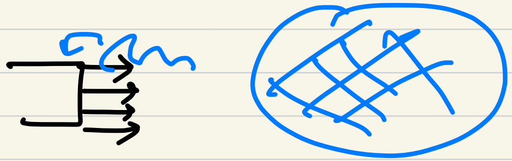
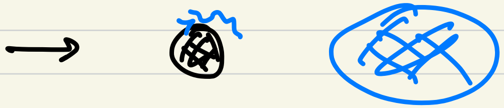
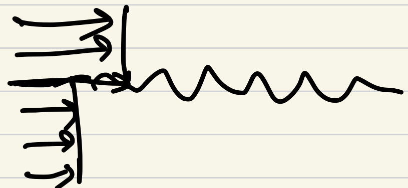
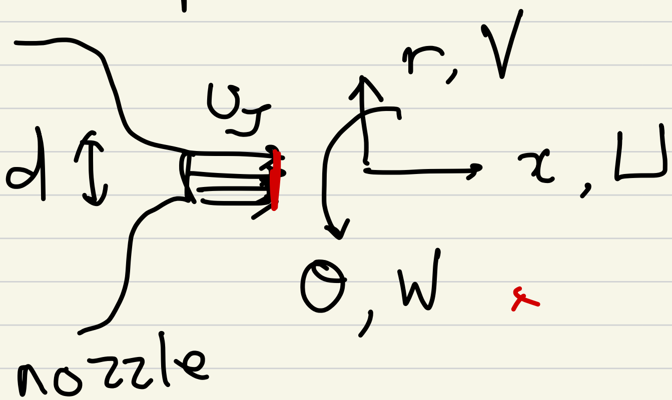
Ch. 5 Free shear flows

remote from wall

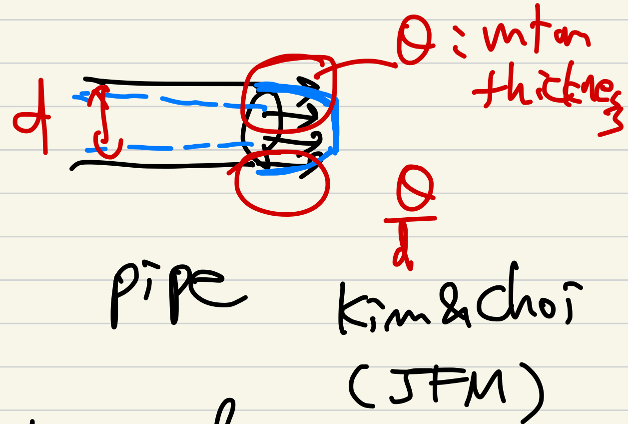
- jet : 
- wake : 
- mixing layer : 

5.1 Round jet : experimental observations

◦ Description of flow

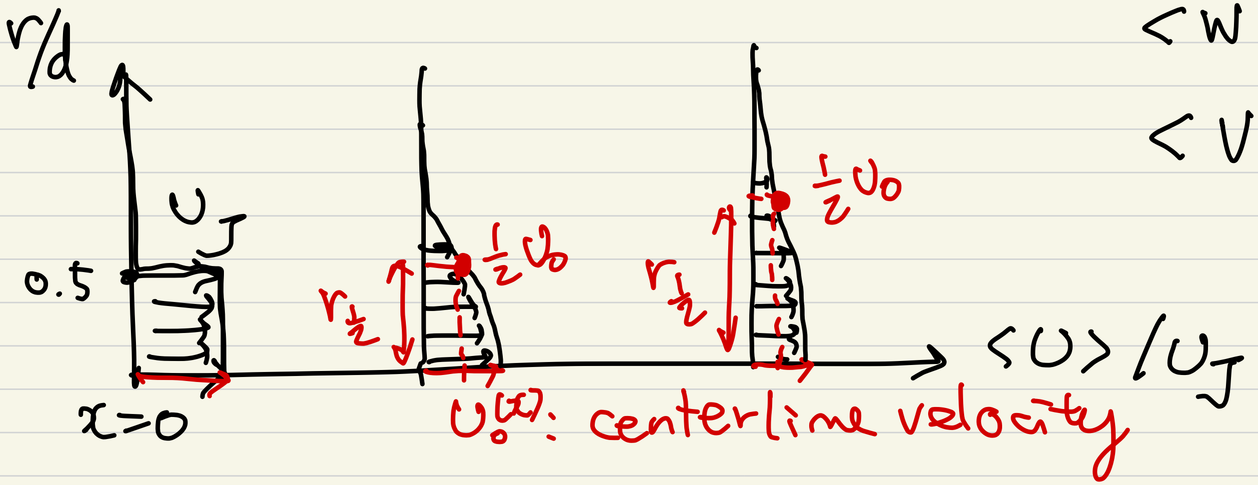


statistically stationary & axisymmetric



$$Re \equiv U_J d/\nu$$: jet Reynolds number

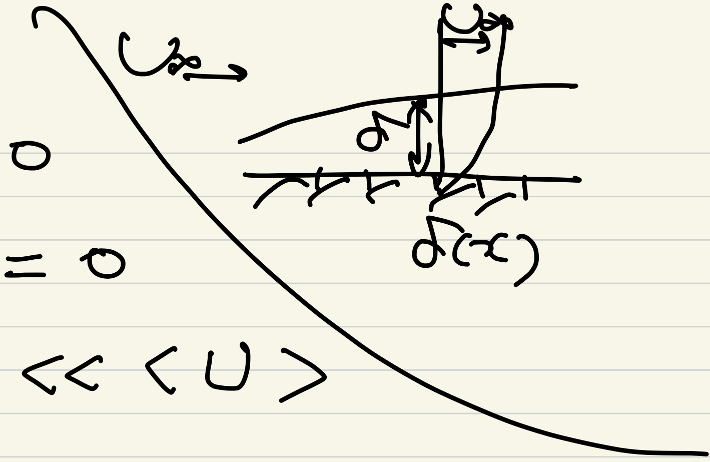
⊙ Mean velocity field



$W \neq 0$

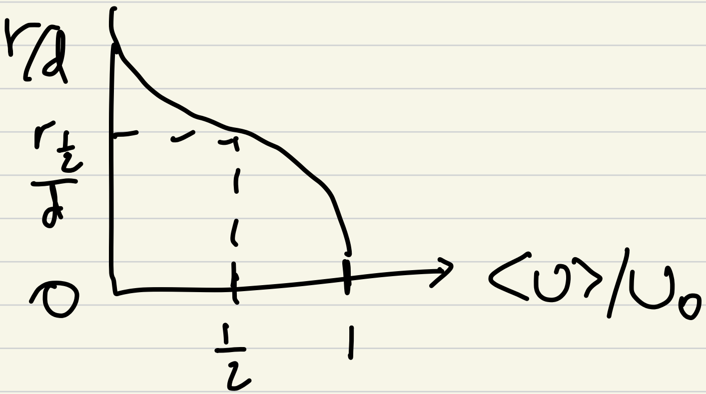
$\langle W\rangle = 0$

$\langle V\rangle \ll \langle U\rangle$



$U(x, r, \theta, t)$

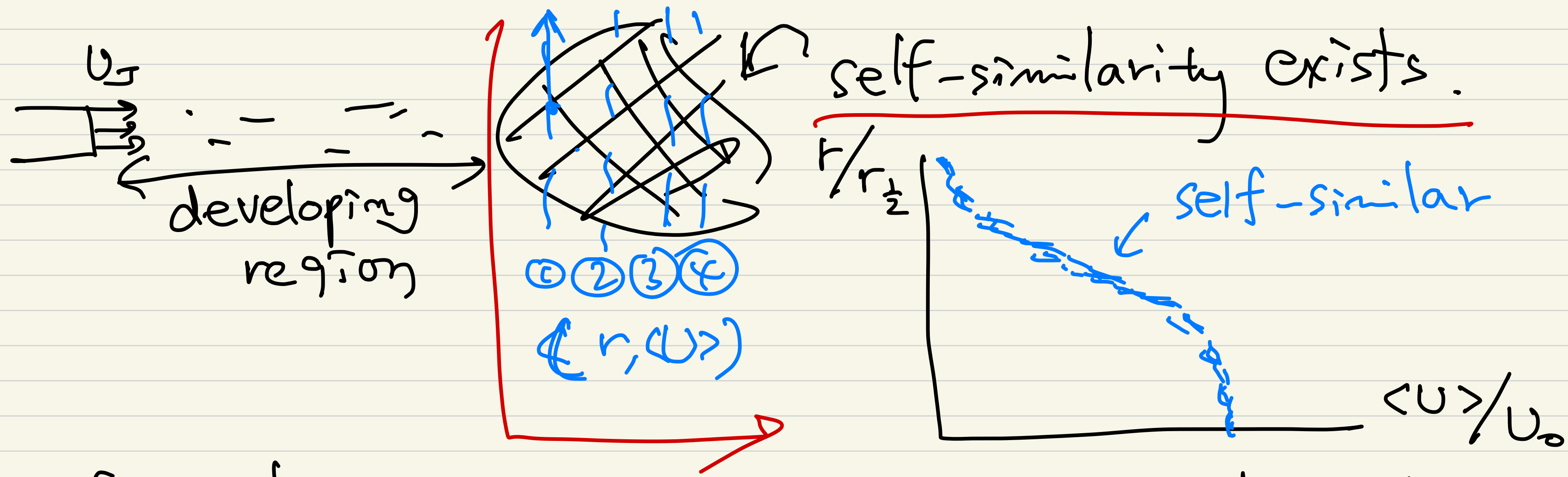
- centerline velocity : $U_0(x) = \langle U(x, 0, 0, t)\rangle$
- Jet half-width $r_{\frac{1}{2}}(x)$: $\langle U(x, r_{\frac{1}{2}}(x), \theta, t)\rangle = \frac{1}{2}U_0(x)$



As $x\uparrow$, jet decays (i.e. $U_0(x)$ decreases)

& jet spreads (i.e. $r_{\frac{1}{2}}(x)$ increases).

- Beyond the developing region ($x > 30d$)



- self-similarity : important concept in turbulent flow

  $Q(x,y)$: a quantity

  $Q_o(x)$ and $\delta(x)$: characteristic scales

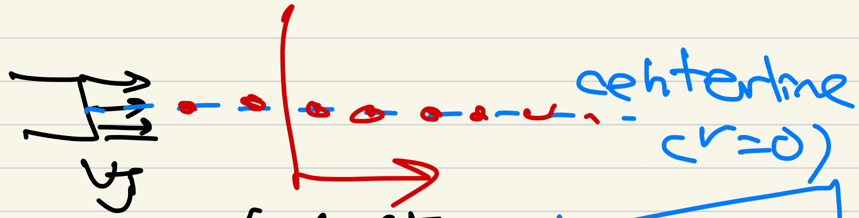
  ↑ length scale

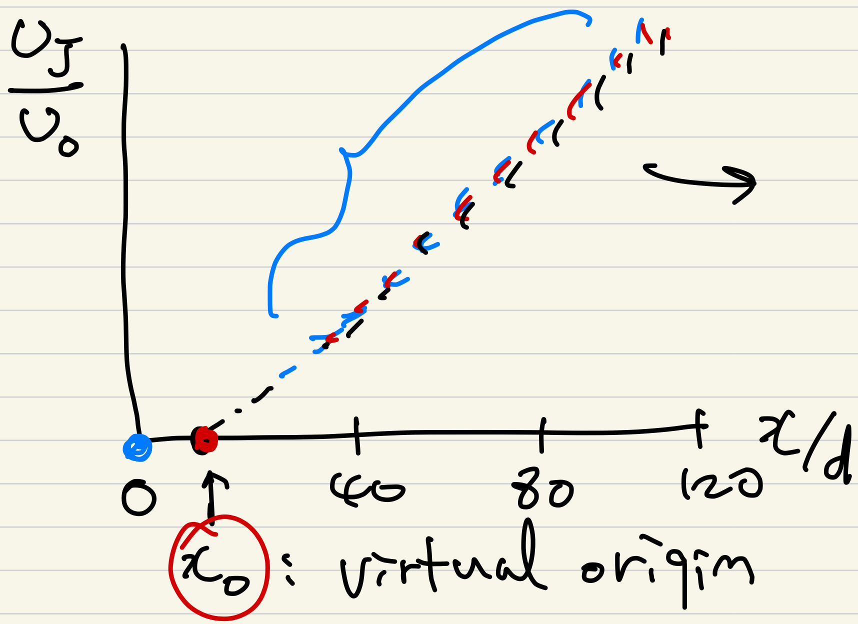
  Define $\xi \equiv y/\delta(x)$

  $$\tilde{Q}(\xi, x) \equiv Q(x,y)/Q_o(x)$$

If $\tilde{Q}(\xi, x) = \hat{Q}(\xi)$, $Q(x,y)$ is self-similar.

• Axial variation of scales





$$\frac{U_0(x)}{U_J} = \frac{\overset{\leftarrow \text{const}}{B}}{(x-x_0)/d} \to \boxed{U_0 \sim x^{-1}}$$

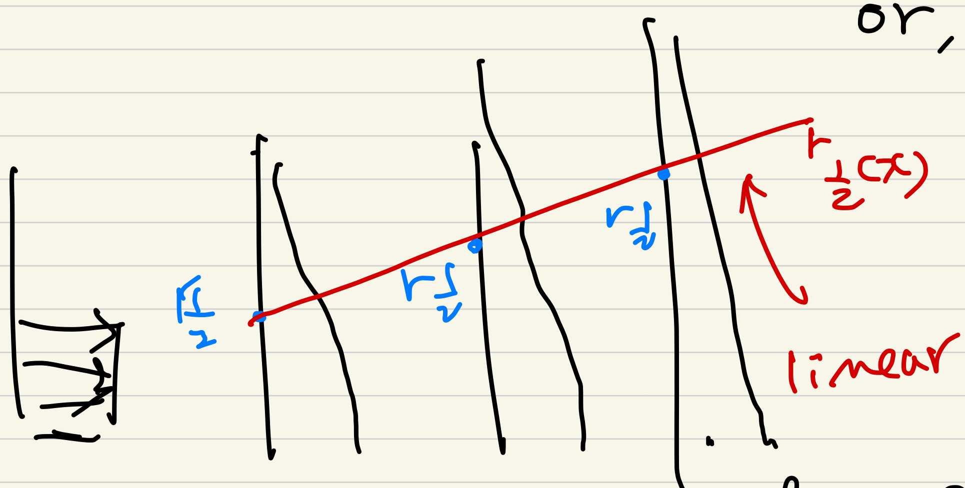
It is also found that the spreading rate S

$$S \equiv dr_{\frac{1}{2}}(x)/dx = \text{const}$$

or, $r_{\frac{1}{2}} = S(x-x_0) \to \boxed{r_{\frac{1}{2}} \sim x}$

$\Downarrow$

$r_{\frac{1}{2}} \cdot U_0$ is indep. of x



Red local Reynolds number $Re_0(x) = \frac{U_0(x)\, r_{\frac{1}{2}}(x)}{\nu}$ is indep. of x!

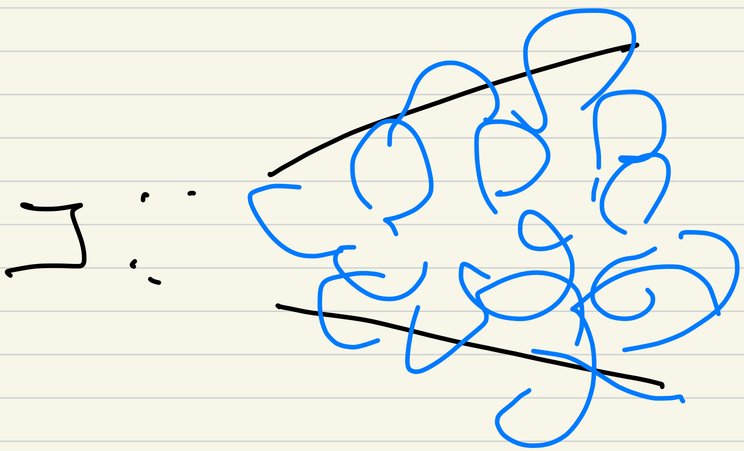
- S and B have no dependence on $Re \rightarrow Re_d$

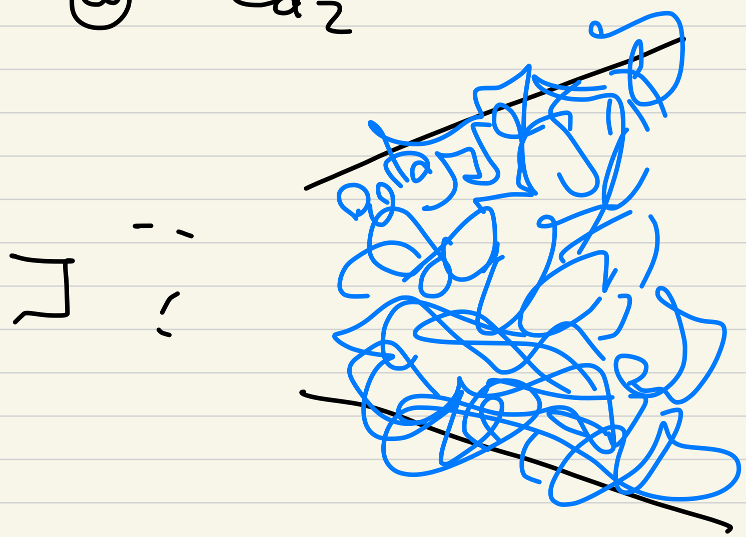
(Table 5.1) $Re_d = 11000 \sim 95500$

$B = 5.8 \sim 6.0$ & $S = 0.094 \sim 0.1$

$\rightarrow$ the mean vel. profile and spreading rate are indep. of Re_d, although small-scale structures are smaller at larger Re #

@ Re_{d_1} < @ Re_{d_2}





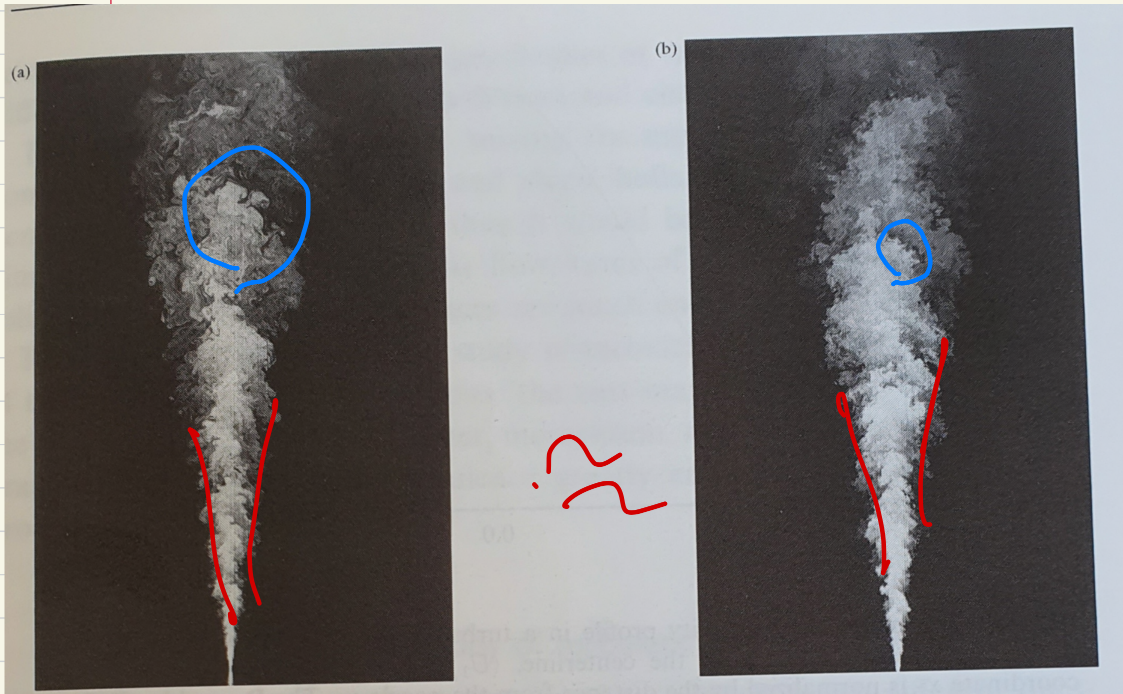


Fig. 1.2. Planar images of concentration in a turbulent jet: (a) Re = 5,000 and (b) Re = 20,000. From Dahm and Dimotakis (1990) .

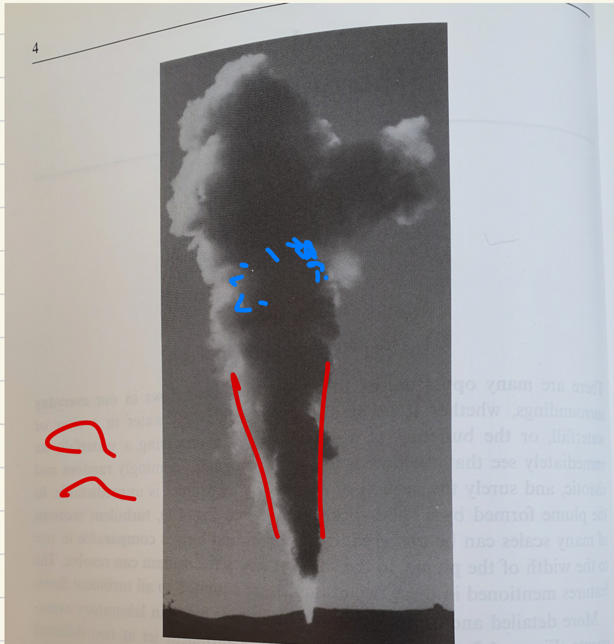
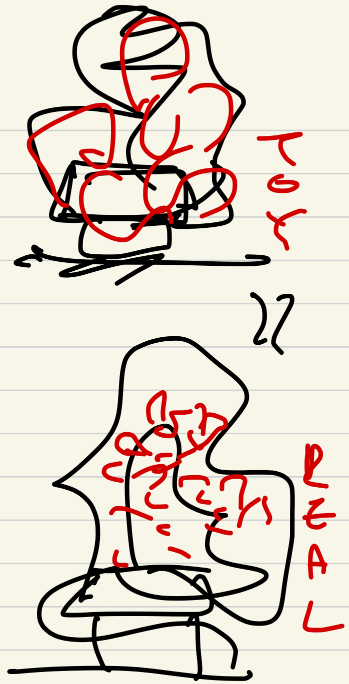


Fig. 1.1. A photograph of the turbulent plume from the ground test of a Titan IV rocket motor. The nozzle's exit diameter is 3 m, the estimated plume height is 1,500 m, and the estimated Reynolds number is 200×10^6. For more details see Mungal and Hollingsworth (1989). With permission of San Jose Mercury & News.

$Re_d = 5,000$ $\qquad$ $Re_d = 20,000$ $\qquad$ $Re_d = 200 \times 10^6$



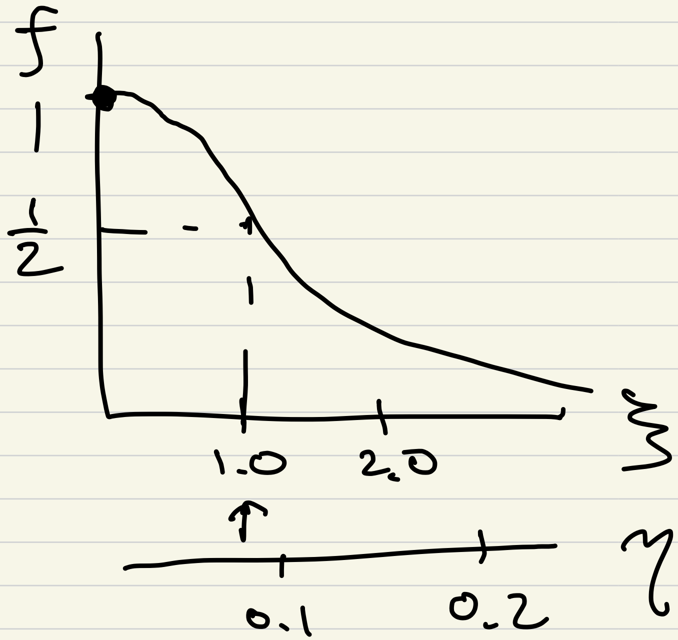
$r_{\frac{1}{2}} = S(x - x_0)$

- $\xi = r / r_{\frac{1}{2}}$ or $\eta = r / (x - x_0)$

$\rightarrow \quad \eta = S \xi$

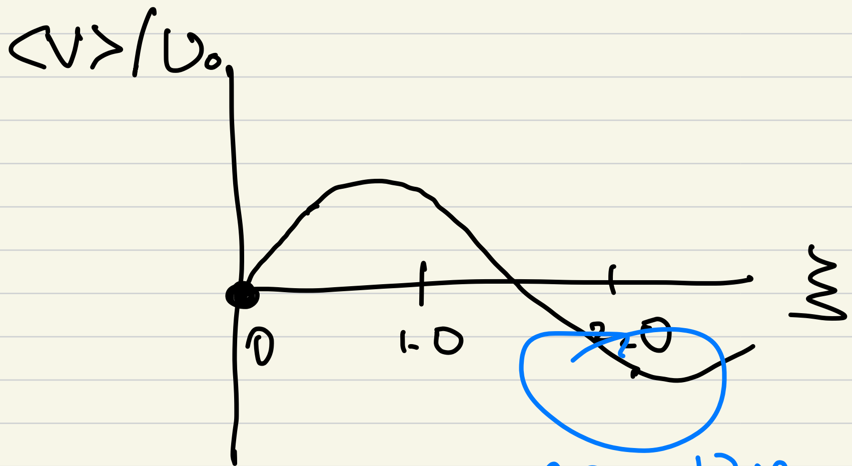
$$f(\eta) = \bar{f}(\xi) = \frac{\langle U(x, r, 0)\rangle}{U_0(x)}$$

self-similar mean velocity profile



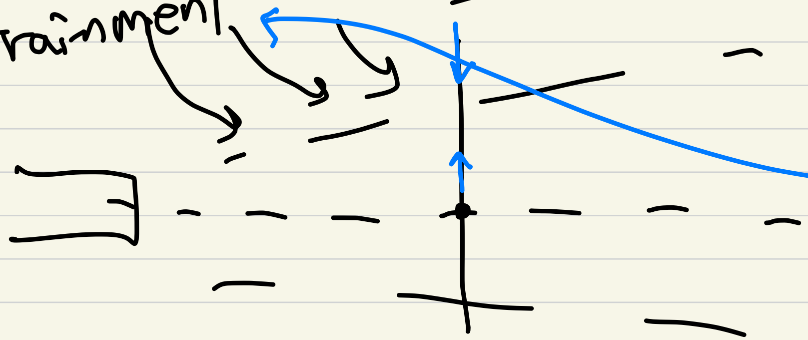
$\langle U \rangle$, $\langle V \rangle \ll \langle U \rangle$

mean lateral velocity

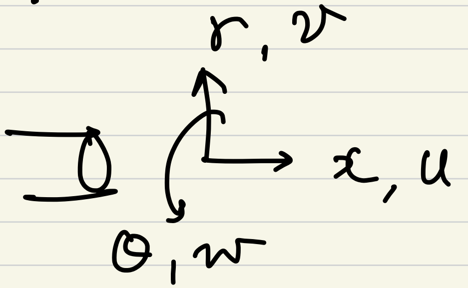


negative

entrainment



- Reynolds stresses



$$\begin{bmatrix} \langle u^2\rangle & \langle uv\rangle & 0 \\ \langle uv\rangle & \langle v^2\rangle & 0 \\ 0 & 0 & \langle w^2\rangle \end{bmatrix}$$

due to circumferential symmetry

$u_0'(x) \equiv \langle u^2\rangle^{\frac{1}{2}}_{r=0}$: rms axial centerline vel. fluctuations

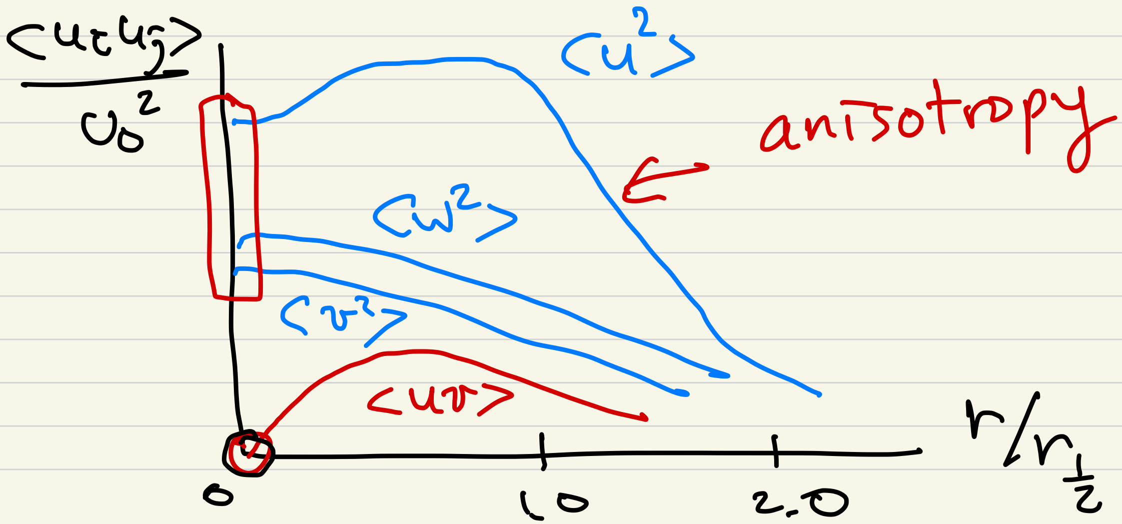
in self-similar region, $u_0'(x)/U_0(x) \sim 0.25$

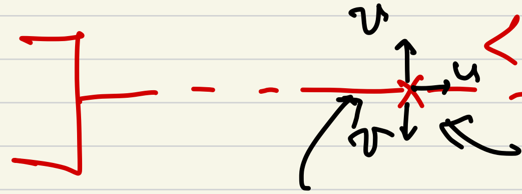
$\rightarrow u_0' \sim U_0 \sim x^{-1}$

$\frac{\partial \langle U\rangle}{\partial r} = 0$

$\langle U\rangle \neq 0$

$\langle V\rangle = \langle W\rangle = 0$





$\langle uv\rangle = 0$

$u \neq 0$

$v \neq 0$

$w \neq 0$

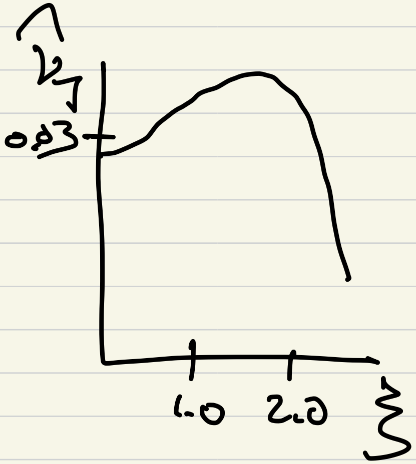
$\rightarrow \langle u^2\rangle = \langle v^2\rangle = \langle w^2\rangle \neq 0$

$$\langle uv\rangle = -\underbrace{\nu_T}_{} \frac{\partial\langle U\rangle}{\partial r} > 0 \qquad \frac{\partial\langle U\rangle}{\partial r} < 0$$

cf. bdry layer $\langle uv\rangle < 0$

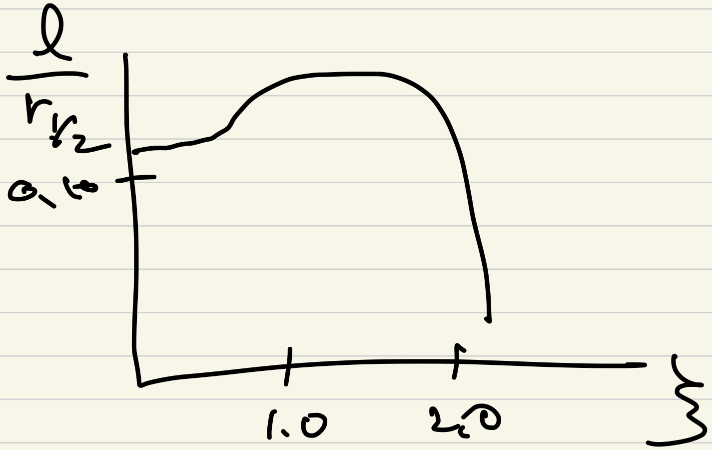
$\hookrightarrow \nu_T \sim u\cdot \ell$

$$\nu_T(x,r) = U_0(x)\, r_{\frac{1}{2}}(x)\, \hat{\nu}_T(\eta) \qquad \hat{\nu}_T(\xi)$$



$\left.\begin{array}{l}\langle uv\rangle\\ \langle U\rangle\end{array}\right\}$ self-similar $\rightarrow \nu_T$ self similar

$$\nu_T = u\cdot \ell = u'\cdot \ell$$



$V = \langle V \rangle + v$ vs $v = \bar{v} + v'$

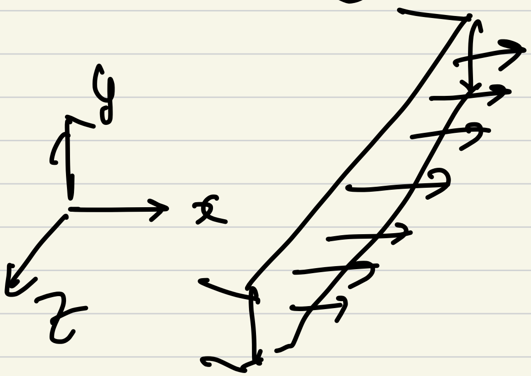
5.2 Round jet : mean momentum $\langle V \rangle \ll \langle W \rangle$

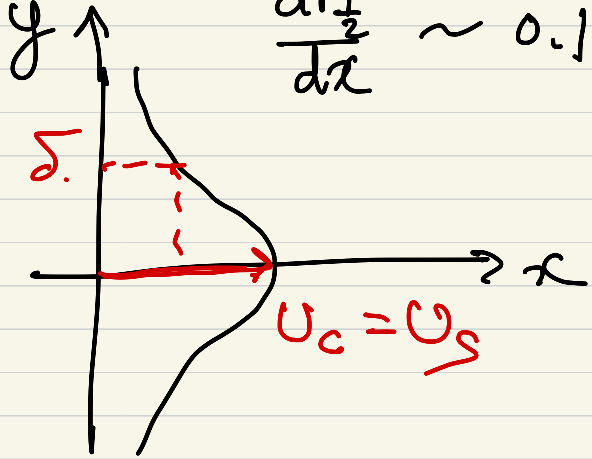
ⓑ Boundary-layer eqs.

in turbulent jet, $|\langle V \rangle| \simeq 0.03 |\langle U \rangle|$

$\frac{dr_{\frac{1}{2}}}{dx} \sim 0.1 \Leftarrow$ axial grad is very small

plane jet



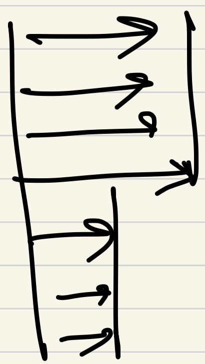


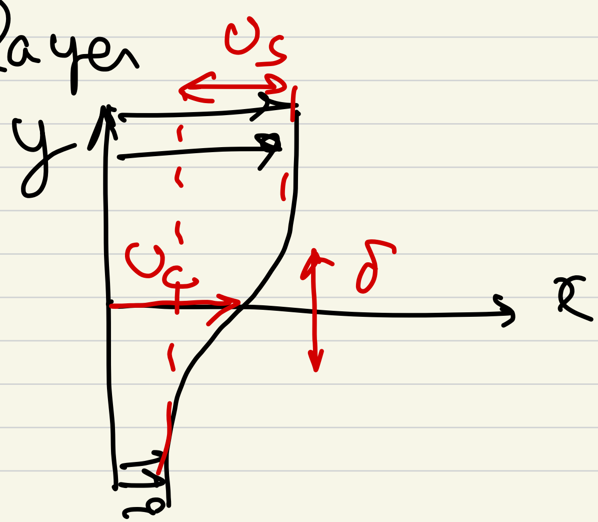
δ : char. flow width

U_c : char. convection vel.

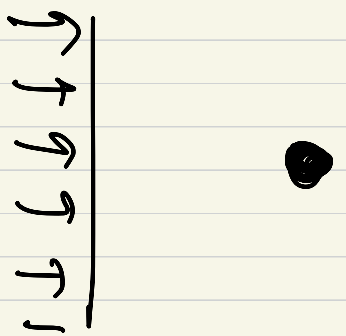
U_s : char. vel. difference

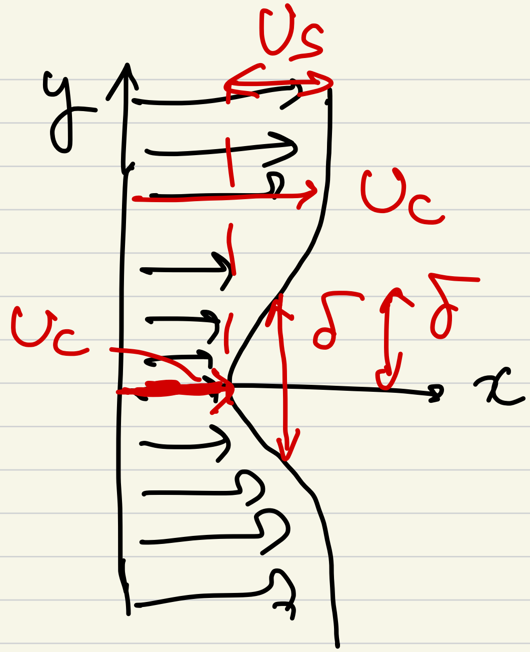
plane mixing layer





plane wake





⇐ boundary-layer approx.

$$|\langle u \rangle| \gg |\langle v \rangle|$$

$$\frac{\partial}{\partial y}(\cdot) \gg \frac{\partial}{\partial x}(\cdot)$$

boundary layer flow