Chapter 6

Vertical Photographs

Elements of Photogrammetry with Applications in GIS

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1. Geometry of Vertical Photographs

- Definition: photographs taken from an aircraft with the optical axis of the camera vertical (truly vertical) or as nearly vertical as possible.
- Equations are developed assuming truly vertical photographs.
- Most of vertical photos have tilts usually less than 1° and rare exceed 3°.
- Near-vertical or tilted photographs can be analyzed using the equations for truly vertical photos without serious error.
- Photo coordinate axis system is assumed to have its origin at the photographic principal point and various distortions are assumed to be corrected throughout this chapter.

1. Geometry of Vertical Photographs

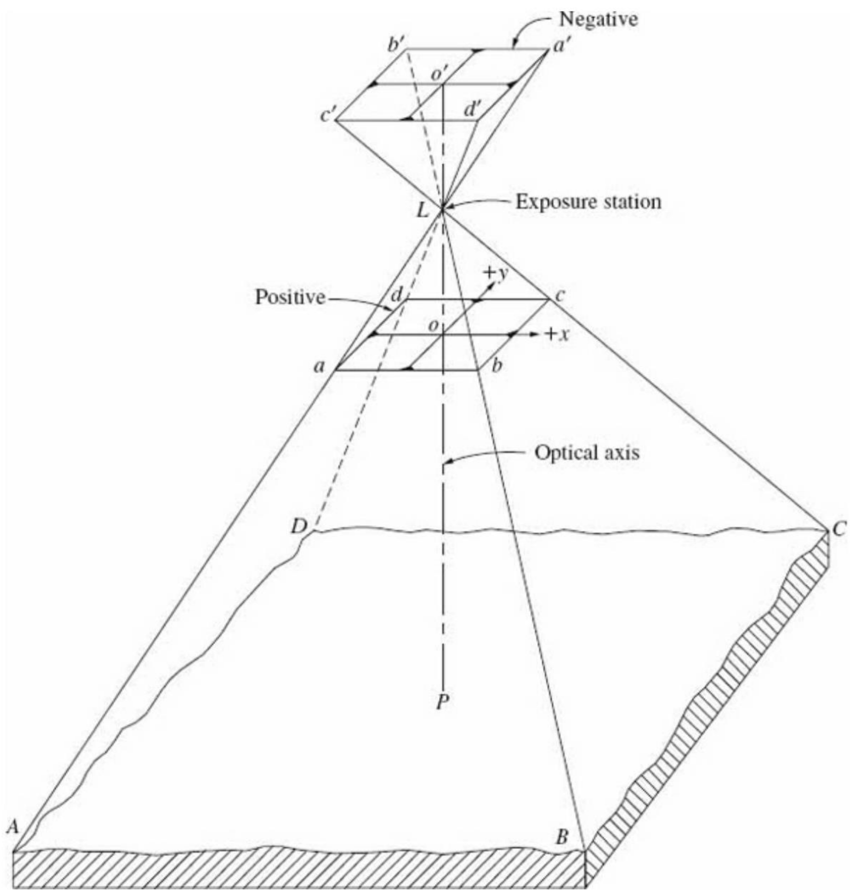

Figure 6-1 The geometry of a vertical photograph.

- The positive can be obtained from the negative having reversed tone and geometry in a focal length distance below the front nodal point of the camera lens

2. Scale

- Map (photograph) scale: ratio of a map (photograph) distance to the corresponding distance on the ground.

- A vertical aerial photograph is a perspective projection where the scale varies according to terrain elevation while the map scale is not influenced by terrain variations.

- Expression of scales: 1 in = 1,000 ft (unit equivalents), 1 in/1000 ft (unit fraction), 1/12,000 (dimensionless representative fraction), or 1:12000 (dimensionless ratio)

- A large number in a scale expression denotes a small scale, and vice versa:

  1:1000 is larger scale than 1:5000.

3. Scale of a Vertical Photograph over Flat Terrain

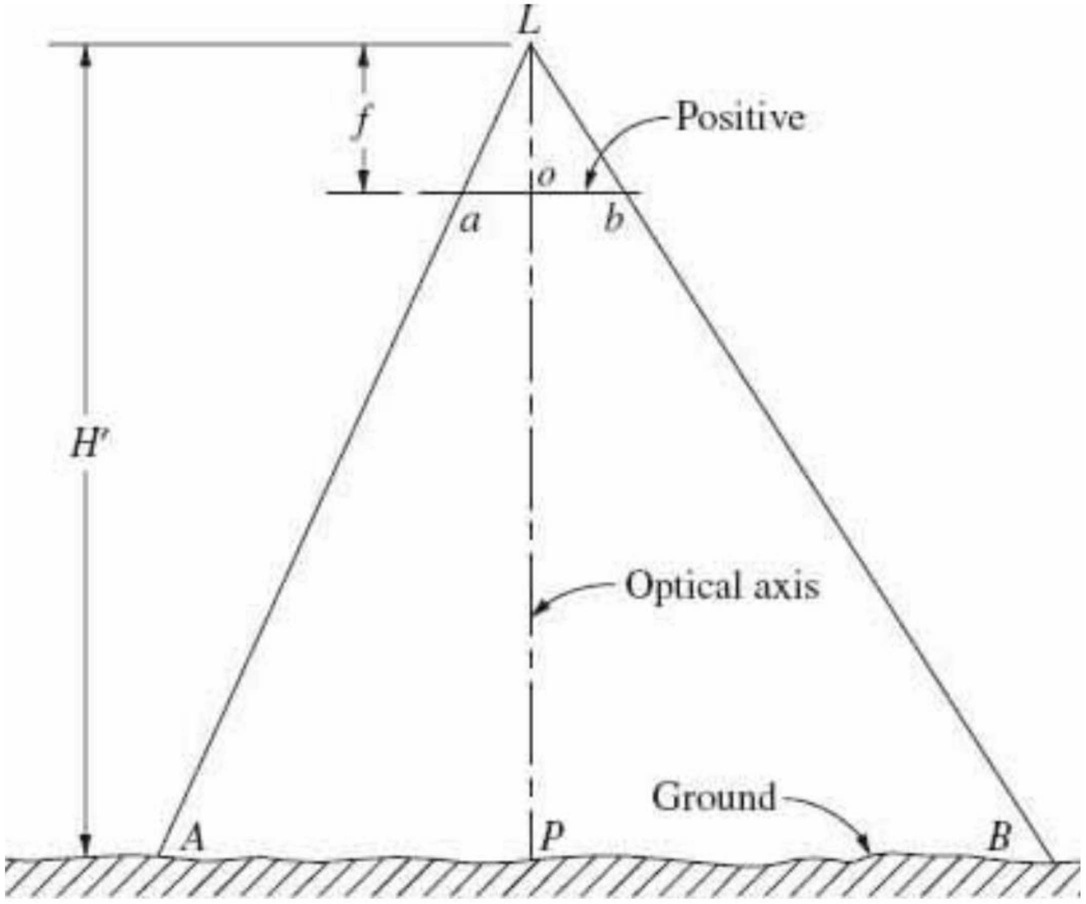

FIGURE 6-2 Two-dimensional view of a vertical photograph taken over flat terrain.

- The scale of a vertical photograph over flat terrain is the ratio of photo distance *ab* to ground distance *AB*, or of focal length *f* to height *H'*:

$$S = \frac{ab}{AB} = \frac{f}{H'}$$

[Example 6-1] A vertical aerial photo is taken over flat terrain with 152.4 mm of focal length and 1,830 m of height. Photo scale?

$$S = \frac{f}{H'} = \frac{0.1524}{1830} = 1/12{,}000 = 1{:}12{,}000$$

4. Scale of a Vertical Photograph over Variable Terrain

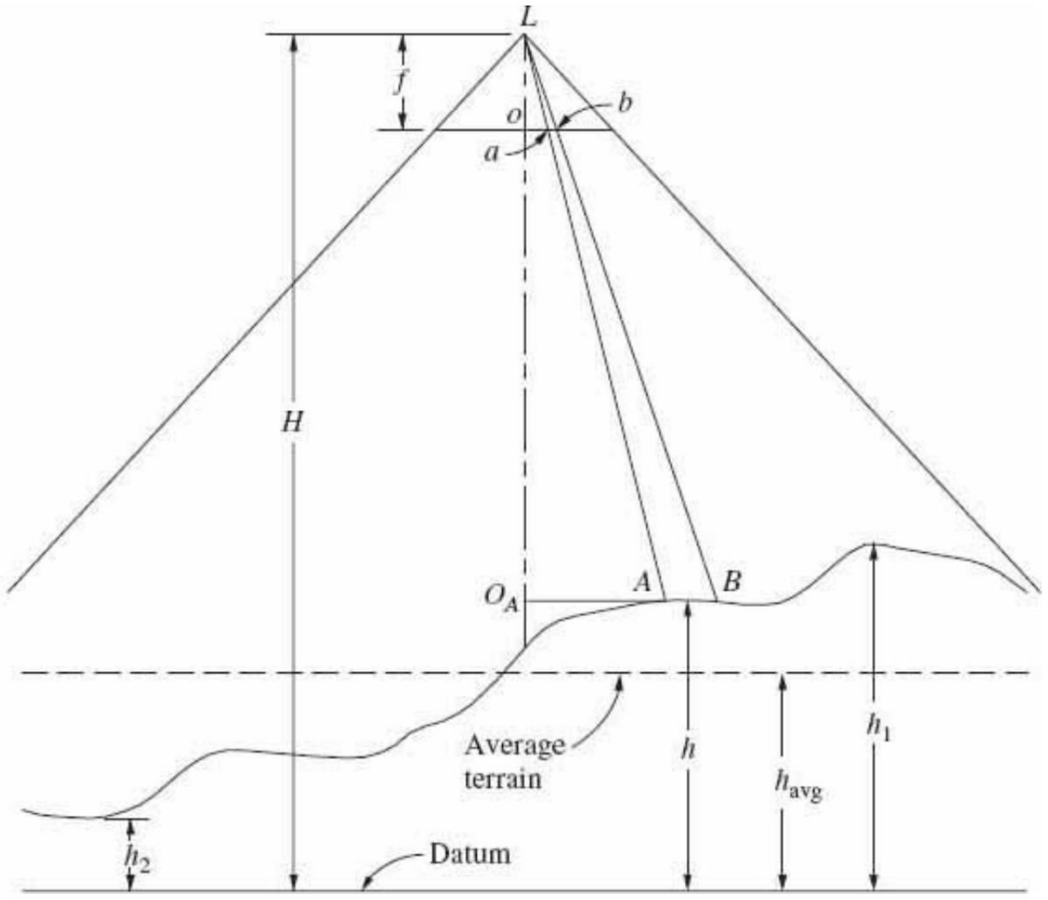

FIGURE 6-3 Scale of a vertical photograph over variable terrain.

- The vertical photo scale increases with increasing terrain elevation:

$$S_{AB} = \frac{ab}{AB} = \frac{La}{LA} = \frac{f}{H-h}$$

5. Average Photo Scale

- Average scale is the scale at the average elevation of the terrain:

$$S_{avg} = \frac{f}{H - h_{avg}}$$

[Example 6-2] The highest terrain h_1, average terrain h_{avg}, and lowest terrain h_2 are 610, 460, 310 m above mean sea level, respectively. Calculate the maximum, minimum, and average scales if flying height above mean sea level is 3000 m and the camera focal length is 152.4 mm.

$S_{max} = \frac{0.1524}{3000-610} = {}^{1}/_{15,700} = 1{:}\,15,700,$ $S_{min} = \frac{0.1524}{3000-310} = {}^{1}/_{17,700} = 1{:}\,17,700$

$S_{avg} = \frac{0.1524}{3000-460} = {}^{1}/_{16,700} = 1{:}\,16,700$

6. Other Methods of Determining Scale of Vertical Photographs

- The scale can be simply determined by the ratio of the photo distance to the ground distance.

[Example 6-3] The horizontal distance AB between the centers of two street intersections was measured on the ground as 402 m. Corresponding line ab on a vertical photograph was 95.8 mm. Photo scale of the line?

$$S = \frac{0.0958}{402} = \frac{1}{4200} = 1:4200$$

- The scale of a vertical aerial photograph can be determined if a map of the corresponding area is available

$$S = \frac{\text{photo distance}}{\text{map distance}} \times \text{map scale}$$

6. Other Methods of Determining Scale of Vertical Photographs

[Example 6-4] An airport runway measures 160 mm on a vertical photograph and 103 mm on a map whose scale is 1:24,000. Scale of the photograph?

$$S = \frac{160}{103} \times \frac{1}{24{,}000} = 1/15{,}400 = 1:15{,}400$$

7. Ground Coordinates from a Vertical Photograph

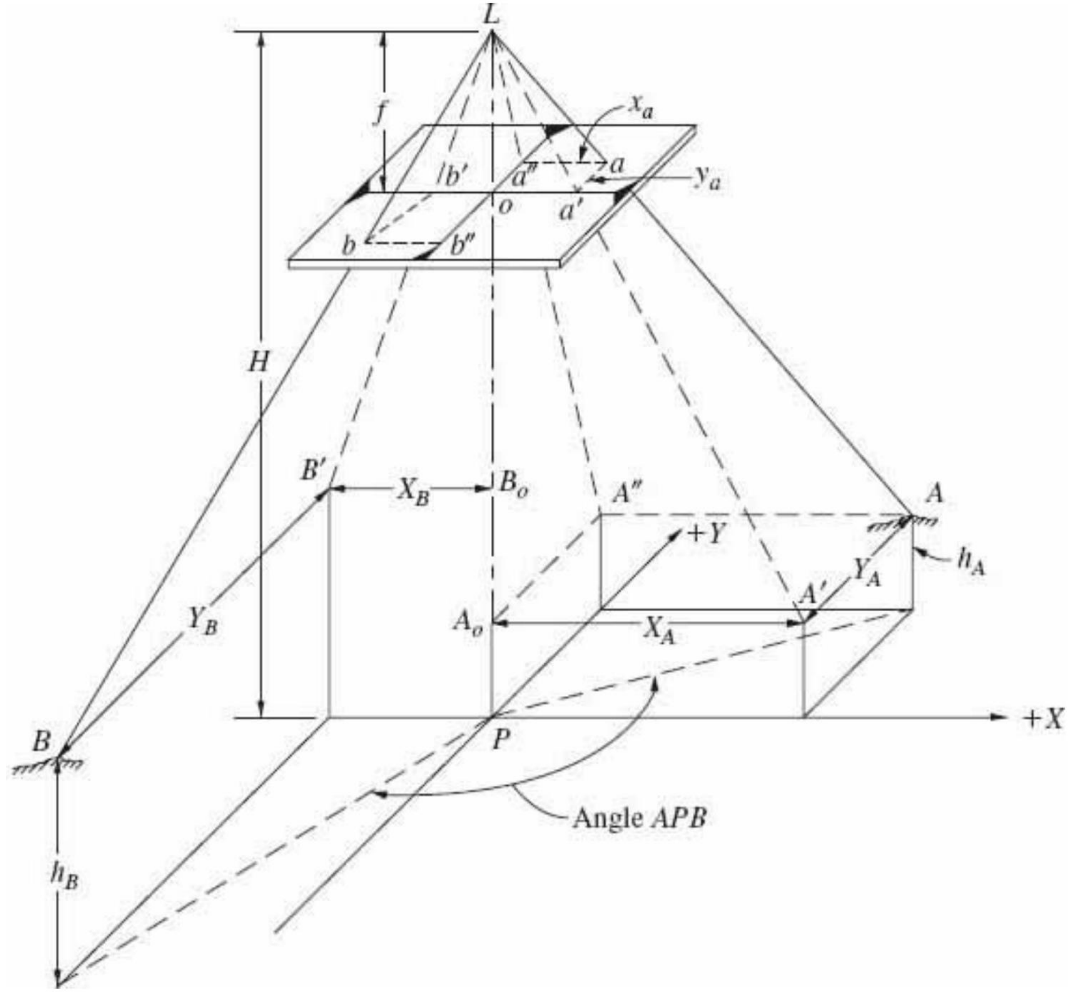

Figure 6-5 Ground coordinates from a vertical photograph.

- The ground coordinates of points whose images appear in a vertical photograph can be determined with respect to an arbitrary ground coordinate system.

$$\frac{oa'}{A_oA'} = \frac{f}{H - h_A} = \frac{x_a}{X_A} \rightarrow X_A = x_a\left(\frac{H - h_A}{f}\right)$$

- Similarly

$$Y_A = y_a\left(\frac{H-h_A}{f}\right),$$

$$X_B = x_b\left(\frac{H-h_B}{f}\right), Y_B = y_b\left(\frac{H-h_B}{f}\right)$$

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$$

$$APB = 90° + \tan^{-1}\left(\frac{X_B}{Y_B}\right) + \tan^{-1}\left(\frac{Y_A}{X_A}\right)$$

7. Ground Coordinates from a Vertical Photograph

[Example 6-6]

A vertical aerial photograph was taken with a 152.4-mm-focal-length camera from a flying height of 1385 m above datum. Images *a* and *b* of two ground points *A* and *B* appear on the photograph, and their measured photo coordinates (corrected for shrinkage and distortions) are x_a = –52.35 mm, y_a = –48.27 mm, x_b = 40.64 mm, and y_b = 43.88 mm. Determine the horizontal length of line *AB* if the elevations of points *A* and *B* are 204 and 148 m above datum, respectively.

$$X_A = \frac{-52.35}{152.4}(1385-204) = -405.7 \text{ m}$$

$$Y_A = \frac{-48.27}{152.4}(1385-204) = -374.1 \text{ m}$$

$$X_B = \frac{40.64}{152.4}(1385-148) = 329.9 \text{ m}$$

$$Y_B = \frac{43.88}{152.4}(1385-148) = 356.2 \text{ m}$$

$$AB = \sqrt{(329.9+405.7)^2 + (356.2+374.1)^2} = 1036 \text{ m}$$

8. Relief Displacement on a Vertical Photograph

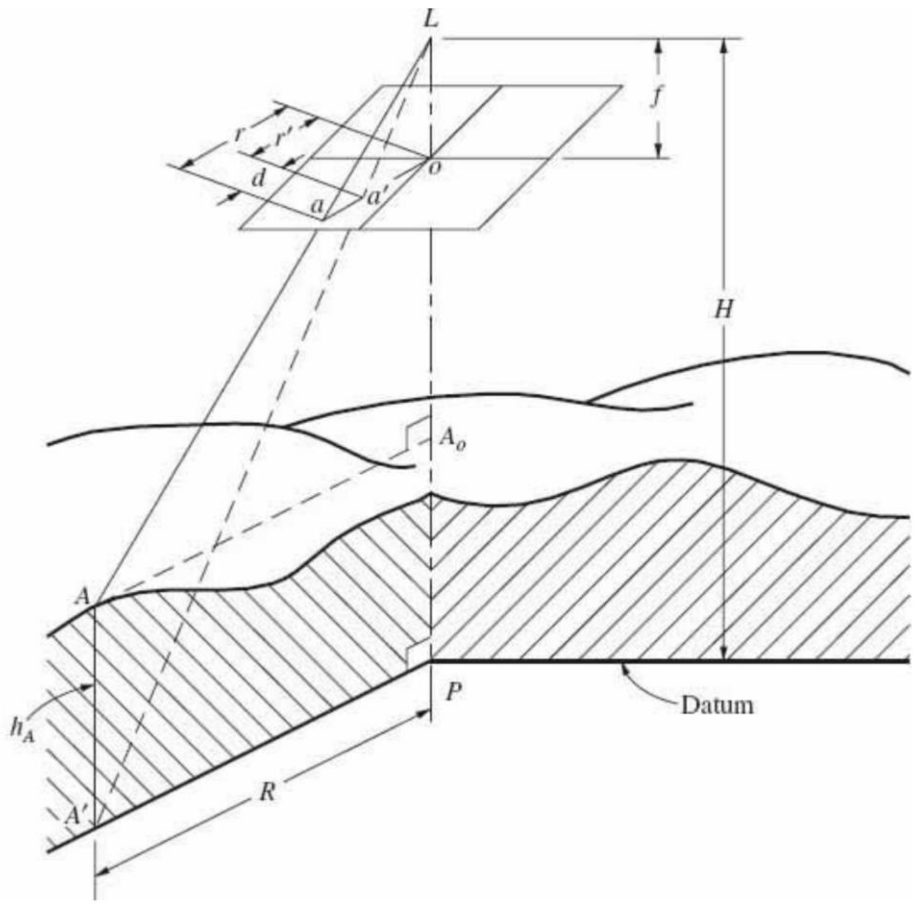

FIGURE 6-6 Relief displacement on a vertical photograph.

- Relief displacement is the shift/displacement in the photograph position of an image caused by the elevation of the object above or below a selected datum.

- Derivation of formulas

$$\frac{r}{R} = \frac{f}{H-h_A} \rightarrow r(H-h_A) = fR,$$

$$\frac{r'}{R} = \frac{f}{H} \quad \text{or} \quad r'H = fR$$

$$\rightarrow r(H-h_A) = r'H$$

$$d = \frac{rh}{H}, \quad h = \frac{dH}{r}$$

8. Relief Displacement on a Vertical Photograph

FIGURE 6-7 Vertical photograph of Tampa, Florida, illustrating relief displacements.

- Relief displacement occurs radially from the center of the photograph (principal point).

[Example 6-7] A vertical photo taken from an elevation of 535 m above mean sea level. The elevation at the base of a tower in the photo is 259 m above MSL. The relief distance d of the tower is 54.1 mm, and the radial distance to the top of the tower from the photo center was 121.7 mm. What is the height of the tower?

$\rightarrow H = 535 - 259 = 276 \text{ m}$

$$\text{h} = \frac{dH}{r} = \frac{54.1(276)}{121.7} = 123 \text{ m}$$

9. Flying height of a Vertical Photograph

- Flying height above datum is an important quantity for solving photogrammetric equations such as in scale, ground coordinate, and relief displacement.

[Example 6-8] A section line whose length is 5,280 ft on fairly level terrain is measured 94.0 mm on photograph. Find the flying height if the camera focal length is 88.9 mm.

$$S = \frac{f}{H} \rightarrow H = \frac{f}{S} = \frac{88.9}{94.0/5{,}280} = 4{,}993.5\text{ft} = 1{,}522\text{ m}$$

9. Flying height of a Vertical Photograph

[Example 6-9] Ground point A and B have elevation 437.4 m and 445.3 m above sea level, respectively, and the horizontal length of line AB is 584.9 m. The measured photo coordinates of a and b which are images of A and B are $x_a = 18.21$ mm, $y_a = -61.32$ mm, $x_b = 109.65$ mm, and $y_b = -21.21$ mm. Calculate the flying height of the photograph above sea level.

$$(AB)^2 = \left[\frac{x_b}{f}(H - h_B) - \frac{x_a}{f}(H - h_A)\right]^2 + \left[\frac{y_b}{f}(H - h_B) - \frac{y_a}{f}(H - h_A)\right]^2$$

$$(584.9)^2 = \left[\frac{109.65}{152.3}(H - 445.3) - \frac{18.21}{152.3}(H - 437.4)\right]^2$$

$$+\left[\frac{-21.21}{152.3}(H - 445.3) + \frac{61.32}{152.3}(H - 437.4)\right]^2$$

9. Flying height of a Vertical Photograph

$$(584.9)^2 = (0.6004H - 268.3)^2 + (0.2634H - 114.1)^2$$

$$0.4298H^2 - 382.3H - 257{,}100 = 0$$

$$H = \frac{382.3 \pm \sqrt{(-382.3)^2 - 4(0.4298)(-257{,}100)}}{2(0.4298)}$$

$$= \frac{382.3 \pm 766.9}{2(0.4298)} = 1337 \text{ m}$$

10. Error Evaluation

- Error caused by lens distortions and atmospheric refraction are relatively small and can be ignored.

- Significant sources of errors in calculated values using the equations of this chapter are:

  1) Errors in photographic measurement, e.g., line lengths or photo coordinates
  2) Errors in ground control
  3) Shrinkage and expansion of film and paper
  4) Tilted photographs where vertical photographs were assumed

- Error minimization: 1) & 2) – using precise equipment, 3) – corrections as in Sec4-9, 4) – adoption of analytical methods as in Chap. 11

10. Error Evaluation

- Calculation of combined effect of several random errors is to use statistical error propagation as in Sec.A-4:

$$F = f(x_1, x_2, \ldots, x_n), \qquad \sigma_F = \pm\sqrt{\left(\frac{\partial F}{\partial x_1}\right)^2 {\sigma_1}^2 + \left(\frac{\partial F}{\partial x_2}\right)^2 {\sigma_2}^2 + \cdots + \left(\frac{\partial F}{\partial x_n}\right)^2 {\sigma_n}^2}$$

-Example: A ground distance AB on flat terrain of 1,524 m (error σ_{AB} of $\pm$ 0.50 m) is measured as 127.0 mm (error σ_{ab} of $\pm$ 0.20 mm) on a vertical photograph whose camera focal length is 152.4 mm. Obtain the flying height, H', and the error in H'.

$$H' = f\left(\frac{AB}{ab}\right) = 152.4\,\frac{1524}{127.0} = 1{,}829 \text{ m}$$

$$\frac{\partial H\prime}{\partial AB} = \frac{f}{ab} = \frac{152.4 \text{ mm}}{127.0 \text{ mm}} = 1.200, \quad \frac{\partial H\prime}{\partial ab} = -\frac{f(AB)}{ab^2} = -\frac{152.4 \text{ mm}(1524 \text{ m})}{(127.0 \text{ mm})^2} = -14.40 \text{ m/mm}$$

10. Error Evaluation

- $\sigma_{H'} = \pm\sqrt{(1.200)^2 0.50^2 + (-14.40\text{m/mm})^2 0.20\text{mm}^2} = \pm\sqrt{0.36\text{m}^2 + 8.29\text{m}^2}$
$= \pm 2.9\ \text{m}$