Lecture Note of Naval Architectural Calculation

Ship Stability

Ch. 7 Inclining Test

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Myung-Il Roh

Department of Naval Architecture and Ocean Engineering
Seoul National University

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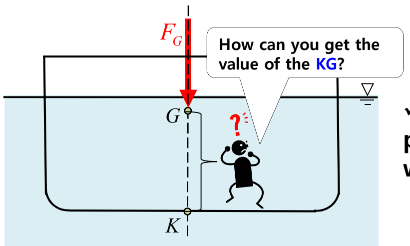
The Problem of Finding an Accurate Vertical Center of Gravity (KG)

The problem of finding an accurate KG for a ship is a serious one for the ship's designer.



K: Keel
G: Center of gravity

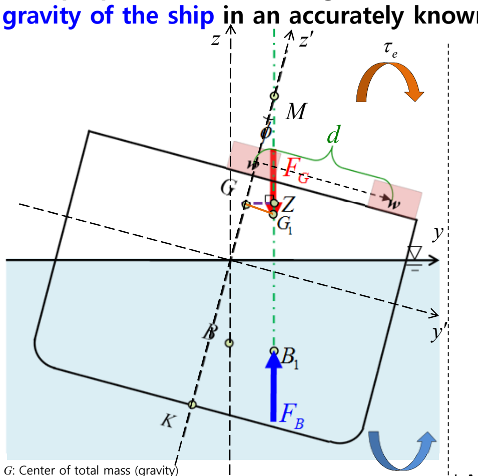
✓ Any difference in the weight of structural parts, equipment, or welds in different ship will produce a different KG.

⇨ There is an accurate method of finding KG for any particular ship and that is the inclining test.

Required Values to Find the KG (1/3)

$\tau_r = F_B \cdot GZ$
$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)
$GM = KB + BM - KG$

The purpose of the inclining test is to determine the position of the center of gravity of the ship in an accurately known condition.



G: Center of total mass (gravity)
B: Center of buoyancy at initial position
G_1: New position of center of total mass (gravity) after the weight has been moved
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of vertical line through the center of buoyancy with the transversally parallel line to a waterline through center of total mass (gravity)
M: The intersection of a vertical line through the center of buoyancy at previous position(B) with a vertical line through the present position of the center of buoyancy(B_1) after the ship has been inclined transversally through a small angle

τ_e : Heeling moment
τ_r : Righting moment

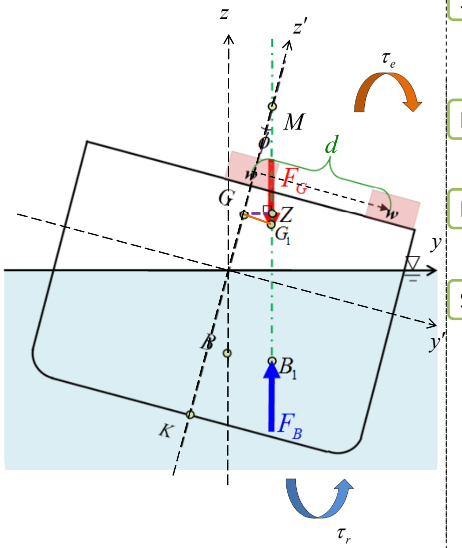
Required values to find the KG

- Draft
- Total weight (F_G)
- Hydrostatic values (KB, BM)
- Weight (w)
- Distance (d)
- Angle of inclination (ϕ)*

*** Angle of inclination should be within the range of inclinations where the metacenter (M) is not changed (about 7°~10°).**

Required Values to Find the KG (2/3)

$\tau_r = F_B \cdot GZ$
$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)
$GM = KB + BM - KG$



Shift of center of total mass (gravity)

$$GG_1 = \frac{w \cdot d}{F_G}$$

Heeling moment produced by total weight

$$\tau_h = F_G \cdot GG_1 \cos\phi$$

Righting moment produced by buoyant force

$$\tau_r = F_B \cdot GZ \approx F_B \cdot GM \sin\phi$$

Static equilibrium of moment

$$\cancel{F_G} \cdot GG_1 \cos\phi = \cancel{F_B} \cdot GM \sin\phi$$

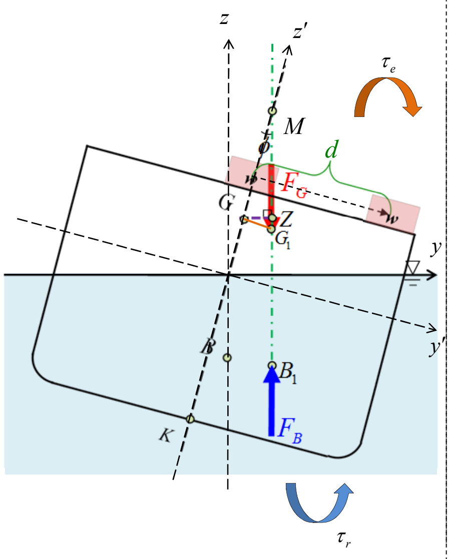
$$(\because \text{Static equilibrium}: F_G = F_B)$$

$$\therefore GM = \frac{GG_1}{\tan\phi} = \frac{w \cdot d}{F_G \cdot \tan\phi}$$

Inclining test formula

Required Values to Find the KG (3/3)

$\tau_r = F_B \cdot GZ$
$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)
$GM = KB + BM - KG$



Inclining experiment formula

$$GM = \frac{w \cdot d}{F_G \cdot \tan\phi}$$

KG

$$KG = KB + BM - GM$$

$$= KB + BM - \frac{w \cdot d}{F_G \cdot \tan\phi}$$

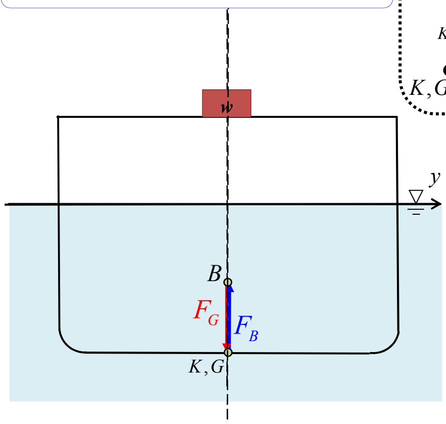
Known (KB), Known (BM), Known ($w \cdot d$, F_G)

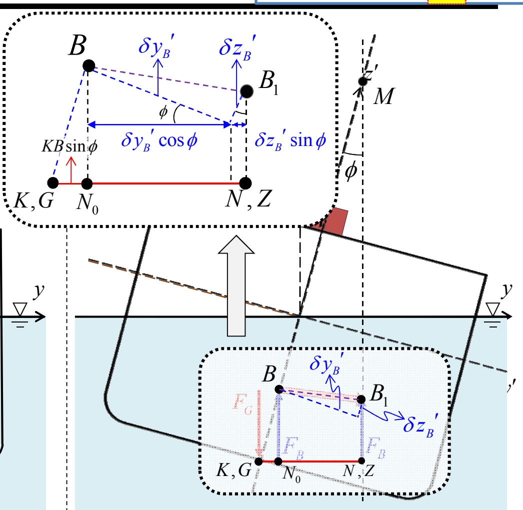
The angle of inclination can be measured when we perform the inclining test.

Derivation of Inclining Test Formula (1/5)

$\tau_r = F_B \cdot GZ$
$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)
$GM = KB + BM - KG$

Suppose that the center of gravity is located at K. Then the KN represents **the righting arm**.





$$GZ = KN$$
$$= KN_0 + N_0N$$
$$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$$

These terms have positive effect to the restoring moment arm.

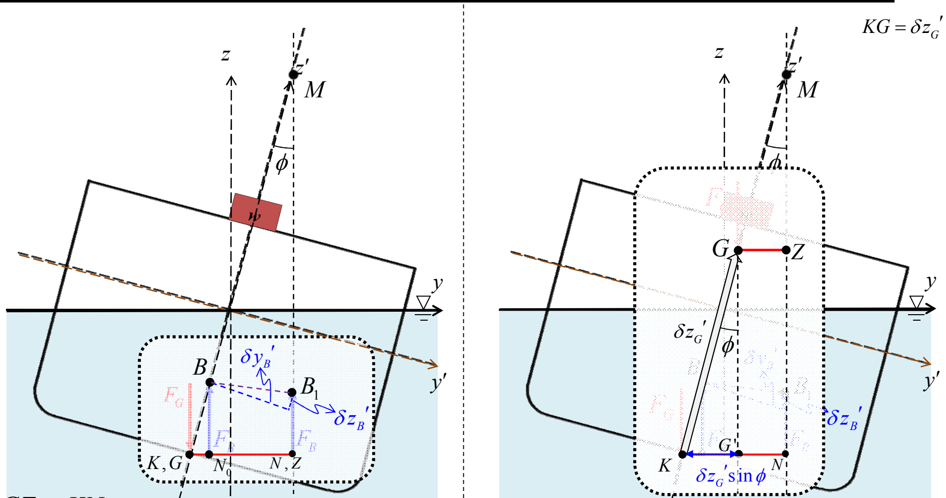
Derivation of Inclining Test Formula (2/5)

$\tau_r = F_B \cdot GZ$

$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)

$GM = KB + BM - KG$

$KG = \delta z_G'$



$GZ = KN$

$= KN_0 + N_0N$

$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$

$GZ = KN - KG'$

$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$

$-\delta z_G' \cdot \sin\phi$

This term has negative effect to the restoring moment arm.

Derivation of Inclining Test Formula (3/5)

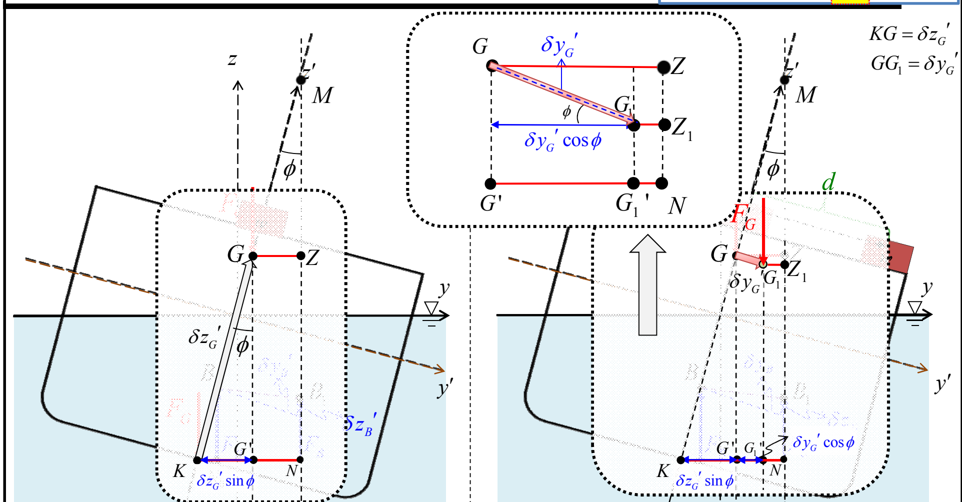
$\tau_r = F_B \cdot GZ$

$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)

$GM = KB + BM - KG$

$KG = \delta z_G'$

$GG_1 = \delta y_G'$



$GZ = KN - KG'$

$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$

$-\delta z_G' \cdot \sin\phi$

$G_1Z_1 = KN - KG' - G'G_1'$

$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$

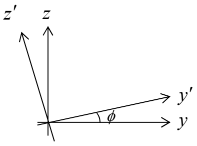
$-\delta z_G' \cdot \sin\phi$ $-\delta y_G' \cdot \cos\phi$

This term has negative effect to the restoring moment arm.

Derivation of Inclining Test Formula (4/5)

$\tau_r = F_B \cdot GZ$
$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)
$GM = KB + BM - KG$

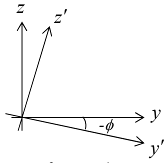
✓ Rotational transformation matrix (b to n frame)



$${}^{n}\mathbf{r}_P = {}^{n}\mathbf{R}_b \, {}^{b}\mathbf{r}_P$$

$$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} = {}^{n}\mathbf{R}_b(\phi)$$

✓ Rotational transformation matrix (b to n frame)



$$\begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} = {}^{n}\mathbf{R}_b(-\phi)$$

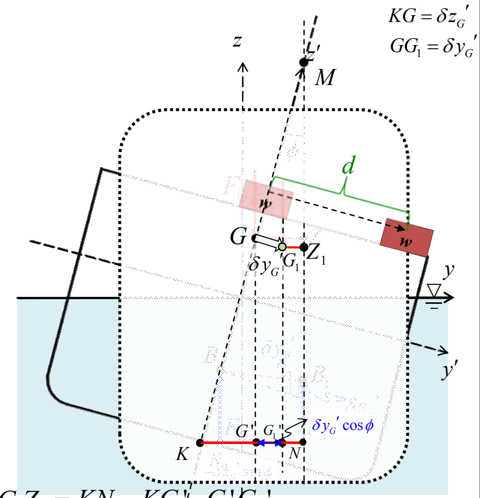
✓ In n-frame, because the forces are acting on vertical direction, moment arm is $\mathbf{y_n}$**-component**.

$$\begin{bmatrix} \delta y_B \\ \delta z_B \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \delta y_B' \\ \delta z_B' \end{bmatrix}$$

⇨ $\delta y_B = \delta y_B' \cos\phi + \delta z_B' \sin\phi$

⇨ $\delta y_G = \delta y_G' \cos\phi + \delta z_G' \sin\phi$

$KG = \delta z_G'$
$GG_1 = \delta y_G'$



$$G_1Z_1 = KN - KG' - G'G_1'$$
$$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$$
$$- \delta z_G' \cdot \sin\phi - \delta y_G' \cdot \cos\phi$$

(δy_B; $-\delta y_G$)

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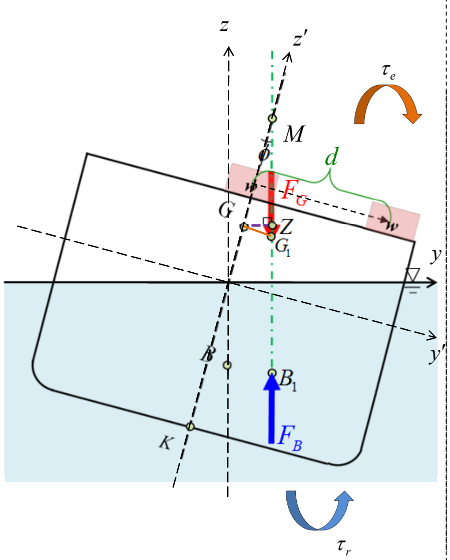
Derivation of Inclining Test Formula (5/5)

$KG = \delta z_G'$ $\quad GG_1 = \dfrac{w \cdot d}{F_G}$
$GG_1 = \delta y_G'$

$\tau_r = F_B \cdot GZ$
$GZ \approx GM \cdot \sin\phi$ (at small angle ϕ)
$GM = KB + BM - KG$



Calculation of moment using "GZ"

$$G_1Z_1 = KN - KG' - G'G_1'$$
$$= KB \cdot \sin\phi + \delta y_B' \cdot \cos\phi + \delta z_B' \cdot \sin\phi$$
$$- \delta z_G' \cdot \sin\phi - \delta y_G' \cdot \cos\phi$$
$$= \underline{KN - KG\sin\phi} - GG_1\cos\phi$$

$= GZ$
$= GM \sin\phi$: Assume that $\phi << 1$

$$= GM\sin\phi - GG_1\cos\phi$$

Moment produced by total weight & buoyant force

$$\delta M = \Delta \cdot G_1Z_1$$
$$= \Delta \cdot (GM\sin\phi - GG_1\cos\phi)$$

Static equilibrium of moment

$$GM\sin\phi = GG_1\cos\phi$$

$$\therefore GM = \frac{GG_1}{\tan\phi} = \frac{w \cdot d}{F_G \cdot \tan\phi}$$: **Inclining test formula**

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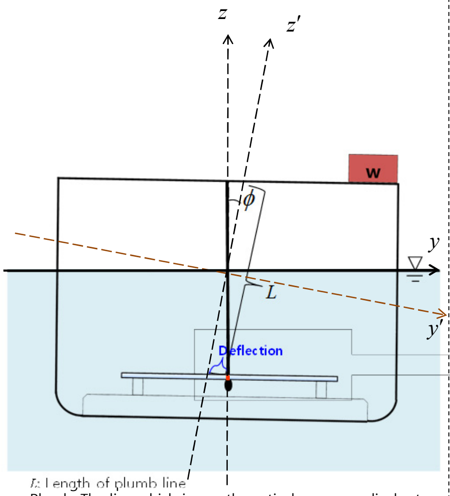
Method of Measuring the Angle of Inclination (1/2)

$GM = \frac{w \cdot d}{F_G \cdot \tan\phi}$ **Inclining test formula**

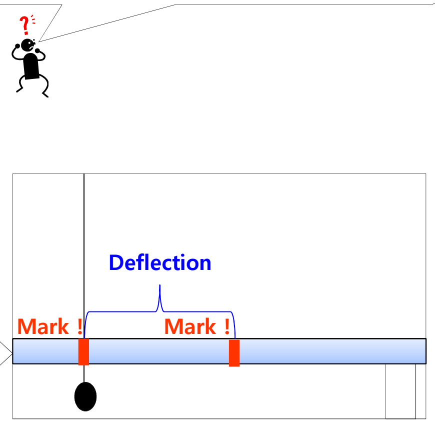


L: Length of plumb line

Plumb: The line which is exactly vertical or perpendicular to a level horizontal line

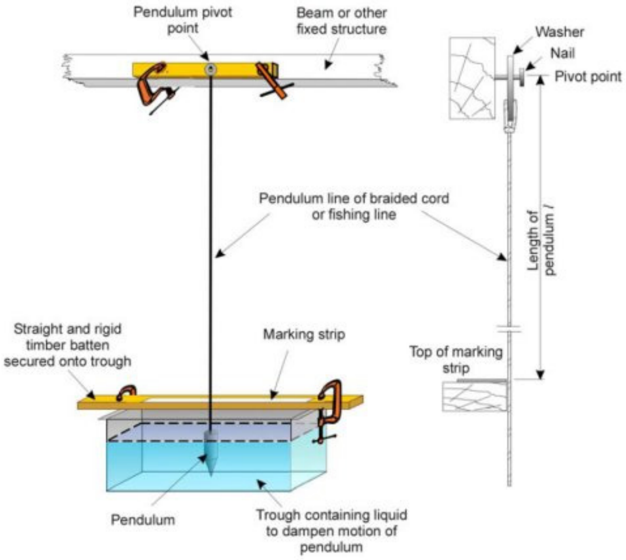
Batten: Long and thin strip of wood

How can you measure the angle of inclination when you perform the inclining test?



$$\therefore \tan\phi = \frac{\text{Deflection}}{L}$$

Method of Measuring the Angle of Inclination (2/2)



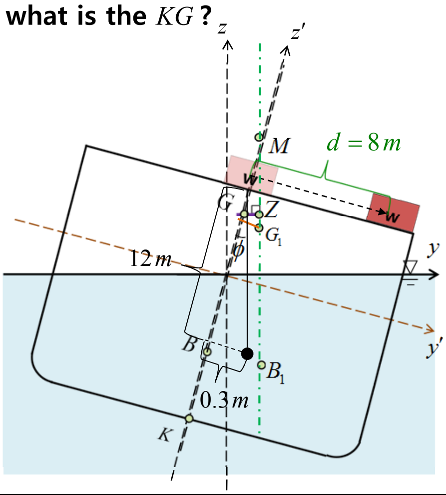
* Reference: http://www.mchl.fr

Example

$GM = \frac{w \cdot d}{F_G \cdot \tan\phi}$ Inclining test formula

$KG = KB + BM - GM$

A ship is inclined by moving a weight of *40 tons* a distance *8 m* from the center line. A *12 m* pendulum shows a deflection of *0.3 m*. Displacement of the ship is *3,700 tons*. If the KB is *5 m* and BM is *14 m*, what is the KG?



Solution)

$$\tan\phi = \frac{0.3}{12} = 0.025$$

$$GM = \frac{w \cdot d}{F_G \cdot \tan\phi} = \frac{40 \cdot 8}{3700 \cdot 0.025} = 3.46$$

$$KG = KB + BM - GM$$
$$= 5 + 14 - 3.46$$
$$= 14.54$$

$$\therefore KG = 14.54[m]$$

Precautions to be Taken During the Inclining Test

Certain precautions must be taken to ensure that accuracy is obtained.

(1) The ship must be floating upright and freely without restraint from ropes.

(2) There should be no wind on the beam.

(3) All loose weights should be fixed.

(4) All cross-connections between tanks should be closed.

(5) Tanks should be empty or pressed full. If neither of these conditions is possible, the level of liquid in the tank should be such that the free surface effect is readily calculable and will remain sensibly constant through the experiment.

(6) The number of men on board should be kept to a minimum and they should be on the center line.

(7) Any mobile equipment used to move the weights across the deck must return to a known positions for each set of readings.

Various Problems Using the Inclining Test Formula

$GM = \frac{w \cdot d}{F_G \cdot \tan\phi}$ Inclining test formula

The inclining test formula can be used in various problems as follows:

(1) To find the angle of heel ϕ, a ship will take by moving a weight a transverse distance d.

$$\tan\phi = \frac{w \cdot d}{GM \cdot F_G}$$

(2) To find the weight w necessary to remove or produce a heel by moving it a transverse distance d.

$$w = \frac{GM \cdot \tan\phi \cdot F_G}{d}$$

(3) To find the distance d necessary to move a weight in order to remove or produce a heel.

$$d = \frac{GM \cdot \tan\phi \cdot F_G}{w}$$