

Interaction Control of Robotic Manipulators

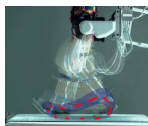
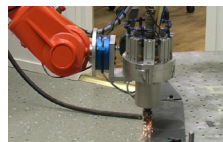
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Interaction Control



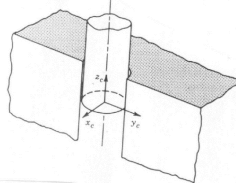
- Many robot tasks require physical interaction (i.e., via force-velocity with power-exchange) with environment, object, robot, human, etc.
- Peg-in-hole, assembly, deburring, walking, tactile exploration.
- Surgical robots, exoskeleton, rehabilitation robots.
- Telemanipulation, multirobot cooperative manipulation.

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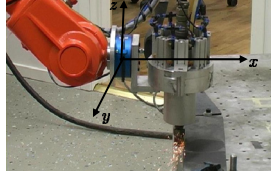
Natural and Artificial Constraints

peg-in-hole assembly



natural	artificial
$V_x=0$	$F_x=0$
$V_y=0$	$F_y=0$
$F_z=0$	$V_z=V_d$
$W_x=0$	$T_x=0$
$W_y=0$	$T_y=0$
$T_z=0$	$W_z=W_d$

deburring operation



natural	artificial
$V_x=0$	$F_x=F_d$
$F_y=0$	$V_y=V_d$
$F_z=0$	$V_z=0$
$T_x=0$	$W_x=0$
$W_y=0$	$T_y=0$
$T_z=0$	$W_z=W_d$

- Robot motion directions are decomposed into **position-controlled direction** and **wrench-controlled directions**.
- Rigid (i.e., stiff/high-impedance) control for position-controlled direction to precisely track desired motion command.
- Compliant (i.e., soft/low-impedance) control for force-controlled direction to avoid excessive build-up of contact force.
- **Impedance/admittance control**: impose desired dynamics behavior between robot and environment (e.g., asymmetric impedance/compliance).
- **Hybrid position-force control**: decouple force-control and position-control directions and control them separately.

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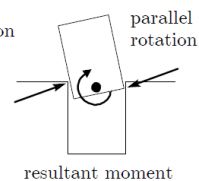
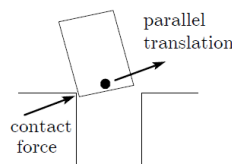
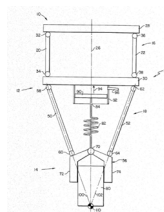
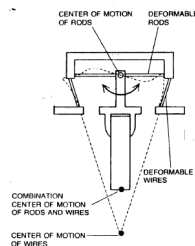
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Remote Compliance Center

- Remote compliance center (RCC): point where linear stiffness and rotational stiffness are decoupled, i.e.,

$$F = \begin{pmatrix} f \\ \tau \end{pmatrix} \approx \begin{bmatrix} K_T & 0 \\ 0 & K_R \end{bmatrix} \begin{pmatrix} \Delta x \\ \Delta \phi \end{pmatrix}$$

- This RCC point can be located at the contact tip by adjusting the geometric design and relative stiffnesses.
- At RCC, contact force causes only translation with no rotation; contact torque causes only rotation with no translation.
- RCC is equivalent to elastic center in beam theory.

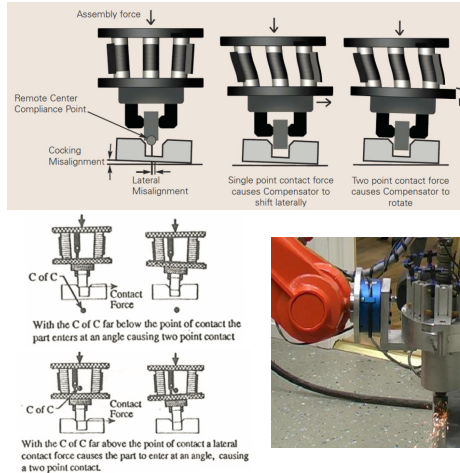
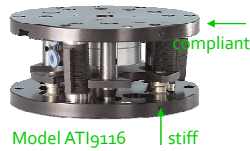


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Passive Compliance Control

- Passive compliance control utilizes RCC to achieve peg-in-hole task while avoiding jamming via sequential transition from lateral translation and aligning rotation (all mechanical, thus, very fast/rugged).

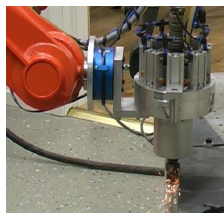


- Active compliance control utilizes F/T sensor and actuation to emulate the desired compliance (yet, with sensing/control delay).

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Network Representation



- Joint-space robot dynamics:

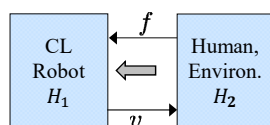
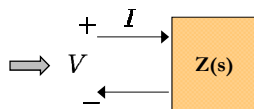
$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + J^T f_e$$

- Workspace robot dynamics:

$$D(q)\ddot{x} + Q(q, \dot{x})\dot{x} + g_x(q) = u + f_e, \quad \tau = J^T(q)u$$

Want to achieve desired workspace **dynamic behavior**.

- From mechanical-electrical analogy,
 - velocity \approx current (flow); force \approx voltage (effort)
- We may control robot to behave with different causality:



- **Impedance:** flow-input, effort-output (e.g., spring)

$$F = Z(s)V \approx V = Z(s)I$$

- **Admittance:** effort-input, flow-output (e.g., inertia)

$$V = A(s)F \approx I = A(s)V$$

- Can't control both force and position at the same time.

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Impedance Control

- Workspace robot dynamics:

$$D(q)\ddot{x} + Q(q, \dot{q})\dot{x} + g_x(q) = u + f_e$$

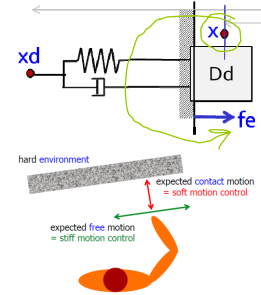
- Desired dynamics behavior: with $\tilde{x} = x - x_d$,

$$D_d\ddot{\tilde{x}} + B_d\dot{\tilde{x}} + K_d\tilde{x} = f_e$$

- Mimic human-arm motion behavior:

- Compliant/slow control along force-control axis: small K_d , large D_d .
- Fast/stiff control along position-control axis: large K_d , small D_d .
- B_d to shape transient behavior.
- Smooth transition from motion control to force control.

- Motion input, force output: force f_e generated by initiating motion \tilde{x} via the specified desired impedance.
- For impedance control, the robot should be **backdrivable** with low friction (i.e., perceive friction instead of desired impedance) and low backlash (i.e., motion but no force).



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Impedance Control

- Workspace robot dynamics: $D(q)\ddot{x} + Q(q, \dot{q})\dot{x} + g_x(q) = u + f_e$.

- Desired impedance: $D_d\ddot{\tilde{x}} + B_d\dot{\tilde{x}} + K_d\tilde{x} = f_e$.

- Feedback linearization (or inverse dynamics):

$$u = Q(q, \dot{q})\dot{x} + g_x(q) - f_e + D(q)a_x$$

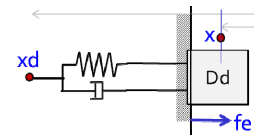
so that $\tilde{x} = a_x$. Thus, the desired acceleration $a_x \in \mathbb{R}^n$ is designed s.t.,

$$a_x = \ddot{x}_d - D_d^{-1}[B_d\dot{\tilde{x}} + K_d\tilde{x} + f_e]$$

- Total impedance control:

$$u = Q(q, \dot{q})\dot{x} + g_x(q) - f_e + D(q)[\ddot{x}_d - D_d^{-1}(B_d\dot{\tilde{x}} + K_d\tilde{x}) + f_e]$$

- Kinetic energy shaping: $\frac{1}{2}\dot{x}^T D(q)\dot{x}$ to $\frac{1}{2}\dot{\tilde{x}}^T D_d\dot{\tilde{x}}$. This kinetic energy shaping (or inertia scaling) requires force sensing (cf. $D_d = D(q)$).
- Potential energy shaping: $V_g(q)$ to $\frac{1}{2}\tilde{x}^T K_d\tilde{x}$. This can be done even without force sensing.



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Admittance Control

- Workspace robot dynamics:

$$D(q)\ddot{x} + Q(q, \dot{q})\dot{x} + g_x(q) = u + f_e$$

- Desired dynamics behavior: with reference position x_r ,

$$D_d(\ddot{x}_r - \ddot{x}_d) + B_d(\dot{x}_r - \dot{x}_d) + K_d(x_r - x_d) = f_e$$

- Admittance causality: force input, motion output

1. Measure interaction force f_e .
2. Compute x_r by **simulating** the desired dynamics.
3. Low-level control to drive $x \rightarrow x_r$ robustly.

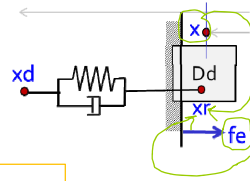
- Free motion: with $f_e = 0$, $x \rightarrow x_r \rightarrow x_d$ regardless of friction, inertia, etc.

- Contact control: behaves similar to the case of impedance control.

- Admittance control based on feedback linearization:

$$u = Q(q, \dot{q})\dot{x}(q) + g_x(q) - f_e + D(q)[\ddot{x}_r - B_d(\dot{x} - \dot{x}_r) - K_d(x - x_r)]$$

to ensure $x \rightarrow x_r$, where x_r is the output from the simulation.



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Compliance Control

Desired dynamics behavior:

$$D(q)\ddot{x} + Q(q, \dot{q})\dot{x} + B_d\dot{x} + K_d(x - x_d) = f_e$$

where K_d^{-1} is desired compliance with intrinsic inertia $D(q)$ intact.

- Impedance control: with $\dot{x}_d = 0$ and $D_d = D(q)$,

$$u = g_x(q) - B_d\dot{x} - K_d(x - x_d)$$

where force sensing is not necessary with no kinetic energy shaping.

- Admittance control: measure f_e and simulate x_r by integrating

$$D(q)\ddot{x}_r + Q(q, \dot{q})\dot{x}_r + B_d\dot{x}_r + K_d(x_r - x_d) = f_e$$

Then, control x to track this x_r (e.g., robust control).

- Impedance control: robot must be backdrivable; low inertia/friction/backlash; force sensing may not be necessary.
- Admittance control: robot can have large friction/inertia; interaction with even small force possible; only slow interaction; force sensor necessary.



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Joint Torque Sensing

- Impedance control desired for interaction, yet, requires backdrivability.
- For safety, robots need to have low inertia and detect whole-body collision.
- **Direct-driven robot** (strong motors with no gear reduction): difficult to make in small form-factor and light weight for safety.
- **Typical multi-DOF arm** (small motors with high gear reduction): small inertia/form-factor, yet, not backdrivable w/ high friction.
- **Joint torque sensing:**
 - Joint torque feedback to address poor backdrivability of high-reduction motors, while also reducing apparent motor inertia.
 - Whole-arm collision detection possible for safety.
 - Flexibility due to joint torque sensing needs to be addressed via control.

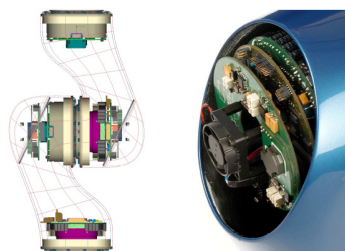
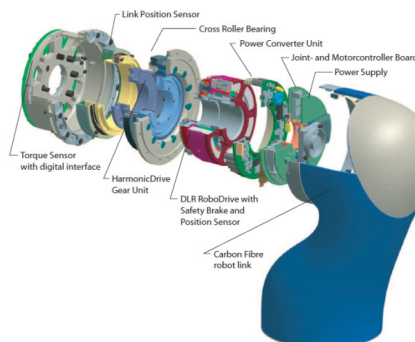


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DLR LWR III

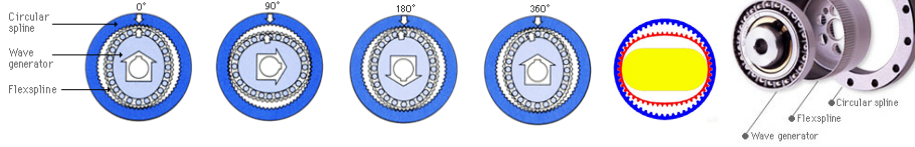
- Invented by DLR, commercialized by KUKA.
- 7-DOF with DLR RoboDrive DC brushless motors.
- Light weight 15kg arms with 1.5m workspace and 15kg payload.
- Harmonic drive (high torque/precision) with strain gauge torque sensing.
- Motor position encoder, link position potentiometer.
- 3kHz low-level control servo-rate; 1kHz high-level control servo-rate.



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Harmonic Drives



- Based upon metal elastic dynamics and flexibility (Walton Musser 1955).
- Wave generator: input shaft attached to elliptical cam with thin-raced ball bearings fitted onto its periphery.
- Flexspline: thin-wall steel circular cup, with output shaft attached on its diaphragm and n gear teeth machined on its outer surface, experience elastic deformation.
- Circular spline: rigid steel ring, attached to casing, with $n + 2$ teeth on its inner diameter.
- Advantages: high torque capacity w/ high reduction ($\approx 1/500$); precise positioning w/ no backlash; compact, light, easy assembly; efficient, quiet.
- Disadvantages: high friction, nonlinear torsional compliance with hysteresis at reversal points.

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https://www.hds.co.jp/english/products/hd_theory/

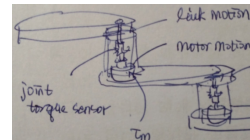


DLR LWR Dynamics

- Dynamics of DLR LWR with joint elasticity:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + DK^{-1}\dot{\tau} + \tau_{ext}$$

$$B\ddot{\theta} + \tau + DK^{-1}\dot{\tau} = \tau_m - \tau_f, \quad \tau = K(\theta - q)$$



where $q, \theta \in \mathbb{R}^n$ are link and motor angles, τ_m, τ_f motor torque command and friction; τ_{ext} external disturbance; $B, D, K \in \mathbb{R}^{n \times n}$ are diagonal mass, and joint damping/stiffness (cf: VSA, flexible robot \rightarrow under-actuation).

- Suppose we want to control link positions $q \rightarrow q_d$. Then, in steady-state,

$$g(q) = \tau = K(\theta - q) = \tau_m$$

suggesting $\theta_d = q_d + K^{-1}g(q_d)$ with $\tau_m \rightarrow g(q_d) = K(\theta_d - q_d) = \tau$.

- For typical robot only with **motor encoders**, we can implement the simple control τ_m s.t.,

$$\tau_m = -K_d\dot{\theta} - K_p(\theta - \theta_d) + g(q_d) + \hat{\tau}_f$$

for $\theta \rightarrow \theta_d$, thereby, $q \rightarrow q_d$, which yet often produces excessive joint vibration due to joint flexibility (cf., input shaping).

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DLR LWR Motion Control

- Dynamics of DLR LWR with joint elasticity:

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) &= \tau + DK^{-1}\dot{\tau} + \tau_{ext} \\ B_d\ddot{\theta} + \tau + DK^{-1}\dot{\tau} &= \tau_m - \tau_f, \quad \tau = K(\theta - q) \end{aligned}$$

- Low-level control w/ **joint torque feedback** (S/G):

$$\tau_m = BB_d^{-1}u + (I - BB_d^{-1}) \cdot (\tau + DK^{-1}\dot{\tau})$$

with $u \in \mathbb{R}^n$ high-level control. Closed-loop motor dynamics is then:

$$B_d\ddot{\theta} + \tau + DK^{-1}\dot{\tau} = u + B_dB^{-1}\tau_f$$

with inertia shaping B_d and friction scaling $B_d < B$.

- High-level link position stabilization:

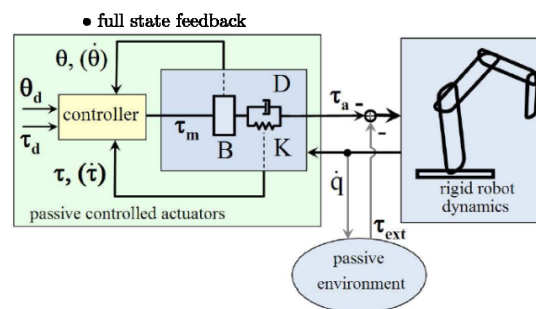
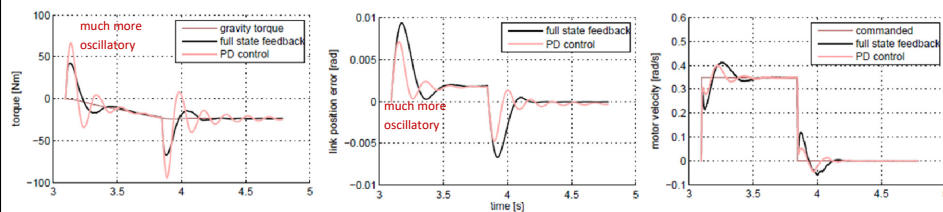
$$u = -K_d\dot{\theta} - K_p(\theta - \theta_d) + g(q_d)$$

- In contrast to previous one, this is **full state feedback** w/ $(\tau, \dot{\tau})$. Reduced motor inertia & friction also desirable for safety/performance (e.g., $B_d \rightarrow 0 \approx$ no flexibility).

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DLR LWR Motion Control



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DLR LWR Impedance Control

- Dynamics of DLR LWR with joint elasticity:

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) &= \tau + DK^{-1}\dot{\tau} + \tau_{ext} \\ B_d\ddot{\theta} + \tau + DK^{-1}\dot{\tau} &= u + B_dB^{-1}\tau_f, \quad \tau = K(\theta - q) \end{aligned}$$

- Workspace impedance control: with $x = f(q) \in \mathbb{R}^6$ as EF pose,

$$u = -J^T(q)[K_d\dot{x} + K_p(x(q) - x_d)] + g(q)$$

where $J(q) = \frac{\partial f(q)}{\partial q}$. Then, in steady-state at equilibrium (θ_o, q_o) ,

$$g(q_o) = K(\theta_o - q_o) + J^T F_{ext}, \quad K(\theta_o - q_o) = -J^T K_p \tilde{x} + g(q_o)$$

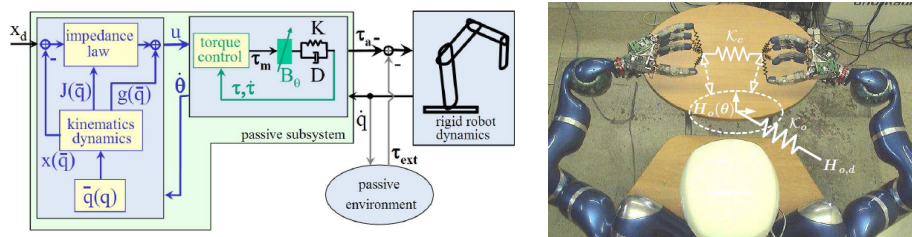
i.e., desired compliance achieved with $F_{ext} = K_p \tilde{x}$.

- Joint-space impedance control: $u = -K_d\dot{\theta} - K_p(q - q_d) + g(q_o)$. At steady-state equilibrium: $g(q_o) = K(\theta_o - q_o) + \tau_{ext}$ and $K(\theta_o - q_o) = -K_p(q_o - q_d) + g(q_o)$, implying desired stiffness achieved with $\tau_{ext} = K_p(q_o - q_d)$.
- Instead of q, \dot{q} , DLR uses $\bar{q}(\theta)$ and $\dot{\theta}$ to enforce closed-loop **passivity** for robust interaction stability with unknown environment.

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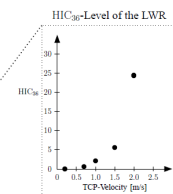
DLR LWR Impedance Control



- Collision safety by stopping actuation when measured joint torque exceeds limit or collision is detected by using τ_{ext} observer with dynamics model and τ -measurement.



Injury Level	HIC Level
Very high	1000
High	20% AIS ≥ 1
Medium	serious, but not life-threatening
Low	5% AIS ≥ 1
Very low	650



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