Chapter 8

Stereoscopic Parallax

Elements of Photogrammetry with Applications in GIS

Wolf, Paul R.; Wolf, Paul R.; DeWitt, Bon A.; DeWitt, Bon A.; Wilkinson, Benjamin E.; Wilkinson, Benjamin E.. Elements of Photogrammetry with Application in GIS, Fourth Edition McGraw-Hill Education. Kindle Edition.

1. Introduction

- Parallax (시차): the apparent displacement in the position of an object with respect to a frame of reference caused by a shift in the positon of observation.

- As aircraft moves forward, images of objects would be seen to move across the viewfinder: The closer an object is to the camera, the more its image will appear to move.

- The change in position of an image from one photograph to the next caused by the aircraft's motion is termed *stereoscopic parallax*, *x parallax*, or simply *parallax*.

- Important aspects of stereoscopic parallax: (1) The parallax of any point is directly related to the elevation of the point, and (2) parallax is greater for high points than for low points. → Basis for determining elevation (X,Y, Z) of points using parallax

1. Introduction

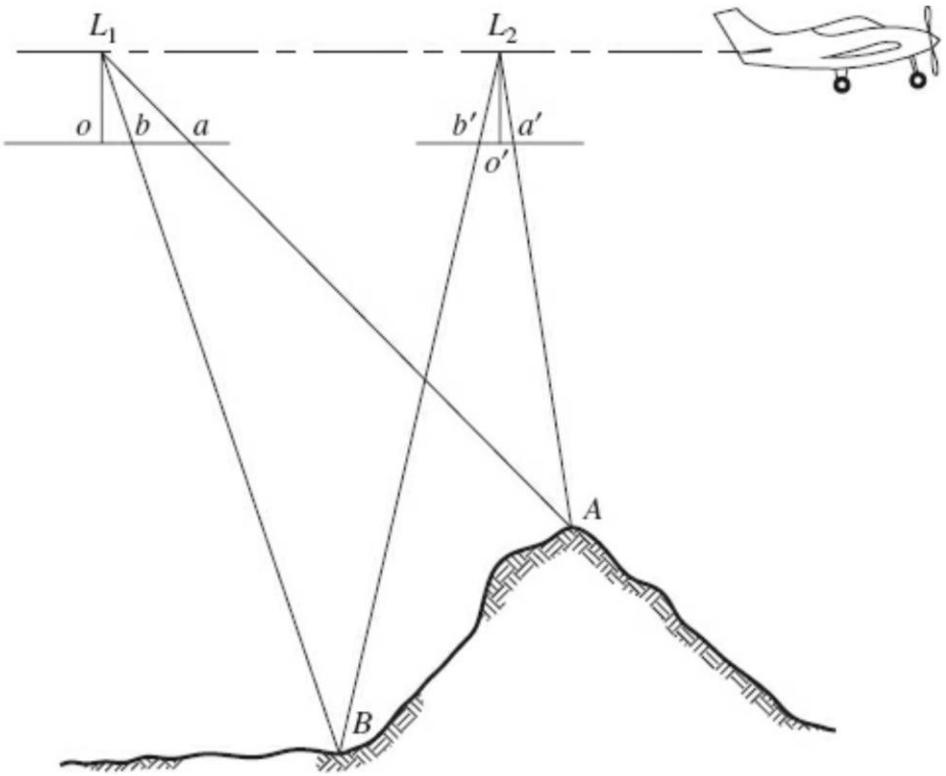
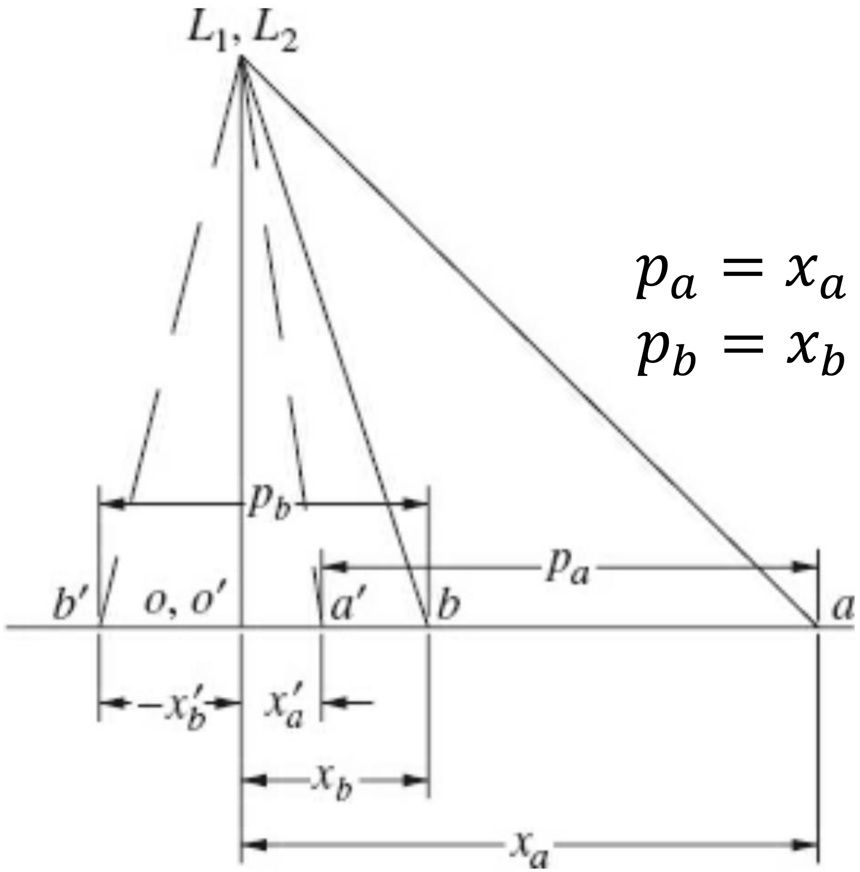


Figure 8-1 Stereoscopic parallax of vertical aerial photographs.



$$p_a = x_a - x'_a$$
$$p_b = x_b - x'_b$$

Figure 8-2 The two photographs of Fig. 8-1 are shown in superposition.

1. Introduction

FIGURE 8-3 Overlapping vertical photographs taken over the University of Florida campus illustrating stereoscopic parallax.

$$x_t = 48.2 \text{ mm}$$
$$x'_t = -53.2 \text{ mm}$$
$$p_t = 101.4 \text{ mm}$$

$$x_b = 42.7 \text{ mm}$$
$$x'_b = -47.9 \text{ mm}$$
$$p_b = 90.6 \text{ mm}$$

2. Photographic Flight – Line Axes for Parallax Measurement

- Since parallax occurs parallel to the direction of flight, the photographic x and x′ axes (right-hand photo) for parallax measurement must be parallel with the flight line (straight line) for each of the photographs of a stereopair.

- All photographs except those on the ends of a flight strip may have two sets of flight axes for parallax measurement.

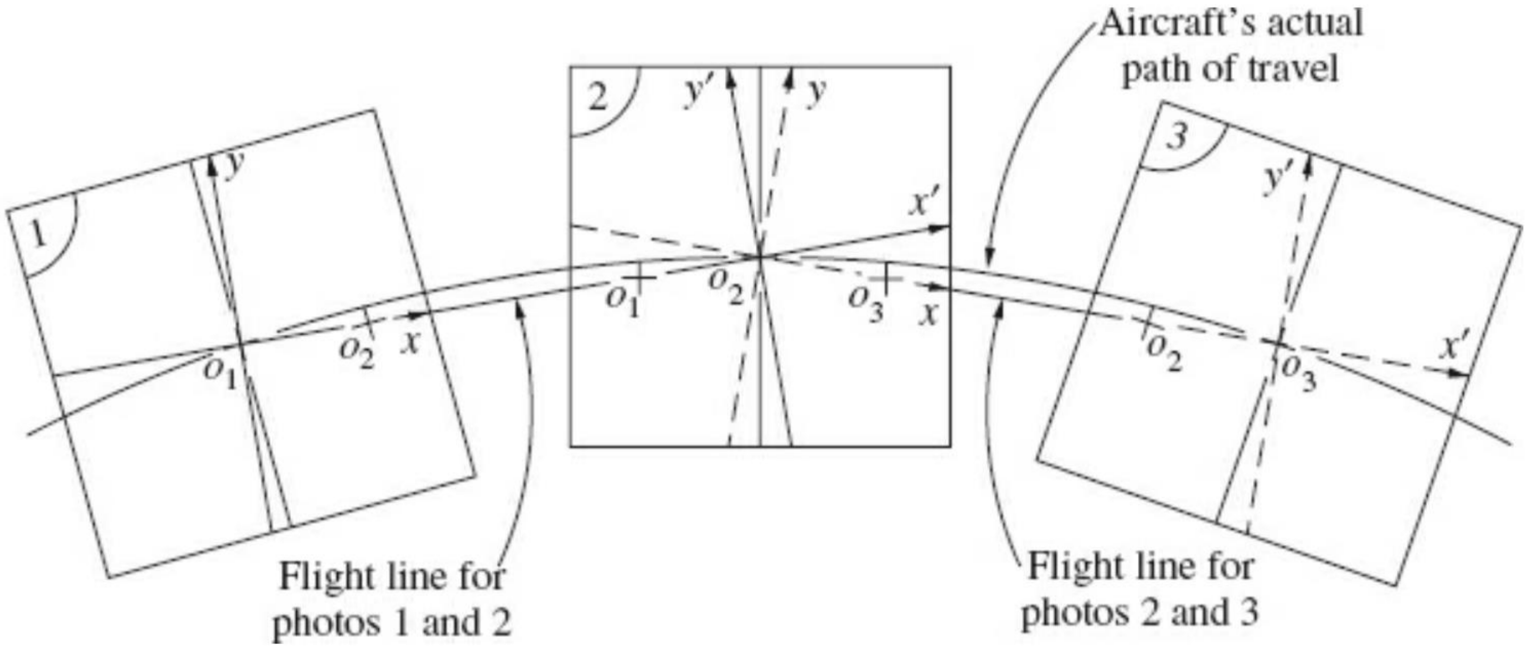


FIGURE 8-4 Flight-line axes for measurement of stereoscopic parallax.

3. Monoscopic Methods of Parallax Measurement

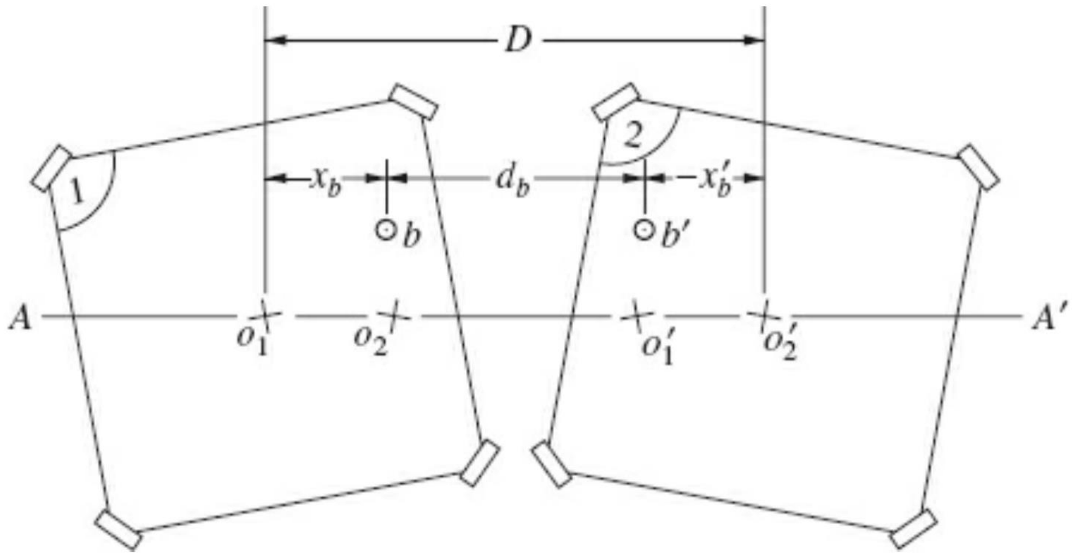


FIGURE 8-5 Parallax measurement using a simple scale.

- Two kinds of monoscopic methods: 1) direct measurement of x and x' on the left and right photos, respectively, 2) Fastening the photographs down on a table so that photographic flight lines coincide.

$$p_b = D - d_b$$

- With the second monoscopic method the parallax of a point can be obtained from a single measurement of d (d_b).

6. Parallax Equations

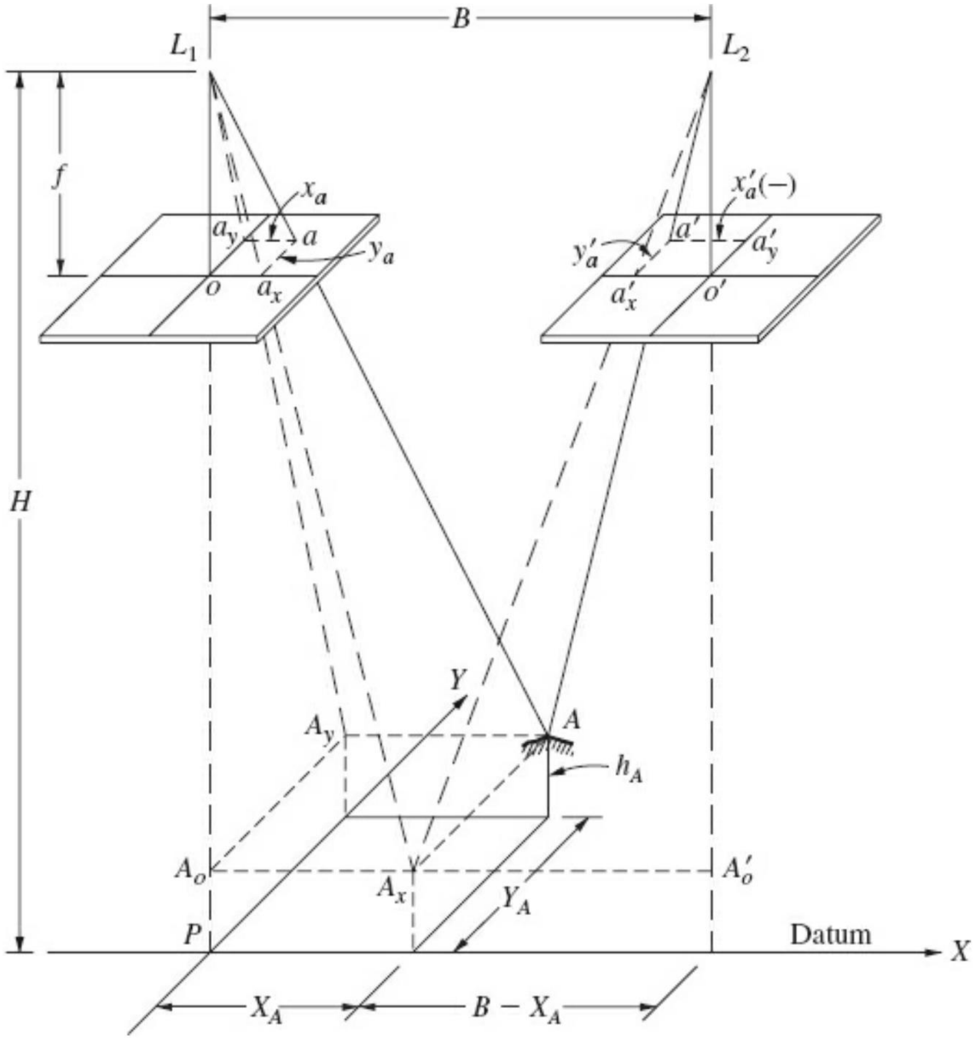


Figure 8-10 Geometry of an overlapping pair of vertical photographs.

- From each stereopair of photographs 3D ground coordinates can be calculated using parallax equations which are derived using similar triangles:

$$X_A = B\frac{x_a}{p_a(= x_a - x'_a)}$$

$$Y_A = B\frac{y_a}{p_a(= x_a - x'_a)}$$

$$h_A = H - \frac{Bf}{p_a(= x_a - x'_a)}$$

6. Parallax Equations

[Example 8-1] A pair of overlapping vertical photographs was taken from a flying height of 1,233 m above sea level with a 152.4-mm of focal length. The air base was 390 m. Fight-line coordinates for points a and b are $x_a = 53.4$ mm, $y_a = 50.8$ mm, $x'_a = -38.3$ mm, $y'_a = 50.9$ mm, $x_b = 88.9$ mm, $y_b = -46.7$ mm, $x'_b = -7.1$ mm, $y'_b = -46.7$ mm. Elevation of points A and B, and the horizontal length of line AB?

$$h_A = H - \frac{Bf}{p_a(=x_a - x'_a)} = 1{,}233 - \frac{390(152.4)}{91.7} = 585 \text{ m above sea level}$$

$$h_B = H - \frac{Bf}{p_b(=x_b - x'_b)} = 1{,}233 - \frac{390(152.4)}{96.0} = 614 \text{ m above sea level}$$

$$X_A = B\frac{x_a}{p_a(=x_a - x'_a)} = 390\left(\frac{53.4}{91.7}\right) = 227 \text{ m},\ Y_A = B\frac{y_a}{p_a(=x_a - x'_a)} = 390\left(\frac{50.8}{91.7}\right) = 216 \text{ m}$$

$$X_B = B\frac{x_b}{p_b(=x_b - x'_b)} = 390\left(\frac{88.9}{96.0}\right) = 316 \text{ m},\ Y_B = B\frac{y_b}{p_b(=x_b - x'_b)} = 390\left(\frac{-46.7}{96.0}\right) = -190 \text{ m}$$

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} = \sqrt{(361 - 227)^2 + (-190 - 216)^2} = 427 \text{ m.}$$

7. Elevations by Parallax Differences

- Parallex differences between two parallex values can be used to obtain parameters including elevation, flying height, etc.

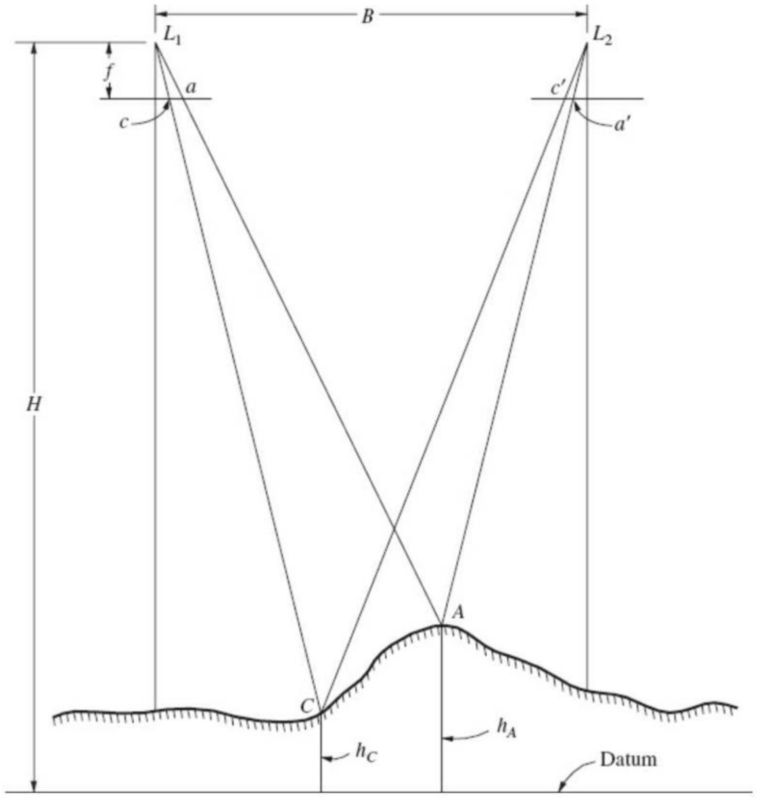


FIGURE 8-11 Elevations by parallax differences.

$$P_c = \frac{fB}{H-h_c}, \quad P_a = \frac{fB}{H-h_A}$$

$$P_a \text{-} P_c = \Delta p = \frac{fB(h_A-h_C)}{(H-h_A)(H-h_C)}$$

$$\rightarrow h_A \text{-} h_C = \frac{\Delta p(H-h_C)}{p_a}$$

7. Elevations by Parallax Differences

[Example 8-2] In Example 8-1, flight-line axis x, and x' coordinates for the images of a vertical control point C were measured as x_c=14.3 mm and x'_c=-78.3 mm. If the elevation of point C is 591 m above sea level, calculate the elevation of points A and B of that example, using parallax difference.

$$P_c = x_c - x'_c = 14.3 - (-78.3) = 92.6\ \text{mm}$$

$$\Delta p = p_a - p_c = 91.7 - 92.6 = -0.9\ \text{mm}$$

$$h_A = h_C + \frac{\Delta p(H - h_C)}{p_a} = 591 + \frac{(-0.9)(1{,}233 - 591)}{91.7} = 585\ \text{m}$$

For point B,

$$\Delta p = p_b - p_c = 96.0 - 92.6 = 3.4\ \text{mm}$$

$$h_B = h_C + \frac{\Delta p(H - h_C)}{p_b} = 591 + \frac{(3.4)(1{,}233 - 591)}{96.0} = 614\ \text{m}$$

8. Simplified Equations for Heights of Objects from Parallax Differences

- Utilizing parallax differences for height determination is particularly useful when application of relief displacement is not possible because either the feature is not vertical (e.g., a construction crane) or the base of the feature is obscured (e.g., trees in a forest).

- When a point on the ground is at the same elevation as the base of the feature height can be obtained using a simplified equation of parallax difference:

$$h_A = \frac{\Delta p H}{p_a}$$

- When the photo base b is used as the parallax of the ground point the parallax equation above can be expressed as $h_A = \frac{\Delta p H}{b + \Delta p}$ ($\Delta p = p_a - b$)

8. Simplified Equations for Heights of Objects from Parallax Differences

[Example 8-3] The parallax difference between the top and the bottom of a tree is measured as 1.3 mm on a stereopair of photos taken at 915 m above ground. Average photo base is 88.2 mm. How tall is the tree?

$$h_A = \frac{\Delta p H}{b + \Delta p} = \frac{1.3 \times 915}{88.2 + 1.3} = 13 \text{ m}$$

10. Computing Flying Height and Air Base

- Generally flying height and air base are necessary to use parallax equations.

- If the air base is known and if one vertical control point is available in the overlap area, flying height for the stereopair may be calculated.

[Example 8-4] An overlapping pair of vertical photographs taken with a 152.4-mm-focal-length camera has an air base of 548 m. The elevation of control point A is 283 m above sea level, and the parallax of point A is 92.4 mm. What is the flying height above sea level for this stereopair?

$$H = h + \frac{Bf}{p} = 283 + \frac{548 \times 152.4}{92.4} = 1187 \text{ m}$$

10. Computing Flying Height and Air Base

[Example 8-5] An overlapping pair of vertical photos was exposed with a 152.4-mm-focal-length camera from a flying height of 1622 m above datum. Control point C has an elevation of 263 m above datum, and the parallax of its images on the stereopair is 86.3 mm. Calculate the air base.

$$B = (H - h)\frac{p}{f} = (1622 - 263)\frac{86.3}{152.4} = 770 \text{ m}$$

If a line of known horizontal length appears in the overlap area, then the air base can be readily calculated. The horizontal length of the line may be expressed in terms of rectangular coordinates, according to the pythagorean theorem, as

$$AB = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2} = B\sqrt{\left(\frac{x_b}{p_b} - \frac{x_a}{p_a}\right)^2 + \left(\frac{y_b}{p_b} - \frac{y_a}{p_a}\right)^2}$$

$$B = \frac{AB}{\sqrt{(x_b/p_b - x_a/p_a)^2 + (y_b/p_b - y_a/p_a)^2}}$$

10. Computing Flying Height and Air Base

[Example 8-6] Images of the endpoints of ground line AB, whose horizontal length is 650.47 m, appear on a pair of overlapping vertical photographs. Photo coordinates measured with respect to the flight axis on the left photo were x_a = 33.3 mm, y_a = 13.5 mm, x_b = 41.8 mm, and y_b = –95.8 mm. Photo coordinates measured on the right photo were x'_a = –52.3 mm and x'_b = –44.9 mm. Calculate the air base for this stereopair.

$$p_a = x_a - x'_a = 33.3 - (-52.3) = 85.6 \text{ mm}$$

$$p_b = x_b - x'_b = 41.83 - (-44.9) = 86.7 \text{ mm}$$

$$B = \frac{AB}{\sqrt{(x_b/p_b - x_a/p_a)^2 + (y_b/p_b - y_a/p_a)^2}}$$

$$= \frac{650.47}{\sqrt{(41.8/86.7 - 33.3/85.6)^2 + (-95.8/86.7 - 13.5/85.6)^2}} = 514 \text{ m}$$

11. Error Evaluation

- Locating and marking the flight lines on photos
- Orienting stereopairs for parallax measurement
- Parallax and photo coordinate measurements
- Shrinkage or expansion of photographs
- Unequal flying heights for the two photos of stereopairs
- Tilted photographs
- Errors in ground control
- Other errors of lesser consequence such camera lens distortion and atmospheric refraction distortion

11. Error Evaluation

[Example 8-7] In the computation of the elevation of point A in Example 8-1, suppose that the random errors were ± 2 m in H, ± 2 m in B, and ± 0.1 mm in p_a. Compute the resulting error in h_A due to the presence of these errors.

From parallex equation, $h_A = H - \frac{Bf}{p_a}$

$$\frac{\partial h_A}{\partial H} = 1, \qquad \frac{\partial h_A}{\partial B} = -\frac{f}{p_a}, \qquad \frac{\partial h_A}{\partial p_a} = \frac{Bf}{{p_a}^2}$$

$$\sigma_{h_A} = \pm\sqrt{1^2\sigma_H^2 + \left(\frac{-f}{p_a}\right)^2 \sigma_B^2 + \left(\frac{Bf}{p_a^2}\right)^2 \sigma_{p_a}^2}$$

$$= \pm\sqrt{1^2 2^2 + \left(\frac{-152.4}{91.7}\right)^2 2^2 + \left(\frac{390(152.4)}{91.7^2}\right)^2 0.1^2} = \pm 3.9 \text{ m}$$