Design Theories of Ship and Offshore Plant

Part I. Ship Design

Ch. 6 Structural Design

Fall 2017

Myung-Il Roh

Department of Naval Architecture and Ocean Engineering
Seoul National University

Contents

Ch. 6 Structural Design

6.1 Generals & Materials
6.2 Global Hull Girder Strength (Longitudinal Strength)
6.3 Local Strength (Local Scantling)
6.4 Buckling Strength
6.5 Structural Design of Midship Section of a 3,700 TEU Container Ship

6.1 Generals & Materials

(1) Stress Transmission
(2) Principal Dimensions
(3) Criteria for the Selection of Plate Thickness, Grouping of Longitudinal Stiffener
(4) Material Factors

(1) Stress Transmission

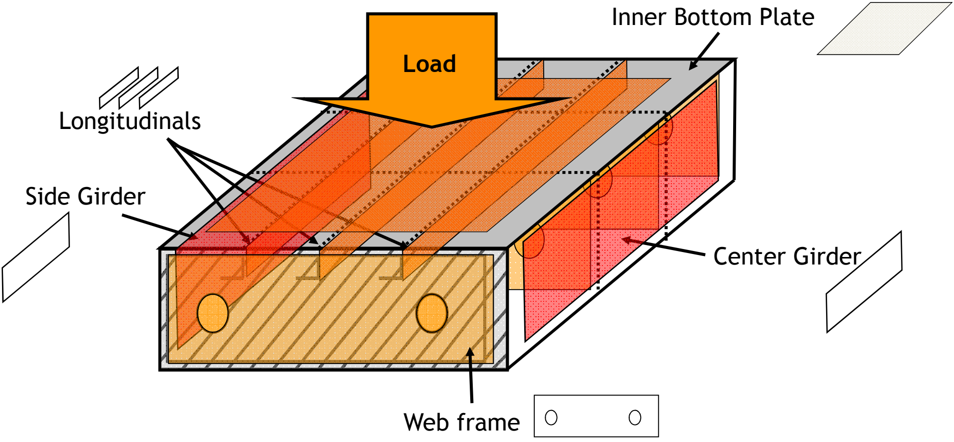


Plate → Longi. → Web frame → Girder

➡ : Stress Transmission

---

(2) Principal Dimensions

DNV Rules, Jan. 2004,Pt. 3 Ch. 1 Sec. 1 101

The following principal dimensions are used in accordance with DNV rule.

1) Rule length (L or L_s)

: Length of a ship used for rule scantling procedure

$$\boxed{0.96 \cdot L_{WL} < L < 0.97 \cdot L_{WL}}$$

- Distance on the summer load waterline (L_{WL}) from the fore side of the stem to the axis of the rudder stock
- Not to be taken less than 96%, and need not be taken greater than 97%, of the extreme length on the summer load waterline (L_{WL})

Example of the calculation of rule length

L_{BP}	L_{WL}	$0.96 \cdot L_{WL}$	$0.97 \cdot L_{WL}$	L
250	261	250.56	253.17	250.56
250	258	247.68	250.26	250.00
250	255	244.80	247.35	247.35

2) Breadth

: Greatest moulded breadth in [m], measured at the summer load waterline

(2) Principal Dimensions

DNV Rules, Jan. 2004,Pt. 3 Ch. 1 Sec. 1 101

3) Depth (D)

: Moulded depth defined as the vertical distance in [m] from baseline to moulded deck line at the uppermost continuous deck measured amidships

4) Draft (T)

: Mean moulded summer draft (**scantling draft**) in [m]

5) Block coefficient (C_B)

: To be calculated based on the rule length

$$C_B = \frac{\Delta}{1.025 \cdot L \cdot B \cdot T}$$, (Δ : Moulded displacement in sea water on draft T)

---

(3) Criteria for the Selection of Plate Thickness, Grouping of Longitudinal Stiffener

DNV Rules, Jan. 2004,Pt. 3 Ch. 1 Sec. 1 101

1) Criteria for the selection of plate thickness

➡ When selecting plate thickness, use the provided plate thickness.

(1) 0.5 mm interval
(2) Above 0.25 mm: 0.5 mm
(3) Below 0.25 mm: 0.0 mm

Ex) 15.75 mm ➡ 16.0 mm
15.74 mm ➡ 15.5 mm

2) Grouping of longitudinal stiffener

For the efficiency of productivity, each member is arranged by grouping longitudinal stiffeners. The grouping members should satisfy the following rule.

Average value but not to be taken less than 90% of the largest individual requirement (DNV).

Ex) The longitudinal stiffeners have design thickness of 100, 90, 80, 70, 60 mm. The average thickness is given by 80 mm×5. However, the average value is less than 100mm×90% = 90 mm of the largest individual requirement, 100 mm.
Therefore, the average value should be taken 90 mm×5.

(4) Material Factors

[1] DNV Rules, Jan. 2004, Pt. 3 Ch. 1 Sec.2

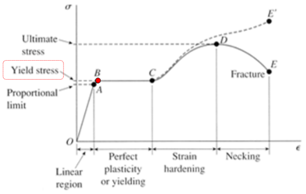
[2] James M. Gere, Mechanics of Materials 7th Edition, Thomson, Chap.1, pp.15~26

- The material factor f_1 is included in the various formulae for scantlings and in expressions giving allowable stresses.[1]

Material Designation	Yield Stress (N/mm^2)	$\dfrac{\sigma}{\sigma_{NV-NS}}$	Material Factor (f_1)
NV-NS	235	235/235 = 1.00	1.00
NV-27	265	265/235 = 1.13	1.08
NV-32	315	315/235 = 1.34	1.28
NV-36	355	355/235 = 1.51	1.39
NV-40	390	390/235 = 1.65	1.47

* NV-NS: Normal Strength Steel (Mild Steel)

* NV-XX: High Tensile Steel



* Yield Stress (σ_y) [N/mm^2] or [MPa]: The magnitude of the load required to cause yielding in the beam.[2]

* High tensile steel: A type of alloy steel that provides better mechanical properties or greater resistance to corrosion than carbon steel. They have a carbon content between 0.05-0.25% to retain formability and weldability, including up to 2.0% manganese, and other elements are added for strengthening purposes.

* A: 'A' grade 'Normal Strength Steel'

* AH: 'A' grade 'High Tensile Steel'

6.2 Global Hull Girder Strength (Longitudinal Strength)

(1) Generals
(2) Still Water Bending Moment (Ms)
(3) Vertical Wave Bending Moment (Mw)
(4) Section Modulus

(1) Generals

Interest of "Ship Structural Design"

- Ship Structural Design

What is designer's major interest?

- Safety:
  Won't 'it' fail under the load?

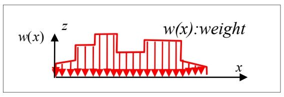
a ship } global

a stiffener
a plate } local

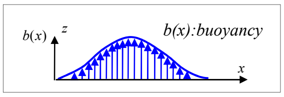
Let's consider the safety of the ship from the point of global strength first.

Dominant Forces Acting on a Ship

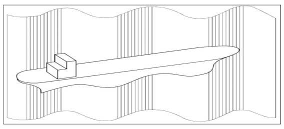
What are dominant forces acting on a ship in view of the longi. strength?



weight of light ship, weight of cargo, and consumables



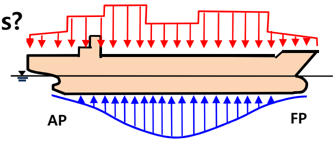
hydrostatic force (buoyancy) on the submerged hull



hydrodynamic force induced by the wave

What is the direction of the dominant forces?

The forces act in vertical (lateral) direction along the ship's length.



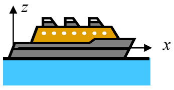
Longitudinal Strength

: Overall strength of ship's hull which resists the bending moment, shear force, and torsional moment acting on a hull girder.

Longitudinal strength loads

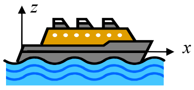
: Load concerning the overall strength of the ship's hull, such as the bending moment, shear force, and torsional moment acting on a hull girder

Static longitudinal loads



Loads are caused by differences between weight and buoyancy in longitudinal direction in the still water condition

Hydrodynamic longitudinal loads



Loads are induced by waves

[1] Okumoto,Y., Design of Ship Hull Structures, Springers, 2009, P.17

Idealization of the Ship Hull Girder Structure

How can we idealize a ship as a structural member?

- **Structural member according to the types of loads**

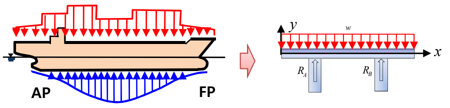
① Axially loaded bar: structural member which supports forces directed along the axis of the bar

② Bar in torsion: structural member which supports torques (or couples) having their moment about the longitudinal axis

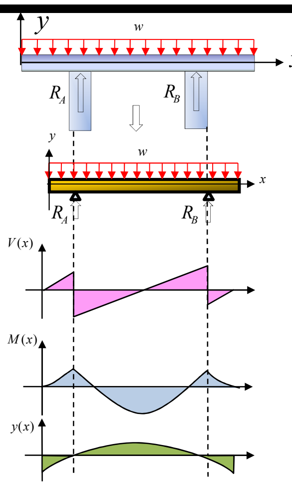
③ **Beam**: structural members subjected to **lateral loads**, that is, forces or moments perpendicular to the axis of the bar

Since a ship has a slender shape and subject to lateral loads, it will behave like a beam from the point view of structural member.

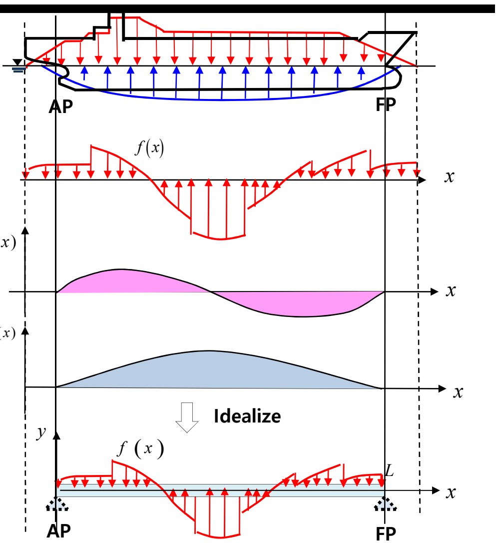
Ship is regarded as a beam.



Applying Beam Theory to a Ship



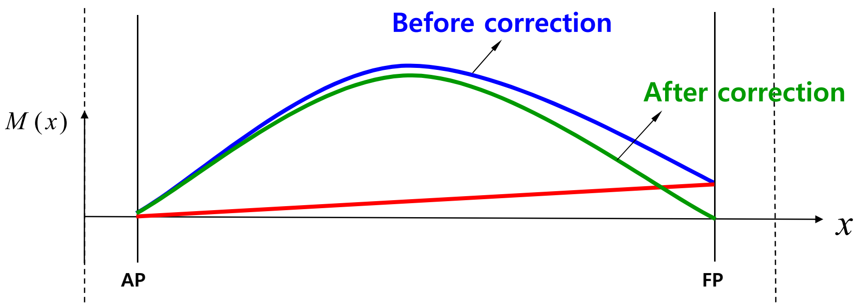
If there are supports at the ends, deflection and slope of the beam occur.



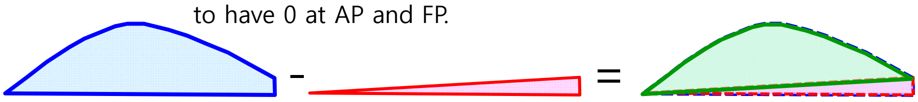
Actually, there are no supports at the ends of the ship. However, the deflection and slope could occur due to inequality of the buoyancy and the weight of a ship. For this problem, we assume that there are simple supports at the A.P and the F.P.

* James M. Gere, Mechanics of Materials, 6th Edition, Thomson, Ch. 4, p. 292

Correction of a Bending Moment Curve



What if the bending moment is not zero at FP?
➡ The deflection and slope of the beam occur at FP.
➡ Thus, we correct the bending moment curve
to have 0 at AP and FP.



* James M. Gere, Mechanics of Materials, 6th Edition, Thomson, Ch. 4, p. 292

Actual Stress ≤ Allowable Stress
- Bending Stress and Allowable Bending Stress

1) DNV Rules, Pt. 3 Ch. 1 Sec. 5, Jan. 2004

The actual bending stress ($\sigma_{act.}$) shall not be greater than the allowable bending stress (σ_l).

M_S : Largest SWBM among all loading conditions and class rule
M_W : Calculated by class rule or direct calculation

$$\boxed{\sigma_{act.} = \frac{|M_S + M_W|}{Z} 10^3 \quad [kg/cm^2]}$$

$$\boxed{\sigma_{act.} \le \sigma_l}$$

(DNV Pt. 3 Ch. 1 Sec. 5 C303)

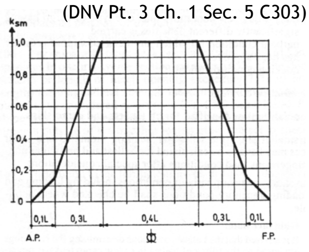


Fig. 2
Stillwater bending moment

$$\sigma_l = \sigma_{allow} = 175 f_1 \ [N/mm^2] \text{ within 0.4L amidship}$$

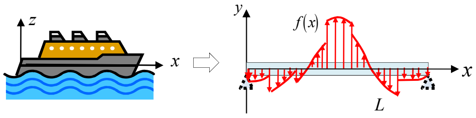
$$= 125 f_1 \ [N/mm^2] \text{ within 0.1L from A.P. or F.P.}$$

(f_1: Material factor. Ex. Mild steel 1.0, HT-32 1.28, HT-36 1.39)

Criteria of Structural Design (1/2)

- **Ship Structural Design**

a ship



The **actual bending stress** ($\sigma_{act.}$) shall not be greater than the **allowable bending stress** (σ_l).

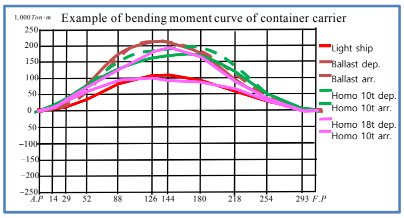
$$\boxed{\sigma_{act.} \le \sigma_l}\ ,\ \sigma_{act.} = \frac{M}{Z} = \frac{|M_S + M_W|}{Z}$$

M_S : Largest SWBM among all loading conditions and class rule
M_W : calculated by class rule or direct calculation

σ_l : **allowable stress**

For instance, allowable bending stresses by DNV rule are given as follows:

$\sigma_l = 175 f_1\ [N/mm^2]$ within 0.4L amidship
$\quad = 125 f_1\ [N/mm^2]$ within 0.1L from A.P. or F.P.



Actual bending moments at aft and forward area are smaller than that at the midship.

What is, then, the f_1?

Criteria of Structural Design (2/2)

$$\boxed{\sigma_{act.} \le \sigma_l}$$

$$\sigma_{act.} = \frac{M}{Z} = \frac{|M_S + M_W|}{Z}$$

(1) Still Water Bending Moment (Ms)
(2) Vertical Wave Bending Moment (Mw)
(3) Section Modulus (Z)

(2) Still Water Bending Moment (Ms)

Still Water Bending Moment (Ms)

$$\boxed{\sigma_{act.} \le \sigma_l} \quad , \sigma_{act.} = \frac{M}{Z} = \frac{|M_S + M_W|}{Z}$$

, M_S: Still water bending moment
M_W: Vertical wave bending moment

Hydrostatic loads along ship's length
caused by the weight & the buoyancy

$f_S(x)$: distributed loads in longitudinal direction in still water

$\downarrow$

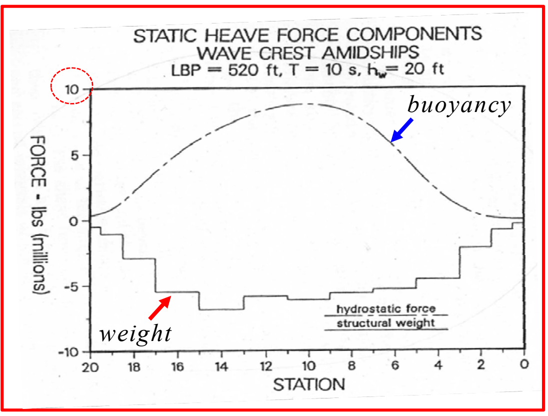
$V_S(x)$: still water shear force

$$V_S(x) = \int_0^x f_S(x)dx$$

$\downarrow$

$M_S(x)$: still water bending moment

$$M_S(x) = \int_0^x V_S(x)dx$$



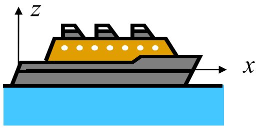
Distributed Loads in Longitudinal Direction

$$f(x) = f_S(x) + f_W(x)$$

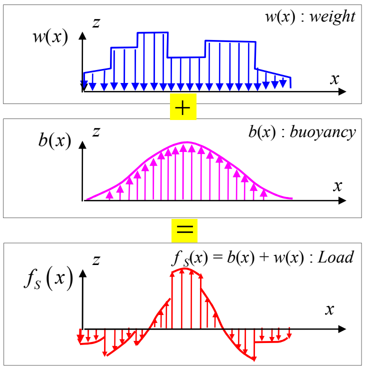
$f(x)$: Distributed loads in longitudinal direction

$f_S(x)$: **Static longitudinal loads** in longitudinal direction
$f_W(x)$: **Hydrodynamic longitudinal loads** induced by wave

In still water



$$f_S(x) = b(x) + w(x)$$



$b(x)$: Distributed buoyancy in longitudinal direction
$w(x) = LWT(x) + DWT(x)$
- $w(x)$: Weight distribution in longitudinal direction
- $LWT(x)$: Lightweight distribution
- $DWT(x)$: Deadweight distribution

Distributed Loads in Still Water

Load Curve, $f_S(x)$ → **Actual Still Water Shear Force, $V_S(x)$** → **Actual Still Water Bending Moment, $M_S(x)$**

Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$

$M_S(x) = \int_0^x V_S(x)dx$

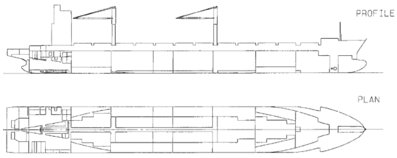
✓ Example of a 3,700 TEU Container Ship in Homogeneous 10 ton Scantling Condition

- Principal Dimensions & Plans

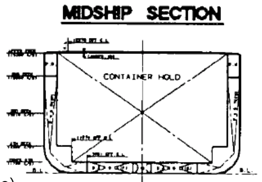
Principal dimension

LENGTH	O. A.	257. 368	M
LENGTH	B. P.	245. 240	M
BREADTH	MOULDED	32. 20	M
DEPTH	MOULDED	19. 30	M
DESIGNED DRAUGHT	MOULDED	10. 10	M
SCANTLING DRAUGHT	MOULDED	12. 50	M

Profile & plan



Midship section



- Loading Condition: Homogeneous 10 ton Scantling Condition (Sailing state)

* Frame space: 800mm

```
                         SAILING    STATE

DRAUGHT  F.P          =        12.260 M   K.M.T                      =      14.889 M
DRAUGHT  MIDSHIP      =        12.457 M   KG  (SOLID)                =      13.586 M
DRAUGHT  A.P          =        12.654 M   GM  (SOLID)                =       1.303 M
TRIM BY STERN         =          .394 M   FREE SURF. CORR.(GGo)      =        .059 M
PROPELLER I/D         =         160.3 %   GoM  (FLUID)               =       1.244 M
DISPLACEMENT          =       66813.6 T   KGo ACTUAL  (FLUID)        =      13.645 M

DRAUGHT AT LCF        =        12.483 M   TRIM  (DIS*A)/(MTC*100)    =        .394 M
LCB FROM A.P          =       115.677 M   FREE SURF. MOM.            =        3921 T-M
LCG FROM A.P          =       115.045 M   M.T.C.                     =      1072.0 T-M
TRIM LEVER : A        =          .632 M   LCF FROM A.P               =     106.275 M

DEGREE    =   .0     5.0    10.0    15.0    20.0    30.0    40.0    50.0    60.0    75.0
KN        = .000   1.296   2.591   3.882   5.168   7.614   9.592  10.930  11.697  11.959
KGo*SINθ  = .000   1.189   2.369   3.532   4.667   6.823   8.771  10.453  11.817  13.180
GZ        = .000    .107    .222    .350    .501    .791    .821    .477   -.120  -1.221
```

Distributed Loads in Still Water - Lightweight

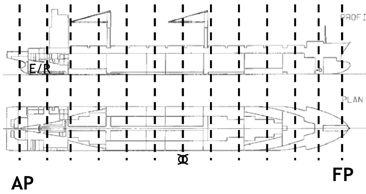
Load Curve, $f_S(x)$ → **Actual Still Water Shear Force, $V_S(x)$** → **Actual Still Water Bending Moment, $M_S(x)$**

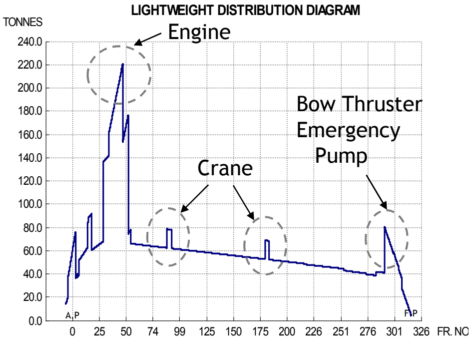
Weight, $w(x)$
Buoyancy, $b(x)$

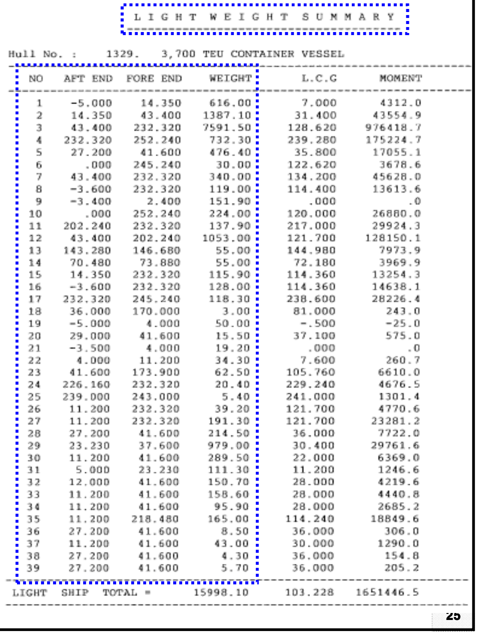
$$V_S(x) = \int_0^x f_S(x)dx$$

$$M_S(x) = \int_0^x V_S(x)dx$$

Example of a 3,700 TEU Container Ship







LIGHT WEIGHT SUMMARY

Hull No. : 1329. 3,700 TEU CONTAINER VESSEL

NO	AFT END	FORE END	WEIGHT	L.C.G	MOMENT
1	-5.000	14.350	616.00	7.000	4312.0
2	14.350	43.400	1387.10	31.400	43554.9
3	43.400	232.320	7591.50	128.620	976418.7
4	232.320	252.240	732.30	239.280	175224.7
5	27.200	41.600	476.40	35.800	17055.1
6	.000	245.240	30.00	122.620	3678.6
7	43.400	232.320	340.00	134.200	45628.0
8	-3.600	232.320	119.00	114.400	13613.6
9	-3.400	2.400	151.90	.000	.0
10	.000	252.240	224.00	120.000	26880.0
11	202.240	232.320	137.90	217.000	29924.3
12	43.400	202.240	1053.00	121.700	128150.1
13	143.280	146.680	55.00	144.980	7973.9
14	70.480	73.880	55.00	72.180	3969.9
15	14.350	232.320	115.90	114.360	13254.3
16	-3.600	232.320	128.00	114.360	14638.1
17	232.320	245.240	118.30	238.600	28226.4
18	36.000	170.000	3.00	81.000	243.0
19	-5.000	4.000	50.00	-.500	-25.0
20	29.000	41.600	15.50	37.100	575.0
21	-3.500	4.000	19.20	.000	.0
22	4.000	11.200	34.30	7.600	260.7
23	41.600	173.900	62.50	105.760	6610.0
24	226.160	232.320	20.40	229.240	4676.5
25	239.000	243.000	5.40	241.000	1301.4
26	11.200	232.320	39.20	121.700	4770.6
27	11.200	232.320	191.30	121.700	23281.2
28	27.200	41.600	214.50	36.000	7722.0
29	23.230	37.600	979.00	30.400	29761.6
30	11.200	41.600	289.50	22.000	6369.0
31	5.000	23.230	111.30	11.200	1246.6
32	12.000	41.600	150.70	28.000	4219.6
33	11.200	41.600	158.60	28.000	4440.8
34	11.200	41.600	95.90	28.000	2685.2
35	11.200	218.480	165.00	114.240	18849.6
36	27.200	41.600	8.50	36.000	306.0
37	11.200	41.600	43.00	30.000	1290.0
38	27.200	41.600	4.30	36.000	154.8
39	27.200	41.600	5.70	36.000	205.2
LIGHT SHIP TOTAL =			15998.10	103.228	1651446.5

25

Distributed Loads in Still Water - Deadweight

Load Curve, $f_S(x)$ → **Actual Still Water Shear Force, $V_S(x)$** → **Actual Still Water Bending Moment, $M_S(x)$**

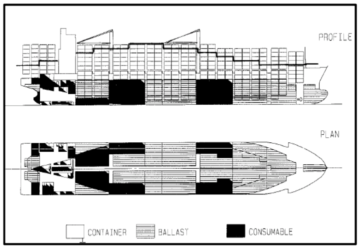
Weight, $w(x)$
Buoyancy, $b(x)$

$$V_S(x) = \int_0^x f_S(x)dx$$

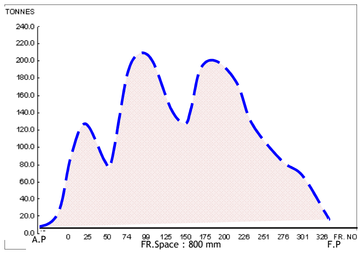
$$M_S(x) = \int_0^x V_S(x)dx$$

Example of a 3,700 TEU Container Ship

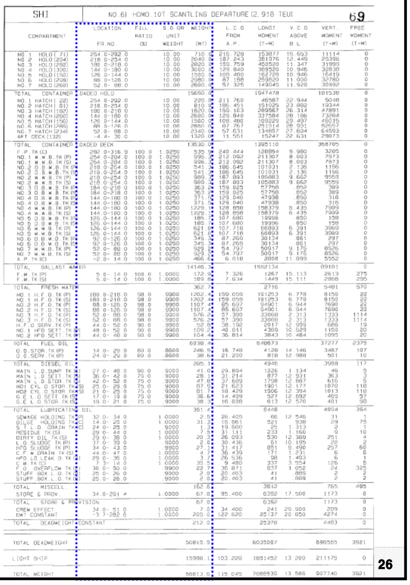
- Loading plan in homogenous 10 ton scantling condition



- Deadweight distribution curve in homogenous 10 ton scantling condition



Deadweight distribution in longitudinal direction in homogenous 10 ton scantling condition



SHI | NO.6) HOMO.10T SCANTLING DEPARTURE (2,918 TEU) 69

COMPARTMENT	LOCATION FR.NO.	FILL. RATIO (%)	S.G OR UNIT WEIGHT	WEIGHT (MT)	L.C.G FROM A.P.	LONGI. MOMENT (T-M)	V.C.G ABOVE B.L	VERT. MOMENT (T-M)	FREE MOMENT (T-M)
NO.1 HOLD(71)	254.0-292.0		10.00	710.0	216.728	153877	15.653	11114	0
NO.2 HOLD(204)	218.0-254.0		10.00	2040.0	187.243	381976	12.449	25396	0
NO.3 HOLD(282)	180.0-218.0		10.00	2820.0	159.759	450520	11.347	31999	0
NO.4 HOLD(300)	144.0-180.0		10.00	3000.0	129.840	389520	10.946	32838	0
NO.5 HOLD(150)	126.0-144.0		10.00	1500.0	108.480	162720	10.946	16419	0
NO.6 HOLD(298)	88.0-126.0		10.00	2980.0	87.188	259820	11.000	32780	0
NO.7 HOLD(260)	52.0- 88.0		10.00	2600.0	57.325	149045	11.920	30992	0
TOTAL CONTAINER LOADED HOLD				15650.0		1947478		181538	0
NO.1 HATCH(22)	254.0-292.0		10.00	220.0	211.760	46587	22.944	5048	0
NO.2 HATCH(81)	218.0-254.0		10.00	810.0	186.451	151025	23.882	19344	0
NO.3 HATCH(182)	180.0-218.0		10.00	1820.0	159.103	289567	26.314	47891	0
NO.4 HATCH(260)	144.0-180.0		10.00	2600.0	129.840	337584	28.186	73284	0
NO.5 HATCH(156)	126.0-144.0		10.00	1560.0	108.480	169229	29.497	46015	0
NO.6 HATCH(288)	88.0-126.0		10.00	2880.0	87.767	251014	28.901	82657	0
NO.7 HATCH(234)	52.0- 88.0		10.00	2340.0	57.631	134857	27.604	64593	0
AFT DECK(132)	-4.4- 30.0		10.00	1320.0	11.551	15247	22.631	29873	0
TOTAL CONTAINER LOADED DECK				13530.0		1395110		368705	0
F.P.TK(C)	292.0-316.9	100.0	1.0250	535.9	240.444	128854	5.980	3205	0
NO.1 W.W.B.TK(P)	254.0-284.0	100.0	1.0250	996.3	212.092	211307	8.003	7973	0
NO.1 W.W.B.TK(S)	254.0-284.0	100.0	1.0250	996.3	212.092	211307	8.003	7973	0
NO.2 O.B.W.B.TK(P)	218.0-254.0	100.0	1.0250	541.3	186.645	101031	2.136	1156	0
NO.2 O.B.W.B.TK(S)	218.0-254.0	100.0	1.0250	541.3	186.645	101031	2.136	1156	0
NO.2 W.W.B.TK(P)	218.0-254.0	100.0	1.0250	989.3	187.893	185883	9.662	9559	0
NO.2 W.W.B.TK(S)	218.0-254.0	100.0	1.0250	989.3	187.893	185883	9.662	9559	0
NO.3 O.B.W.B.TK(P)	184.0-218.0	100.0	1.0250	363.2	159.025	57758	.852	309	0
NO.3 O.B.W.B.TK(S)	184.0-218.0	100.0	1.0250	363.2	159.025	57758	.852	309	0
NO.4 O.B.W.B.TK(P)	144.0-180.0	100.0	1.0250	371.5	129.040	47938	.850	316	0
NO.4 O.B.W.B.TK(S)	144.0-180.0	100.0	1.0250	371.5	129.040	47938	.850	316	0
NO.4 W.W.B.TK(P)	144.0-180.0	100.0	1.0250	1229.1	128.858	158379	6.435	7909	0
NO.4 W.W.B.TK(S)	144.0-180.0	100.0	1.0250	1229.1	128.858	158379	6.435	7909	0
NO.5 O.B.W.B.TK(P)	126.0-144.0	100.0	1.0250	185.7	107.680	19996	.850	158	0
NO.5 O.B.W.B.TK(S)	126.0-144.0	100.0	1.0250	185.7	107.680	19996	.850	158	0
NO.5 W.W.B.TK(P)	126.0-144.0	100.0	1.0250	621.0	107.718	66893	6.391	3969	0
NO.5 W.W.B.TK(S)	126.0-144.0	100.0	1.0250	621.0	107.718	66893	6.391	3969	0
NO.6 O.B.W.B.TK(P)	92.0-126.0	100.0	1.0250	345.3	87.269	30134	.861	297	0
NO.6 O.B.W.B.TK(S)	92.0-126.0	100.0	1.0250	345.3	87.269	30134	.861	297	0
NO.7 W.W.B.TK(P)	52.0- 88.0	100.0	1.0250	929.2	54.797	50917	9.176	8526	0
NO.7 W.W.B.TK(S)	52.0- 88.0	100.0	1.0250	929.2	54.797	50917	9.176	8526	0
A.P.TK(C)	-2.0- 14.0	100.0	1.0250	466.6	6.018	2808	11.899	5552	0
TOTAL BALLAST WATER				14146.3		1952134		89101	0
F.W.TK(P)	5.0- 14.0	100.0	1.0000	172.9	7.326	1267	15.113	2613	275
F.W.TK(S)	5.0- 14.0	100.0	1.0000	189.8	7.634	1449	15.111	2868	295
TOTAL FRESH WATER				362.7		2716		5481	570
NO.1 H.F.O.TK(P)	180.0-218.0	98.0	.9900	1202.4	159.059	191253	6.778	8150	22
NO.1 H.F.O.TK(S)	180.0-218.0	98.0	.9900	1202.4	159.059	191253	6.778	8150	22
NO.2 H.F.O.TK(P)	88.0-126.0	98.0	.9900	1107.4	85.697	94901	6.944	7690	22
NO.2 H.F.O.TK(S)	88.0-126.0	98.0	.9900	1107.4	85.697	94901	6.944	7690	22
NO.3 H.F.O.TK(P)	52.0- 88.0	98.0	.9900	576.2	57.390	33068	2.313	1333	1114
NO.3 H.F.O.TK(S)	52.0- 88.0	98.0	.9900	576.2	57.390	33068	2.313	1333	1114
H.F.O SERV.TK(P)	44.0- 52.0	90.0	.9900	52.8	38.192	2017	12.999	686	19
NO.1 HFO SETT.TK(P)	48.0- 52.0	90.0	.9900	109.2	40.011	4369	10.529	1150	20
NO.2 HFO SETT.TK(P)	44.0- 48.0	90.0	.9900	104.4	36.814	3843	10.484	1095	20
TOTAL FUEL OIL				6038.4		648673		37277	2375
D.O.STOR.TK(P)	14.0- 29.0	80.0	.8600	246.5	16.748	4128	14.146	3487	107
D.O.SERV.TK(P)	24.0- 29.0	80.0	.8600	38.6	21.200	818	12.988	501	10
TOTAL DIESEL OIL				285.1		4946		3988	117
MAIN L.O.SUMP TK(C)	27.0- 48.0	90.0	.9000	41.0	29.894	1226	1.134	46	5
MAIN L.O.SETT.TK(S)	36.0- 42.0	75.0	.9000	28.1	31.214	877	12.931	363	3
MAIN L.O.STOR.TK(S)	42.0- 52.0	75.0	.9000	47.8	37.609	1798	12.887	616	5
NO1 CYL.O.STOR.TK(S)	25.0- 29.0	75.0	.9000	87.9	21.623	1901	12.177	1070	118
NO2 CYL.O.STOR.TK(S)	21.0- 25.0	75.0	.9000	81.7	18.429	1506	12.394	1013	118
G.E L.O.SETT.TK(S)	17.0- 19.0	75.0	.9000	36.6	14.409	527	12.692	465	57
G.E L.O.STOR.TK(S)	19.0- 21.0	75.0	.9000	38.3	16.008	613	12.570	481	58
TOTAL LUBRICATING OIL				361.4		8448		4054	364
SEWAGE HOLDING TK(P)	32.0- 34.0		1.0000	2.5	26.405	66	12.546	31	1
BILGE HOLDING TK(C)	14.0- 25.0		1.0000	31.3	16.661	521	.938	29	75
S.T L.O. DRAIN TK(C)	24.0- 25.0		.9000	1.3	19.600	25	1.313	2	1
RESIDUE TK(S)	29.0- 44.0		1.0000	7.5	31.111	233	1.160	9	10
DIRTY OIL TK(S)	29.0- 36.0		1.0000	20.3	26.093	530	12.389	251	4
L.O.SLUDGE TK(P)	37.0- 39.0		.9000	2.0	30.436	61	10.195	20	2
HFO SLUDGE TK(P)	34.0- 43.0		.9900	27.1	31.417	851	9.490	257	60
C.F.W.DRAIN TK(S)	44.0- 47.0		1.0000	4.7	36.439	171	1.231	6	6
HFO.LO.LEAK O.TK(P)	29.0- 36.0		1.0000	3.7	26.536	98	1.493	6	1
C.W.TK(C)	7.3- 14.0		1.0000	35.5	9.480	337	3.554	126	8
F.O. OVERFLOW TK(C)	36.0- 50.0		.9900	22.7	36.871	837	1.052	24	325
STUFF.BOX.L.O.TK(P)	25.0- 26.0		.9000	2.0	20.403	41	.889	2	2
STUFF.BOX.L.O.TK(S)	25.0- 26.0		.9000	2.0	20.403	41	.889	2	2
TOTAL MISCELL.				162.6		3812		765	495
STORE & PROV.	34.0-251.4		1.0000	67.0	95.400	6392	17.500	1173	0
TOTAL STORE & PROVISION				67.0		6392		1173	0
CREW EFFECT	34.0- 51.0		1.0000	7.0	34.400	241	29.900	209	0
DWT CONSTANT	-3.7-282.6		1.0000	205.0	122.620	25137	20.850	4274	0
TOTAL DEADWEIGHT CONSTANT				212.0		25378		4483	0
TOTAL DEADWEIGHT				50815.5		6035087		696565	3921
LIGHT SHIP				15998.1	103.228	1651452	13.200	211175	0
TOTAL WEIGHT				66813.6	115.045	7686539	13.586	907740	3921

26

Distributed Loads in Still Water - Buoyancy Curve

Load Curve, $f_S(x)$ → **Actual Still Water Shear Force, $V_S(x)$** → **Actual Still Water Bending Moment, $M_S(x)$**

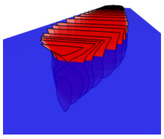
Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x)=\int_0^x f_S(x)dx$

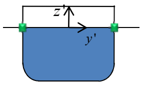
$M_S(x)=\int_0^x V_S(x)dx$

Example of a 3,700 TEU Container Ship

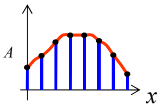
- ✓ Calculation of buoyancy



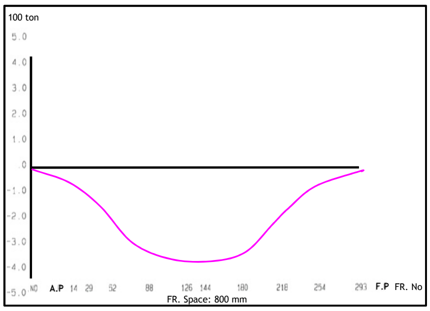
(1) Calculation of sectional area below waterline



(2) Integration of sectional area over the ship's length



- ✓ Buoyancy curve in homogeneous 10 ton scantling condition



DRAUGHT F.P	=	12.260 M	K.M.T	=	14.889 M
DRAUGHT MIDSHIP	=	12.457 M	KG (SOLID)	=	13.586 M
DRAUGHT A.P	=	12.654 M	GM (SOLID)	=	1.303 M
TRIM BY STERN	=	.394 M	FREE SURF. CORR. (GGo)	=	.059 M
PROPELLER I/D	=	160.3 %	GoM (FLUID)	=	1.244 M
DISPLACEMENT	=	66813.6 T	KGo ACTUAL (FLUID)	=	13.645 M
DRAUGHT AT LCF	=	12.483 M	TRIM (DIS*A)/(MTC*100)	=	.394 M
LCB FROM A.P	=	115.677 M	FREE SURF. MOM.	=	3921 T-M
LCG FROM A.P	=	115.045 M	M.T.C.	=	1072.0 T-M
TRIM LEVER : A	=	.632 M	LCF FROM A.P	=	106.275 M

Distributed Loads in Still Water - Load Curve

Load Curve, $f_S(x)$ → **Actual Still Water Shear Force, $V_S(x)$** → **Actual Still Water Bending Moment, $M_S(x)$**

Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x)=\int_0^x f_S(x)dx$

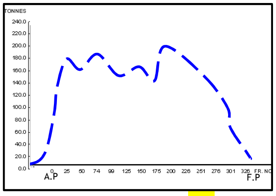
$M_S(x)=\int_0^x V_S(x)dx$

Load Curve

$\boxed{f_S(x)}$: **Loads Curve = Weight + Buoyancy**

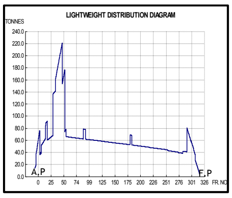
Weight Curve = Lightweight + Deadweight

in homogenous 10ton scantling condition



=

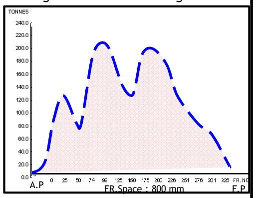
Lightweight Distribution Curve



+

Deadweight Distribution Curve

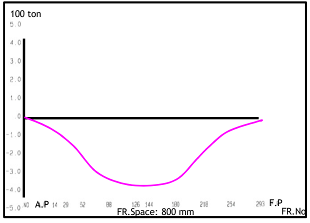
in homogenous 10 ton scantling condition



+

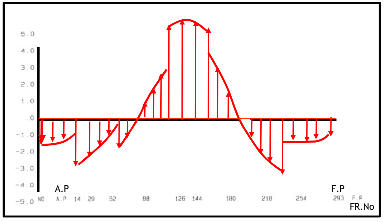
Buoyancy Curve

in homogenous 10 ton scantling condition



Load Curve = Weight $w(x)$ + Buoyancy $b(x)$

in homogenous 10 ton scantling condition



Actual Still Water Shear Force

Actual Still Water Shear Force & Actual Still Water Bending Moment

Load Curve, $f_S(x)$ → **Actual Still Water Shear Force, $V_S(x)$** → **Actual Still Water Bending Moment, $M_S(x)$**

Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$

$M_S(x) = \int_0^x V_S(x)dx$

Example of a 3,700TEU Container Ship

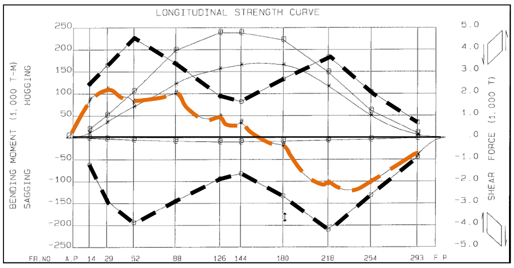
Load Curve $f_S(x) = b(x) + w(x)$

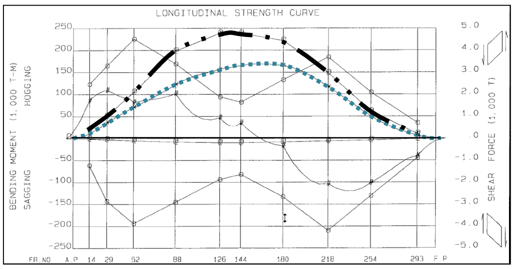
Actual Still Water Shear Force $V_S(x) = \int_0^x f_S(x)dx$

▬ Actual still water shear force
▬ ▪ Design still water shear force

Actual Still Water Bending Moment $M_S(x) = \int_0^x V_S(x)dx$

▪▪▪▪ Actual still water bending moment
— · · Design still water bending moment





Rule Still Water Bending Moment by the Classification Rule

Recently, actual still water bending moment based on the loading conditions is used for still water bending moment, because the rule still water bending moment is only for the tanker.

- The design still water bending moments to be taken less than

(DNV Pt. 3 Ch. 1 Sec. 5 A105)

$$\boxed{M_S = M_{SO}\ [kNm]}$$ (**rule still water bending moment**)

$$M_{SO} = \underline{-0.065C_{WU}}L^2B(C_B + 0.7) \quad \text{[kNm] in sagging}$$

$$= C_{WU}L^2B(0.1225 - 0.015C_B) \quad \text{[kNm] in hogging}$$

C_{WU}: Wave coefficient for unrestricted service

The design still water bending moment shall not be less than the large of: the largest actual still water bending moment based on the loading conditions and the rule still water bending moment.

Design SWBM = Max(Actual SWBM, Rule SWBM) + margin

Rule Still Water Shear Force by the Classification Rule

- The design values of still water shear forces along the length of the ship are normally not to be taken less than

(Dnv Pt.3 Ch.1 Sec. 5 B107)

(**rule still water shear force**)

$$\boxed{Q_S = k_{sq} Q_{SO} \, (kN)}$$

$$Q_{SO} = 5 \frac{M_{SO}}{L} (kN)$$

k_{sq} = 0 at A.P. and F.P.
= 1.0 between 0.15L and 0.3L from A.P.
= 0.8 between 0.4L and 0.6L from A.P.
= 1.0 between 0.7L and 0.85L from A.P.

$M_{SO} = -0.065 C_{WU} L^2 B (C_B + 0.7)$ [kNm] in sagging
$= C_{WU} L^2 B (0.1225 - 0.015 C_B)$ [kNm] in hogging

C_{WU}: wave coefficient for unrestricted service

The design still water shear force shall not be less than the large of: the largest actual still water shear forces based on loading conditions and the rule still water shear force.

Design SWSF = Max(Actual SWSF, Rule SWSF) + margin

---

(3) Vertical Wave Bending Moment (Mw)

Vertical Wave Bending Moment (Mw)

$$\boxed{\sigma_{act.} \le \sigma_l} \quad , \sigma_{act.} = \frac{M}{Z} = \frac{|M_S + M_W|}{Z}$$

, { M_S: Still water bending moment
M_W: Vertical wave bending moment

Hydrodynamic loads induced by waves along ship's length

$f_W(x)$: distributed loads induced by waves
= Froude-Krylov force + diffraction force
+ added mass force + damping force

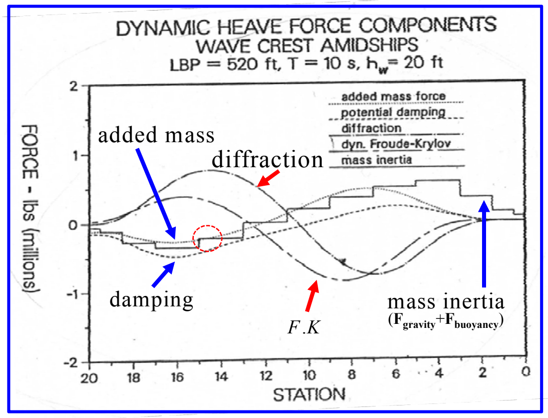
$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = (Body\ \ Force) + (Surface\ \ Force)$$
$$= \mathbf{F}_{gravity}(\mathbf{r}) \qquad + \qquad \mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r})$$
$$+ \mathbf{F}_{R,Damping}(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}})$$

$V_W(x)$: vertical wave shear force

$$V_W(x) = \int_0^x f_W(x)dx$$

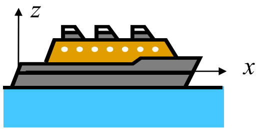
$M_W(x)$: vertical wave bending moment

$$M_W(x) = \int_0^x V_W(x)dx$$



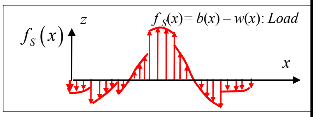
Dynamic Longitudinal Loads

In still water



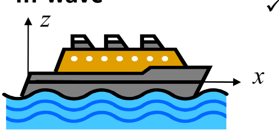
$$f_S(x) = b(x) + w(x)$$

$f(x)$: Distributed loads in longitudinal direction
$f_S(x)$: **Static longitudinal loads** in longitudinal direction
$f_W(x)$: **Hydrodynamic longitudinal loads** induced by wave



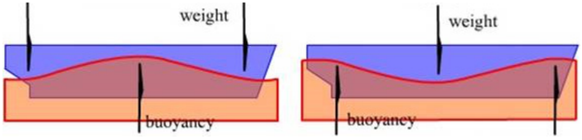
+

In wave



✓ Dynamic longitudinal loads
: Loads are induced by waves

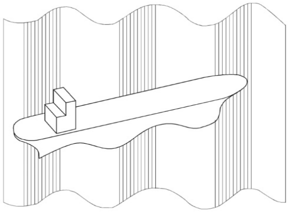
Vertical bending due to waves



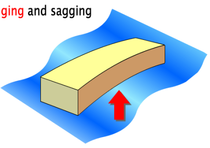
Hogging

Sagging

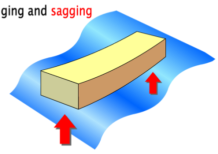
- Ship in oblique waves



Hogging and sagging

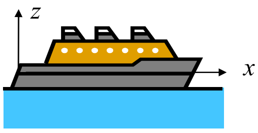


Hogging and sagging



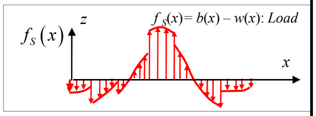
Dynamic Longitudinal Loads
- Direct Calculation of Dynamic Longitudinal Loads (1/2)

In still water



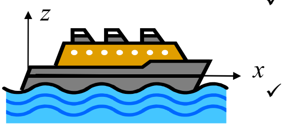
$$f_S(x) = b(x) + w(x)$$

$f(x)$: Distributed loads in longitudinal direction
$f_S(x)$: **Static longitudinal loads** in longitudinal direction
$f_W(x)$: **Hydrodynamic longitudinal loads** induced by wave

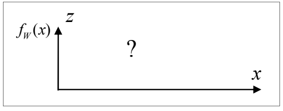


+

In wave



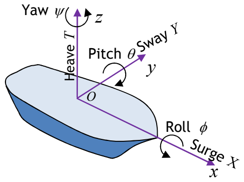
- ✓ Dynamic longitudinal loads
  : Loads are induced by waves



- ✓ Direct calculation of dynamic longitudinal loads
  - from 6DOF motion of ship

$$\mathbf{x} = [X, Y, T, \phi, \theta, \psi]^T$$

✓ Ref.> 6 DOF motion of ship



$$\begin{aligned} f(x) &= f_S(x) \quad \boxed{+ f_W(x)} \\ &= b(x) + w(x) + \boxed{f_D(x) + f_{F.K}(x) + f_R(x)} \end{aligned}$$

additional loads in wave

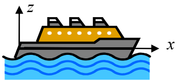
where,

$$f_R(x) = -a(x)\boxed{\ddot{\mathbf{x}}} - b(x)\boxed{\dot{\mathbf{x}}}$$

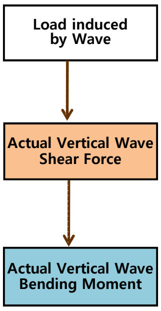
$f_D(x)$: Diffraction force at x
$f_R(x)$: Radiation force at x by damping and added mass
$f_{F.K}(x)$: Froude-Krylov force at x

Dynamic Longitudinal Loads
- Direct Calculation of Dynamic Longitudinal Loads (2/2)

In wave



✓ Direct calculation of dynamic longitudinal loads



Load induced by Wave

$$f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$$

where, $f_R(x) = -a(x)\,\ddot{\mathbf{x}} - b(x)\,\dot{\mathbf{x}}$

Actual Vertical Wave Shear Force

$$Q_W(x) = \int_0^x f_W(x)dx$$

Actual Vertical Wave Bending Moment

$$M_W(x) = \int_0^x Q_W(x)dx$$

Rule Values of Vertical Wave Bending Moments

✓ Direct calculation of dynamic longitudinal loads

- Loads are induced by waves

Actual Vertical Wave Shear Force $Q_W(x)=\int_0^x f_W(x)dx$

↓

Actual Vertical Wave Bending Moment $M_W(x)=\int_0^x Q_W(x)dx$

Recently, rule values of vertical wave moments are used, because of the uncertainty of the direct calculation values of vertical wave bending moments.

Design VWBM = Min(Actual VWBM, Rule VWBM) + margin

The rule vertical wave bending moments amidships are given by:

$$\boxed{M_W = M_{WO} \quad [kNm]}$$

(DNV Pt.3 Ch.1 Sec.5 B201)

L	C_W
$L \le 100$	$0.0792 \cdot L$
$100 < L < 300$	$10.75 - [(300-L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L-350)/150]^{3/2}$

$M_{WO} = -0.11\alpha C_W L^2 B(C_B + 0.7)$ [kNm] in sagging

$= 0.19\alpha C_W L^2 B C_B$ [kNm] in hogging

$\alpha = 1.0$ for seagoing condition

$= 0.5$ for harbor and sheltered water conditions (enclosed fiords, lakes, rivers)

C_W: wave coefficient

C_B: block coefficient, not be taken less than 0.6

Direct calculation values of vertical wave bending moments can be used for design vertical wave bending moment instead of the rule values of vertical wave moments, if the value of the direct calculation is smaller than that of the rule value.

Rule Values of Vertical Wave Shear Forces

✓ Direct calculation of dynamic longitudinal loads

- Loads are induced by waves

Load induced by Wave $f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$

where, $f_R(x) = -a(x)\,\ddot{\mathbf{x}} - b(x)\,\dot{\mathbf{x}}$

↓

Actual Vertical Wave Shear Force $Q_W(x)=\int_0^x f_W(x)dx$

Design VWSF = Min(Actual VWSF, Rule VWSF) + margin

The rule values of vertical wave shear forces along the length of the ship are given by:

(DNV Pt.3 Ch.1 Sec.5 B203)

Positive shear force: $\boxed{Q_{WP} = 0.3\beta k_{wqp} C_W LB(C_B + 0.7)}$

Negative shear force: $\boxed{Q_{WN} = -0.3\beta k_{wqn} C_W LB(C_B + 0.7)}$

β: coefficient according to operating condition

k_{wqp}, k_{wqn}: coefficients according to location in lengthwise

C_W: wave coefficient

Direct calculation values of vertical wave shear forces can be used for vertical wave shear force instead of the rule values of vertical shear forces, if the value of the direct calculation is smaller than that of the rule value.

[Example] Rule Values of Still Water Bending Moments (Ms) and Vertical Wave Bending Moment (Mw)

Calculate L_S, $C_{B,SCANT}$, and vertical wave bending moment (M_W) at amidships (0.5L) of a ship in hogging condition for sea going condition.

Given: $L_{OA} = 332.0\,m, L_{BP} = 317.2\,m, L_{EXT} = 322.85\,m, B = 43.2m, T_s = 14.5\,m,\ \Delta = 140,960\,ton$

(Sol.) $L_s = 0.97 \times L_{EXT} = 0.97 \times 322.85 = 313.16$

$$C_{B,SCANT} = \Delta / \left(1.025 \times L_s \times B \times T_s\right) = \frac{140,906}{1.025 \times 313.16 \times 43.2 \times 14.5} = 0.701$$

$\alpha = 1.0$, for sea going condition,

$C_W = 10.75$, if $300 \le \mathrm{L} \le 350$ (wave coefficient)

$k_{wm} = 1.0$ between 0.4L and 0.65 L from A.P(=0.0) and F.P

$$M_{WO} = 0.19 \times \alpha \times C_W \times L^2 \times B \times C_{B,SCANT}\ \left(kNm\right)$$

$$= 0.19 \times 1.0 \times 10.75 \times 313.16^2 \times 43.2 \times 0.701 = 6,066,303\ \left(kNm\right)$$

at 0.5L, $k_{wm} = 1.0$

$M_W = 1.0 \times M_{WO}$

So, $M_W = 1.0 \times M_{WO} = 6,066,303\ \left(kNm\right)$

$$\boxed{M_S = M_{SO}(kNm)}$$

$$M_{SO} = -0.065 C_{WU} L^2 B(C_B + 0.7),\ _{(in\,sagging)}$$

$$= C_{WU} L^2 B(0.1225 - 0.015 C_B),\ _{(in\,hogging)}$$

$$\boxed{M_w = M_{WO}\quad (kNm)}$$

$$M_{WO} = -0.11\alpha C_W L^2 B(C_B + 0.7),\ _{(in\,sagging)}$$

$$= 0.19\alpha C_W L^2 B C_B,\ _{(in\,hogging)}$$

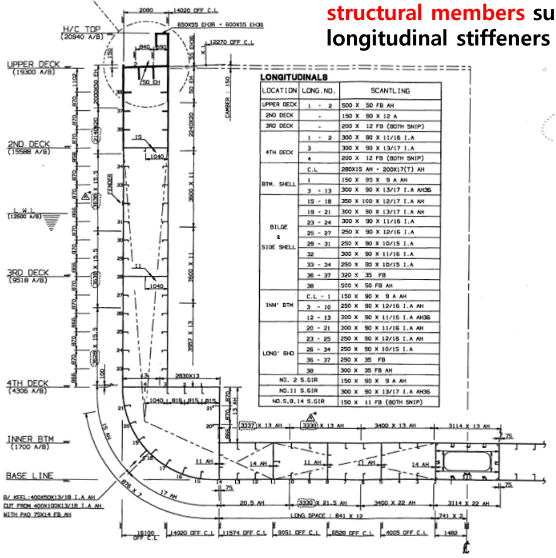
---

1) DSME, Ship Structural Design, 5-2 Load on Hull Structure, Example 4, 2005

(4) Section Modulus

Example of Midship Section of a 3,700 TEU Container Ship

1) First, determine the dimensions of the longitudinal structural members such as longitudinal plates and longitudinal stiffeners by rule local scantling.



LONGITUDINALS

LOCATION	LONG.NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/18 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/18 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

Vertical Location of Neutral Axis about Baseline

2) Second, calculate the moment of sectional area about the base line.

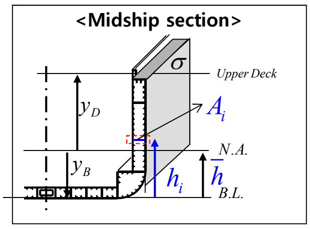
$$\sum h_i A_i$$

h_i : vertical center of structural member
A_i : area of structural member

3) Vertical location of neutral axis from base line ($\bar{h}$) is, then, calculated by dividing the moment of area by the total sectional area.

$$\bar{h} = \frac{\sum h_i A_i}{A}$$

$\bar{h}$: vertical location of neutral axis
A : total area



By definition, neutral axis pass through the centroid of the cross section.

Midship Section Moment of Inertia about N.A

- The midship section moment of inertia about base line ($I_{B.L}$)

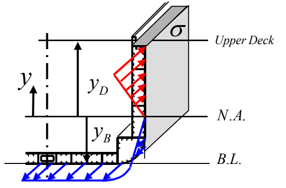
$$I_{B.L} = I_{N.A.} + A\,\overline{h}^2$$

- then calculate the midship section moment of inertia about neutral axis ($I_{N.A}$) using $I_{B.L}$.

$$I_{N.A.} = I_{B.L} - A\,\overline{h}^2$$

Calculation of Section Modulus and Actual Stress at Deck and Bottom

<Midship section>



σ : bending stress
M_T : Total bending moment
A : Total Area
$I_{N.A.}$: Inertia *moment* of the midship section area about neutral axis (N.A.)
B.L : Base Line

Section modulus

$$Z_D = \frac{I_{N.A.}}{y_D}, \quad Z_B = \frac{I_{N.A.}}{y_B}$$

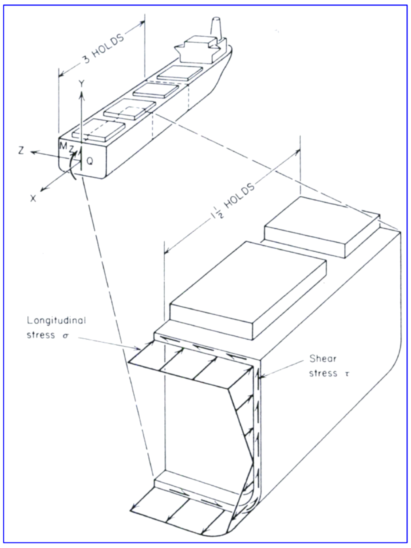
Calculation of Actual Stress at Deck and Bottom

$$\sigma_{Deck} = \frac{M}{Z_D} = \frac{M}{I_{N.A} / y_D}$$

$$\sigma_{Bottom} = \frac{M}{Z_B} = \frac{M}{I_{N.A} / y_B}$$

$$\boxed{\sigma \le \sigma_l}\,, \quad \sigma = \frac{M}{Z} = \frac{M}{I_{N.A} / y}$$

Global Hull Girder Strength (Longitudinal Strength) - Definition of the Longitudinal Strength Members



Application of hull girder load effects

※ Example of Requirement for Longitudinal Structural Member

DNV Rules for Classification of Ships
Part 3 Chapter 1 HULL STRUCTUREALDESIGN SHIPS WITH LENGTH 100 METERS AND ABOVE
Sec. 5 Longitudinal Strength
C 300 Section modulus

301 The requirements given in 302 and 303 will normally be satisfied when calculated for the midship section only, provided the following rules for tapering are complied with:

a) **Scantlings of all continuous longitudinal strength members shall be maintained within 0.4 L amidships**.

b) **Scantlings outside 0.4 L amidships are gradually reduced** to the local requirements at the ends, and the same material strength group is applied over the full length of the ship.
The section modulus at other positions along the length of the ship may have to be specially considered for ships with small block coefficient, high speed and large flare in the fore body or when considered necessary due to structural arrangement, see A106.

* Hughes, Ship Structural Design, John Wiley & Sons, 1983

The Minimum Required Midship Section Modulus and Inertia Moment by DNV Rule

DNV Rules, Jan. 2004, Pt. 3 Ch. 1 Sec. 5

The **midship section modulus** about the transverse neutral axis shall not be less than:
(Pt.3 Ch.1 Sec.5 C302)

$$Z_O = \frac{C_{WO}}{f_1} L^2 B(C_B + 0.7) \quad [\mathrm{cm}^3]$$

C_{WO}: wave coefficient

L	C_{WO}
$L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

C_B is in this case not to be taken less than 0.60.

The midship section moment of **inertia** about the transverse neutral axis shall not be less than:
(Pt.3 Ch.1 Sec.5 C400)

$$I_{ship} = 3 C_W L^3 B(C_B + 0.7) \quad [cm^4]$$

* DNV Rules, Jan. 2004, Pt. 3 Ch. 1 Sec. 5

Material Factors (f_1)

[1] DNV Rules, Jan. 2004,Pt. 3 Ch. 1 Sec.2

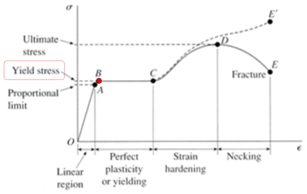
[2] James M. Gere, Mechanics of Materials 7th Edition, Thomson, Chap.1, pp.15~26

- The material factor f_1 is included in the various formulae for scantlings and in expressions giving allowable stresses.[1]

Material Designation	Yield Stress (N/mm^2)	$\dfrac{\sigma}{\sigma_{NV-NS}}$	Material Factor (f_1)
NV-NS	235	235/235 = 1.00	1.00
NV-27	265	265/235 = 1.13	1.08
NV-32	315	315/235 = 1.34	1.28
NV-36	355	355/235 = 1.51	1.39
NV-40	390	390/235 = 1.65	1.47

* NV-NS: Normal Strength Steel (Mild Steel)

* NV-XX: High Tensile Steel



* Yield Stress (σ_y) [N/mm^2] or [MPa]: The magnitude of the load required to cause yielding in the beam.[2]

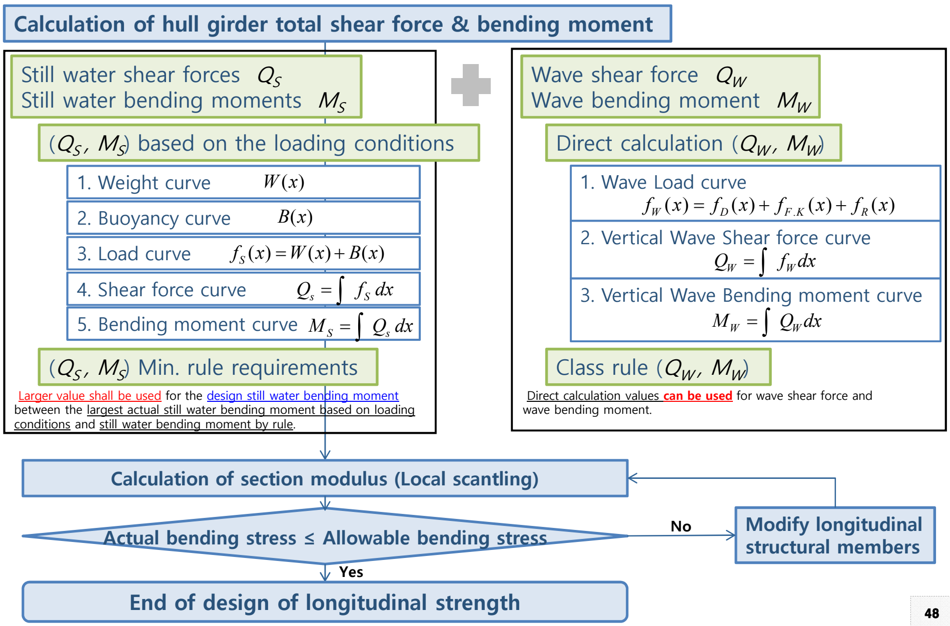
* High tensile steel: A type of alloy steel that provides better mechanical properties or greater resistance to corrosion than carbon steel. They have a carbon content between 0.05-0.25% to retain formability and weldability, including up to 2.0% manganese, and other elements are added for strengthening purposes.

* A: 'A' grade 'Normal Strength Steel'

* AH: 'A' grade 'High Tensile Steel'

Summary of Longitudinal Strength

$$\boxed{\sigma \le \sigma_l}, \quad \sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$$



Calculation of hull girder total shear force & bending moment

Still water shear forces Q_S
Still water bending moments M_S

(Q_S, M_S) based on the loading conditions

1. Weight curve $W(x)$
2. Buoyancy curve $B(x)$
3. Load curve $f_S(x) = W(x) + B(x)$
4. Shear force curve $Q_s = \int f_S\, dx$
5. Bending moment curve $M_S = \int Q_s\, dx$

(Q_S, M_S) Min. rule requirements

Larger value shall be used for the design still water bending moment between the largest actual still water bending moment based on loading conditions and still water bending moment by rule.

Wave shear force Q_W
Wave bending moment M_W

Direct calculation (Q_W, M_W)

1. Wave Load curve $f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$
2. Vertical Wave Shear force curve $Q_W = \int f_W dx$
3. Vertical Wave Bending moment curve $M_W = \int Q_W dx$

Class rule (Q_W, M_W)

Direct calculation values **can be used** for wave shear force and wave bending moment.

Calculation of section modulus (Local scantling)

Actual bending stress ≤ Allowable bending stress

No

Modify longitudinal structural members

Yes

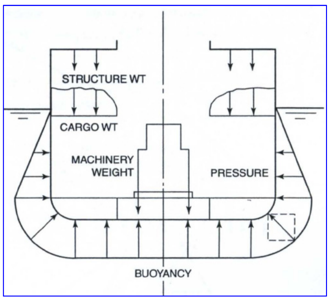
End of design of longitudinal strength

6.3 Local Strength (Local Scantling)

(1) Procedure of Local Scantling
(2) Local Strength & Allowable Stress
(3) Design Loads
(4) Scantling of Plates
(5) Scantling of Stiffeners
(6) Sectional Properties of Steel Sections

Local Scantling

- Ship structure members are designed to endure the loads acting on the ship structure such as hydrostatic and hydrodynamic loads[1].



[2]

- For instance, the structural member is subjected to:

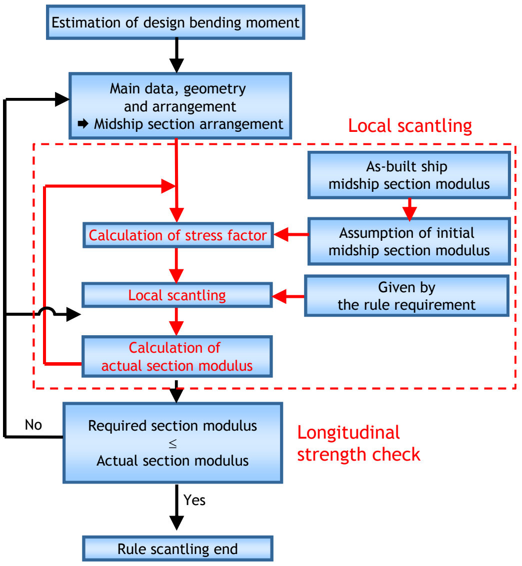
Hydrostatic pressure due to surrounding water
Internal loading due to self weight and cargo weight
Hydrodynamic load due to waves
Inertia force of cargo or ballast due to ship motion

[1] Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures - a Practical Guide for Engineers, Springer, pp. 17-32, 2009

[2] Mansour, A., Liu, D., The Principles of Naval Architecture Series – Strength of Ships and Ocean Structures, The Society of Naval Architects and Marine Engineers, 2008

(1) Procedure of Local Scantling

Procedure of Local Scantling
- Design Procedure of Structures



Estimation of design bending moment

Main data, geometry and arrangement
➡ Midship section arrangement

Local scantling

As-built ship midship section modulus

Calculation of stress factor

Assumption of initial midship section modulus

Local scantling

Given by the rule requirement

Calculation of actual section modulus

No

Required section modulus ≤ Actual section modulus

Longitudinal strength check

Yes

Rule scantling end

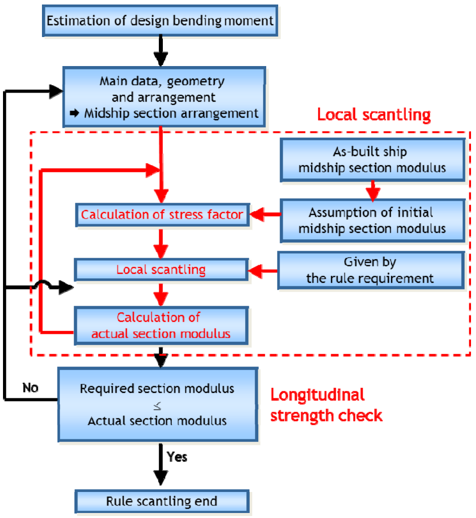
Ship structure design is carried out in accordance with the procedure shown in the figure.

Each member is adjusted to have enough local strength given by the rule of Classification Societies based on the mechanics of materials.

This is called the "local scantling".

Design Procedure of Structures
- Stress Factor

Why iteration is needed for the calculation of local scantling?



The actual midship section modulus at bottom or deck is needed.

However, the section modulus can be calculated after the scantlings of the members are determined.

➡ **Assumption!**

Therefore, the actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

---

1) DNV Rules, Pt. 3 Ch. 1 Sec. 6 C800, Jan. 2004

Design Procedure of Structures
- Stress Factor

Why iteration is needed for the calculation of local scantling?

Example) Inner bottom longitudinals1)

- **Minimum plate thickness**

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} \ (mm)$$

σ_{db}: Mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f_1 for cargo holds in general cargo vessel
= 50 f_1 for holds for ballast
= 85 f_1 b/B for tanks for liquid cargo

- **Minimum section modulus for longitudinal stiffener**

$$\boxed{Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)}$$

l: Stiffener span in m
s: Stiffener spacing in m
p: Design loads
w_k: Section modulus corrosion factor in tanks, Sec.3 C1004

Where, $\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$: **Allowable stress of this structural part**

f_1 : **Material factor as defined in DNV Rules Pt. 3 Ch. 1 Sec.2**

f_{2b} **: stress factor**

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

Required midship section modulus [cm3] at bottom or deck

Actual midship section modulus [cm3] at bottom or deck as-built

M_S: Largest design SWBM2) [kN·m]
M_W: VWBM by class rule or direct calculation in [kN·m]

2) Largest SWBM among all loading conditions and class rule

The actual midship section modulus at bottom or deck is needed.

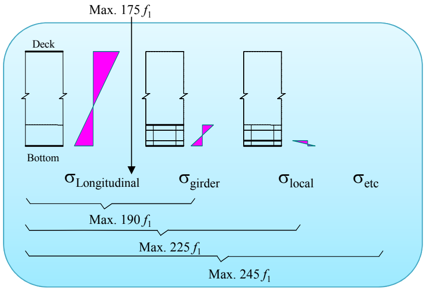
However, the section modulus can be calculated after the scantlings of the members are determined.

➡ **Assumption!**

Therefore, the actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

(2) Local Strength & Allowable Stress

Local Strength & Allowable Stresses
- Allowable Stress for Local Strength



In the figure above, the meaning of the coefficients of the maximum allowable stresses is as follows:

245 f_1: Maximum Yield Stress
235 f_1: Proportional Limit

➡ 225 f_1: The maximum allowable stress for the local strength uses the value less than the maximum yield stress.
In other words, 225 f_1 is used for the yield stress, except for the other effects.

Relationship between load and stress

1) Longitudinal load induced by waves (Hogging or Sagging)

$\downarrow$

$\sigma_{Longitudinal}$

2) Cargo load

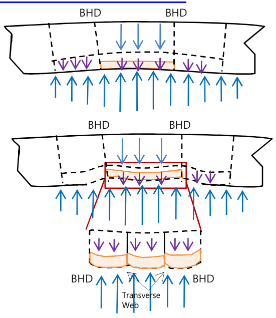
$\downarrow$

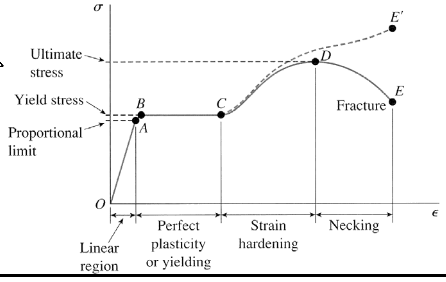
σ_{girder} (σ_{db})

3) Ballasting load

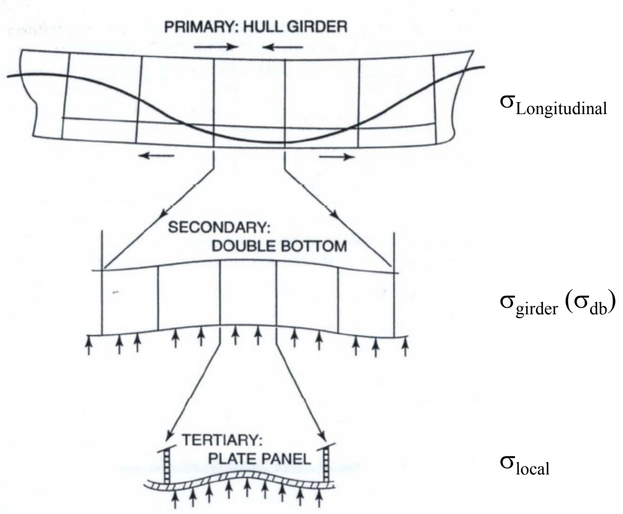
$\downarrow$

σ_{local}





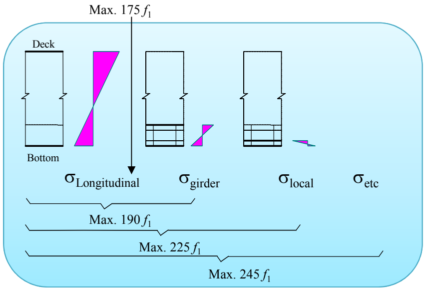
Local Strength & Allowable Stresses

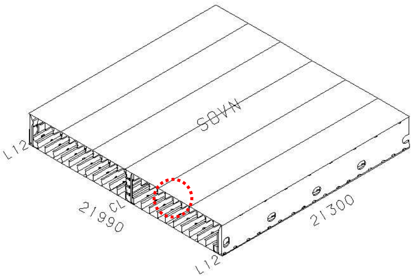


Primary, secondary, and tertiary structure

* Mansour, A., Liu, D., The Principles of Naval Architecture Series – Strength of Ships and Ocean Structures, The Society of Naval Architects and Marine Engineers, 2008

Allowable Stresses
- Allowable Stress for Local Strength





Another interpretation of the figure

Example) Inner bottom longitudinals[1)]

The section modulus requirement is given by:

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

where, p is the local pressure on bottom structure.

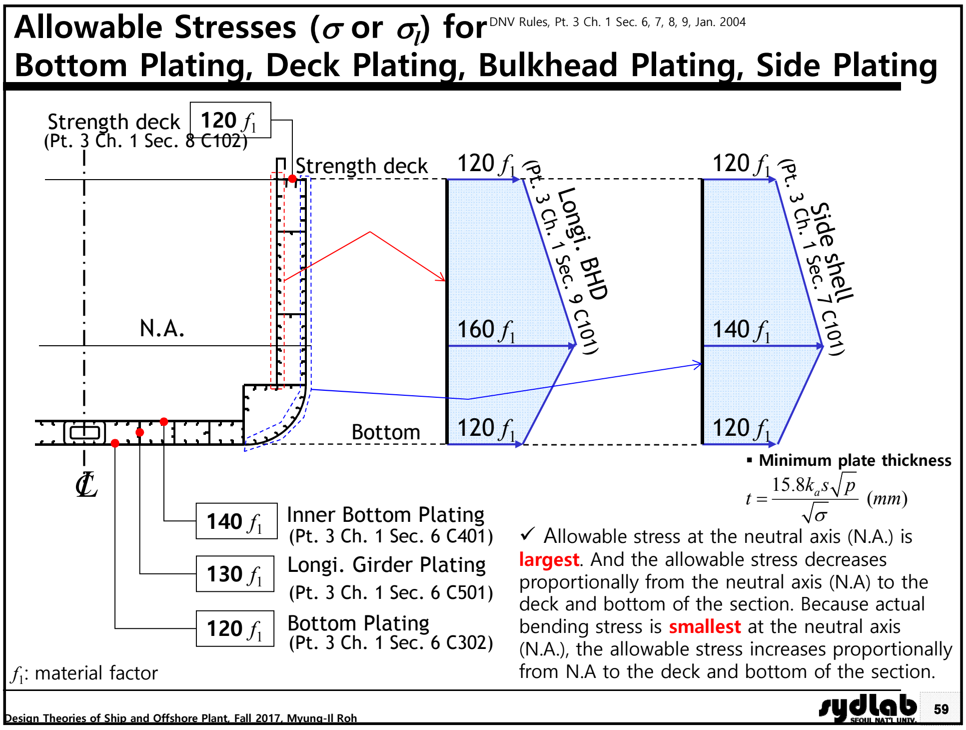
The nominal allowable bending stress due to lateral pressure is used except for the longitudinal stress and the double bottom stress.

$$\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

The longitudinal stress is given by the stress factor. And the double bottom stress is given by:

σ_{db}: Mean double bottom stress at plate flanges, normally not to be taken less than
- = 20 f_1 for cargo holds in general cargo vessel
- = 50 f_1 for holds for ballast
- = 85 f_1 b/B for tanks for liquid cargo

1) DNV Rules, Pt. 3 Ch. 1 Sec. 6 C800, Jan. 2004



Allowable Stresses (σ or σ_l) for Bottom Plating, Deck Plating, Bulkhead Plating, Side Plating

DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004

Strength deck **120 f_1** (Pt. 3 Ch. 1 Sec. 8 C102)

140 f_1 Inner Bottom Plating (Pt. 3 Ch. 1 Sec. 6 C401)

130 f_1 Longi. Girder Plating (Pt. 3 Ch. 1 Sec. 6 C501)

120 f_1 Bottom Plating (Pt. 3 Ch. 1 Sec. 6 C302)

f_1: material factor

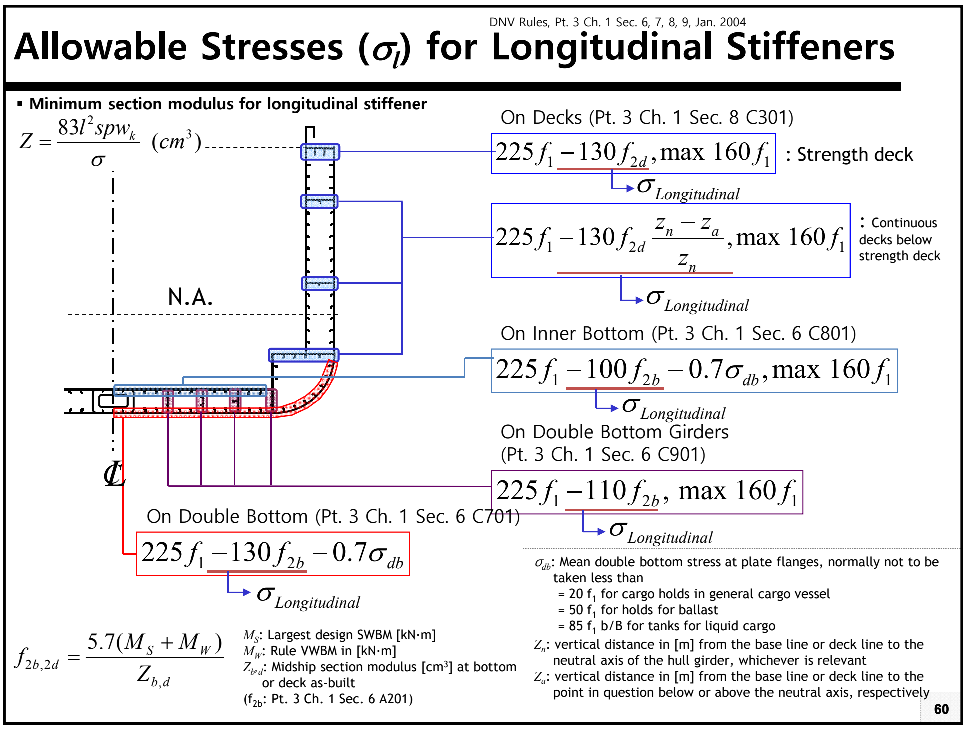
- **Minimum plate thickness**

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} \ (mm)$$

✓ Allowable stress at the neutral axis (N.A.) is **largest**. And the allowable stress decreases proportionally from the neutral axis (N.A) to the deck and bottom of the section. Because actual bending stress is **smallest** at the neutral axis (N.A.), the allowable stress increases proportionally from N.A to the deck and bottom of the section.

Design Theories of Ship and Offshore Plant, Fall 2017, Myung-Il Roh

sydlab SEOUL NAT'L UNIV. 59



DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004

Allowable Stresses (σ_l) for Longitudinal Stiffeners

- **Minimum section modulus for longitudinal stiffener**

$$Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)$$

On Decks (Pt. 3 Ch. 1 Sec. 8 C301)

$$225 f_1 \underbrace{-130 f_{2d}}_{\sigma_{Longitudinal}}, \max 160 f_1$$: Strength deck

$$225 f_1 \underbrace{-130 f_{2d} \frac{z_n - z_a}{z_n}}_{\sigma_{Longitudinal}}, \max 160 f_1$$: Continuous decks below strength deck

On Inner Bottom (Pt. 3 Ch. 1 Sec. 6 C801)

$$225 f_1 \underbrace{-100 f_{2b}}_{\sigma_{Longitudinal}} - 0.7 \sigma_{db}, \max 160 f_1$$

On Double Bottom Girders (Pt. 3 Ch. 1 Sec. 6 C901)

$$225 f_1 \underbrace{-110 f_{2b}}_{\sigma_{Longitudinal}}, \max 160 f_1$$

On Double Bottom (Pt. 3 Ch. 1 Sec. 6 C701)

$$225 f_1 \underbrace{-130 f_{2b}}_{\sigma_{Longitudinal}} - 0.7 \sigma_{db}$$

σ_{db}: Mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f_1 for cargo holds in general cargo vessel
= 50 f_1 for holds for ballast
= 85 f_1 b/B for tanks for liquid cargo

Z_n: vertical distance in [m] from the base line or deck line to the neutral axis of the hull girder, whichever is relevant

Z_a: vertical distance in [m] from the base line or deck line to the point in question below or above the neutral axis, respectively

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

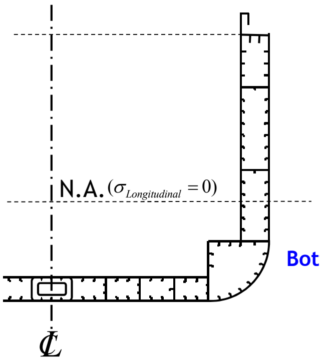
M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm^3] at bottom or deck as-built
(f_{2b}: Pt. 3 Ch. 1 Sec. 6 A201)

60

Allowable Stresses
- Longitudinal Stiffeners (1/6)

DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004
DSME, DNV Rule Commentary Book, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



✓ Calculation of $f_{2b,2d}$

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as-built

For example, 3,700 TEU Container Carrier: $I = 2.343e^{10}\ cm^4$

Bottom: $y_B = 9.028e^2\ cm$

$$Z_B = 2.595e^7\ cm^3 \longrightarrow f_{2b} = \frac{5.7(M_S + M_W)}{Z_b} = 1.030$$

Deck: $y_D = 10.272e^2\ cm$

$$Z_D = 2.345e^7\ cm^3 \longrightarrow f_{2d} = \frac{5.7(M_S + M_W)}{Z_d} = 1.140$$

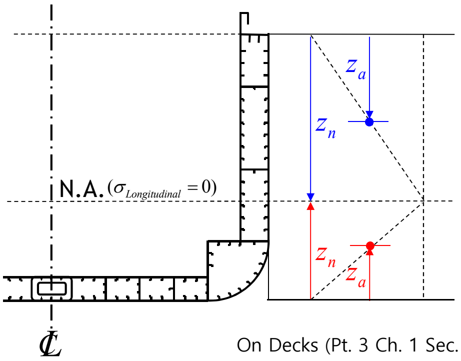
Section modulus of bottom is larger than that of deck, and thus the stress factor f_{2b} is smaller than f_{2d}.

Allowable Stresses
- Longitudinal Stiffeners (2/6)

DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004
DSME, DNV Rule Commentary Book, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$

✓ Calculation of z_n, z_a



Z_n: Vertical distance in [m] from the base line or deck line to the neutral axis of the hull girder, whichever is relevant

Z_a: Vertical distance in [m] from the base line or deck line to the point in question below or above the neutral axis

On Decks (Pt. 3 Ch. 1 Sec. 8 C301)

$$\boxed{\sigma_l = 225 f_1 - \underbrace{130 f_{2d} \frac{z_n - z_a}{z_n}}_{\sigma_{Longitudinal}}}$$

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as-built

Allowable Stresses - Longitudinal Stiffeners (3/6)

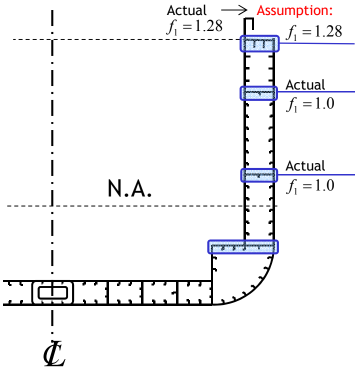
DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004
DSME, DNV Rule Commentary Book, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$

On Decks (Pt. 3 Ch. 1 Sec. 8 C301) $\boxed{\sigma_l = 225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}}$ → $\sigma_{Longitudinal}$

For example, 3,700 TEU Container Carrier:

$$f_{2d} = 1.140$$



$$z_n = 10.272,\ z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \quad \rightarrow \sigma_l = 139.800\ [MPa]$$

$$z_n = 10.272,\ z_a = 3.712 \rightarrow \frac{z_n - z_a}{z_n} = 0.639, \rightarrow \sigma_l = 130.300\ [MPa]$$

$$z_n = 10.272,\ z_a = 9.782 \rightarrow \frac{z_n - z_a}{z_n} = 0.048, \rightarrow \sigma_l = \cancel{217.886}\ [MPa] = 160\ [MPa]$$

(Maximum: 160)

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as-built

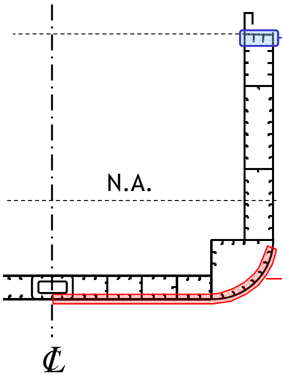
Z_n: Vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a: Vertical distance in m from the baseline or deck line to the point in question below or above the neutral axis

---

Allowable Stresses - Longitudinal Stiffeners (4/6)

DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004
DSME, DNV Rule Commentary Book, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



On Decks (Pt. 3 Ch. 1 Sec. 8 C301)

$$\boxed{225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}}$$

For example, 3,700 TEU Container Carrier:

$$f_1 = 1.28 \quad f_{2d} = 1.140$$

$$z_n = 10.272,\ z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \rightarrow \boxed{\sigma_l = 139.8 [MPa]}$$

On Double Bottom (Pt. 3 Ch. 1 Sec. 6 C701)

$$\boxed{225 f_1 - 130 f_{2b} - 0.7 \sigma_{db}}$$

For example, 3,700 TEU Container Carrier, Assumption: $\sigma_{db} = 0$

$$f_1 = 1.28 \quad f_{2b} = 1.030$$

$$z_n = 9.208,\ z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \rightarrow \boxed{\sigma_l = 154.1 [MPa]}$$

Allowable stresses at deck are **smaller** than those at bottom, because the distance from N.A to deck is longer than N.A to bottom.

If the mean double bottom stress (σ_{db}) is considered as 20,

$$\sigma_l = 225 f_1 - 130 f_{2b} - 0.7 \sigma_{db}$$
$$= 225 \times 1.28 - 130 \times 1.030 - 0.7 \times 20 = \boxed{140.1 [MPa]}$$

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

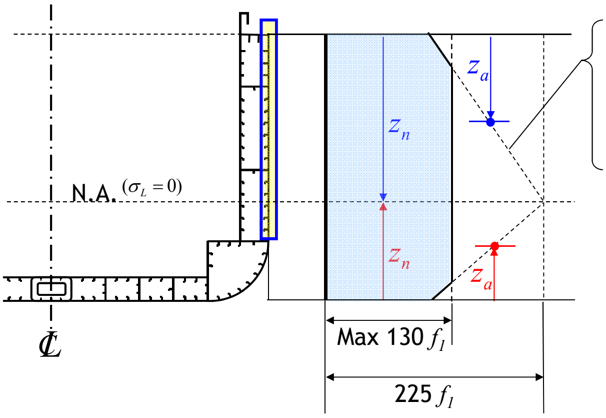
M_S : Largest design SWBM [kN·m]
M_W : Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as-built

Zn : vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the base line or deck line to the point in question below or above the neutral axis
σ_{db} : mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f_1 for cargo holds in general cargo vessel
= 50 f_1 for holds for ballast
= 85 f_1 b/B for tanks for liquid cargo

Allowable Stresses - Longitudinal Stiffeners (5/6)

DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004
DSME, DNV Rule Commentary Book, 1991.8

$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$



On Side Shell
(Pt. 3 Ch. 1 Sec. 7 C301)

$$225 f_1 - \underbrace{130 f_2 \frac{z_n - z_a}{z_n}}_{\sigma_{Longitudinal}}$$

$\boxed{130}$

which is lesser.

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as-built

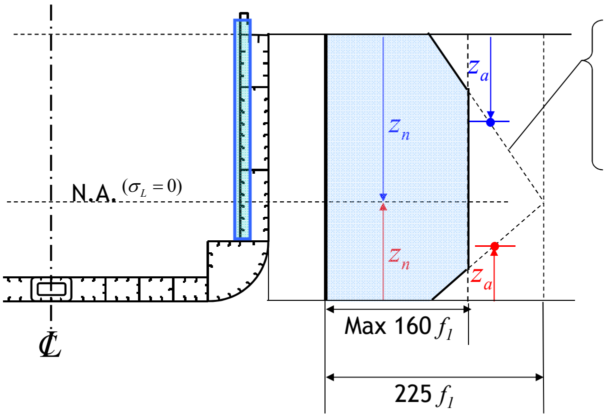
Z_n: Vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a: Vertical distance in m from the baseline or deck line to the point in question below or above the neutral axis

---

Allowable Stresses - Longitudinal Stiffeners (6/6)

DNV Rules, Pt. 3 Ch. 1 Sec. 6, 7, 8, 9, Jan. 2004
DSME, DNV Rule Commentary Book, 1991.8

$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$



On Longitudinal Bulkhead
(Pt. 3 Ch. 1 Sec. 9 C201)

$$225 f_1 - \underbrace{130 f_2 \frac{z_n - z_a}{z_n}}_{\sigma_{Longitudinal}}$$

$\boxed{160}$

which is lesser.

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as-built

Z_n: Vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a: Vertical distance in m from the baseline or deck line to the point in question below or above the neutral axis

- **Minimum plate thickness**

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} \ (mm)$$

- **Minimum section modulus for longitudinal stiffener**

$$Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)$$

(3) Design Loads

Contents

☑ Ship Motion and Acceleration

☑ Combined Acceleration

☑ Design Probability Level

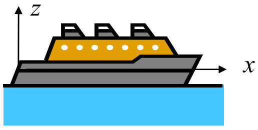
☑ Load Point

☑ Pressure & Force

- Sea Pressure
- Liquid Tank Pressure

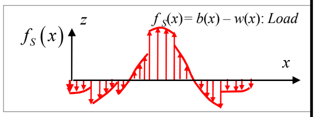
[Review] Loads in Wave

In still water



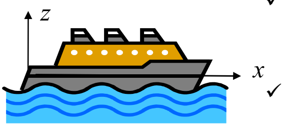
$$f_S(x) = b(x) + w(x)$$

$f(x)$: Distributed loads in longitudinal direction
$f_S(x)$: **Static longitudinal loads** in longitudinal direction
$f_W(x)$: **Hydrodynamic longitudinal loads** induced by wave



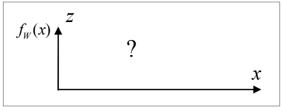
+

In wave



✓ Dynamic longitudinal loads

: Loads are induced by waves

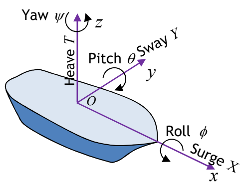


✓ Direct calculation of dynamic longitudinal loads

- from 6DOF motion of ship

$$\mathbf{x} = \left[X, Y, T, \phi, \theta, \psi\right]^T$$

✓ Ref.> 6 DOF motion of ship



$$f(x) = f_S(x) \boxed{+ f_W(x)}$$
$$= b(x) + w(x) \boxed{+ f_D(x) + f_{F.K}(x) + f_R(x)}$$

additional loads in wave

where,

$$f_R(x) = -a(x)\boxed{\ddot{\mathbf{x}}} - b(x)\boxed{\dot{\mathbf{x}}}$$

$f_D(x)$: Diffraction force at x
$f_R(x)$: Radiation force at x by damping and added mass
$f_{F.K}(x)$: Froude-Krylov force at x

[Review] 6 DOF Equation of Motion of Ship

How to know $\ddot{\mathbf{x}}, \dot{\mathbf{x}}$?

By solving equations of motion, we can get the velocities and accelerations.

✓ Pressure acting on hull

Linearized Bernoulli Eq. $P_{Fluid} = -\rho g z - \rho \frac{\partial \Phi}{\partial t} = -\rho g z - \rho\left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t}\right)$

✓ Fluid force acting on hull

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = -\iint_{S_B} \rho g z\,\mathbf{n}dS - \rho\iint_{S_B}\left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t}\right)dS$$
$$= \mathbf{F}_{Static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

✓ 6 D.O.F equations of motion of a ship in waves

Pressure force acting as surface force on hull

Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \boxed{\mathbf{F}_{Body}} + \boxed{\mathbf{F}_{Surface}}$$
$$= \mathbf{F}_{Gravity} + \mathbf{F}_{Fluid} + \mathbf{F}_{External}$$

External force excluding wave exciting force (ex. control force)

Body force Surface force

$$\mathbf{M}\ddot{\mathbf{x}} = \underbrace{\boxed{\mathbf{F}_{Gravity}} + \boxed{\mathbf{F}_{Static}}_{\mathbf{F}_{Restoring}} + \underbrace{\mathbf{F}_{F.K} + \mathbf{F}_D}_{\mathbf{F}_{Wave\ exciting}} + \mathbf{F}_R} + \boxed{\mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}}$$

$$\mathbf{F}_R = -\boxed{\mathbf{A}}\ddot{\mathbf{x}} - \boxed{\mathbf{B}}\dot{\mathbf{x}}$$

Added mass
Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = (\mathbf{F}_{Gravity} + \mathbf{F}_{Static}) + \mathbf{F}_{Wave\ exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}} + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

Linearization, $(F_{Restoring} = (\mathbf{F}_{Gravity} + \mathbf{F}_{Static}) \approx -\mathbf{C}\mathbf{x})$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{Wave\ exciting} + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

By solving equations of motion, we can get the velocities and accelerations of the ship!

$F_{F.K}$: Froude-Krylov force
F_D: Diffraction force
F_R: Radiation force

Φ_I : Incident wave velocity potential

Φ_D: Diffraction wave velocity potential

Φ_R: Radiation wave velocity potential

$\mathbf{M}_A$: 6×6 added mass matrix

$\mathbf{B}$: 6×6 damping coeff. matrix

$\mathbf{C}$: 6×6 restoring coeff. matrix

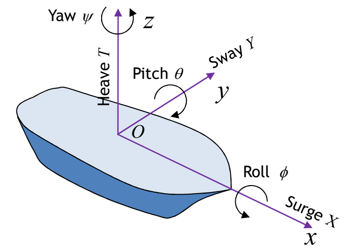
(1) Ship Motion and Acceleration - Empirical Formula of DNV Rule

DNV Rules, Pt. 3 Ch. 1 Sec. 4, Jan. 2004

Common Acceleration Parameter	$a_0 = \frac{3C_w}{L} + C_v C_{v1}$
Surge Acceleration	$a_x = 0.2 g_0 a_0 \sqrt{C_b}$
Combined Sway/Yaw Acceleration	$a_y = 0.3 g_0 a_0$
Heave Acceleration	$a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$
Tangential Roll Acceleration	$a_r = \phi \left(\frac{2\pi}{T_r} \right)^2 R_r$
Tangential Pitch Acceleration	$a_p = \theta \left(\frac{2\pi}{T_p} \right)^2 R_p$

g_0 : standard acceleration of gravity =9.81m/s^2

✓ Ref. 6 DOF motion of ship



Common Acceleration Parameter, a_0

$$\boxed{a_0 = \frac{3C_W}{L} + C_V C_{V1}}$$

$C_V = \frac{\sqrt{L}}{50}$, maximum 0.2

$C_{V1} = \frac{V}{\sqrt{L}}$, minimum 0.8

C_W = Wave coefficient

L	C_W
$L \le 100$	$0.0792 \cdot L$
$100 < L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

---

(1) Ship Motions and Accelerations - Roll Angle & Roll Period

DNV Rules, Pt. 3 Ch. 1 Sec. 4, Jan. 2004

✓ Roll angle

$$\boxed{\phi = \frac{50c}{B + 75} \quad \text{(rad)}}$$

c = $(1.25 - 0.025\ T_R)$ k
k = 1.2 for ships without bilge keel
= 1.0 for ships with bilge keel
= 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

✓ Roll period

$$\boxed{T_R = \frac{2k_r}{\sqrt{GM}} \quad \text{(s)}}$$

k_r= 0.39B for ships with even transverse distribution of mass
= 0.35B for tankers in ballast
= 0.25B for ships loaded with ore between longitudinal bulkheads
GM= 0.07B in general
= 0.12B for tankers and bulk carriers

✓ Pitch angle

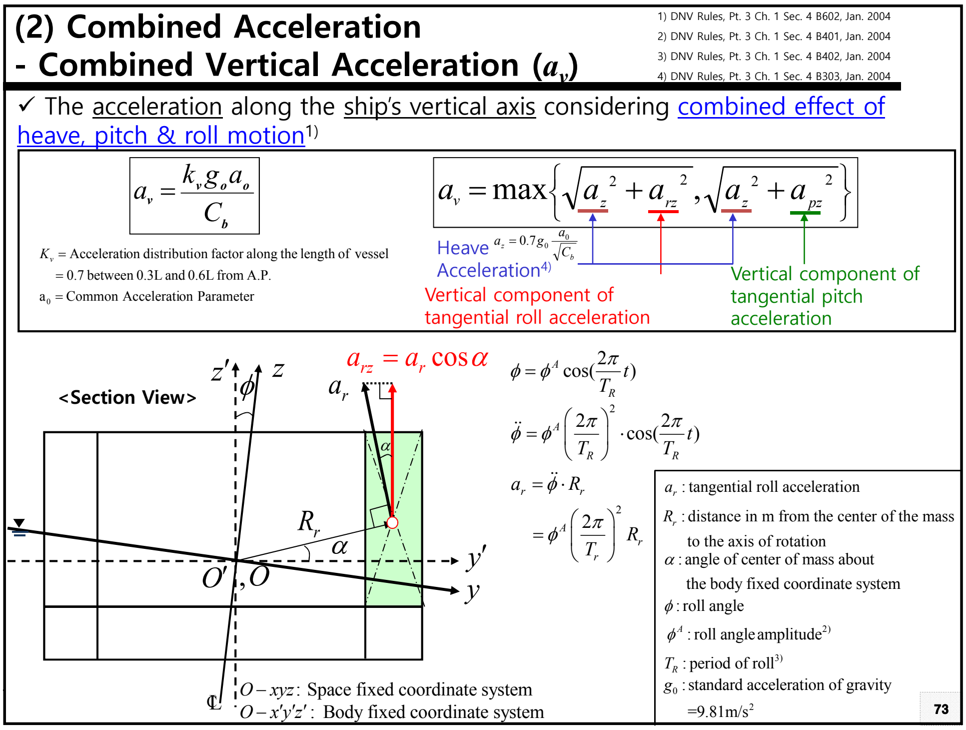
$$\boxed{\theta = 0.25 \frac{a_0}{C_B} \quad \text{(rad)}}$$

$$a_0 = \frac{3C_W}{L} + C_V C_{V1}$$

✓ Pitch period

$$\boxed{T_P = 1.8 \sqrt{\frac{L}{g_0}} \quad \text{(s)}}$$

g_0 : standard acceleration of gravity =9.81m/s^2



(2) Combined Acceleration

- Combined Vertical Acceleration (a_v)

1) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B602, Jan. 2004
2) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B401, Jan. 2004
3) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B402, Jan. 2004
4) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B303, Jan. 2004

✓ The acceleration along the ship's vertical axis considering combined effect of heave, pitch & roll motion1)

$$a_v = \frac{k_v g_o a_o}{C_b}$$

K_v = Acceleration distribution factor along the length of vessel
= 0.7 between 0.3L and 0.6L from A.P.
a_0 = Common Acceleration Parameter

$$a_v = \max\left\{\sqrt{\underline{a_z}^2 + \underline{a_{rz}}^2}, \sqrt{\underline{a_z}^2 + \underline{a_{pz}}^2}\right\}$$

Heave Acceleration4) $a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$

Vertical component of tangential roll acceleration

Vertical component of tangential pitch acceleration

$$a_{rz} = a_r \cos\alpha$$

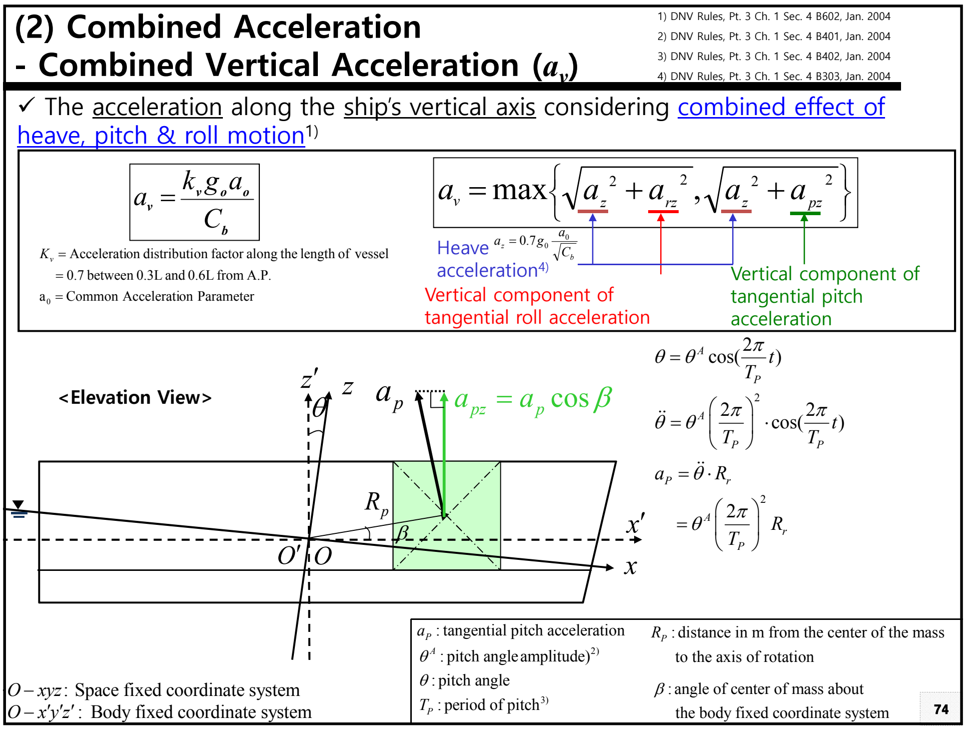
$$\phi = \phi^A \cos(\frac{2\pi}{T_R} t)$$

$$\ddot{\phi} = \phi^A \left(\frac{2\pi}{T_R}\right)^2 \cdot \cos(\frac{2\pi}{T_R} t)$$

$$a_r = \ddot{\phi} \cdot R_r$$
$$= \phi^A \left(\frac{2\pi}{T_r}\right)^2 R_r$$

$O-xyz$: Space fixed coordinate system
$O-x'y'z'$: Body fixed coordinate system

a_r : tangential roll acceleration
R_r : distance in m from the center of the mass to the axis of rotation
α : angle of center of mass about the body fixed coordinate system
ϕ : roll angle
ϕ^A : roll angle amplitude2)
T_R : period of roll3)
g_0 : standard acceleration of gravity =9.81m/s^2



(2) Combined Acceleration

- Combined Vertical Acceleration (a_v)

1) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B602, Jan. 2004
2) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B401, Jan. 2004
3) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B402, Jan. 2004
4) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B303, Jan. 2004

✓ The acceleration along the ship's vertical axis considering combined effect of heave, pitch & roll motion1)

$$a_v = \frac{k_v g_o a_o}{C_b}$$

K_v = Acceleration distribution factor along the length of vessel
= 0.7 between 0.3L and 0.6L from A.P.
a_0 = Common Acceleration Parameter

$$a_v = \max\left\{\sqrt{\underline{a_z}^2 + \underline{a_{rz}}^2}, \sqrt{\underline{a_z}^2 + \underline{a_{pz}}^2}\right\}$$

Heave acceleration4) $a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$

Vertical component of tangential roll acceleration

Vertical component of tangential pitch acceleration

$$\theta = \theta^A \cos(\frac{2\pi}{T_P} t)$$

$$\ddot{\theta} = \theta^A \left(\frac{2\pi}{T_P}\right)^2 \cdot \cos(\frac{2\pi}{T_P} t)$$

$$a_P = \ddot{\theta} \cdot R_r$$
$$= \theta^A \left(\frac{2\pi}{T_P}\right)^2 R_r$$

$$a_{pz} = a_p \cos\beta$$

$O-xyz$: Space fixed coordinate system
$O-x'y'z'$: Body fixed coordinate system

a_P : tangential pitch acceleration
θ^A : pitch angle amplitude)2)
θ : pitch angle
T_P : period of pitch3)
R_P : distance in m from the center of the mass to the axis of rotation
β : angle of center of mass about the body fixed coordinate system

(2) Combined Acceleration
- Combined Transverse Acceleration (a_t)

1) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B700, Jan. 2004

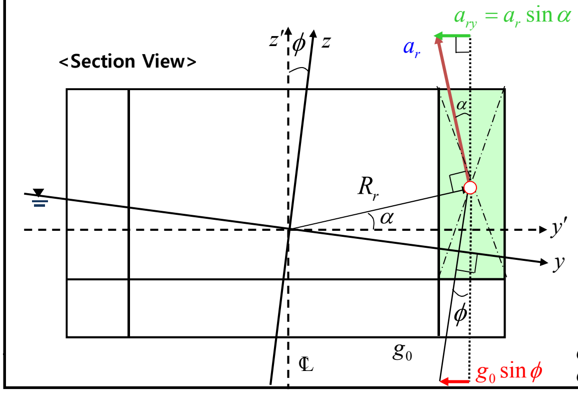
✓ The acceleration along the ship's transverse axis considering combined effect of sway, yaw & roll motion1)

$$a_t = \sqrt{\underline{a_y}^2 + (\underline{g_o \sin\phi} + \underline{a_{ry}})^2}$$

Combined sway & yaw acceleration
$a_y = 0.3 g_0 a_0$

Transverse component of acceleration of gravity by roll angle

Transverse component of the tangential roll acceleration



$$a_r = \phi^A \left(\frac{2\pi}{T_r}\right)^2 R_r$$

a_r : tangential roll acceleration

R_r : distance in m from the center of the mass to the axis of rotation

ϕ : roll angle

ϕ^A : roll angle amplitude

T_R : period of roll3)

g_0 : standard acceleration of gravity =9.81m/s^2

$O-xyz$: Space fixed coordinate system
$O-x'y'z'$: Body fixed coordinate system

(2) Combined Acceleration
- Combined Longitudinal Acceleration (a_l)

1) DNV Rules, Pt. 3 Ch. 1 Sec. 4 B800, Jan. 2004

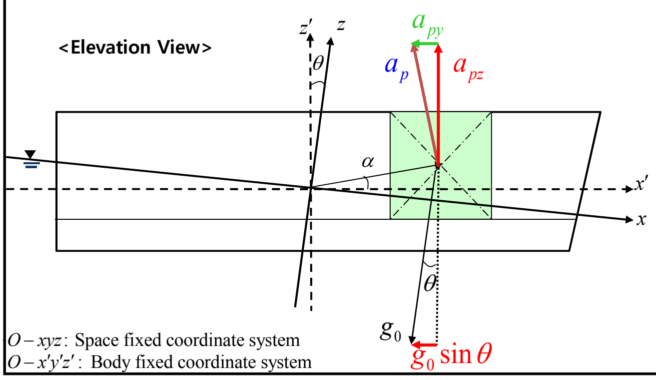
✓ The acceleration along the ship's longitudinal axis considering combined effect of surge & pitch motion1)

$$a_l = \sqrt{\underline{a_x}^2 + (\underline{g_o \sin\theta} + \underline{a_{px}})^2}$$

Surge acceleration
$a_x = 0.2 g_0 a_0 \sqrt{C_b}$

Longitudinal component of gravitational acceleration by pitch angle

Longitudinal component of the pitch acceleration



$O-xyz$: Space fixed coordinate system
$O-x'y'z'$: Body fixed coordinate system

$$a_P = \theta^A \left(\frac{2\pi}{T_P}\right)^2 R_r$$

a_P : tangential pitch acceleration

R_P : distance in m from the center of the mass to the axis of rotation

θ : pitch angle

θ^A : pitch angle amplitude2)

T_P : period of pitch3)

g_0 : standard acceleration of gravity =9.81m/s^2

(2) Combined Acceleration
- [Example] Vertical Acceleration

(Example) Calculate the vertical acceleration of a given ship at 0.5L (amidships) by DNV Rule.

[Dimension] L_s=315.79 m, V=15.5 knots, C_B=0.832

$$\boxed{a_v = \frac{k_v g_o a_o}{C_b}}$$

K_v = Acceleration distribution factor along the length of vessel
= 0.7 between 0.3L and 0.6L from A.P.
a_0 = Common Acceleration Parameter
g_0 = Standard acceleration of gravity (=9.81m/sec^2)

(Sol.) $a_v = \left(k_v\, g_0\, a_0\right) / C_B = \left(0.7\times 9.81\times 0.277\right)/0.832$

$= 2.286 \left(m/\sec^2\right)$

where, $k_v = 0.7$ at mid ship

$a_0 = 3C_W / L + C_v C_{v1} = 3\times 10.75/315.79 + 0.2\times 0.872 = 0.277$

$C_v = L^{0.5}/50 = 315.79^{0.5}/50 = 0.355$ or Max. 0.2

$=0.2$

$C_{v1} = V / L^{0.5} = 15.5/315.79^{0.5} = 0.872$ or Min. 0.8

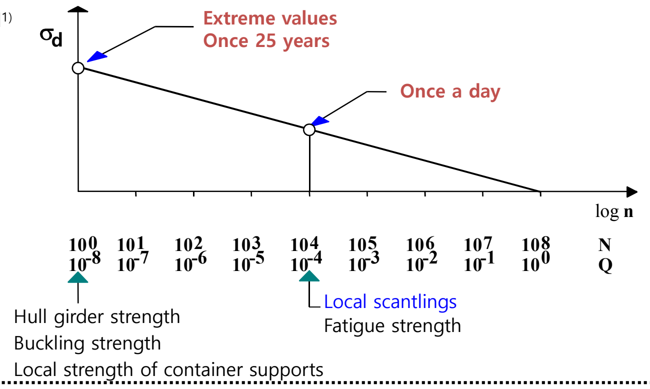
$=0.872$

---

1) DNV, Fatigue Assessment of Ship Structures, p.18, 2003

(3) Design Probability Level

- Probability Level[1)]



- Design Probability Level[2)]
  - ✓ Number of waves that the ship experiences during the ship's life (for 25 years): about 10^8
    - ➨ The ship is designed to endure the extreme wave (10^{-8} probability) which the ship encounters once for 25 years.
      (Extreme condition: Ship motion and acceleration are given as extreme values.)
  - ✓ In case of design pressure, use the reduced value of 10^{-4} (Reduction value = 0.5×Extreme value)

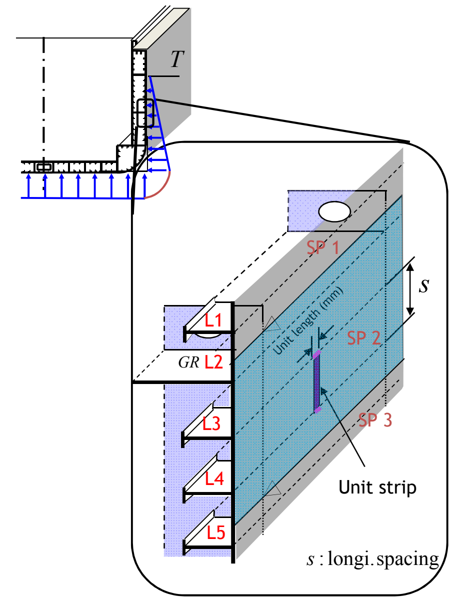
Ex) Liquid Tank Pressure: Pressure, P_1, considering vertical acceleration

$$p_1 = \rho\left(g_o + 0.5a_v\right)h_s$$

(4) Load Point
- Horizontally Stiffened Plate

DNV Rules, Pt. 3 Ch. 1 Sec. 4 A202, Jan. 2004

$$p_2 = \rho\, g_o \left[\, 0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\, \right]$$

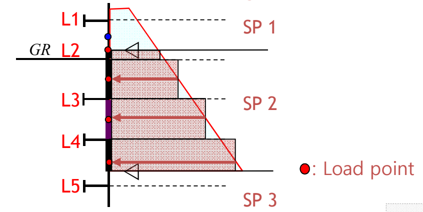


✓ **<u>The pressure at the load point</u>** is considered as **<u>uniform load</u>** of unit strip

✓ Definition of load point

1. General
   : Midpoint of stiffened plate field
2. Seam & butt (In case two plates are welded)
   1) When considered plate includes the midpoint of stiffened plate field
      : Midpoint of stiffened plate field
   2) When considered plate <u>does not include</u> the midpoint of stiffened plate field
      : Nearest seam or butt line from midpoint

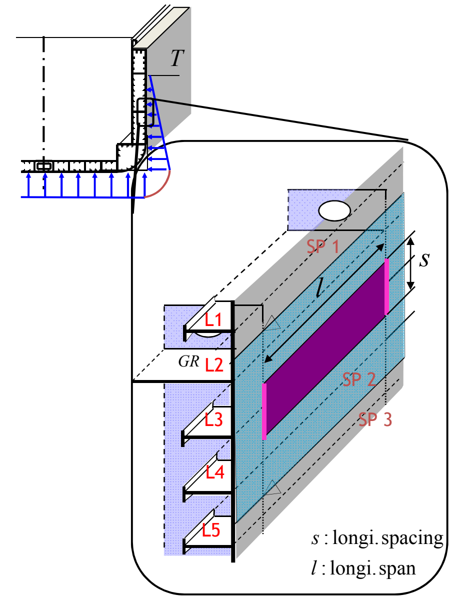
✓ Load point of sea pressure acting on the side plate



---

(4) Load Point
- Longitudinal Stiffeners (1/2)

DNV Rules, Pt. 3 Ch. 1 Sec. 4 A202, Jan. 2004

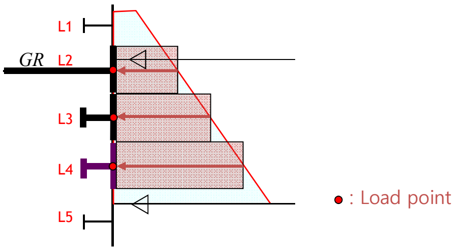


✓ **<u>The pressure at the load point</u>** is considered as **<u>uniform load</u>**

✓ Definition of load point

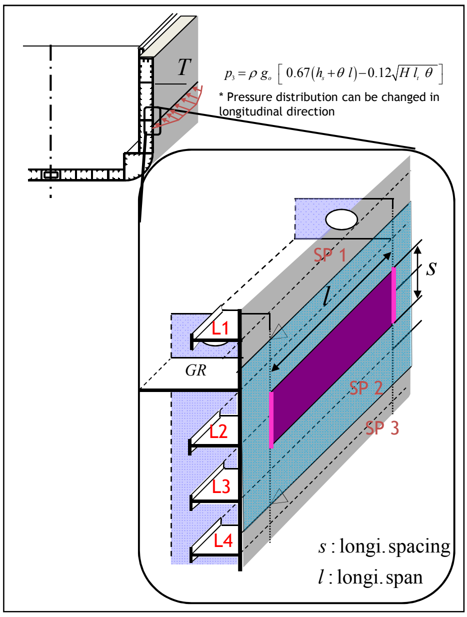
1. In vertical direction
   : The point of intersection between a plate and a stiffener
2. In longitudinal direction
   : Midpoint of span

✓ Load point of sea pressure acting on the side plate
- In vertical direction



(4) Load Point
- Longitudinal Stiffeners (2/2)

DNV Rules, Pt. 3 Ch. 1 Sec. 4 A202, Jan. 2004



$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\, \right]$

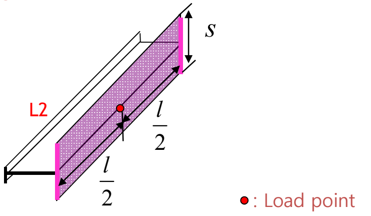
* Pressure distribution can be changed in longitudinal direction

✓ **The pressure at the load point** is considered as **uniform load**

✓ Definition of load point

1. In vertical direction
   : The point of intersection between a plate and a stiffener
2. In longitudinal direction
   : Midpoint of span

✓ Load point of sea pressure acting on the side plate
- In longitudinal direction



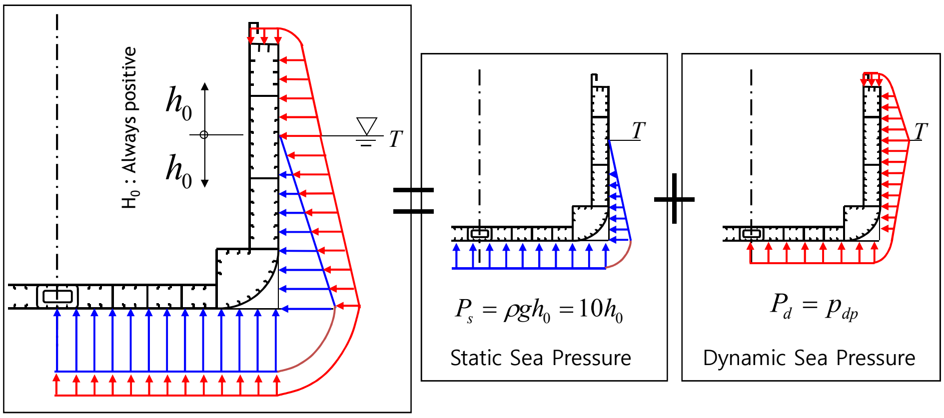
---

(5) Pressure and Force
- Sea Pressure

DNV Rules, Pt. 3 Ch. 1 Sec. 4 C201, Jan. 2004

✓ Sea pressures = Static sea pressure + Dynamic sea pressure

$$P = P_s + P_d$$

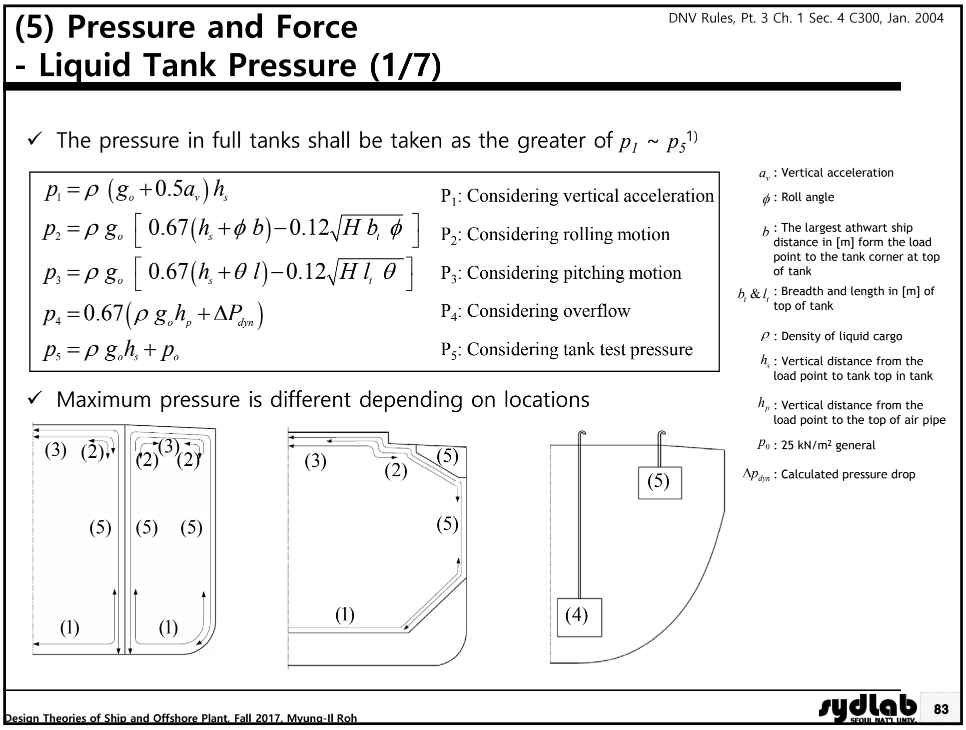


$P_s = \rho g h_0 = 10 h_0$

Static Sea Pressure

$P_d = p_{dp}$

Dynamic Sea Pressure



(5) Pressure and Force
- Liquid Tank Pressure (1/7)

DNV Rules, Pt. 3 Ch. 1 Sec. 4 C300, Jan. 2004

- ✓ The pressure in full tanks shall be taken as the greater of p_1 ~ $p_5$$^{1)}$

$p_1 = \rho\ (g_o + 0.5a_v)\,h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\ g_o\left[\,0.67(h_s + \phi\ b) - 0.12\sqrt{H\ b_t\ \phi}\,\right]$	P_2: Considering rolling motion
$p_3 = \rho\ g_o\left[\,0.67(h_s + \theta\ l) - 0.12\sqrt{H\ l_t\ \theta}\,\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\ g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\ g_o h_s + p_o$	P_5: Considering tank test pressure

a_v : Vertical acceleration

ϕ : Roll angle

b : The largest athwart ship distance in [m] form the load point to the tank corner at top of tank

b_t & l_t : Breadth and length in [m] of top of tank

ρ : Density of liquid cargo

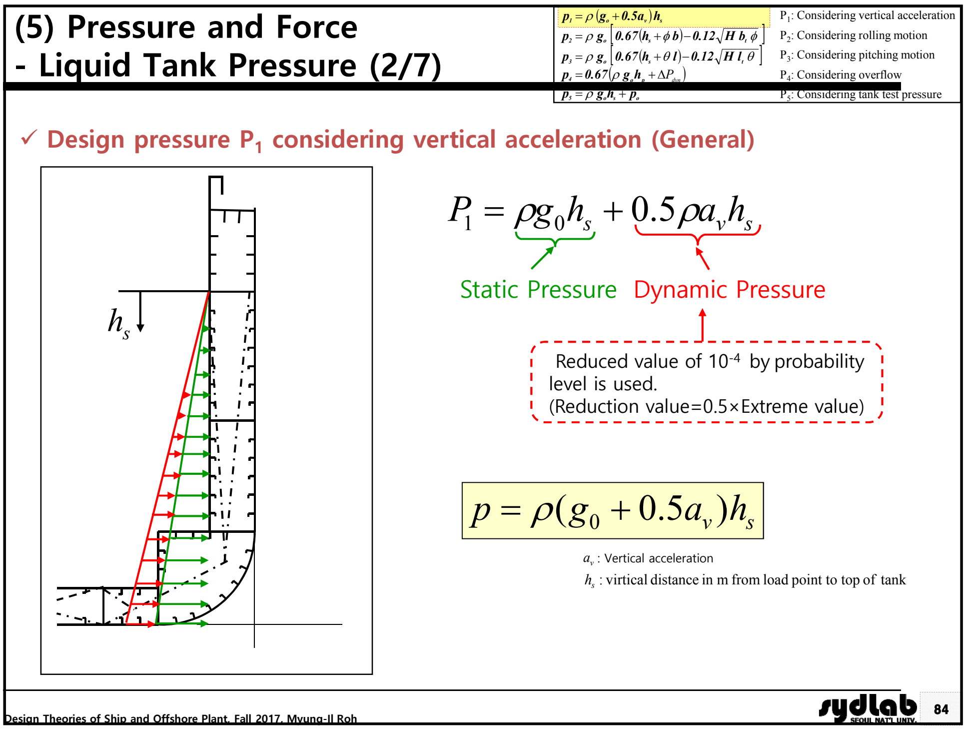
h_s : Vertical distance from the load point to tank top in tank

h_p : Vertical distance from the load point to the top of air pipe

p_0 : 25 kN/m^2 general

Δp_{dyn} : Calculated pressure drop

- ✓ Maximum pressure is different depending on locations



(5) Pressure and Force
- Liquid Tank Pressure (2/7)

$p_1 = \rho\,(g_o + 0.5a_v)\,h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\ g_o\left[0.67(h_s + \phi\ b) - 0.12\sqrt{H\ b_t\ \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\ g_o\left[0.67(h_s + \theta\ l) - 0.12\sqrt{H\ l_t\ \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\ g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\ g_o h_s + p_o$	P_5: Considering tank test pressure

- ✓ **Design pressure P_1 considering vertical acceleration (General)**

$$P_1 = \underbrace{\rho g_0 h_s}_{\text{Static Pressure}} + \underbrace{0.5\rho a_v h_s}_{\text{Dynamic Pressure}}$$

Reduced value of 10^{-4} by probability level is used.
(Reduction value=0.5×Extreme value)

$$\boxed{p = \rho(g_0 + 0.5a_v)h_s}$$

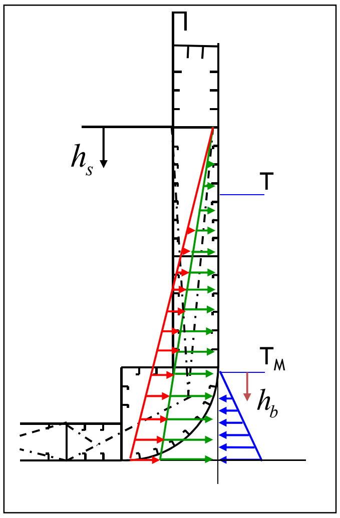
a_v : Vertical acceleration

h_s : virtical distance in m from load point to top of tank

(5) Pressure and Force
- Liquid Tank Pressure (3/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

✓ Design pressure P_1 considering vertical acceleration (In case of side shell)



In case of side shell, the effect of sea pressure is considered.

$$P = \underbrace{\rho g_0 h_s}_{\text{Static Pressure}} + \underbrace{0.5\rho a_v h_s}_{\text{Dynamic Pressure}} \underbrace{\boxed{-10h_b}}_{\text{Sea Pressure}}$$

When we consider the design pressure, the largest value shall be applied. The liquid cargo pressure acting on the side shell is the highest when the sea pressure is the lowest, i.e. in case of minimum draft.

$$\boxed{p = \rho(g_0 + 0.5a_v)h_s - 10h_b}$$

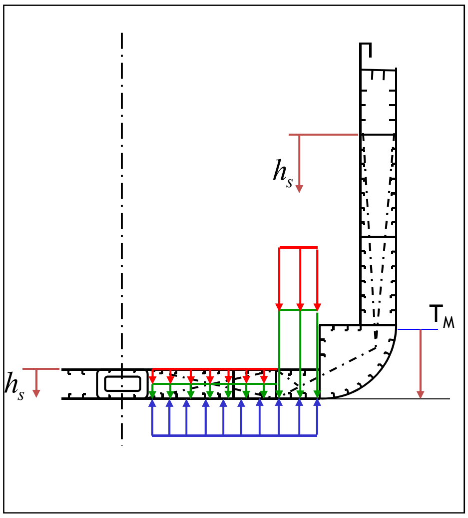
h_b: vertical distance in m from load point to minimum design draft
= 2 + 0.02L for Tanker
= 0.35 T for Dry Cargo
(T: Rule Draft)

---

(5) Pressure and Force
- Liquid Tank Pressure (4/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

✓ Design pressure P_1 considering vertical acceleration (In case of bottom shell)



In case of bottom shell, the effect of sea pressure is considered

$$P = \underbrace{\rho g_0 h_s}_{\text{Static Pressure}} + \underbrace{0.5\rho a_v h_s}_{\text{Dynamic Pressure}} \underbrace{\boxed{-10T_M}}_{\text{Sea Pressure}}$$

When we consider the design pressure, the largest value shall be applied. The liquid cargo pressure acting on the bottom shell is the highest when the sea pressure is the lowest, i.e. in case of minimum draft.

$$\boxed{p = \rho(g_0 + 0.5a_v)h_s - 10T_M}$$

T_M: vertical distance in m from load point to minimum design draft
= 2 + 0.02L for Tanker
= 0.35 T for Dry Cargo
(T: Rule Draft)

(5) Pressure and Force
- Example) Calculation of P_1 Pressure

DNV Rules, Pt. 3 Ch. 1 Sec. 4 B800, Jan. 2004

(Example) When the tank is filled up, calculate the P_1 pressure of inner bottom and deck by using vertical acceleration (a_v=2.286 m/s²) and dimensions of tank which is given below.

[Dimension] Inner bottom height: 3.0 m, Deck height: 31.2m, ρ = 1.025 ton/m³

$$\boxed{P_1 = \rho\left(g_0 + 0.5a_v\right)h_s}$$

ρ = density (ton/m³)
a_v = Vertical acceleration
g_0 = Standard acceleration of gravity (=9.81m/sec²)
h_s : virtical distance in m from load point to top of tank

(Sol.) $a_v = 2.286\ \text{m/s}^2$

① Inner Bottom

$$h_s = 31.2 - 3.0 = 28.8\ m$$

$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$
$$= 1.025\left(9.81 + 0.5\times 2.286\right)\times 28.2$$
$$= 316.6\,kN/m^2$$

② Deck

$$h_s = 31.2 - 31.2 = 0\ m$$

$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$
$$= 1.025\left(9.81 + 0.5\times 2.286\right)\times 0$$
$$= 0\,kN/m^2$$

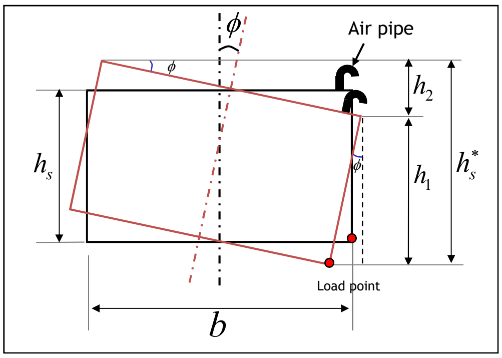
---

(5) Pressure and Force
- Liquid Tank Pressure (5/7)

$p_1 = \rho\left(g_o + 0.5a_v\right)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o\left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o\left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67\left(\rho\, g_o h_p + \Delta P_{dyn}\right)$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3
DNV Rules, Pt. 3 Ch. 1 Sec. 4, Jan. 2004

✓ **Design pressure P_2 considering the rolling motion**



When the ship is rolling, the higher static pressure is applied.

Assumption: $\phi << 1$

$$h_1 = h_s \cos\phi \approx h_s$$
$$h_2 = b\sin\phi \approx b\phi$$

$$\therefore h_s^* = h_1 + h_2$$
$$= \left(h_s + b\phi\right)$$

$$\boxed{p_2 = \rho g_0[\underline{0.67}(h_s + \phi b) - \underline{0.12\sqrt{H\phi b_t}}]}$$

In case of rolling of a ship, two third (=0.67) of actual pressure is applied considering pressure drop by overflow.

The filling ratio of the most tank is about 98%.
That (about 2%) is considered.

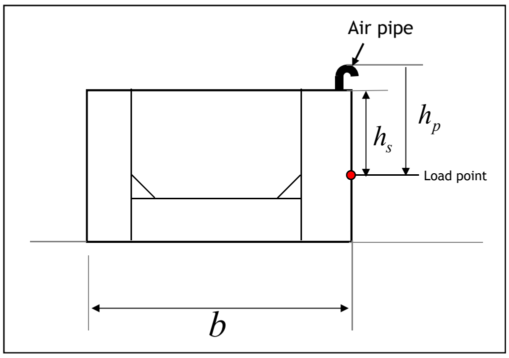
H: Height in m of the tank
b_t: Breadth in m of top of tank

(5) Pressure and Force
- Liquid Tank Pressure (6/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi} \right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta} \right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3
DNV Rules, Pt. 3 Ch. 1 Sec. 4, Jan. 2004

✓ Design pressure P_4 considering the tank overflow



The liquid of tank is filled up to air pipe in case of tank overflow.
So, h_p is used for calculating static pressure.

h_p = vertical distance in m from the load point to the top of air pipe

$$p = 0.67\left(\underline{\rho g_0 h_p} + \underline{\Delta P_{dyn}}\right)$$

Calculated pressure drop
Generally, 25kN/m^2

In case of rolling of a ship, two third (=0.67) of actual pressure is applied considering pressure drop by overflow.

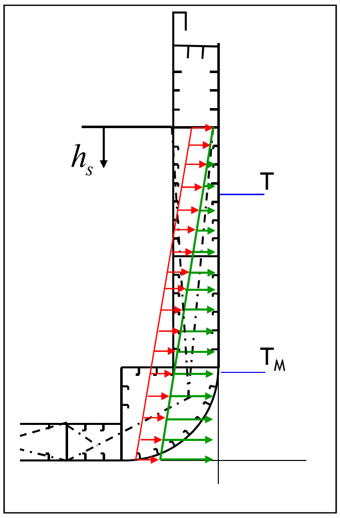
---

(5) Pressure and Force
- Liquid Tank Pressure (7/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi} \right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta} \right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3
DNV Rules, Pt. 3 Ch. 1 Sec. 4, Jan. 2004

✓ Design pressure P_5 considering the tank test pressure



Over-pressure is applied in order to have the water head of 'tank height + 2.5' [m] in case of tank test for leakage.
(Water head of over-pressure of tank test: 2.5m)

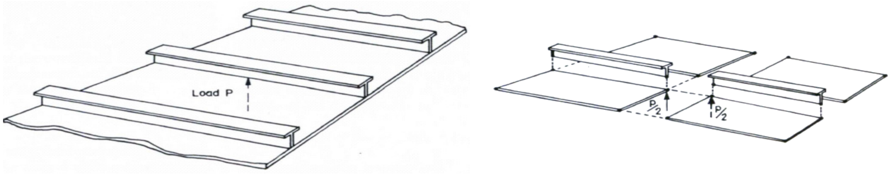
$$p = \rho g_0 h_s + \underline{p_o}$$

$$\begin{aligned} p_o &= \rho g_0 \times 2.5 \\ &= 10 \times 2.5 \\ &= 25kN/m^2 \end{aligned}$$

(4) Scantling of Plates

Scantling of Plates (1/3)

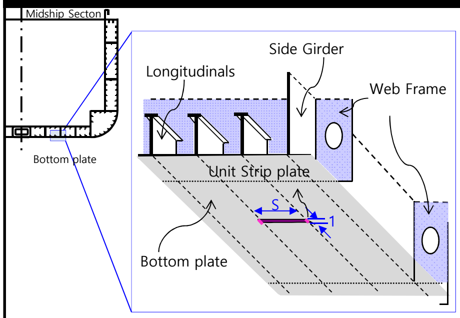
Use of eccentric beam element

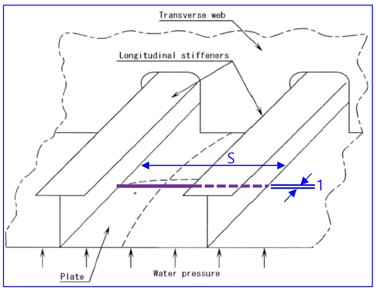


(a) Beams attached to plating (b) Structural model using eccentric beam element

Scantling of Plates (2/3)

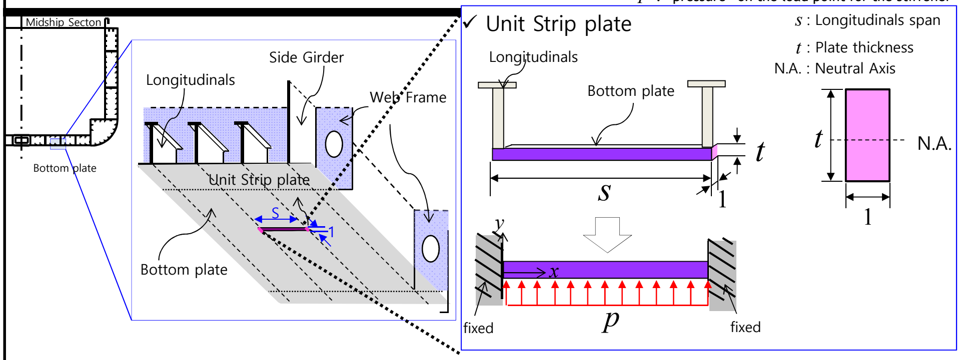
p : "pressure" on the load point for the stiffener





Scantling of Plates (3/3)

p : "pressure" on the load point for the stiffener



s : Longitudinals span

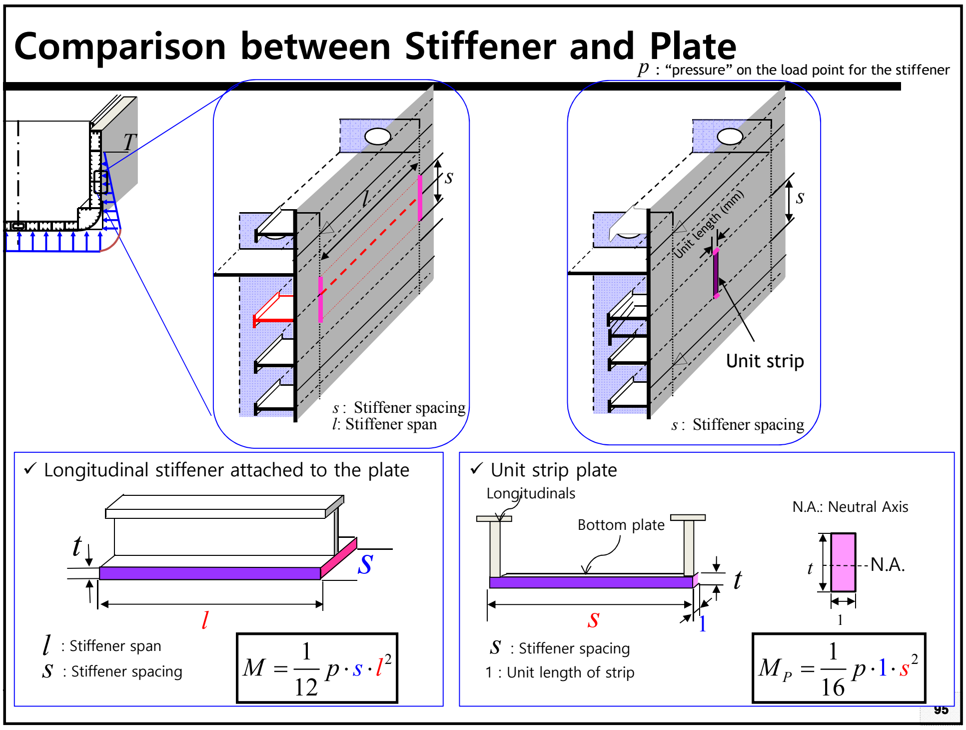
t : Plate thickness

N.A. : Neutral Axis

Assumption 1. Cut off the **unit strip plate** supported by the **longitudinals** or **girder.** And consider the unit strip plate as a **"fixed-end beam"** which has a span 's', thickness 't'.

Assumption 2. Consider the lateral load of the beam as a uniformly distributed load. (Assume the pressure on the load point as an intensity of uniformly distributed load.)

Assumption 3. The design of plates is based on the **plastic design**.



Comparison between Stiffener and Plate

p : "pressure" on the load point for the stiffener

✓ Longitudinal stiffener attached to the plate

l : Stiffener span

s : Stiffener spacing

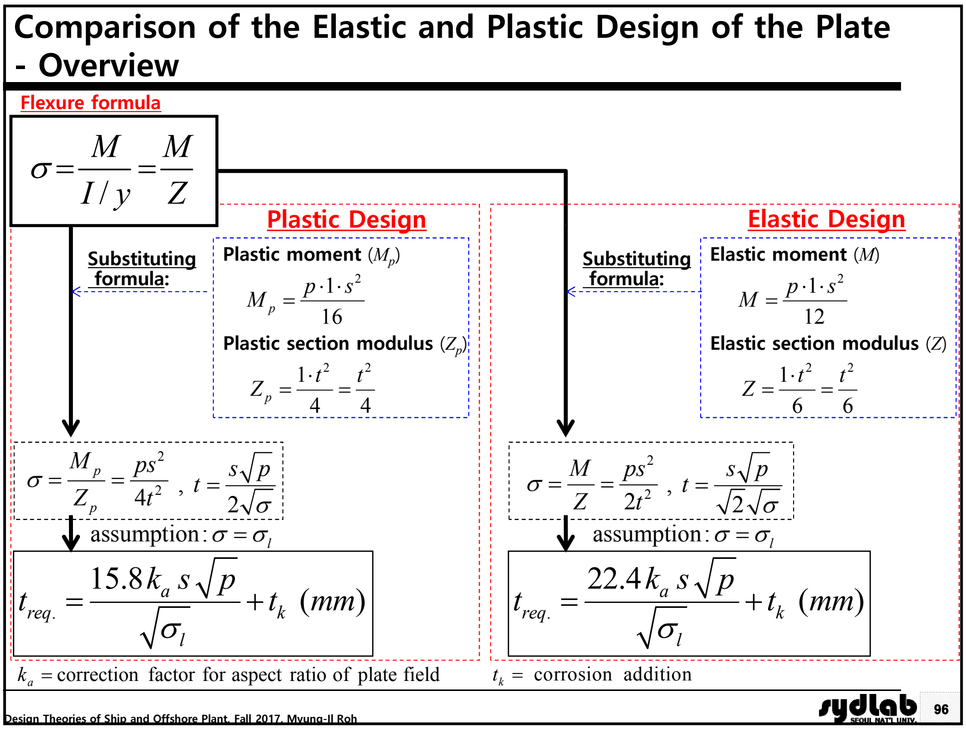
$$M = \frac{1}{12} p \cdot s \cdot l^2$$

✓ Unit strip plate

s : Stiffener spacing

1 : Unit length of strip

$$M_P = \frac{1}{16} p \cdot 1 \cdot s^2$$



Comparison of the Elastic and Plastic Design of the Plate - Overview

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Plastic Design

Substituting formula:

Plastic moment (M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus (Z_p)

$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

$$\sigma = \frac{M_p}{Z_p} = \frac{ps^2}{4t^2}, \quad t = \frac{s\sqrt{p}}{2\sqrt{\sigma}}$$

assumption: $\sigma = \sigma_l$

$$t_{req.} = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

Elastic Design

Substituting formula:

Elastic moment (M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$

$$\sigma = \frac{M}{Z} = \frac{ps^2}{2t^2}, \quad t = \frac{s\sqrt{p}}{\sqrt{2}\sqrt{\sigma}}$$

assumption: $\sigma = \sigma_l$

$$t_{req.} = \frac{22.4 k_a s \sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

k_a = correction factor for aspect ratio of plate field

t_k = corrosion addition

Comparison of the Elastic and Plastic Design - [Example] Thickness Requirements

Plastic moment (M_p)

$$M_p = \frac{p\cdot 1\cdot s^2}{16}$$

Plastic section modulus (Z_p)

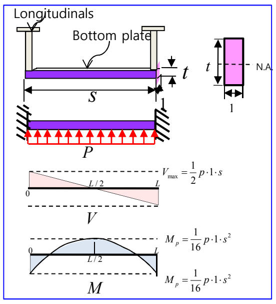
$$Z_p = \frac{1\cdot t^2}{4} = \frac{t^2}{4}$$

Elastic moment (M)

$$M = \frac{p\cdot 1\cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1\cdot t^2}{6} = \frac{t^2}{6}$$



① A mild steel plate carries the uniform pressure of 100 kN/m² on a span length of 800 mm.
Compare the thickness requirement depending on the plastic design and elastic design.

$$t_{\substack{req.\\ plastic}} = \frac{15.8k_a s\sqrt{p}}{\sqrt{\sigma_l}} = \frac{15.8\times 1\times 0.8\times\sqrt{100}}{\sqrt{235}} = \boxed{8.24\ (mm)}$$

$$t_{\substack{req.\\ elastic}} = \frac{22.4k_a s\sqrt{p}}{\sqrt{\sigma_l}} = \frac{22.4\times 1\times 0.8\times\sqrt{100}}{\sqrt{235}} = \boxed{11.69\ (mm)}$$

The thickness requirement of the plate of plastic design is smaller than that of the elastic design at the same pressure and on the same span.

k_a = correction factor for aspect ratio of plate field

Comparison of the Elastic and Plastic Design - [Example] Design Pressure

Plastic moment (M_p)

$$M_p = \frac{p\cdot 1\cdot s^2}{16}$$

Plastic section modulus (Z_p)

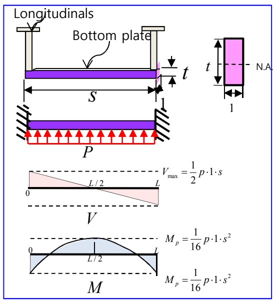
$$Z_p = \frac{1\cdot t^2}{4} = \frac{t^2}{4}$$

Elastic moment (M)

$$M = \frac{p\cdot 1\cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1\cdot t^2}{6} = \frac{t^2}{6}$$



② A mild steel plate has a thickness of 10 mm on a span length of 800 mm.
Compare the design pressure that the maximum stresses of the plate reaches the yield stress depending on the plastic design and elastic design.

$$p_{plastic} = \frac{t^2\,\sigma_l}{15.8^2\,s^2} = \frac{10^2\times 235}{15.8^2\,0.8^2} = \boxed{147\ [kN/m^2]}$$

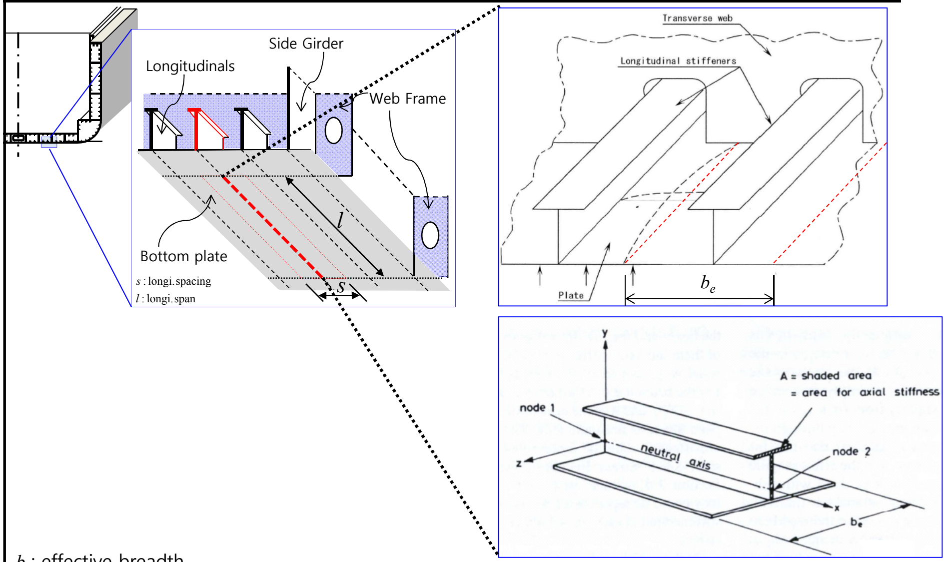
$$p_{elastic} = \frac{t^2\,\sigma_l}{22.4^2\,s^2} = \frac{10^2\times 235}{22.4^2\,0.8^2} = \boxed{73\ [kN/m^2]}$$

The design pressure of plastic design that reaches the yield stress, is higher than that of the elastic design on the same span with the same thickness.

(5) Scantling of Stiffeners

Scantling of Stiffeners (1/3)

p : “pressure” on the load point for the stiffener

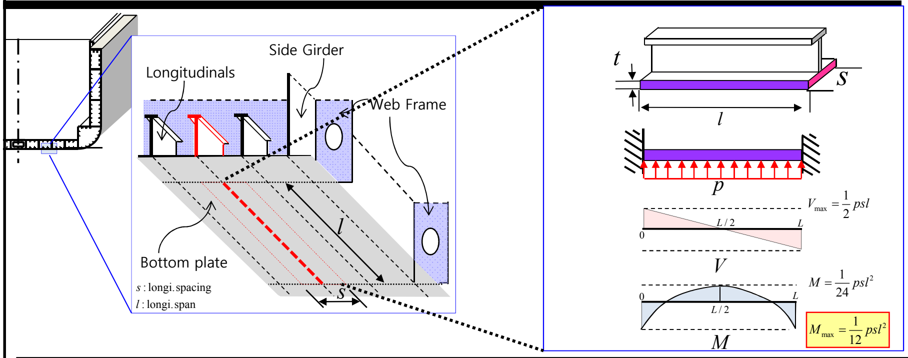


b_e: effective breadth

* Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures - a Practical Guide for Engineers, Springer, pp. 17-32, 2009

Scantling of Stiffeners (2/3)

p : "pressure" on the load point for the stiffener



Assumption 1. Cut off the **stiffener and attached plate with effective breadth**. Sectional properties of stiffener are calculated including attached plate.

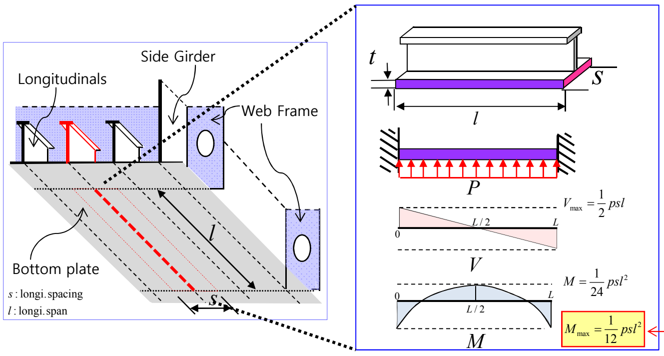
Assumption 2. Consider the stiffener and attached plate as a **"fixed-end beam"** supported by the **web frames**.

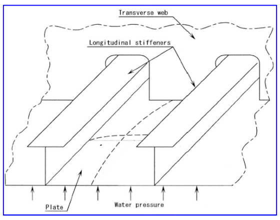
Assumption 3. Consider the **lateral load** of the beam as a **uniformly distributed load.** (Assume the **"pressure"** on the load point as an intensity of uniformly distributed load.)

Assumption 4. The design of stiffener is based on the **elastic design** (when the load is removed, the material returns to its original dimensions)

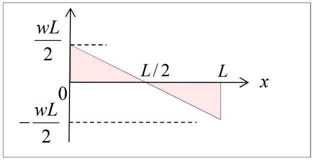
Scantling of Stiffeners (3/3)

p : "pressure" on the load point for the stiffener

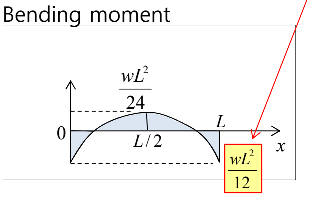




Shear force



Bending moment



Relation between $p \cdot s$ and w

p: pressure
(load per unit area)

$p \cdot s$: distributed load
(load per unit length)

||

w: distributed load
(load per unit length)

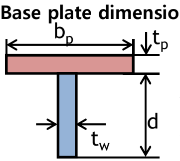
$$\boxed{\frac{wL^2}{12}} \xrightarrow[L = l]{w = p \cdot s} \boxed{\frac{psl^2}{12}}$$ **Same!**

(6) Sectional Properties of Steel Sections

Sectional Properties of Steel Sections for Ship Building[1] (1/12)

<Sectional properties of steel sections including attached plate>

(Base plate dimension : b_p x t_p = 420 x 8)



b_p = breadth of plate (mm)
t_p = thickness of plate (mm)

A = Area including plate(cm^2)
Z = Section modulus including plate (cm^3)
I = Moment of inertia of area including plate(cm^4)

d	t_w	6	9	11	12.7	14
50	A	3.00	4.5	5.50	6.35	7.00
	Z	6.05	8.81	10.6	12.1	13.3
	I	31.2	44.5	53.0	59.7	75.2
65	A	3.90	5.85	7.15	8.26	9.10
	Z	9.55	14.0	16.8	19.3	21.1
	I	62.3	88.8	105	119	129
75	A	4.50	6.75	8.25	9.53	10.5
	Z	12.3	18.1	21.8	25.0	27.3
	I	91.4	130	154	174	189
90	A	5.40	8.10	9.90	11.4	12.6
	Z	17.2	25.3	30.5	34.8	38.0
	I	150	214	252	284	307
100	A	6.00	9.00	11.0	12.7	14.0
	Z	20.9	30.6	37.0	42.2	46.1
	I	200	284	335	376	407
125	A	7.50	11.3	13.8	15.9	17.5
	Z	31.7	46.4	55.8	63.6	69.5
	I	370	521	612	685	738
150	A	9.00	13.5	16.5	19.1	21.0
	Z	44.7	65.2	78.3	89.1	97.2
	I	614	856	1000	1120	1200

d	t_w	16	19	22	25.4	28	32	35	38
200	A	32.0	38.0	44.0	50.8	56.0	64.0	70.0	76.0
	Z	215	259	305	359	401	469	521	576
	I	3900	4730	5600	6640	7460	8790	9830	10900
250	A	40.0	47.5	55.0	63.5	70.0	80.0	87.5	95.0
	Z	325	390	458	536	597	694	769	845
	I	7120	8600	10100	11900	13400	15600	17400	19200
300	A	48.0	57.0	66.0	76.2	84.0	96.0	105.0	114.0
	Z	455	546	639	746	829	961	1060	1160
	I	11700	14000	16500	19300	21600	25100	27800	30700
350	A	56.0	66.5	77.0	88.9	98.0	112.0	122.5	133.0
	Z	606	726	847	988	1100	1270	1400	1530
	I	17700	21200	24800	29100	32400	37600	41600	45700
400	A	64.0	76.0	88.0	101.6	112.0	128.0	140.0	152.0
	Z	776	928	1080	1260	1400	1610	1780	1940
	I	25300	30300	35400	41400	46000	53300	58900	64600
450	A	72.0	85.5	99.0	114.3	126.0	144.02	157.5	171.0
	Z	965	1150	1340	1560	1730	2000	2200	2400
	I	34700	41500	48500	56500	62800	72600	80100	87700
500	A	80.0	95.0	110.0	127.0	140.0	160.0	175.0	190.0
	Z	1170	1400	1630	18907	2100	2420	2660	2900
	I	46000	55000	64200	4700	82900	95700	10500	11500

A = 42*0.8 + 15*1.4 = 21 [cm^2]
Z_{Top} = 349.6 [cm^3]
Z_{Bottom} = 97.2 [cm^3]

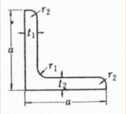
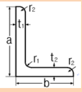
[1] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

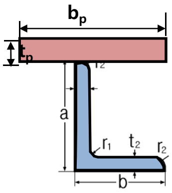
Sectional Properties of Steel Sections for Ship Building1) (2/12)

1) “조선설계편람”, 제 4판 (일본어), 일본관서조선협회, 1996

<Sectional properties of steel sections including attached plate>

- Use the standard dimension of plate depending on “a” (b_p× t_p) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

	Dimension						Area	Including plate	
Symbol	a	b	t_1	t_2	r_1	r_2	A	I	Z
Unit	mm						cm^2	cm^4	cm^3
Equal angle	∟							⫟	
	50 65 65 75 75 75 90 90 100 100 130 130 130 150 150 150 200 200 200	‘’	6 6 8 6 9 12 10 13 10 13 9 12 15 12 15 19 20 25 29	‘’	6.5 8.5 8.5 8.5 8.5 8.5 10 10 10 10 12 12 12 14 14 14 17 17 17	4.5 4 6 4 6 6 7 7 7 7 6 8.5 8.5 7 10 10 12 12 12	5.64 7.53 9.76 8.73 12.69 16.56 17.00 21.71 19.00 24.31 11.74 19.76 36.75 34.77 42.74 53.38 76.00 93.75 107.6	90.1 191 229 284 369 433 767 905 1030 1220	18.7 31.9 39.7 42.5 58.2 71.6 96.0 117 119 147
Unequal angle	L							⫟	
	100	75	7		10	5	11.87	674	72.5
	100	75	10		10	7	16.50	860	96.2
	125	75	7	‘’	10	5	13.62	110	97.2
	125	75	10		10	7	19.00	1420	130
	150	90	9		12	6	20.94	2490	181
	150	90	12		12	8.5	27.36	3060	230

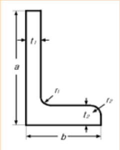
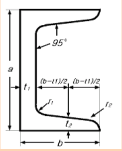
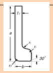


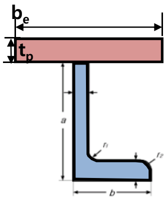
Sectional Properties of Steel Sections for Ship Building1) (3/12)

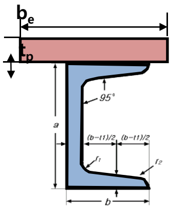
1) “조선설계편람”, 제 4판 (일본어), 일본관서조선협회, 1996

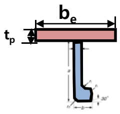
<Sectional properties of steel sections including attached plate>

- Use the standard dimension of plate depending on “a” (b_p× t_p) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

	Dimension						Area	Including plate	
Symbol	a	b	t_1	t_2	r_1	r_2	A	I	Z
Unit	mm						cm^2	cm^4	cm^3
Unequal angle	L							⫟	
	200	90	9	14	14	7.0	29.66	5870	340
	250	90	10	15	17	8.5	37.47	10300	494
	250	90	12	16	17	8.5	42.95	11000	540
	300	90	11	16	19	9.5	46.22	16400	681
	300	90	13	17	19	9.5	52.67	17600	743
	400	100	11.5	16	24	12	61.09	34200	1120
	400	100	13	18	24	12	68.59	36700	1230
	450	125	11.5	18	24	12	73.11	51200	1570
	450	150	11.5	15	24	12	73.45	51700	1590
	500	150	11.5	18	24	12	83.6	70400	2020
	550	150	12	21	24	12	95.91	93300	2520
	600	150	12.5	23	24	12	107.6	118000	3000
Channels	⊏							⊏	
	150	75	6.5	10	10	5	23.71	2160	154
	200	90	8	13.5	14	7	38.65	5650	322
	250	90	9	13	14	7	44.07	9420	439
	250	90	11	14.5	17	8.5	51.17	10500	499
	300	90	9	13	14	7	48.57	14300	567
	300	90	10	15.5	19	9.5	55.74	16000	646
	300	90	12	16	19	9.5	61.90	16900	693
	380	100	10.5	16	18	9	69.39	29900	989
	380	100	13	20	24	12	85.71	34900	1190
Bulb flats	l							T	
	180	32.5	9.5	-	7	2	21.06	2860	172
	200	36.5	10	-	8	2	25.23	4160	231
	230	41	11	-	9	2	31.98	6610	330
	250	45	12	-	10	2	38.13	8960	424







6.4 Buckling Strength

(1) Column Buckling
(2) Buckling Strength of Stiffener
(3) Buckling Strength of Plate
(4) Buckling Strength by DNV Rule
(5) Buckling Strength of Stiffener by DNV Rule
(6) Buckling Strength of Plate by DNV Rule

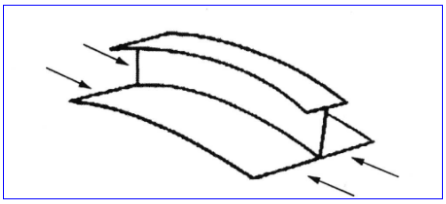
Buckling

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748-762
Rules for classification of ships, Det Norske Veritas, January 2004, Pt. 3 Ch. 1 Sec. 13

- **Definition: The phenomenon where lateral deflection may arise in the athwart direction* against the axial working load**

*선측(船側)에서 선측으로 선체를 가로지르는

- **This section covers buckling control for plate and longitudinal stiffener.**


Flexural buckling of stiffeners plus plating

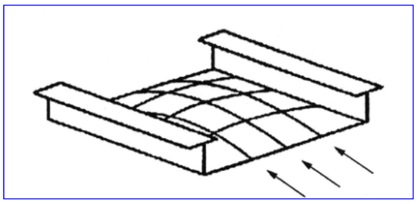


Plate alone buckles between stiffeners

* Mansour, A., Liu, D., The Principles of Naval Architecture Series – Strength of Ships and Ocean Structures, The Society of Naval Architects and Marine Engineers, 2008

(1) Column Buckling
- The Equation of the Deflection Curve

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748~762

- **Differential equation for column buckling:** $\boxed{EIy'' + Py = 0}$

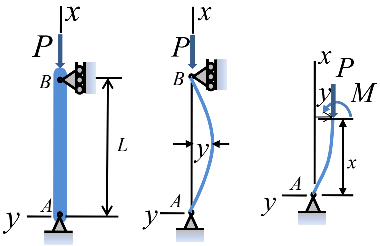
$$\frac{P}{EI} = k^2, k = \frac{n\pi}{l}$$

Using the notation $k^2 = \frac{P}{EI}$, $\boxed{y'' + k^2 y = 0}$

General solution of the equation: $\boxed{y = C_1 \sin kx + C_2 \cos kx}$

Boundary conditions:
$y(0) = 0,\ y(l) = 0$

$$y(0) = C_2 = 0$$
$$y(l) = \underline{C_1}\ \underline{\sin kL} = 0$$



1) If $C_1 = 0,\ y = 0$ **(trivial solution).**

2) If $\sin kl = 0$, $(\sin kl = 0$**: buckling equation)**

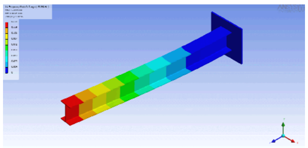
① **If** $kl = 0,\ y = 0$ **(trivial solution).**

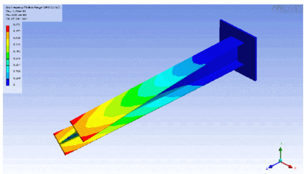
② **If** $kl = n\pi$ **(n=1, 2, 3) or** $P = \left(\frac{n\pi}{l}\right)^2 EI$**, it is nontrivial solution.**

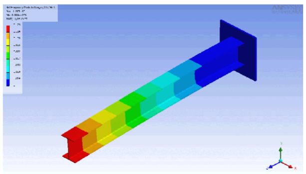
$$\boxed{\therefore y = C_1 \sin kx = C_1 \sin \frac{n\pi x}{L}, n = 1, 2, 3...}$$

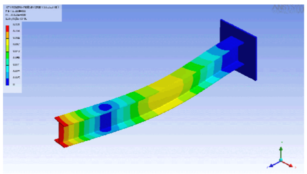
E = modulus of elasticity
I = 2nd moment of the section area
EI = flexural rigidity
P = axial load
v = deflection of column
L = length of column

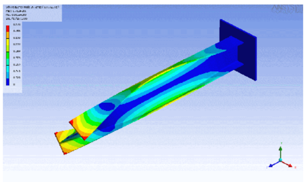
[Example] Mode Shapes of a Cantilevered I-beam

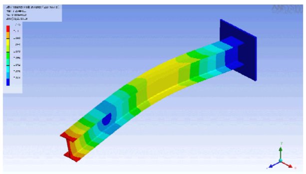

Lateral bending (1st mode)


Torsional bending (1st mode)

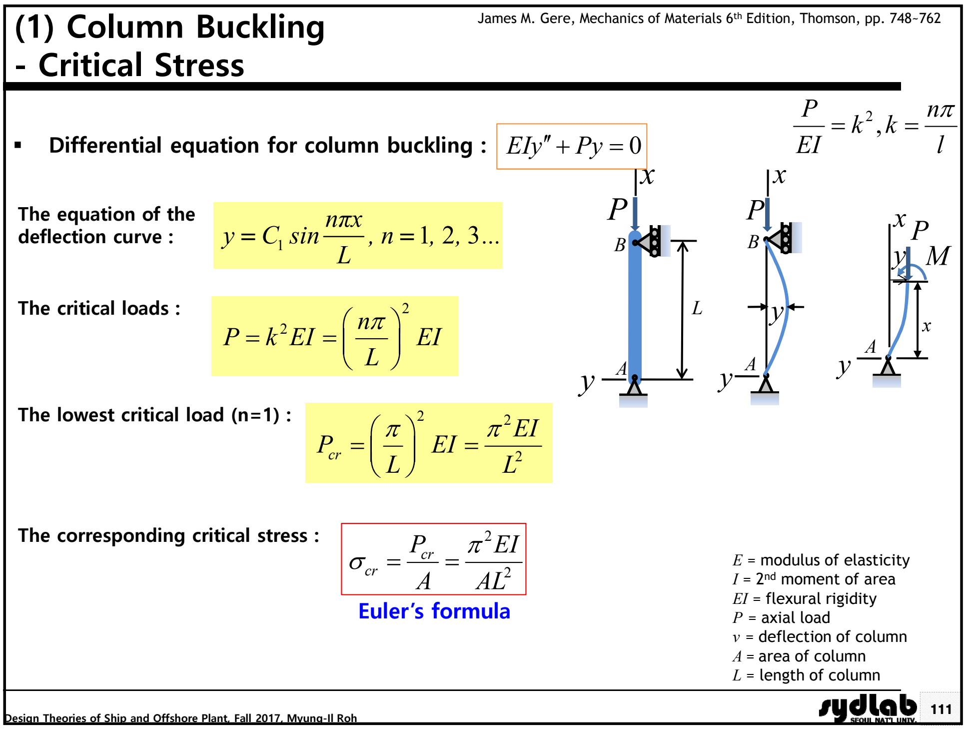

Vertical bending (1st mode)


Lateral bending (2nd mode)


Torsional bending (2nd mode)


Vertical bending (2nd mode)

* Reference: https://en.wikipedia.org/wiki/Bending



(1) Column Buckling - Critical Stress

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748~762

- **Differential equation for column buckling :** $EIy'' + Py = 0$

$$\frac{P}{EI} = k^2, k = \frac{n\pi}{l}$$

The equation of the deflection curve : $y = C_1 \sin\frac{n\pi x}{L},\ n = 1,\ 2,\ 3\ldots$

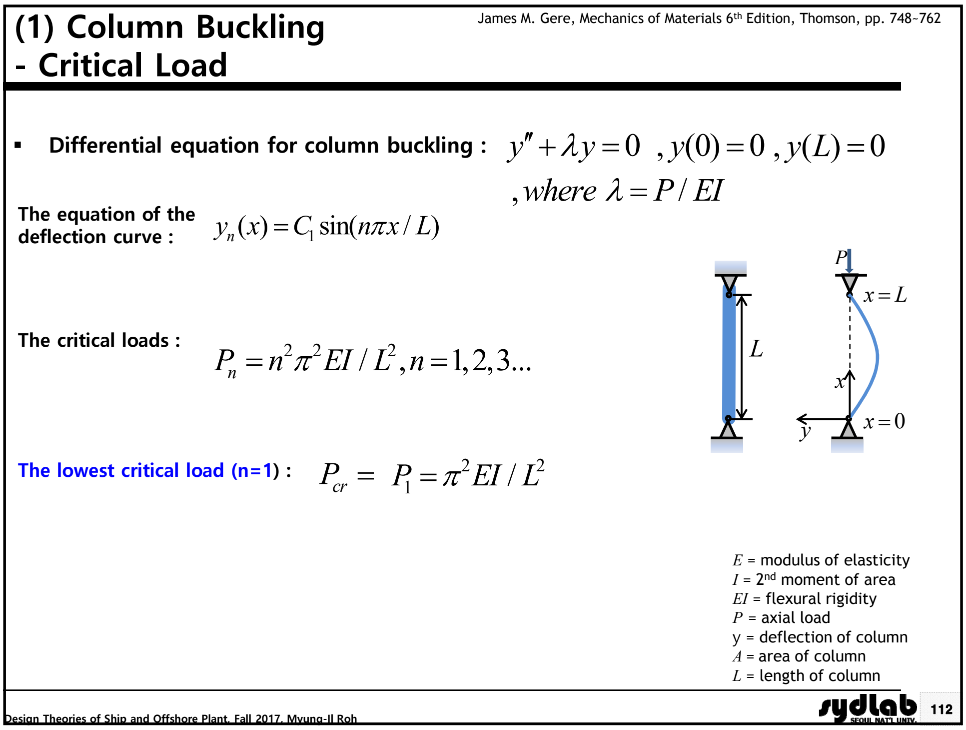
The critical loads : $P = k^2 EI = \left(\frac{n\pi}{L}\right)^2 EI$

The lowest critical load (n=1) : $P_{cr} = \left(\frac{\pi}{L}\right)^2 EI = \frac{\pi^2 EI}{L^2}$

The corresponding critical stress : $\boxed{\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2}}$

Euler's formula

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
v = deflection of column
A = area of column
L = length of column



(1) Column Buckling - Critical Load

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748~762

- **Differential equation for column buckling :** $y'' + \lambda y = 0\ \ , y(0) = 0\ , y(L) = 0$
$, where\ \lambda = P / EI$

The equation of the deflection curve : $y_n(x) = C_1 \sin(n\pi x / L)$

The critical loads : $P_n = n^2\pi^2 EI / L^2, n = 1, 2, 3\ldots$

The lowest critical load (n=1) : $P_{cr} = \ P_1 = \pi^2 EI / L^2$

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
y = deflection of column
A = area of column
L = length of column

(1) Column Buckling
- Critical Buckling Stress

A critical buckling stress is often used instead of a buckling load and it can be derived by dividing P_{cr} by A, the cross sectional area of the column.

Euler's formula

The corresponding critical stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{Al^2} = \pi^2 E\left(\frac{k}{l}\right)^2$$

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
y = deflection of column
A = area of column
l = length of column

, where $k\left(k^2 = I/A\right)$ is the radius of gyration[1] of the section of the column.

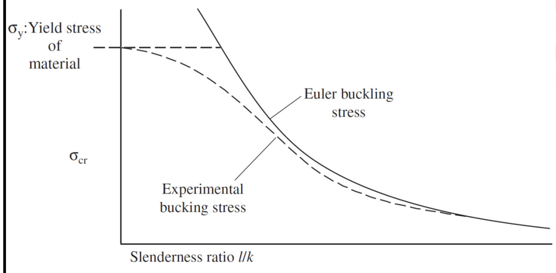
The ratio (l/k), often called the slenderness ratio, is the main factor which governs the critical stress

For large value of l/k the critical stress tends toward zero, and at small values of l/k it tends to infinity. In Euler's formula, the buckling stress may become infinite for a small value of l/k, however, **buckling stress never goes up above the yield stress of the material in actual conditions, because the material would fail if the stress exceeded the yield stress.**

---

[1] The radius of gyration describes a circular ring whose area is the same as the area of interest.

(1) Column Buckling
- Curve of Buckling Stress



by theoretical consideration, a horizontal line of yield stress connected to Euler buckling stress is specified as an upper limit of Euler's buckling curve.

$$\sigma_{cr} = a - b\left(\frac{l}{k}\right)$$ Tetmayer's formula

$$\sigma_{cr} = a - b\left(\frac{l}{k}\right)^2$$ Johnson's formula

$$\sigma_{cr} = \frac{a}{1 + b\left(l/k\right)^2}$$ Rankine's formula

For example, one of the Classification Societies, ABS (American Bureau of Shipping) specifies the permissible load of a pillar or strut of mild steel material in the following equation:

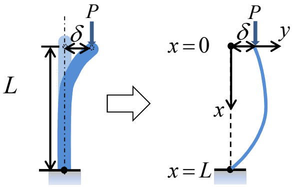
$$\sigma_{cr} = 1.232 - 0.00452\left(\frac{l}{k}\right) \quad [ton \cdot f / cm^2]$$

From the above equation, we can see that the ABS formula is theoretically based on Tetmayer's experimental result.

(1) Column Buckling
- Buckling of Thin Vertical Column Embedded at Its Base and Free at Its Top (1/2)

Suppose that a tin vertical homogeneous column is embedded at its base ($x=0$) and free at its top ($x=L$) and that a constant axial load P is applied to its free end.
The load either causes a small deflection δ, or does not cause such a deflection. In either case the differential equation for the deflection $y(x)$ is

$$EI\frac{d^2y}{dx^2}=P(\delta-y) \Rightarrow EI\frac{d^2y}{dx^2}+Py=P\delta \cdots(1)$$



(1) What is the predicted deflection when $\delta=0$?

- **The general solution of the differential equation (1) is**

$$y=c_1\cos\sqrt{\frac{P}{EI}}x+c_2\sin\sqrt{\frac{P}{EI}}x+\delta$$

- **The boundary conditions of the differential equation (1) are**

$$y(0)=y'(0)=0$$

- **If $\delta=0$, this implies that $c_1=c_2=0$ and $y(x)=0$. That is, there is no deflection.**

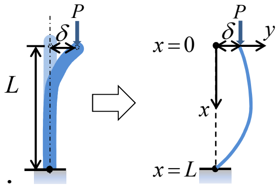
* Zill, D.G., Advanced Engineering Mathematics, 3rd edition, pp.166-174, 2006

---

(1) Column Buckling
- Buckling of Thin Vertical Column Embedded at Its Base and Free at Its Top (2/2)

Suppose that a tin vertical homogeneous column is embedded at its base ($x=0$) and free at its top ($x=L$) and that a constant axial load P is applied to its free end.
The load either causes a small deflection δ, or does not cause such a deflection. In either case the differential equation for the deflection $y(x)$ is

$$EI\frac{d^2y}{dx^2}=P(\delta-y) \Rightarrow EI\frac{d^2y}{dx^2}+Py=P\delta \cdots(1)$$



(2) When $\delta\neq0$, show that the Euler load for this column is one-fourth of the Euler load for the hinged column?

- **If $\delta\neq0$, the boundary conditions give, in turn, $c_1=-\delta,\ c_2=0$. Then**

$$y=\delta\left(1-\cos\sqrt{\frac{P}{EI}}x\right)$$

- **In order to satisfy the boundary condition $y(L)=\delta$, we must have**

$$\delta=\delta\left(1-\cos\sqrt{\frac{P}{EI}}L\right)\longrightarrow\cos\sqrt{\frac{P}{EI}}L=0\longrightarrow\sqrt{\frac{P}{EI}}L=n\pi/2$$

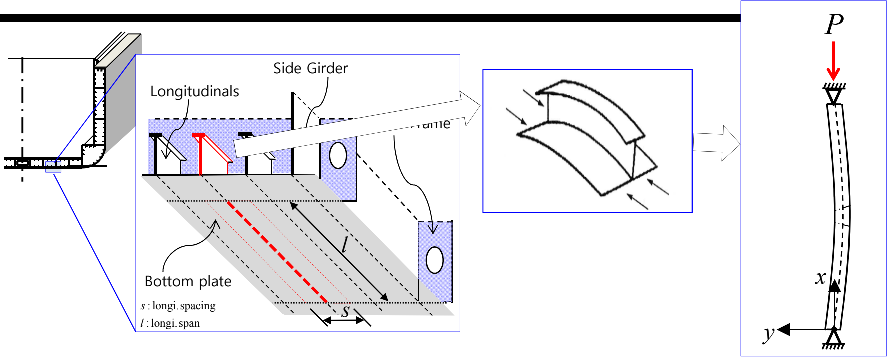
- **The smallest value of P_n, the Euler load, is then**

One-fourth of the Euler load

$$\sqrt{\frac{P_1}{EI}}L=\frac{\pi}{2}\quad\text{or}\quad P_1=\boxed{\frac{1}{4}}\underbrace{\left(\frac{\pi^2EI}{L^2}\right)}_{\text{Euler load}}$$

* Zill, D.G., Advanced Engineering Mathematics, 3rd edition, pp.166-174, 2006

(2) Buckling Strength of Stiffener



It is assumed that the <u>stiffener is a fixed-end column supported by the web frames.</u>

Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation. And it is acting on the each stiffener as axial load from the point view of local deformation.

what is our interest?

- **Safety:**

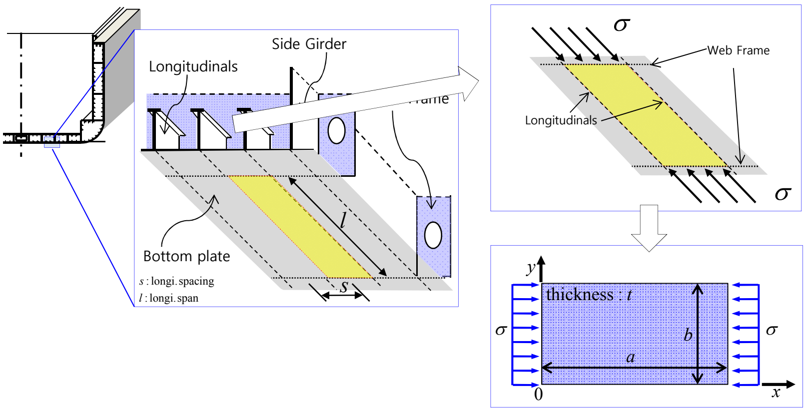
Won't it fail under the load?

The <u>actual compressive stress</u> (σ_a) shall not be great than the <u>critical buckling stress</u> (σ_{cr})

$$\boxed{\sigma_a \le \sigma_{cr}} \quad \text{, where } \sigma_a = \frac{M}{I_{N.A}/y} = \frac{M}{Z} \quad ,Z = Z(y)$$

---

(3) Buckling Strength of Plate (1/7)

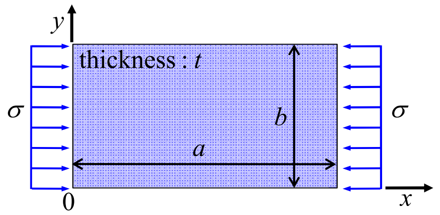


A ship hull is a stiffened-plate structure, the plating supported by a system of transverse or longitudinal stiffeners.

For practical design purpose, it is often assumed that the plate is simply supported at the all edges, since it gives the least critical stress and is on the safe side.

(3) Buckling Strength of Plate (2/7)

Let us consider the rectangular plate with only supported edges as shown in this figure.



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a : plate length
b : plate width
t : thickness of the plate

- **The equation of elastic buckling stress of the plate under uni-axial compressive stress:**

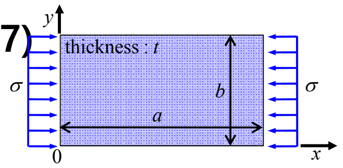
$$\boxed{\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0} \cdots (1)$$

where, $w = w(x, y)$: deflection of the plate

* Okumoto, Y., Design of Ship Hull Structures, pp.57-60, 2009

---

(3) Buckling Strength of Plate (3/7)



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a: plate length
b : plate width
t : thickness of the plate

- **The equation of elastic buckling stress of the plate under uni-axial compressive stress:**

$$\boxed{\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0} \cdots (1)$$

where, $w = w(x, y)$ **: deflection of the plate**

- **Because all four edges are simply supported, the boundary condition can be expressed in the form:**

$$w(0, y) = w(a, y) = 0 \leftarrow$$
$$w(x, 0) = w(x, b) = 0 \leftarrow$$

deformation at the edges are zero

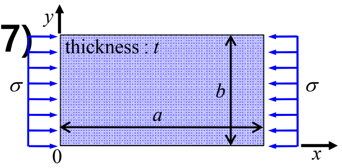
- **Let us assume the following formula for the solution of the equation (1), so that the solution satisfies the boundary conditions.**

$$w = f \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) \cdots (2)$$

where, m, n are integers presenting the number of half-wave of buckles.

* Okumoto, Y., Design of Ship Hull Structures, pp.57-60, 2009

(3) Buckling Strength of Plate (4/7)



σ : the uni-axial compressive stress
v : Poisson's ratio
E : Modulus of elasticity
a : plate length
b : plate width
t : thickness of the plate

- **The equation of elastic buckling stress of the plate under uni-axial compressive stress:**

$$\boxed{\frac{Et^3}{12(1-v^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0} \cdots(1)$$

where, $w = w(x, y)$: deflection of the plate

- **Substituting the formula (2) into the equation (1),**

$$w = f\sin\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right) \cdots(2)$$

$$\boxed{\sigma=\frac{Et^3}{12(1-v^2)}\frac{\pi^2}{b^2 t}\left(\frac{m}{\alpha}+n^2\frac{\alpha}{m}\right)^2} \cdots(3)$$

where, $\alpha = \dfrac{a}{b}$

- **Elastic buckling stress is a minimum critical stress, therefore, we put n=1 in the equation (3),**

Ideal elastic (Euler) compressive buckling stress:

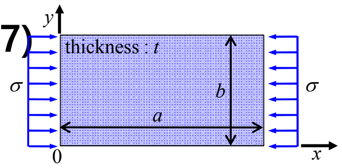
$$\boxed{\sigma_{el}=\frac{\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2 K}$$

where, K = Minimum value of k, $k=\left(\dfrac{m}{\alpha}+\dfrac{\alpha}{m}\right)^2$

---

* Okumoto, Y., Design of Ship Hull Structures, pp.57-60, 2009

(3) Buckling Strength of Plate (5/7)



σ : the uni-axial compressive stress
v : Poisson's ratio
E : Modulus of elasticity
a : plate length
b : plate width
t : thickness of the plate

Ideal elastic (Euler) compressive buckling stress:

$$\boxed{\sigma_{el}=\frac{\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2 K}$$

where, K = Minimum value of k

$$k=\left(\frac{m}{\alpha}+\frac{\alpha}{m}\right)^2, \alpha=\frac{a}{b}$$

- **For the small b in comparison with t, the elastic buckling stress becomes more than the yield stress of the plate material.**
- **Therefore, it is usual to use Johnson's modification factor η_p and the critical buckling stress σ_c for the full range of value of t/b as follows:**

- **Bryan's formula**[1)]

$$\boxed{\frac{\sigma_c}{\eta_p}=\sigma_{el}=\frac{\pi^2 E}{12\left(1-v^2\right)}\cdot\left(\frac{t}{b}\right)^2\cdot K}$$

σ_c : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
K : plate factor (corresponding to the boundary conditions and a/b)
η_p : plasticity reduction factor

$$\eta_p = 1 \quad \text{, when } \sigma_{el} < \frac{\sigma_y}{2}$$

$$\eta_p = \frac{\sigma_y}{\sigma_{el}}\left(1-\frac{\sigma_y}{4\sigma_{el}}\right) \text{, when } \sigma_{el} \geq \frac{\sigma_y}{2}$$

$$\leq 1$$

σ_y = upper yield stress in [N/mm^2]

ex) Coefficient K when all four edges are simply supported

$$K = 4.0 \quad a/b \geq 1.0$$

$$K = \left(a/b+b/a\right)^2, \ a/b < 1.0$$

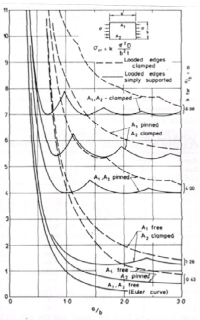


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

---

1) DSME, "선박구조설계" 13-18 Buckling, 2005.8

(3) Buckling Strength of Plate (6/7) - Buckling Strength of Web Plate

1) DSME, "Ship Structural Design", 13-18 Buckling, 2005.8

Web plate of stiffener have to be checked about buckling.

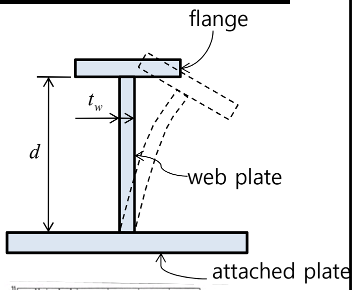
In case of T-bar, it is assumed that the <u>web plate of stiffener is the plate simply supported by flange and attached plate</u>.

$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t}{d}\right)^2 \cdot K$$

, (Bryan's formula)
, K = 4.0

$$\rightarrow \frac{d}{t_w} \leq \sqrt{\frac{\pi^2 EK}{12\left(1-\nu^2\right)} \frac{1}{\sigma_{el}}}$$

σ_{cr} : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
ν : Poisson's ratio
K : Plate factor (corresponding to the boundary conditions and a/b)
d : depth of web plate
t : thickness of web plate
E : Modulus of elasticity



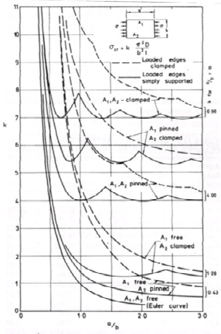


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

---

(3) Buckling Strength of Plate (7/7) - Buckling Strength of Flange Plate

1) DSME, "Ship Structural Design", 13-18 Buckling, 2005.8

Flange of stiffener have to be checked about buckling.

It is assumed that the <u>flange of stiffener is the rectangular plate simply supported on one end by web plate</u>.

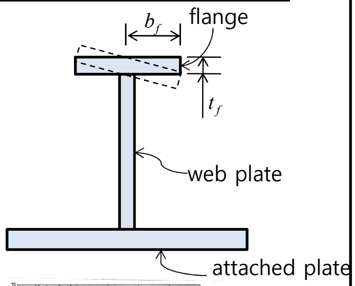
$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t_f}{b_f}\right)^2 \cdot K$$

, (Bryan's formula)
, K = 0.5

$$\rightarrow \frac{b}{t_f} \leq \sqrt{\frac{K\pi^2 E}{12\left(1-\nu^2\right)} \frac{1}{\sigma_{el}}}$$

In general, b/t_f does not exceed 15.

σ_c : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
ν : Poisson's ratio
K : Plate factor (corresponding to the boundary conditions and a/b)
b_f : breadth of flange plate
t_f : thickness of flange plate
E : Modulus of elasticity



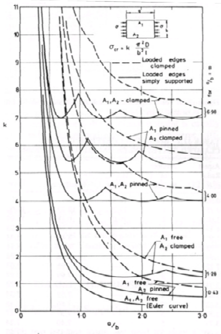


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

(4) Buckling Strength by DNV Rule

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004

- ◆ **Criteria for buckling strength**

$$\boxed{\sigma_c > \frac{\sigma_a}{\eta}}$$

σ_c = critical buckling stress in N/mm^2
σ_a = calculated actual stress in N/mm^2
η = usage factor

◆ **Critical buckling stress** σ_c

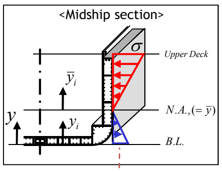
- σ_f is yield stress of material in N/mm^2

$$\sigma_c = \sigma_{el} \quad ,\text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

◆ **Calculated actual stress** σ_a

- σ_a is calculated actual stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,\left(N/mm^2\right)$$
$$= \text{minimum } 30\ f_1 \text{ N/mm}^2 \text{ at side}$$



consider each different stress according to location

◆ σ_{el} **for Plate in uni-axial compression**[1]

Plate

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

◆ σ_{el} **for stiffener in uni-axial compression**[1]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

Stiffener

◆ σ_{el} **for stiffener in lateral buckling mode**

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2}$$

M_S : still water bending moment as given in Sec. 5
M_W : wave bending moment as given in Sec. 5
I_{NA} : moment of inertia in cm^4 of the hull girder
σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_c : critical buckling stress
σ_f : upper yield stress in [N/mm^2]
t : thickness in [mm]
t_k : corrosion addition
t_w : web thickness, h_w : web height
E : modulus of elasticity
s : stiffener spacing in [m]
I_A : moment of inertia in [cm^4] about the axis perpendicular to the expected direction of buckling
A : cross-sectional area in [cm^2]
l : length of member in [m]

---

(5) Buckling Strength of Stiffener by DNV Rule
- Stiffener in Uni-axial Compression (1/2)

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004

◆ **Criteria for Buckling Strength**
(in the same way of plate)

$$\boxed{\sigma_c > \frac{\sigma_a}{\eta}}$$

σ_c : critical buckling stress in [N/mm^2]
σ_a : calculated actual compressive stress in [N/mm^2]
η : usage factor

Usage Factor (η)

η = 1.0	Deck, Single bottom & Side shell (long stiff)
η = 0.9	Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0	Extreme loads (Q = 10^{-8})
η = 0.8	Normal loads (Q = 10^{-4})

◆ **Critical buckling stress** σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_f : yield stress of material in N/mm^2

◆ **Calculated actual stress** σ_a

(Uni-axial compression)

- σ_a is calculated actual compressive stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,\left(N/mm^2\right)$$
$$= \text{minimum } 30\ f_1 \text{ N/mm}^2 \text{ at side}$$

(※ Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation.
And it is acting on the each stiffener as axial load from the point view of local deformation.)

M_S = still water bending moment as given in Sec. 5
M_W = wave bending moment as given in Sec. 5
I_N = moment of inertia in cm^4 of the hull girder

(5) Buckling Strength of Stiffener by DNV Rule
- Stiffener in Uni-axial Compression (2/2)

◆ **Critical buckling stress σ_c**

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004

$$\boxed{\begin{aligned}\sigma_c &= \sigma_{el} && \text{, when } \sigma_{el} < \frac{\sigma_f}{2} \\ &= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) && \text{, when } \sigma_{el} > \frac{\sigma_f}{2}\end{aligned}}$$

σ_f : yield stress of material in [N/mm^2]

'σ_{el}' is determined according to specific load.

◆ **Ideal compressive buckling stress σ_{el} of stiffener in uni-axial compression[1]**

$$\boxed{\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_W}\right)^2}$$

- **Derivation of the coefficient '3.8'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K$,

$$\frac{\pi^2}{12(1-\nu^2)} = 0.9038 (\fallingdotseq 0.9)$$

And substituting K=4(for simply supported plate), the coefficient is approximately equal to 3.8.

σ_{el} : ideal compressive buckling stress
σ_c : critical buckling stress
σ_s : minimum upper yield stress
t_w : web thickness, h_w : web height
E : modulus of elasticity
s : stiffener spacing (m)
ν : 0.3 (Poisson's ratio of steel)

◆ **Ideal compressive buckling stress σ_{el} of stiffener in lateral buckling mode**

$$\boxed{\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2}}$$

- **Derivation of the coefficient '0.001'**

From Euler's formula $\sigma_{cr} = \frac{\pi^2 EI}{Al^2}\ \frac{\pi^2\ N/mm^2\ cm^4}{cm^2\ m^2}$,

$$\frac{\pi^2\ N/mm^2\ cm^4}{cm^2\ m^2} = \frac{\pi^2\ N/mm^2\,(10mm)^4}{(10mm)^2\,(1000mm)^2} \fallingdotseq 0.001\ N/mm^2$$

◆ **Thickness of flange**

For flanges on angles and T-sections of longitudinals and other highly compressed stiffeners, the thickness shall not be less than

$$t_f = 0.1b_f + t_k \quad (mm)$$

b_f = flange width in mm for angles, half the flange width for T-Section(m)
t_k = corrosion addition(DNV Rule : Pt. 3 Ch. 1 Sec.2 - Page15)

(6) Buckling Strength of Plate by DNV Rule
- Plate Panel in Uni-axial Compression (1/4)

◆ **Criteria for buckling strength**

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004

$$\boxed{\sigma_c > \frac{\sigma_a}{\eta}}$$

σ_c : critical buckling stress in [N/mm^2]
σ_a : calculated actual compressive stress in [N/mm^2]
η : usage factor

Usage Factor (η)
η = 1.0: Deck, Single bottom & Side shell (longl. stiff)
η = 0.9: Bottom, Inner bottom & Side shell (trans. stiff)
η = 1.0: Extreme loads (Q = 10^{-8})
η = 0.8: Normal loads (Q = 10^{-4})

◆ **Critical buckling stress σ_c**

$$\boxed{\begin{aligned}\sigma_c &= \underline{\sigma_{el}} && \text{, when } \sigma_{el} < \frac{\sigma_f}{2} \\ &= \underline{\sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)} && \text{, when } \sigma_{el} > \frac{\sigma_f}{2}\end{aligned}}$$

σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_f : upper yield stress in [N/mm^2]

From Bryan's formula,

$$\boxed{\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K}$$

when $\underline{\sigma_{el} < \frac{\sigma_f}{2}}$, $\eta_p = 1$

$$\sigma_c = \eta_p \sigma_{el} \rightarrow \underline{\sigma_c = \sigma_{el}}$$

when $\underline{\sigma_{el} \geq \frac{\sigma_f}{2}}$, $\eta_p = \frac{\sigma_f}{\sigma_{el}}\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)$

$$\sigma_c = \eta_p \sigma_{el} \rightarrow \sigma_c = \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)$$

(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (2/4)

◆ **Criteria for buckling strength**

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c : critical buckling stress in [N/mm^2]
σ_a : calculated actual compressive stress in [N/mm^2]
η : usage factor

Usage Factor (η)
η = 1.0: Deck, Single bottom & Side shell (long stiff)
η = 0.9: Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0: Extreme loads (Q = 10^{-8})
η = 0.8: Normal loads (Q = 10^{-4})

◆ **Critical buckling stress σ_c**

$$\begin{aligned}\sigma_c &= \sigma_{el} &&\text{, when } \sigma_{el} < \frac{\sigma_f}{2}\\ &= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) &&\text{,when } \sigma_{el} > \frac{\sigma_f}{2}\end{aligned}$$

σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_f : upper yield stress in [N/mm^2]

◆ **Calculated actual stress σ_a**

(Uni-axial compression)

- σ_a is calculated actual compressive stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\begin{aligned}\sigma_a &= \frac{Ms+Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,\left(N/mm^2\right)\\ &= \text{minimum } 30\ \text{f}_1\ \text{N/mm}^2 \text{ at side}\end{aligned}$$

(※ Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation. And it is acting on the each plate as axial load from the point view of local deformation.)

M_S : still water bending moment as given in Sec. 5
M_W : wave bending moment as given in Sec. 5
I_N : moment of inertia in cm^4 of the hull girder

---

(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (3/4)

◆ **Critical buckling stress σ_c**

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004

$$\begin{aligned}\sigma_c &= \sigma_{el} &&\text{, when } \sigma_{el} < \frac{\sigma_f}{2}\\ &= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) &&\text{,when } \sigma_{el} > \frac{\sigma_f}{2}\end{aligned}$$

σ_f : minimum upper yield stress of material in [N/mm^2]

'σ_{el}' is determined according to specific load.

◆ **Ideal compressive buckling stress σ_{el} in uni-axial compression[1]**

$$\sigma_{el} = 0.9kE\left(\frac{t-t_k}{1000s}\right)^2$$

- **Derivation of the coefficient '0.9'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12\left(1-\nu^2\right)}\cdot\left(\frac{t}{b}\right)^2\cdot K$,

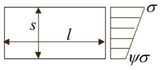
$$\frac{\pi^2}{12\left(1-\nu^2\right)} = \frac{3.141593^2}{12\left(1-0.3^2\right)} = 0.9038 \ (\fallingdotseq 0.9)$$

σ_{el} : ideal compressive buckling stress
σ_c : critical buckling stress
σ_f : upper yield stress in N/mm^2
t : thickness (mm)
t_k : corrosion addition
E : modulus of elasticity
s : stiffener spacing (m)
ν : 0.3 (Poisson's ratio of steel)

◆ **factor k**

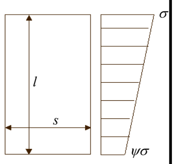
- For plating with longitudinal stiffeners (in direction of compression stress):

$$k = k_l = \frac{8.4}{\psi + 1.1}$$

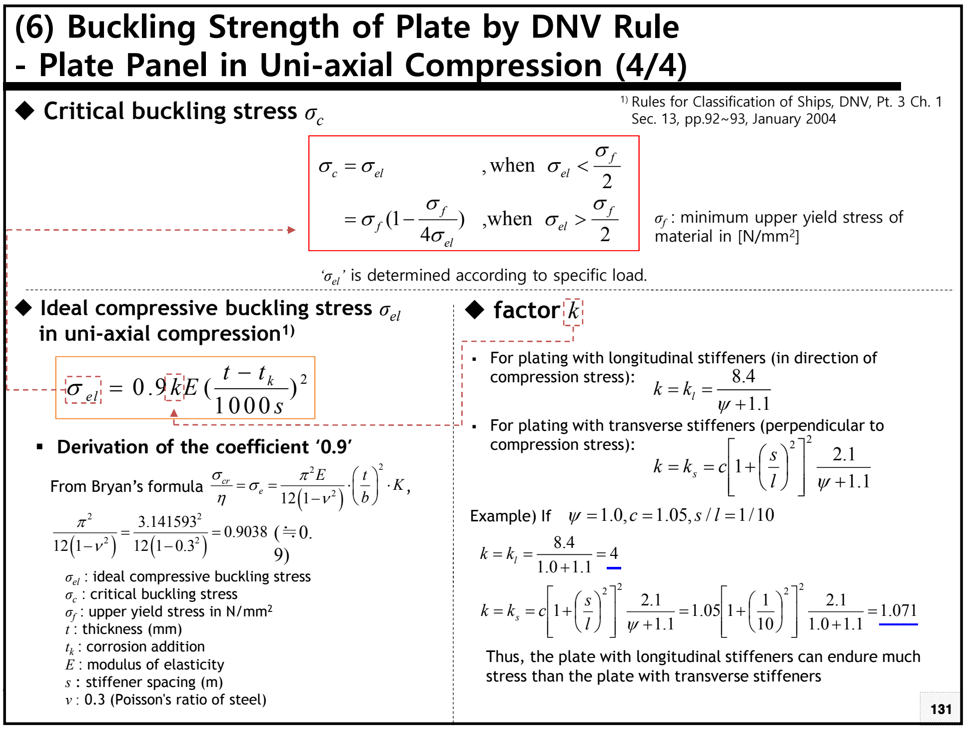


- For plating with transverse stiffeners (perpendicular to compression stress):

$$k = k_s = c\left[1+\left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1}$$



ψ = ratio between the smaller and the larger compressive stress (positive value) $\left(0 \le \psi \le 1\right)$
c =1.21 when stiffeners are angles or T sections
=1.10 when stiffeners are bulb flats
=1.05 when stiffeners are flat bars
=1.30 when plating is supported by deep girders



(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (4/4)

◆ Critical buckling stress σ_c

$$\begin{aligned} \sigma_c &= \sigma_{el} && \text{, when } \sigma_{el} < \frac{\sigma_f}{2} \\ &= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) && \text{,when } \sigma_{el} > \frac{\sigma_f}{2} \end{aligned}$$

σ_f : minimum upper yield stress of material in [N/mm^2]

'σ_{el}' is determined according to specific load.

◆ Ideal compressive buckling stress σ_{el} in uni-axial compression[1]

$$\boxed{\sigma_{el}} = 0.9 \boxed{k} E \left(\frac{t - t_k}{1000s}\right)^2$$

- **Derivation of the coefficient '0.9'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \cdot K$,

$$\frac{\pi^2}{12\left(1-\nu^2\right)} = \frac{3.141593^2}{12\left(1-0.3^2\right)} = 0.9038 \ (\fallingdotseq 0.9)$$

σ_{el} : ideal compressive buckling stress
σ_c : critical buckling stress
σ_f : upper yield stress in N/mm^2
t : thickness (mm)
t_k : corrosion addition
E : modulus of elasticity
s : stiffener spacing (m)
ν : 0.3 (Poisson's ratio of steel)

◆ factor $\boxed{k}$

- For plating with longitudinal stiffeners (in direction of compression stress): $k = k_l = \frac{8.4}{\psi + 1.1}$
- For plating with transverse stiffeners (perpendicular to compression stress): $k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1}$

Example) If $\psi = 1.0, c = 1.05, s/l = 1/10$

$$k = k_l = \frac{8.4}{1.0 + 1.1} = \underline{4}$$

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} = 1.05\left[1 + \left(\frac{1}{10}\right)^2\right]^2 \frac{2.1}{1.0 + 1.1} = \underline{1.071}$$

Thus, the plate with longitudinal stiffeners can endure much stress than the plate with transverse stiffeners

---

[1] Rules for Classification of Ships, DNV, Pt. 3 Ch. 1 Sec. 13, pp.92~93, January 2004