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.

ⓧ : Midship



About which point a box-shaped ship rotates, while the submerged volume and the emerged volume are to be the same?

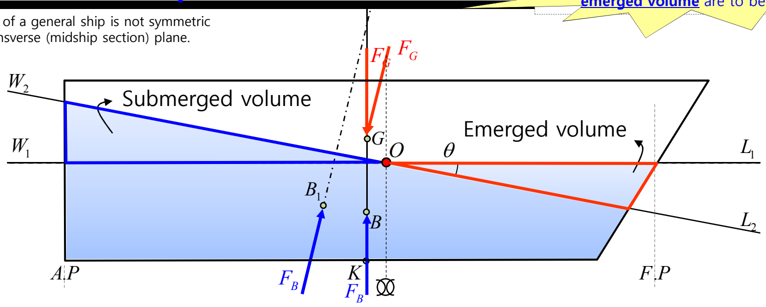
① Apply an external trim moment (τ_e) which results in the ship to incline with a trim angle θ.

② For the **submerged volume** and the **emerged volume** are to be the **same**, the ship rotates **about the transverse axis through the point O**.

Longitudinal Stability for a **General Ship**

Assumption
① **A small angle of inclination** (3~5° for trim)
② The **submerged volume** and the **emerged volume** are to be the **same**.

The hull form of a general ship is not symmetric about the transverse (midship section) plane.

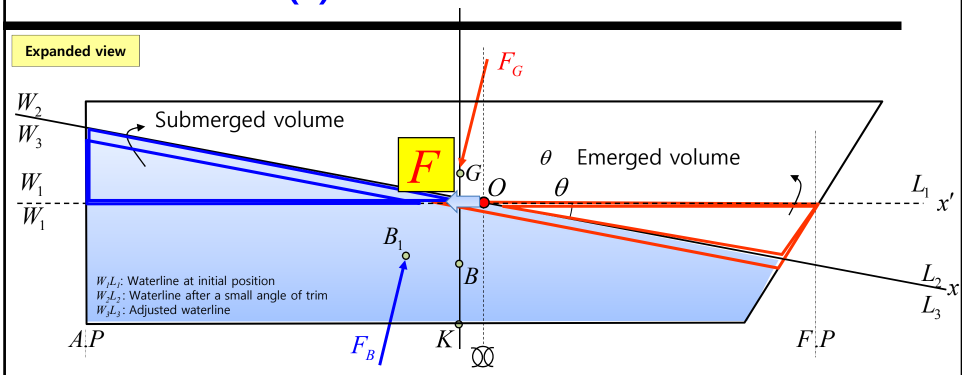


What will happen if the hull form of a ship is **not symmetric** about the transverse (midship section) plane through point O?

The submerged volume and the emerged volume are **not same**!

So, **the draft** must **be adjusted** to maintain same displacement.

Rotation Point (F)



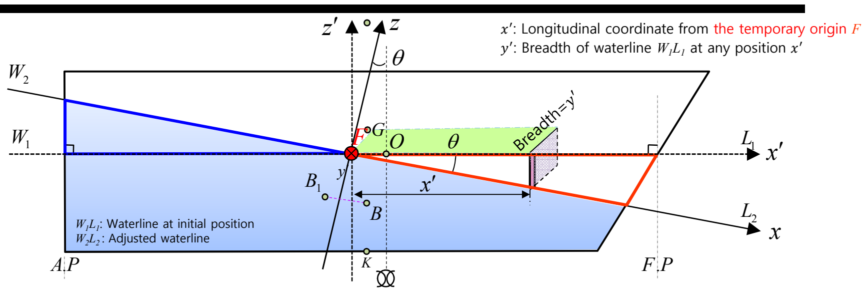
So, **the draft** must **be adjusted** to maintain same displacement.

The intersection of the initial waterline (W_1L_1) with the adjusted waterline (W_3L_3) is a point ***F***, on which the submerged volume and the emerged volume are supposed to be the same.

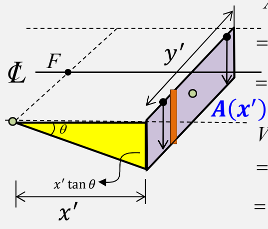
What we want to find out is the point ***F***. How can we find the point ***F*** ?

Longitudinal Center of Floatation (LCF) (1/3)

x': Longitudinal coordinate from the temporary origin F
y': Breadth of waterline W_1L_1 at any position x'



From now on, the adjusted waterline is indicated as W_2L_2.



$$A(x') = (y')\cdot(x'\cdot\tan\theta) = x'\cdot y'\cdot\tan\theta$$

$$V(x') = \int A(x')dx' = \int x'\cdot y'\cdot\tan\theta dx'$$

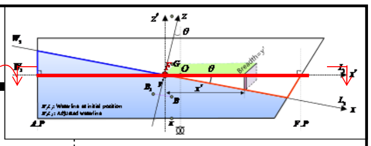
Submerged volume (v_a) = Emerged volume (v_f)

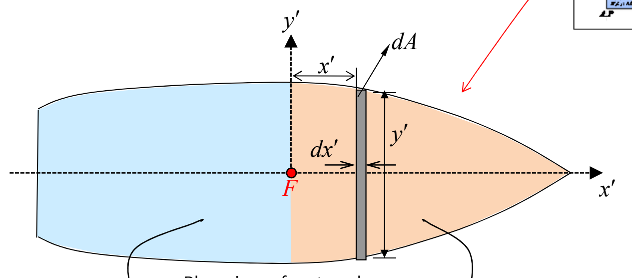
$$\int_{A.P}^{F} y'\cdot(x'\cdot\tan\theta)dx' = \int_{F}^{F.P} y'\cdot(x'\cdot\tan\theta)dx'$$

$$\int_{A.P}^{F} x'\cdot y'\ dx' = \int_{F}^{F.P} x'\cdot y'\ dx'$$

What does this equation mean?

Longitudinal Center of Floatation (LCF) (2/3)





<Plan view of water plane>

Longitudinal moment of the **after** water plane area from F about the transverse axis (y') through the point F

$$\int_{A.P}^{F} x'\,dA = \int_{A.P}^{F} x'\cdot y'\,dx'$$

Longitudinal moment of the **forward** water plane area from F about the transverse axis (y') through the point F

$$\int_{F}^{F.P} x'\,dA = \int_{F}^{F.P} x'\cdot y'\,dx$$

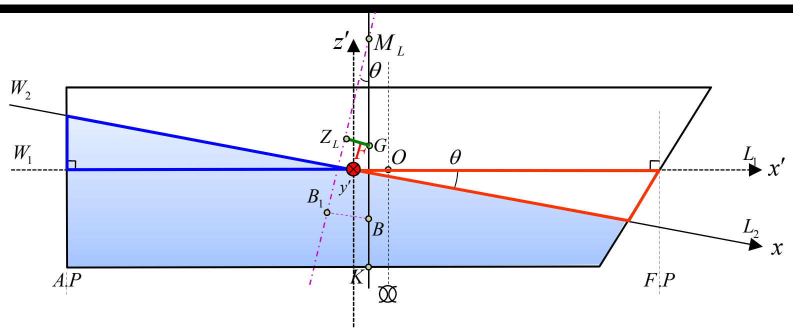
$$v_a = v_f \Rightarrow \int_{A.P}^{F} y'\cdot(x'\cdot\tan\theta)dx' = \int_{F}^{F.P} y'\cdot(x'\cdot\tan\theta)dx'$$

$$\int_{A.P}^{F} x'\cdot y'\,dx' = \int_{F}^{F.P} x'\cdot y'\,dx'$$

Since these moments are **equal and opposite**, the longitudinal moment of the entire water plane area about the transverse axis through point F is **zero**.

That means the point F lies on the transverse axis through the **centroid of the water plane**, called longitudinal center of floatation (LCF).

Longitudinal Center of Floatation (LCF) (3/3)

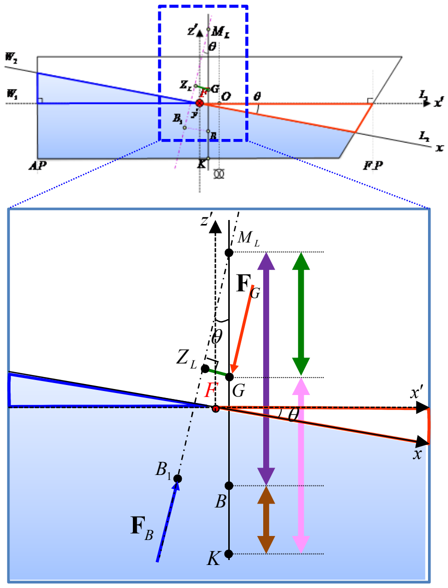


Therefore, for the ship to incline under the condition that the submerged volume and emerged volume are to be the same,
the ship rotates about the transverse axis through the longitudinal center of floatation (LCF).

F ⇨ Longitudinal Center of Floatation (LCF)

3. Longitudinal Righting Moment Arm

Longitudinal Righting Moment Arm (GZ_L)



Longitudinal Righting Moment

$$= \underline{GZ_L} \cdot \mathbf{F}_B$$

From geometrical configuration

$$GZ_L \cong GM_L \cdot \sin\theta$$

with assumption that M_L remains at the same position within a small angle of trim (about 3~5°)

$$GM_L = KB + BM_L - KG$$

KB ≒ 51~52% draft

How can you get the value of the BM_L?

Vertical center of mass of the ship

GM_L: Longitudinal metacentric height

KB: Vertical center of buoyancy at initial position

BM_L: Longitudinal metacentric radius

KG: Vertical center of mass of the ship

K: Keel M_L: Longitudinal Metacenter

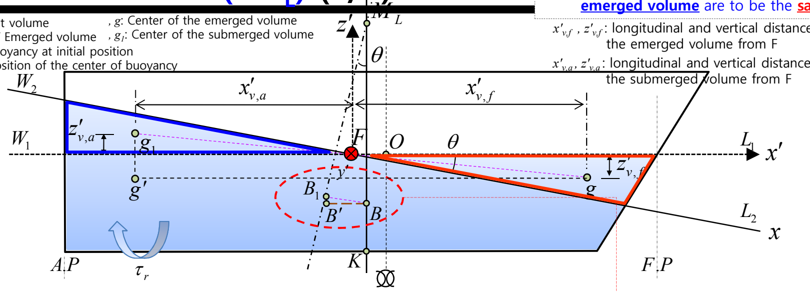
4. Derivation of Longitudinal Metacentric Radius (BM$_L$)

Derivation of Longitudinal Metacentric Radius (BM$_L$) (1/8)

Assumption
① **A small trim angle** (3~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.

∇: Displacement volume, g: Center of the emerged volume
v: Submerged / Emerged volume, g_1: Center of the submerged volume
B: Center of buoyancy at initial position
B_1: Changed position of the center of buoyancy

$x'_{v,f}$, $z'_{v,f}$: longitudinal and vertical distance of the emerged volume from F
$x'_{v,a}$, $z'_{v,a}$: longitudinal and vertical distance of the submerged volume from F



Let's derive longitudinal metacentric radius BM$_L$ when a ship is trimmed.

The center of buoyancy at initial position (B) moves parallel to gg_1.

The distance BB_1 equals to $\frac{v}{\nabla}\cdot gg_1$

$$BB_1 = \frac{v}{\nabla}\cdot gg_1$$

Because the triangle $gg'g_1$ is similar with $BB'B_1$

$$\frac{BB_1}{gg_1} = \frac{BB'}{gg'} = \frac{\delta x'_B}{(x'_{v,a}+x'_{v,f})}$$

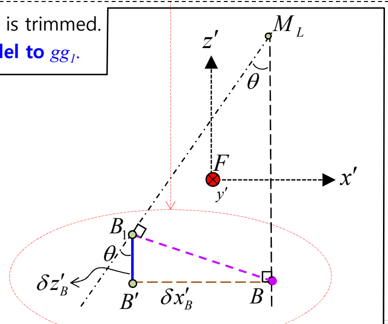
$$\frac{BB_1}{gg_1} = \frac{B'B_1}{g'g_1} = \frac{\delta z'_B}{(z'_{v,a}+z'_{v,f})}$$

$$\delta x'_B = \frac{v}{\nabla}\cdot(x'_{v,a}+x'_{v,f}) \quad ...(1)$$

: **Longitudinal translation** of B to B$_1$

$$\delta z'_B = \frac{v}{\nabla}\cdot(z'_{v,a}+z'_{v,f}) \quad ...(2)$$

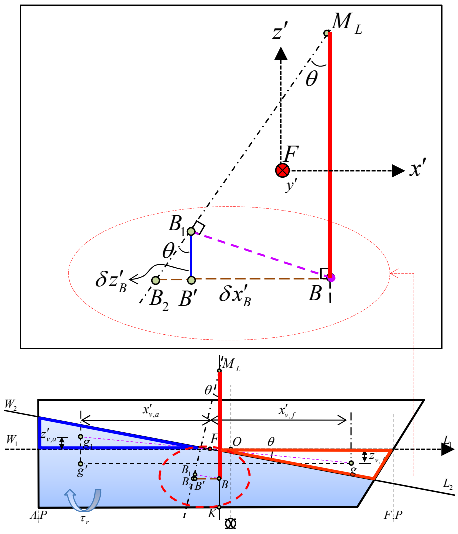
: **Vertical translation** of B to B$_1$



Derivation of Longitudinal Metacentric Radius (BM_L) (2/8)

$$\delta x'_B = \frac{v}{\nabla}\cdot(x'_{v,a} + x'_{v,f}) \quad ...(1)$$

$$\delta z'_B = \frac{v}{\nabla}\cdot(z'_{v,a} + z'_{v,f}) \quad ...(2)$$



$$BM_L \cdot \tan\theta = BB_2$$

$$BM_L = \frac{BB_2}{\tan\theta}$$

$$BB_2 = BB' + B'B_2$$

$$= \frac{1}{\tan\theta}(BB' + B'B_2)$$

$$= \frac{1}{\tan\theta}(\delta x'_B + \delta z'_B \tan\theta)$$

Substituting (1) and (2) into $\delta x'_B$ and $\delta z'_B$

$$= \frac{1}{\tan\theta}\left(\frac{v}{\nabla}\cdot(x'_{v,a} + x'_{v,f}) + \frac{v}{\nabla}\cdot(z'_{v,a} + z'_{v,f})\tan\theta\right)$$

$$BM_L = \frac{1}{\nabla\cdot\tan\theta}\left(\boxed{v\cdot x'_{v,a}} + \boxed{v\cdot x'_{v,f}} + (\boxed{v\cdot z'_{v,a}} + \boxed{v\cdot z'_{v,f}})\tan\theta\right)$$

Find!

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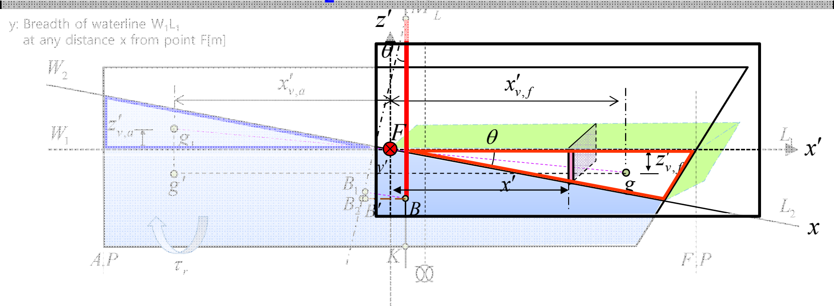
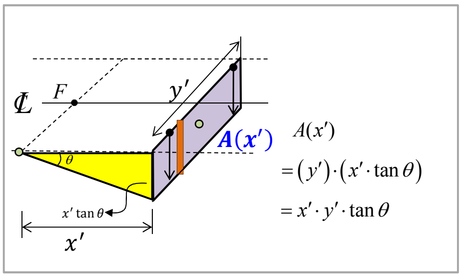
Derivation of Longitudinal Metacentric Radius (BM_L) (3/8)

$$BM_L = \frac{1}{\nabla\cdot\tan\theta}\left(\underbrace{\boxed{v\cdot x'_{v,f}}}_{(A)} + \underbrace{\boxed{v\cdot x'_{v,a}}}_{(B)} + (\underbrace{\boxed{v\cdot z'_{v,f}}}_{(C)} + \underbrace{\boxed{v\cdot z'_{v,a}}}_{(D)})\tan\theta\right)$$





(A) $v\cdot x'_{v,f}$: **Moment** of the emerged volume **about y'-z' plane**

$$= \int_F^{F.P} A(x')\cdot x'\,dx'$$

$$= \int_F^{F.P} x'\cdot y'\cdot\tan\theta\cdot x'\,dx'$$

$$= \tan\theta\int_F^{F.P} (x')^2\cdot y'\,dx'$$

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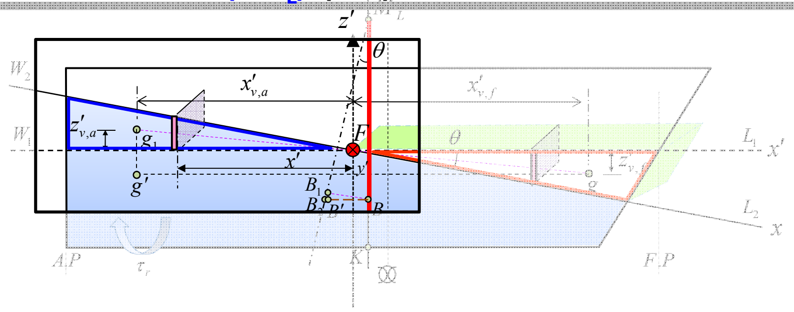
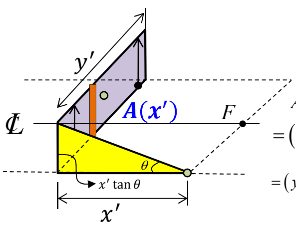
Derivation of Longitudinal Metacentric Radius (BM_L) (4/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v\cdot x'_{v,f} + v\cdot x'_{v,a} + \left(v\cdot z'_{v,f} + v\cdot z'_{v,a}\right)\tan\theta\right)$$

(A) (B) (C) (D)





$A(x')$

$= (y')\cdot(x'\cdot\tan\theta)$

$= (y')\cdot(x'\cdot\tan\theta)$

(B) $v\cdot x'_{v,a}$: **Moment** of the submerged volume **about y'-z' plane**

$$= \int_{A.P}^{F} A(x')\cdot x'\,dx'$$

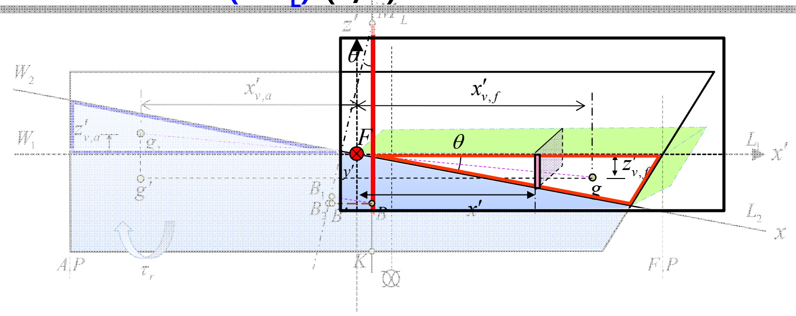
$$= \int_{A.P}^{F} \left(x'\cdot y'\cdot\tan\theta\cdot x'\right)dx'$$

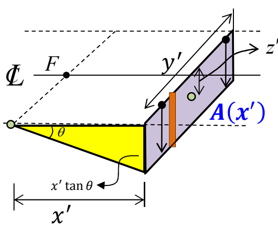
$$= \tan\theta\int_{A.P}^{F} \left(x'\right)^2\cdot y'\,dx'$$

Derivation of Longitudinal Metacentric Radius (BM_L) (5/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v\cdot x'_{v,f} + v\cdot x'_{v,a} + \left(v\cdot z'_{v,f} + v\cdot z'_{v,a}\right)\tan\theta\right)$$

(A) (B) (C) (D)





$z' = \frac{1}{2}x'\cdot\tan\theta$

$A(x')$

$= (y')\cdot(x'\cdot\tan\theta)$

$= x'\cdot y'\cdot\tan\theta$

(C) $v\cdot z'_{v,f}$: **Moment** of the emerged volume **about x'-y' plane**

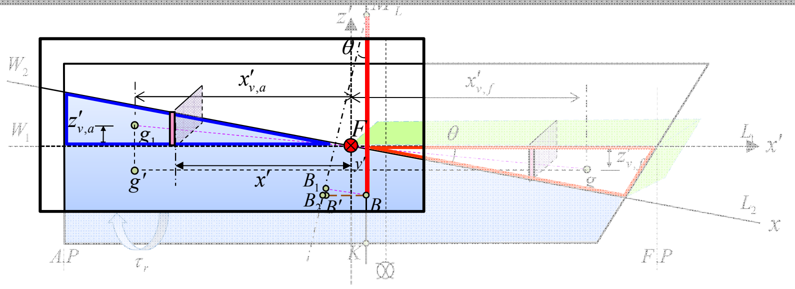
$$= \int_{F}^{F.P} A(x')\cdot z'\,dx$$

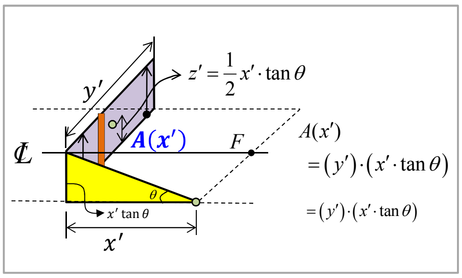
$$= \int_{F}^{F.P} \left(x'\cdot y'\cdot\tan\theta\right)\left(\frac{x'}{2}\tan\theta\right)dx'$$

$$= \frac{\tan^2\theta}{2}\int_{F}^{F.P} \left(x'\right)^2\cdot y'\,dx'$$

Derivation of Longitudinal Metacentric Radius (BM_L) (6/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v \cdot x'_{v,f} + v \cdot x'_{v,a} + (v \cdot z'_{v,f} + v \cdot z'_{v,a})\tan\theta\right)$$
(A) (B) (C) (D)





(D) $v \cdot z'_{v,a}$: **Moment** of the submerged volume **about x'-y' plane**

$$= \int_{A.P}^{F} A(x') \cdot z' dx'$$

$$= \int_{A.P}^{F} (x' \cdot y' \cdot \tan\theta)\left(\frac{x'}{2}\tan\theta\right)dx'$$

$$= \frac{\tan^2\theta}{2}\int_{A.P}^{F} (x')^2 \cdot y' dx'$$

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Derivation of Longitudinal Metacentric Radius (BM_L) (7/8)

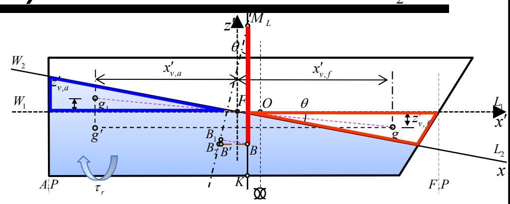
(A) $v \cdot x'_{v,f} = \tan\theta \int_F^{FP} (x')^2 \cdot y' dx'$

(B) $v \cdot x'_{v,a} = \tan\theta \int_{AP}^{F} (x')^2 \cdot y' dx'$

(C) $v \cdot z'_{v,f} = \frac{\tan^2\theta}{2}\int_F^{FP} (x')^2 \cdot y' dx'$

(D) $v \cdot z'_{v,a} = \frac{\tan^2\theta}{2}\int_{AP}^{F} (x')^2 \cdot y' dx'$

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v \cdot x'_{v,f} + v \cdot x'_{v,a} + (v \cdot z'_{v,f} + v \cdot z'_{v,a})\tan\theta\right)$$
(A) (B) (C) (D)



By substituting (A), (B), (C), and (D) into the BM_L equation

$$= \frac{1}{\nabla \cdot \tan\theta}\left(\tan\theta \cdot \int_F^{F.P} (x')^2 \cdot y'dx' + \tan\theta \cdot \int_{A.P}^{F} (x')^2 \cdot y' dx' + \left(\frac{\tan^2\theta}{2}\int_F^{F.P} (x')^2 \cdot y'dx' + \frac{\tan^2\theta}{2}\int_{A.P}^{F} (x')^2 \cdot y' dx'\right)\tan\theta\right)$$

$$= \frac{1}{\nabla \cdot \tan\theta}\left(\tan\theta\left(\int_F^{F.P} (x')^2 \cdot y'dx' + \int_{A.P}^{F} (x')^2 \cdot y' dx'\right) + \frac{\tan^3\theta}{2}\left(\int_F^{F.P} (x')^2 \cdot y'dx' + \int_{A.P}^{F} (x')^2 \cdot y' dx'\right)\right)$$

$$,\left(I_L = \int_F^{F.P} (x')^2 \cdot y'dx' + \int_{A.P}^{F} (x')^2 \cdot y' dx'\right)$$

I_L : Moment of inertia of the entire water plane about transverse axis through its centroid F

$$= \frac{1}{\nabla \cdot \tan\theta}\left(\tan\theta \cdot I_L + \frac{1}{2}\tan^3\theta \cdot I_L\right)$$

$$\boxed{BM_L = \frac{I_L}{\nabla}\left(1 + \frac{1}{2}\tan^2\theta\right)}$$

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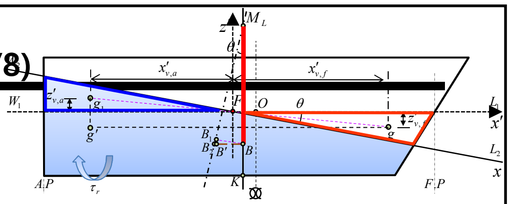
Derivation of Longitudinal Metacentric Radius (BM_L) (8/8)



$$BM_L = \frac{I_L}{\nabla}\left(1+\frac{1}{2}\cancel{\tan^2\theta}\right) \xrightarrow{\text{If } \theta \text{ is small, } \tan^2\theta \approx \theta^2 = 0} BM_L = \frac{I_L}{\nabla}$$

which is generally known as BM_L.

The BM_L does not consider the change of center of buoyancy **in vertical direction**.

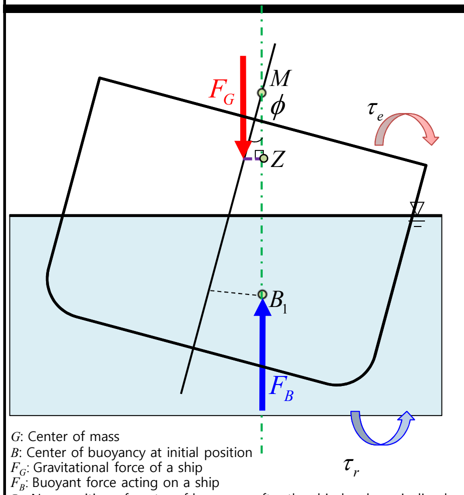
In order to distinguish between them, the two will be indicated as follows:

$\boxed{BM_{L0} = \frac{I_L}{\nabla}(1+\frac{1}{2}\tan^2\theta)}$ (**Considering** the change of center of buoyancy in vertical direction)

$\boxed{BM_L = \frac{I_L}{\nabla}}$ (**Without considering** the change of center of buoyancy in vertical direction)

5. Moment to Trim One Degree and Moment to Trim One Centimeter (MTC)

Moment to Heel One Degree



G: Center of mass
B: Center of buoyancy at initial position
F_G: Gravitational force of a ship
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of vertical line through the center of buoyancy with the transversally parallel line to a waterline through center of mass
M: The intersection point of a vertical line through the center of buoyancy at initial position(B) with a vertical line through the new position of the center of buoyancy(B_1) after the ship has been inclined transversally through a small angle

Definition

It is **the moment of external forces** required to produce **one degree heel**.

Assumption

Small inclination where the metacenter is not changed (about 7~10°)

When the ship is heeled and in static equilibrium,

Moment to heel = Righting moment

$$= F_B \cdot GZ$$

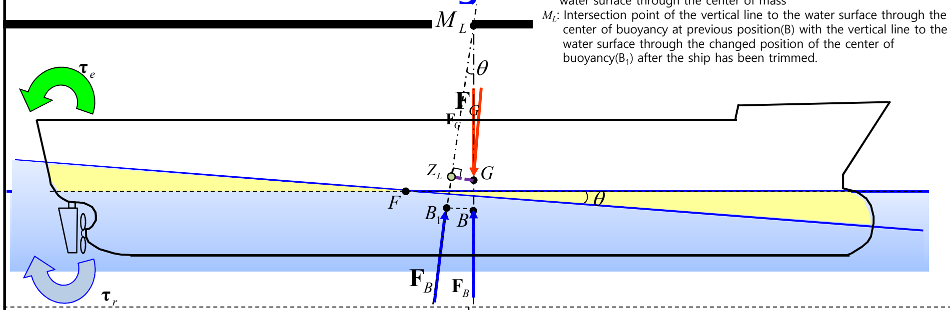
$$(\text{at small angle } \phi) \quad \boxed{\simeq F_B \cdot GM \cdot \sin\phi}$$

Substituting $\phi = 1^{\circ}$

$$\text{Moment to heel one degree} = F_B \cdot GM \cdot \sin 1^{\circ}$$

Moment to Trim One Degree

LCF: Longitudinal center of floatation
Z_L: Intersection point of the vertical line to the water surface through the changed center of buoyancy with the horizontal line parallel to the water surface through the center of mass
M_L: Intersection point of the vertical line to the water surface through the center of buoyancy at previous position(B) with the vertical line to the water surface through the changed position of the center of buoyancy(B_1) after the ship has been trimmed.



Definition

It is **the moment of external forces** required to produce **one degree trim**.

Assumption

Small trim where the metacenter is not changed (about 3~5°)

When the ship is trimmed and in static equilibrium,

Moment to trim = Righting moment

$$= F_B \cdot GZ_L$$

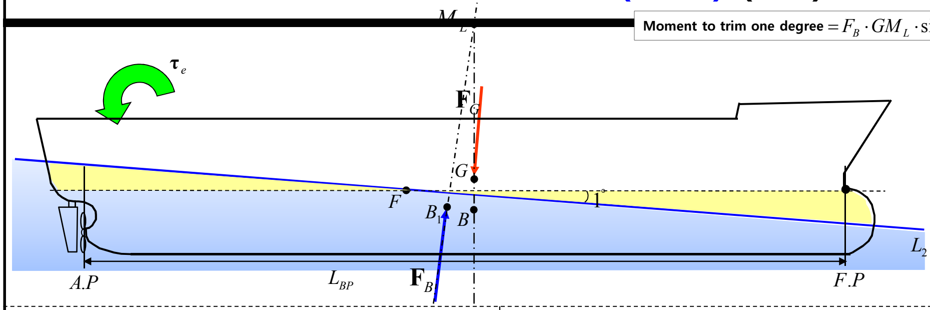
$$(\text{at small angle } \theta) \quad \boxed{\simeq F_B \cdot GM_L \cdot \sin\theta}$$

Substituting $\theta = 1^{\circ}$

$$\text{Moment to trim one degree} = F_B \cdot GM_L \cdot \sin 1^{\circ}$$

Moment to Trim One Centimeter (MTC) (1/2)

Moment to trim one degree $= F_B \cdot GM_L \cdot \sin 1^{\circ}$

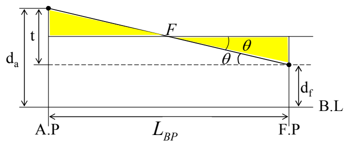


Often, we are more interested in the **changes in draft** produced by a trim moment than the changes in trim angle.

$$MTC = F_B \cdot GM_L \cdot \frac{1}{L_{BP} \cdot 100}$$

MTC: Moment to trim 1 cm

L_{BP}: Length between perpendiculars [m]



$t = 1\ cm$

Trim (t): $d_a - d_f$

$\sin\theta \approx \tan\theta = \frac{t}{L_{BP}}$

d_a: Draft after

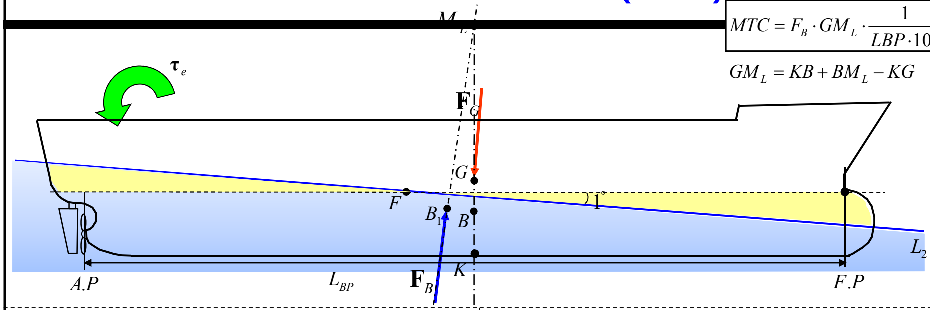
d_f: Draft forward

$(If\ \theta\ is\ small)$

Moment to Trim One Centimeter (MTC) (2/2)

$$MTC = F_B \cdot GM_L \cdot \frac{1}{LBP \cdot 100}$$

$$GM_L = KB + BM_L - KG$$



As practical matter, KB and KG are usually so small compared to GM_L that ***BM_L can be substituted for GM_L***.

$$MTC = F_B \cdot GM_L \cdot \frac{1}{L_{BP} \cdot 100} \approx F_B \cdot BM_L \cdot \frac{1}{L_{BP} \cdot 100}$$

Substituting $F_B = \rho \cdot \nabla,\ BM_L = \frac{I_L}{\nabla}$

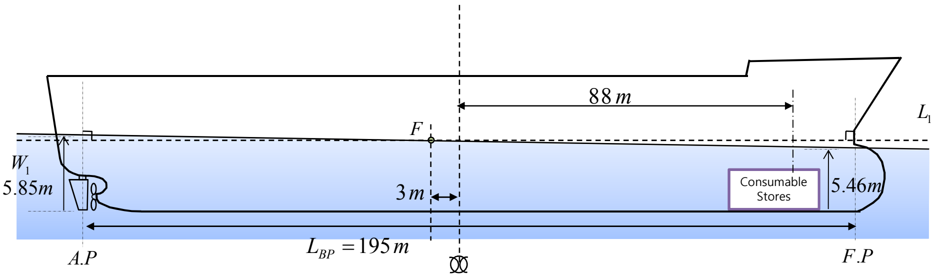
I_L: Moment of inertia of the water plane area about y' axis

$$MTC = \rho \cdot \nabla \cdot \frac{I_L}{\nabla} \cdot \frac{1}{L_{BP} \cdot 100}$$

$$= \frac{\rho \cdot I_L}{L_{BP} \cdot 100}$$

6. Example of Longitudinal Stability

[Example] Calculation of Draft Change Due to Fuel Consumption (1/4)



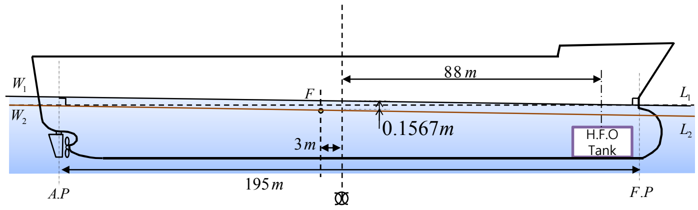
During a voyage, a cargo ship uses up 320ton of consumable stores (H.F.O: Heavy Fuel Oil), located 88m forward of the midships.

Before the voyage, the forward draft marks at forward perpendicular recorded 5.46m, and the after marks at the after perpendicular, recorded 5.85m.

At the mean draft between forward and after perpendicular, the hydrostatic data show the ship to have LCF after of midship = 3m, Breadth = 10.47m, moment of inertia of the water plane area about transverse axis through point F = 6,469,478m^4, Cwp = 0.8.

Calculate the draft mark the readings at the end of the voyage, assuming that there is no change in water density (ρ=1.0ton/m^3).

[Example] Calculation of Draft Change Due to Fuel Consumption (2/4)



① Calculation of parallel rise (draft change)

$$A_{WP} = C_{WP} \cdot L \cdot B = 0.8 \cdot 195 \cdot 10.47 = 1,633.3[m^2]$$

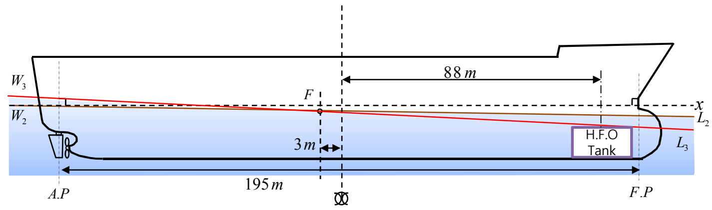
- Tones per 1 cm immersion (TPC)

$$: TPC = \rho \cdot A_{WP} \cdot \frac{1}{100} = 1[ton/m^3] \cdot 1,633.3[m^2] \cdot \frac{1}{100[cm/m]} = 20.4165[ton/cm]$$

- Parallel rise

$$: \delta d = \frac{weight}{TPC} = \frac{320[ton]}{20.4165[ton/cm]} = 15.6736[cm] = 0.1567[m]$$

[Example] Calculation of Draft Change Due to Fuel Consumption (3/4)



② Calculation of trim

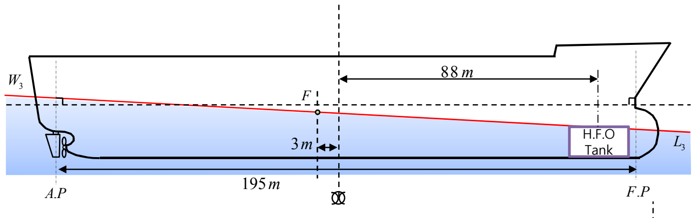
- Trim moment $: \tau_{trim} = 320[ton] \cdot 88[m] = 28,160[ton \cdot m]$
- Moment to trim 1 cm (MTC)

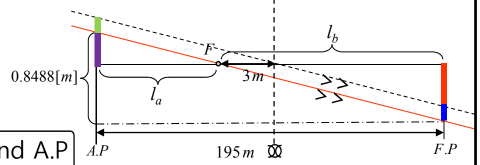
$$: MTC = \frac{\rho \cdot I_L}{100 \cdot L_{BP}} = \frac{1[ton/m^3]}{100[cm/m] \cdot 195[m]} \cdot 6,469,478[m^4] = 331.7949[ton \cdot m/cm]$$

- Trim

$$: Trim = \frac{\tau_{trim}}{MTC} = \frac{28,160[ton \cdot m]}{331.7949[ton \cdot m/cm]} = 84.8785[cm] = 0.8488[m]$$

[Example] Calculation of Draft Change Due to Fuel Consumption (4/4)





③ Calculation of changed draft at F.P and A.P

- Draft change at F.P due to trim $= -\dfrac{195/2+3}{195} \times 0.8488 = -0.4375[m]$
- Draft change at A.P due to trim $= \dfrac{195/2-3}{195} \times 0.8488 = 0.4113[m]$
- Changed Draft at F.P : draft – parallel rise - draft change due to trim
  $= 5.46[m] - 0.1567[m] - 0.4375[m] = 4.8658[m]$
- Changed Draft at A.P : draft – parallel rise + draft change due to trim
  $= 5.85[m] - 0.1567[m] + 0.4113[m] = 6.1046[m]$