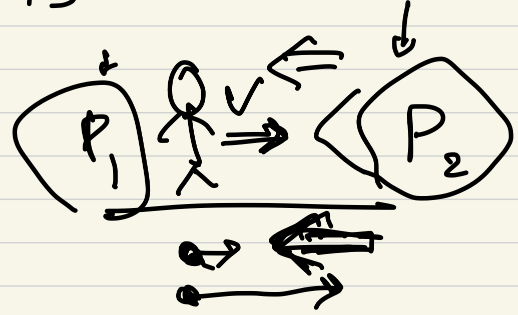
7.5 Boundary layers with pressure gradients



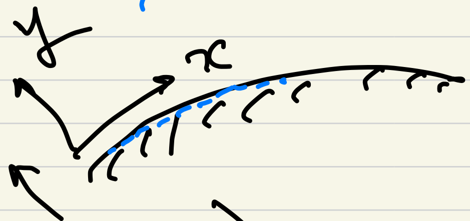
flat plate : $U = \text{const} \rightarrow dp/dx = 0$

$\dfrac{dp}{dx} > 0$: adverse pres. grad
역 압력 구배

$\dfrac{dp}{dx} < 0$: favorable " "
순

$$\text{N-S eq.}: \cancel{\frac{\partial u}{\partial t}}^{\,0} + \cancel{u}^{\,0}\frac{\partial u}{\partial x} + \cancel{v}^{\,0}\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{\,0} + \frac{\partial^2 u}{\partial y^2}\right)$$

@ $y=0$ (wall), $u = v = 0$ (no slip)



$$\text{N-S} \Rightarrow 0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \left.\frac{\partial^2 u}{\partial y^2}\right|_w$$

$\left(\dfrac{\partial p}{\partial y} = 0\right)$

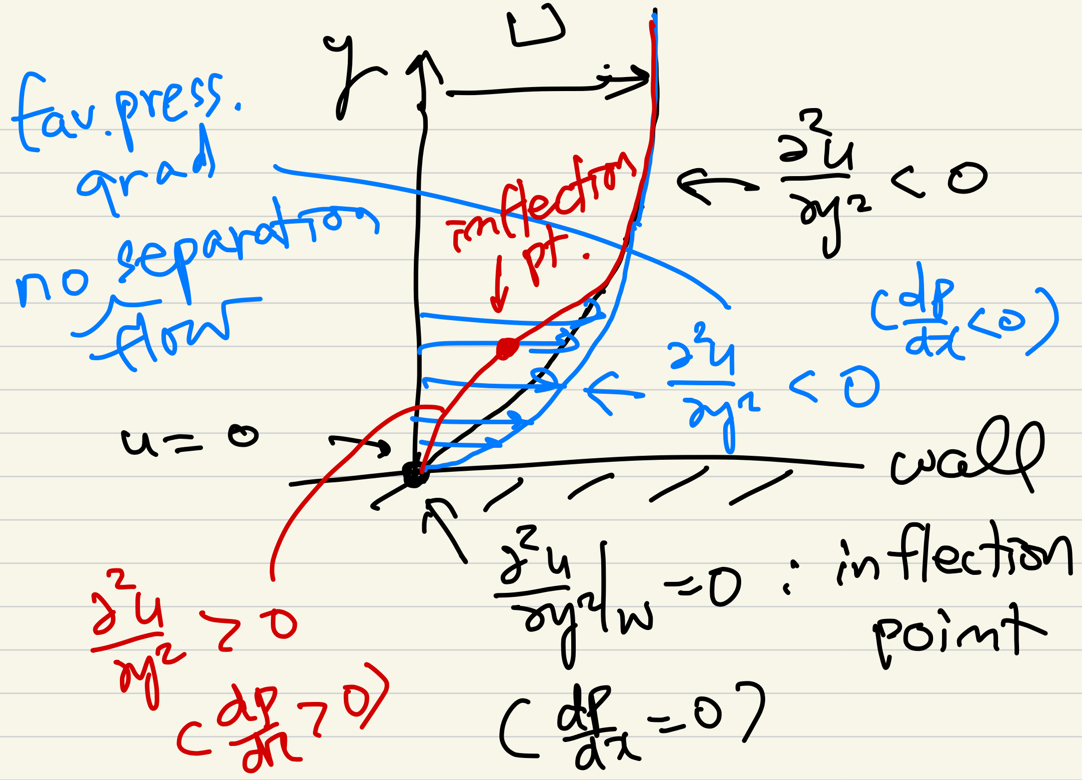
$$\rightarrow \boxed{\left.\frac{\partial^2 u}{\partial y^2}\right|_w = \frac{1}{\mu}\frac{dp}{dx}}$$

↳ curvature

if $\frac{dp}{dx} = 0$, $\left.\frac{\partial^2 u}{\partial y^2}\right|_w = 0$

$\frac{dp}{dx} < 0$, $\left.\frac{\partial^2 u}{\partial y^2}\right|_w < 0$

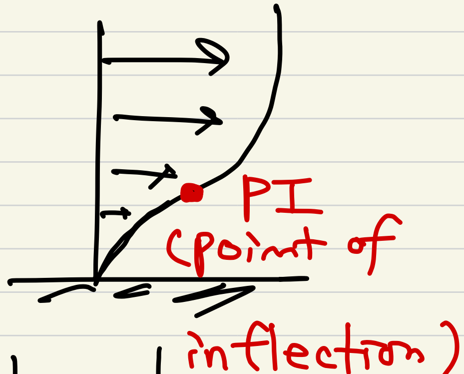
$\frac{dp}{dx} > 0$, $\left.\frac{\partial^2 u}{\partial y^2}\right|_w > 0$



→ Any boundary layer profile in an adverse press. grad. exhibits a characteristic S shape.

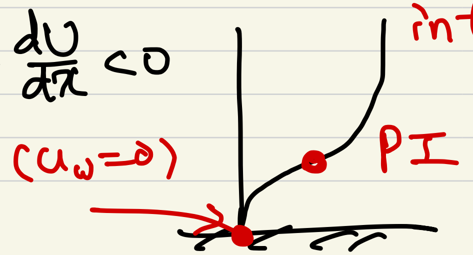
zero press. grad. → $\frac{dp}{dx} = 0$, $\frac{dU}{dx} = 0$, no sep.

weak adverse press. grad → $\frac{dp}{dx} > 0$ → $\frac{dU}{dx} < 0$
no separation



"critical" adverse press. grad. → $\frac{dp}{dx} > 0$ → $\frac{dU}{dx} < 0$

(separation pt. 박리점. → $\left.\frac{\partial u}{\partial y}\right|_w = 0$



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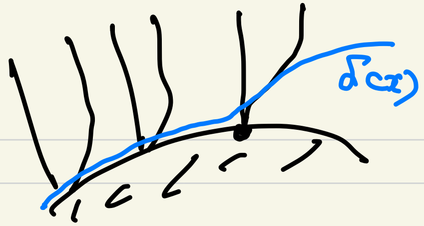
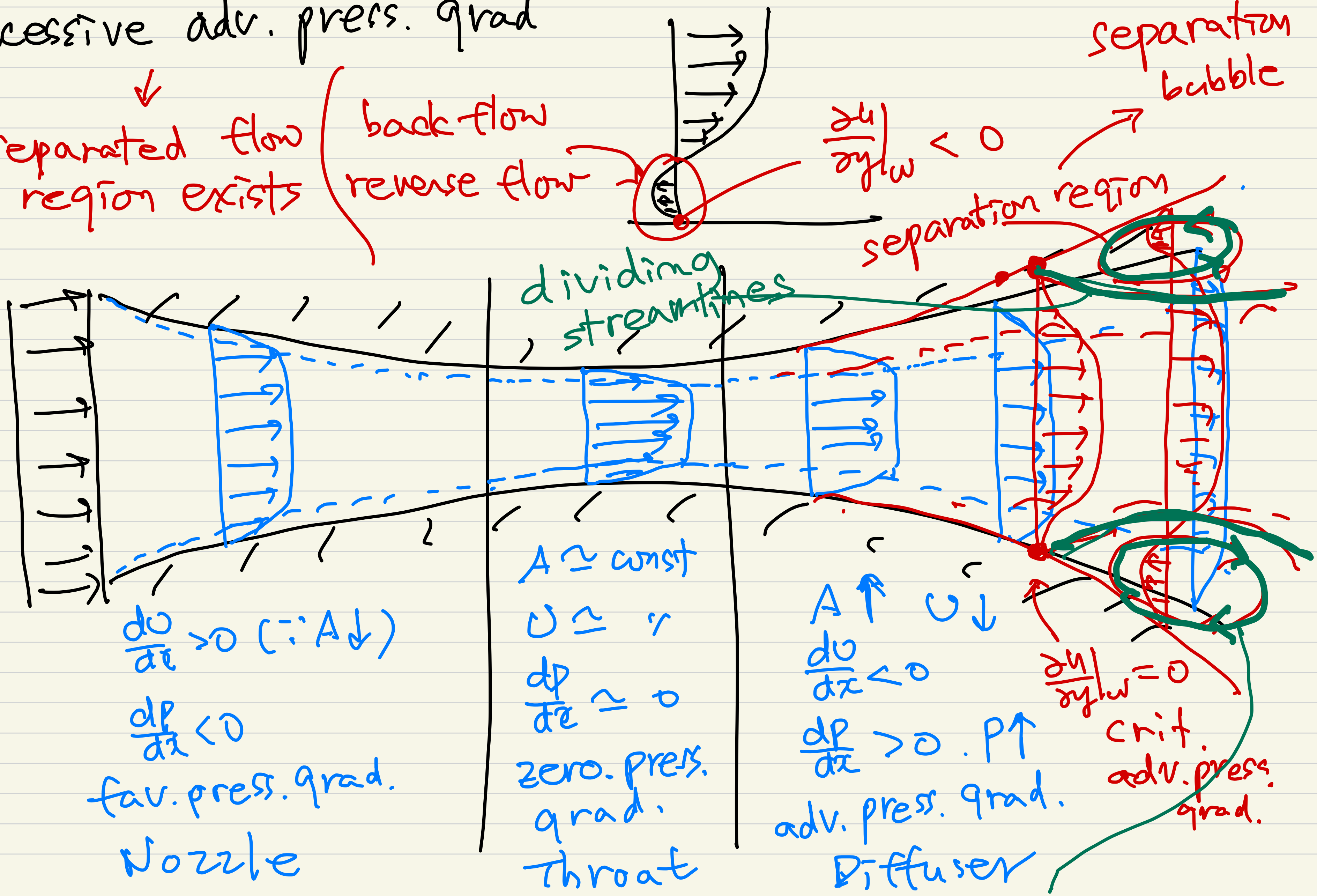
separation starts → boundary layer thickness ↑



excessive adv. pres. grad

↓

separated flow region exists (back flow / reverse flow



$\frac{dU}{dx} > 0$ ($\because A \downarrow$)

$\frac{dp}{dx} < 0$

fav. press. grad.

Nozzle

$A \simeq$ const

$U \simeq$ "

$\frac{dp}{dx} \simeq 0$

zero. pres. grad.

Throat

$A \uparrow$ $U \downarrow$

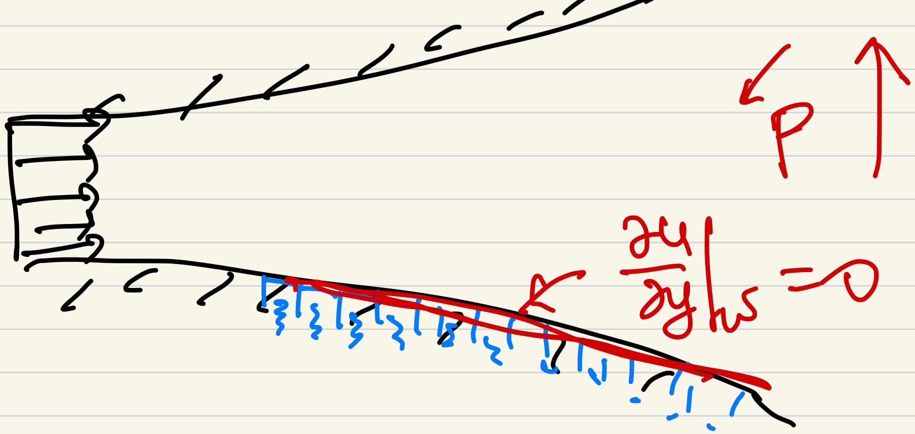
$\frac{dU}{dx} < 0$

$\frac{dp}{dx} > 0$. $P \uparrow$

adv. press. grad.

Diffuser

"Stratford" diffuser



stall
↓
heavy flow loss

Boundary layer theory
($u \gg v$, $\frac{\partial}{\partial y}(\cdot) \gg \frac{\partial}{\partial x}(\cdot)$)
is valid only up to the separation pt.

• Laminar bdry layer integral theory (Karman)

$$\frac{\tau_w}{\rho U^2} = \boxed{\frac{c_f}{2} = \frac{d\theta}{dx} + (2+H)\frac{\theta}{U}\frac{dU}{dx}} \qquad \left(\frac{dU}{dx} = -\frac{1}{\rho}\frac{1}{U}\frac{dp}{dx}\right)$$

c_f, θ, H open form.

separation occurs @ $H = 3.5$ laminar flow
↳ $c_f = 0$ $H = 2.4$ turbulent "

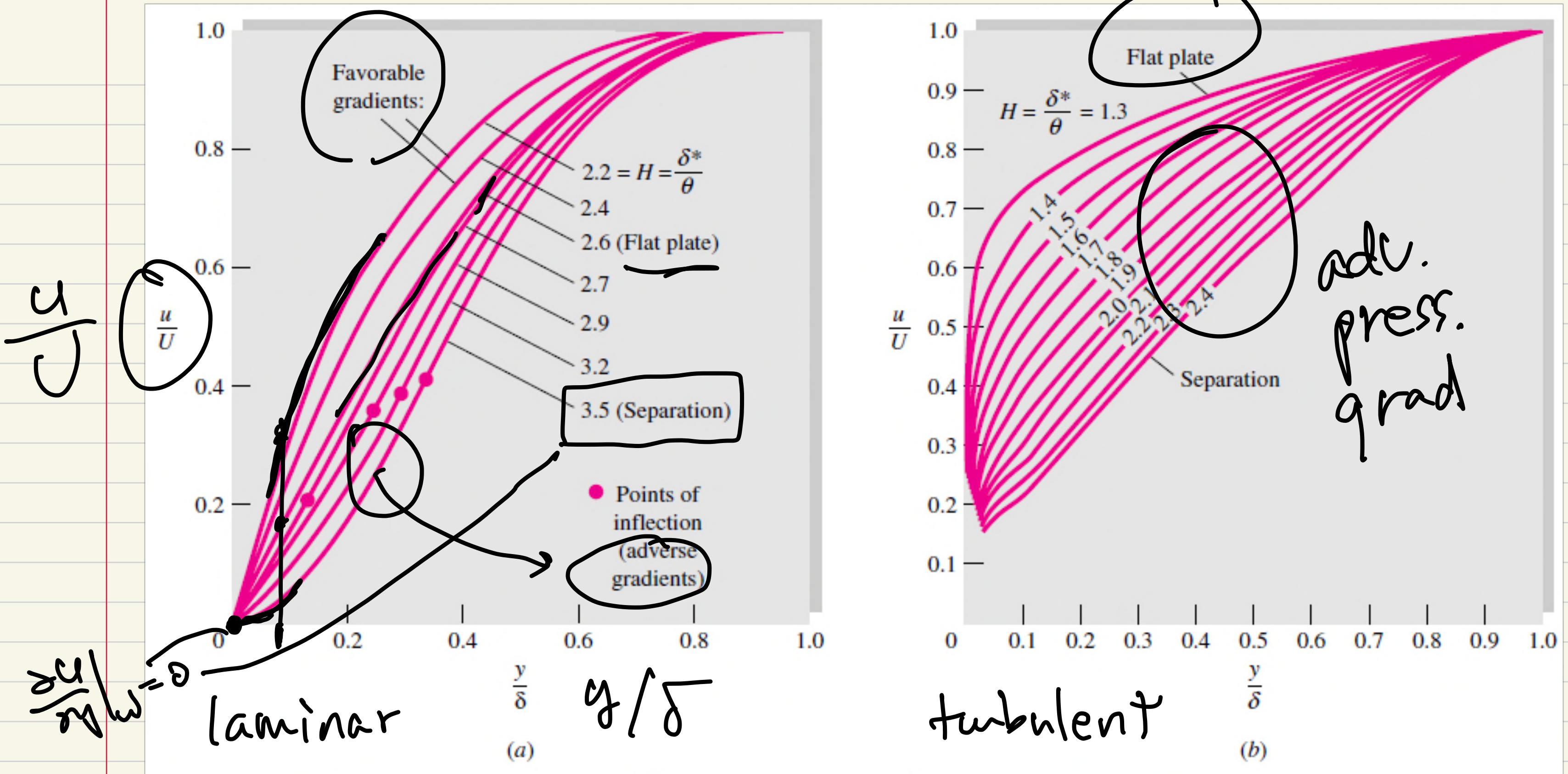
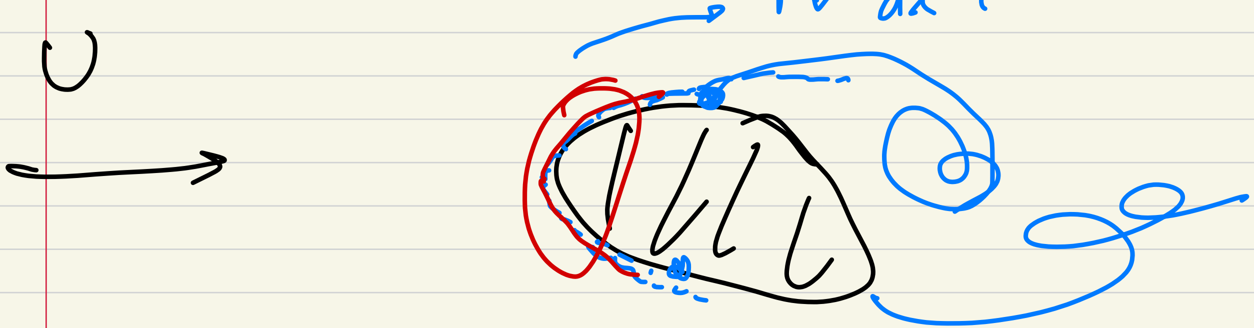


Fig. 7.9 Velocity profiles with pressure gradient: (a) laminar flow; (b) turbulent flow with adverse gradients.

7.6 Experimental external flows

$V\uparrow$ $p\downarrow$ $\frac{dp}{dx}\uparrow$



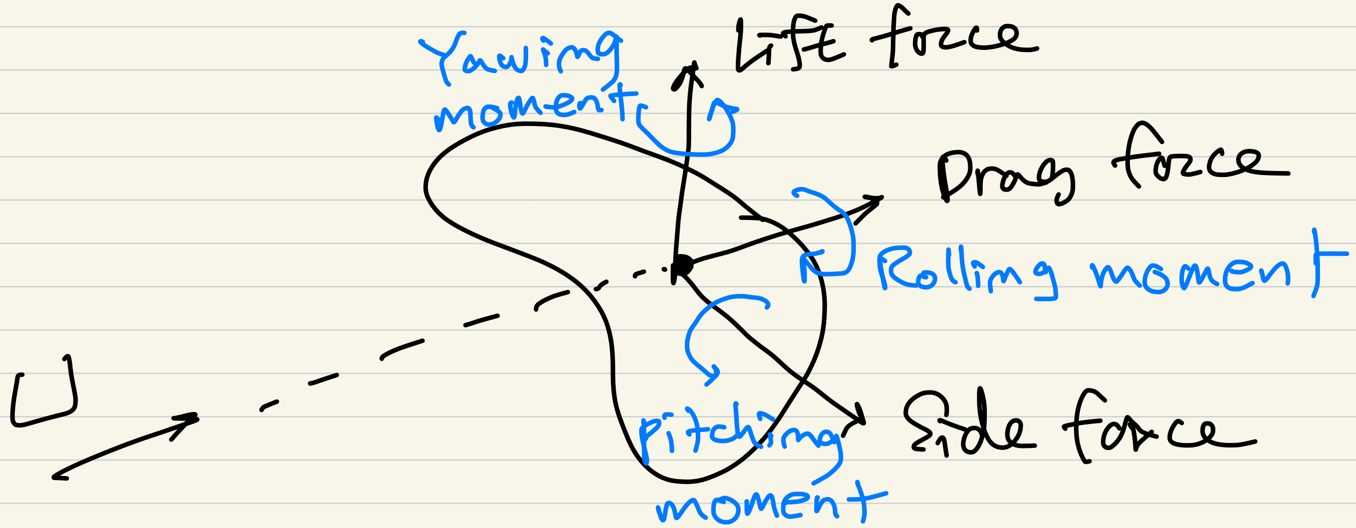
No theory ← flow separation

↓

experiment or CFD

- Drag of 2D & 3D bodies

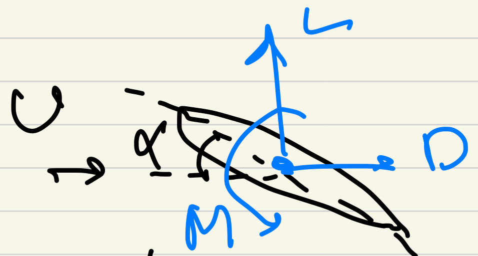
Performance of lifting bodies → Lift, Drag



Symmetry about the lift-drag axis

→ side force, yaw, roll mtm = 0

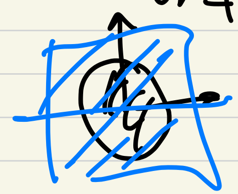
drag, lift, pitching mtm ≠ 0



α : angle of attack

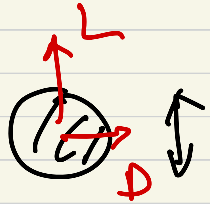
two planes of symmetry → Drag ≠ 0

$L(t) \neq 0$ 영각
$L = 0$



- similarity (geometric, kinematic, dynamic)
  [L] [L, T] [L, T, M]

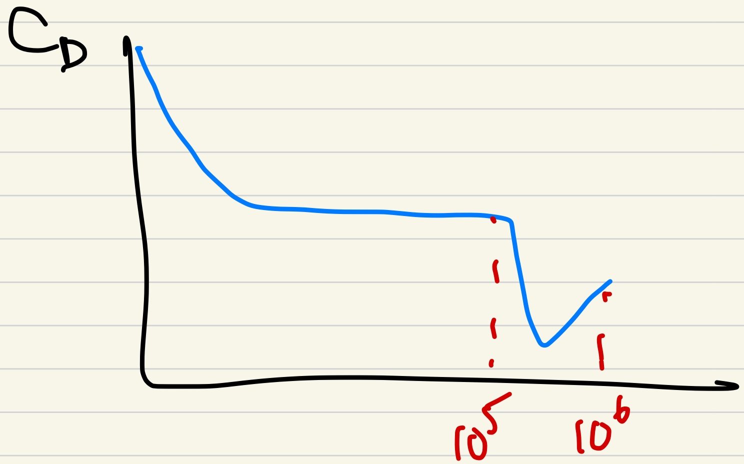
$U \rightarrow$



$\updownarrow d$

$D = f(U, d, \rho, \mu)$

dimensional analysis



$\rightarrow C_D = f(Re)$ $\quad Re = Ud/\nu$

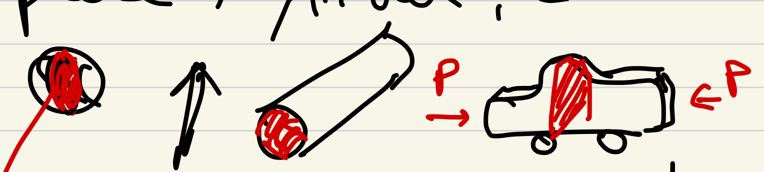
항력계수

$\dfrac{D}{\frac{1}{2}\rho U^2 A}$

$\boxed{A\ ?}$

sphere, cylinder, car

$U \rightarrow$

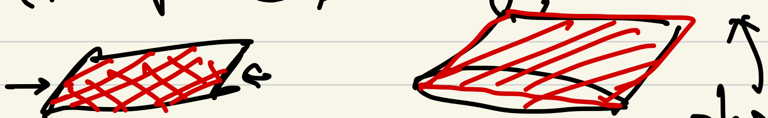


form drag (press. drag)

A : frontal area ΔP!

bluff bodies

flat plate, wing, airfoil



skin friction drag

A : planform area

friction!

streamlined bodies

surface ship ← skin friction

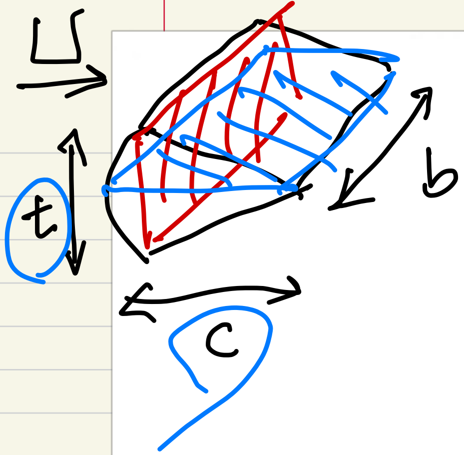
A : wetted surface area

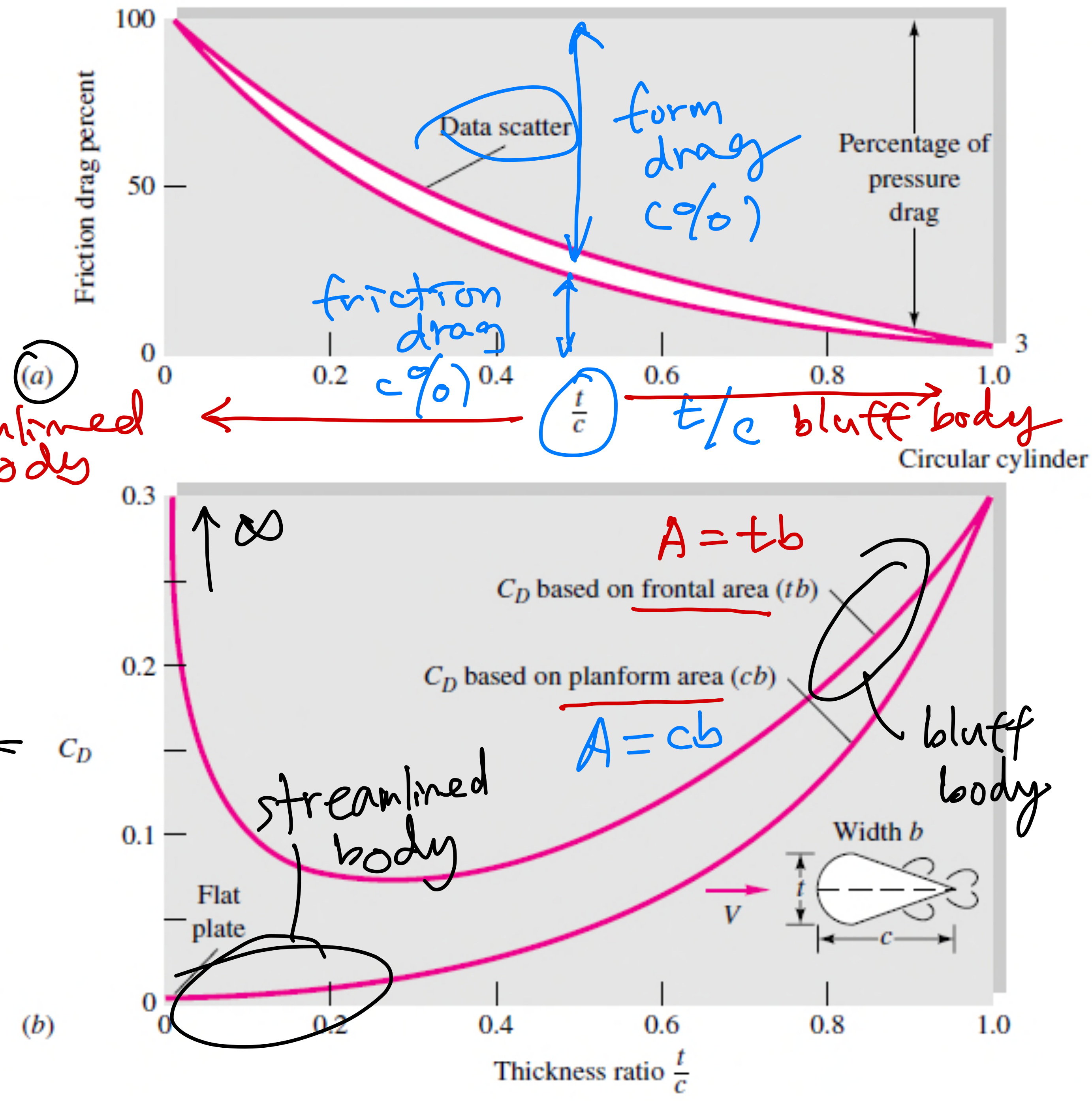
press.

Total drag = Friction drag + Form drag (+ wave drag)

$$C_D = C_{D_f} + C_{D_p}$$

$$A = A$$





$$\frac{D}{\frac{1}{2}\rho U^2 A} = C_D$$

Fig. 7.12 Drag of a streamlined two-dimensional cylinder at $Re_c = 10^6$: (*a*) effect of thickness ratio on percentage of friction drag; (*b*) total drag versus thickness when based on two different areas.