Chapter 11

Introduction to Analytical Photogrammetry

Elements of Photogrammetry with Applications in GIS

Wolf, Paul R.; Wolf, Paul R.; DeWitt, Bon A.; DeWitt, Bon A.; Wilkinson, Benjamin E.; Wilkinson, Benjamin E.. Elements of Photogrammetry with Application in GIS, Fourth Edition McGraw-Hill Education. Kindle Edition.

1. Introduction

- Analytical photogrammetry (해석적 사진측정학): mathematical calculation of coordinates of points in object space based upon camera parameters, measured photo coordinates, and ground control

- Analytical photogrammetry forms the basis of many modern hardware and software systems, including: stereoplotters (analytical and softcopy), digital terrain model generation, orthophoto production, digital photo rectification, and aerotriangulation.

- This chapter presents an introduction to some fundamental topics and elementary applications in analytical photogrammetry.

2. Image Measurements

- A fundamental type of measurement used in analytical photogrammetry is an x and y photo coordinate pair.
- The analytical photogrammetry assumes that light rays are straight and focal plane of a frame camera is flat. This requires various correction of measured coordinates.
- In many analytical photogrammetry methods, it is necessary to measure image coordinates of common object points that appear in more than one photograph.

3. Control Points

- Object space coordinates of ground control points are generally required for analytical photogrammetry (A.P. afterward) where GPS is frequently employed.

4. Collinearity Condition

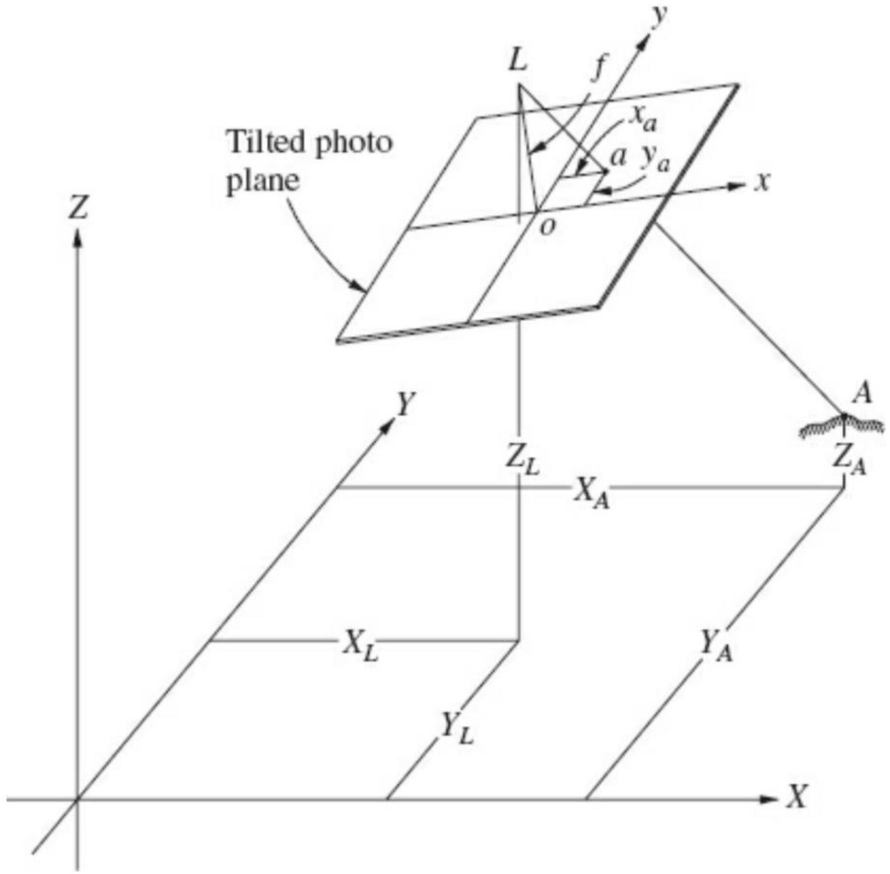

FIGURE 11-1 The *collinearity* condition.

- Collinearity condition is the most fundamental and useful relationship in A.P.

- Collinearity is the condition that the exposure station, any object point, and its photo image all lie along a straight line in 3D space.

C-7 3D Conformal Coordinate Transformation

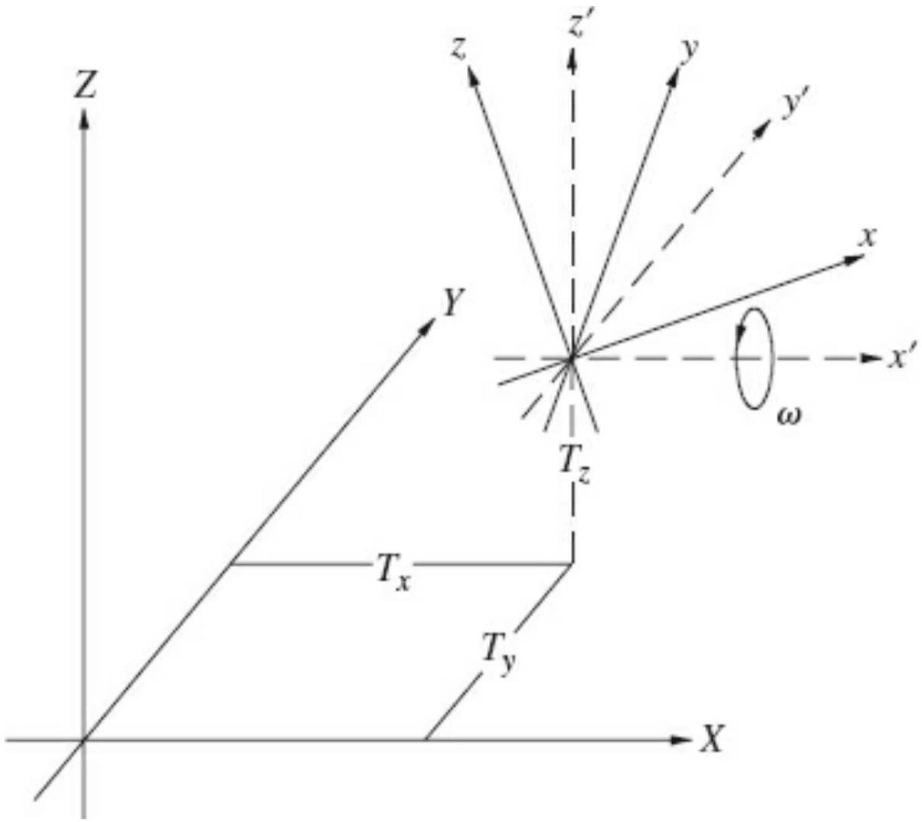

Figure C-6 *XYZ* and *xyz* right-handed three-dimensional coordinate systems.

- Transformation equations are developed in two steps: (1) rotation and (2) scaling and translation

- Step 1: Rotation from $x'\ y'\ z'$ to $x\ y\ z$ system

  1) ω rotation about $x'(x_1)$ axis:
  $y' \rightarrow y_1,\ z' \rightarrow z_1$

  2) ϕ rotation about $y_1(y_2)$ axis:
  $x'(x_1) \rightarrow x_2,\ z_1 \rightarrow z_2$

  3) κ rotation about $z_2(z)$ axis:
  $x_2 \rightarrow x,\ y_2 \rightarrow \mathrm{y}$

C-7 3D Conformal Coordinate Transformation

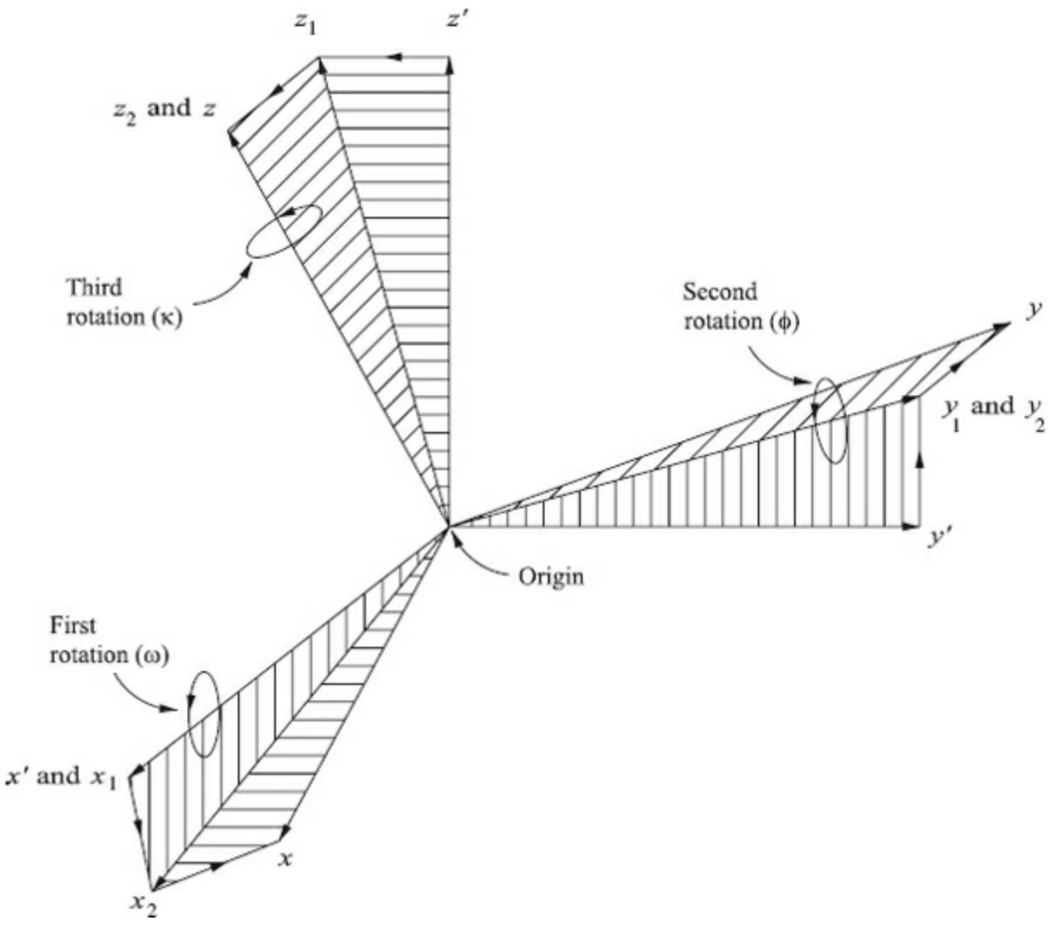

Figure C-7 The three sequential angular rotations.

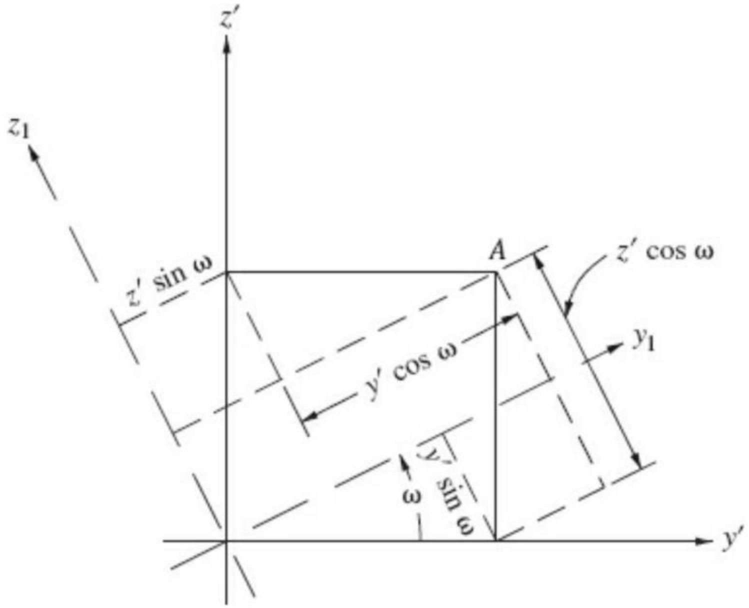

Figure C-8 Omega rotation about the x' axis.

$$x_1 = x'$$

$$y_1 = y' \cos \omega + z' \sin \omega$$

$$z_1 = -y' \sin \omega + z' \cos \omega$$

C-7 3D Conformal Coordinate Transformation

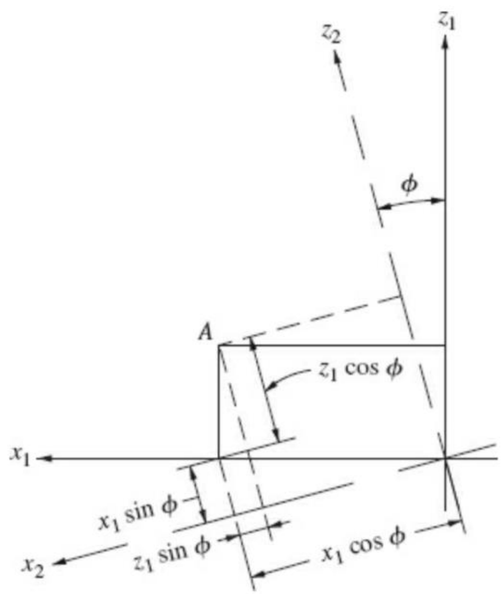

FIGURE C-9 Phi rotation about the y_1 axis.

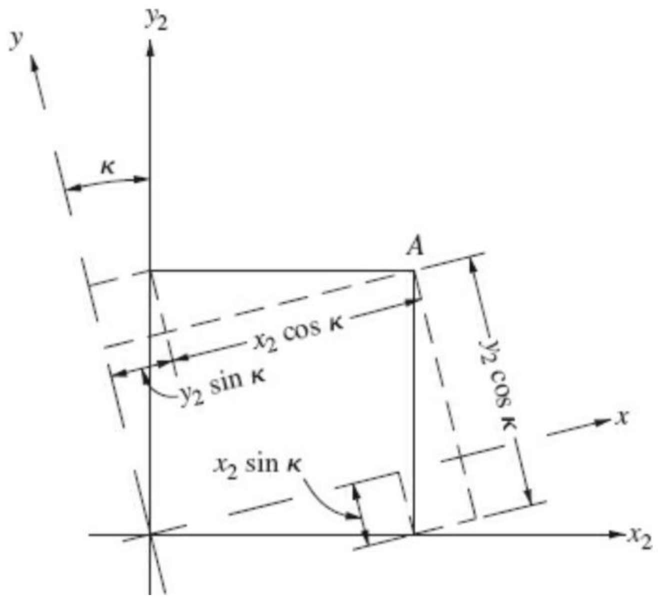

FIGURE C-10 Kappa rotation about the z_2 axis.

$$
\begin{aligned}
x_2 &= -z_1 \sin \phi + x_1 \cos \phi \\
&= (-y' \sin \omega + z' \cos \omega) \sin \phi + x' \cos \phi \\
y_2 &= y_1 = y' \cos \omega + z' \sin \omega \\
z_2 &= z_1 \cos \phi + x_1 \sin \phi \\
&= (-y' \sin \omega + z' \cos \omega) \cos \phi + x_1 \sin \phi
\end{aligned}
$$

$$
\begin{aligned}
x &= x_2 \cos \kappa + y_2 \sin \kappa \\
y &= -x_2 \sin \kappa + y_2 \cos \kappa \\
z &= z_2
\end{aligned}
$$

C-7 3D Conformal Coordinate Transformation

$$x = x'(\cos\phi\cos\kappa) + y'(\sin\omega\sin\phi\cos\kappa + \cos\omega\sin\kappa) + z'(-\cos\omega\sin\phi\cos\kappa + \sin\omega\sin\kappa)$$

$$y = x'(-\cos\phi\sin\kappa) + y'(-\sin\omega\sin\phi\sin\kappa + \cos\omega\cos\kappa) + z'(\cos\omega\sin\phi\sin\kappa + \sin\omega\cos\kappa)$$

$$z = x'(\sin\phi) + y'(-\sin\omega\cos\phi) + z'(\cos\omega\cos\phi)$$

$$\begin{aligned} x &= m_{11}x' + m_{12}y' + m_{13}z' \\ y &= m_{21}x' + m_{22}y' + m_{23}z' \\ z &= m_{31}x' + m_{32}y' + m_{33}z' \end{aligned} \quad \rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = M \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$M = \begin{bmatrix} \cos xx' & \cos xy' & \cos xz' \\ \cos yx' & \cos yy' & \cos yz' \\ \cos zx' & \cos zy' & \cos zz' \end{bmatrix}$$

C-7 3D Conformal Coordinate Transformation

The rotation matrix is an orthogonal matrix, which has the property that its inverse is equal to its transpose:

$$M^{-1} = M^{T}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \rightarrow \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{21} & m_{31} \\ m_{12} & m_{22} & m_{32} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Step 2: Scaling (s) and Translation (T_x, T_y, T_z)

$$X = sx' + T_x = s(m_{11}x + m_{21}y + m_{31}z) + T_x$$
$$Y = sy' + T_y = s(m_{12}x + m_{22}y + m_{23}z) + T_y$$
$$Z = sz' + T_z = s(m_{13}x + m_{23}y + m_{33}z) + T_z$$

: 7 unknowns – s, ω, ϕ, κ, T_x, T_y, T_z

→ 3 points having x, y, z coordinates are required?

C-7 3D Conformal Coordinate Transformation

$\bar{X} = sM^T X + T$: Nonlinear in terms of the unknowns s, ω, ϕ, κ

→ Linearization by using a Taylor series expansion including only the first-order terms

$$X_p = \left(X_p\right)_0 + \left(\frac{\partial X_p}{\partial s}\right)_0 ds + \left(\frac{\partial X_p}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial X_p}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial X_p}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial X_p}{\partial T_x}\right)_0 dT_x + \left(\frac{\partial X_p}{\partial T_y}\right)_0 dT_y + \left(\frac{\partial X_p}{\partial T_z}\right)_0 dT_z$$

$$\rightarrow \left[X_p - (X_p)_0\right] + v_{X_p} = a_{11} ds + a_{12} d\omega + a_{13} d\phi + a_{14} d\kappa + a_{15} dT_x + a_{16} dT_y + a_{17} dT_z$$

$$Y_p = \left(Y_p\right)_0 + \left(\frac{\partial Y_p}{\partial s}\right)_0 ds + \left(\frac{\partial Y_p}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial Y_p}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial Y_p}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial Y_p}{\partial T_x}\right)_0 dT_x + \left(\frac{\partial Y_p}{\partial T_y}\right)_0 dT_y + \left(\frac{\partial Y_p}{\partial T_z}\right)_0 dT_z$$

$$\rightarrow \left[Y_p - (Y_p)_0\right] + v_{Y_p} = a_{21} ds + a_{22} d\omega + a_{23} d\phi + a_{24} d\kappa + a_{25} dT_x + a_{26} dT_y + a_{27} dT_z$$

$$Z_p = \left(Z_p\right)_0 + \left(\frac{\partial Z_p}{\partial s}\right)_0 ds + \left(\frac{\partial Z_p}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial Z_p}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial Z_p}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial Z_p}{\partial T_x}\right)_0 dT_x + \left(\frac{\partial Z_p}{\partial T_y}\right)_0 dT_y + \left(\frac{\partial Z_p}{\partial T_z}\right)_0 dT_z$$

$$\rightarrow \left[Z_p - (Z_p)_0\right] + v_{Z_p} = a_{31} ds + a_{32} d\omega + a_{33} d\phi + a_{34} d\kappa + a_{35} dT_x + a_{36} dT_y + a_{37} dT_z$$

C-7 3D Conformal Coordinate Transformation

$$a_{11} = m_{11}x_P + m_{21}y_P + m_{31}z_P$$

$$a_{12} = 0$$

$$a_{13} = [(-\sin\phi\cos\kappa)\,x_p + \sin\phi\sin\kappa\,(y_p) + \cos\phi\,(z_p)]\,s$$

$$a_{14} = (m_{21}x_P - m_{11}y_P)\,s$$

$$a_{15} = a_{26} = a_{37} = 1$$

$$a_{16} = a_{17} = a_{25} = a_{27} = a_{35} = a_{36} = 0$$

$$a_{21} = m_{12}x_P + m_{22}y_P + m_{32}z_P$$

$$a_{22} = (-m_{13}x_P - m_{23}y_P - m_{33}z_P)\,s$$

$$a_{23} = [(\sin\omega\, m_{11})\,x_P + (-\sin\omega\, m_{21})\,y_P + (\sin\omega\, m_{31})\,z_P]\,s$$

$$a_{24} = (m_{22}x_P - m_{12}y_P)\,s$$

$$a_{31} = m_{13}x_P + m_{23}y_P + m_{33}z_P$$

$$a_{32} = (m_{12}x_P + m_{22}y_P + m_{32}z_P)\,s$$

$$a_{33} = [(-\cos\omega\, m_{11})x_P + (-\cos\omega\, m_{21})y_P + (-\cos\omega\, m_{31})z_P]\,s$$

$$a_{34} = (m_{23}x_P - m_{13}y_P)\,s$$

D-3 Development of the Collinearity Condition Equations

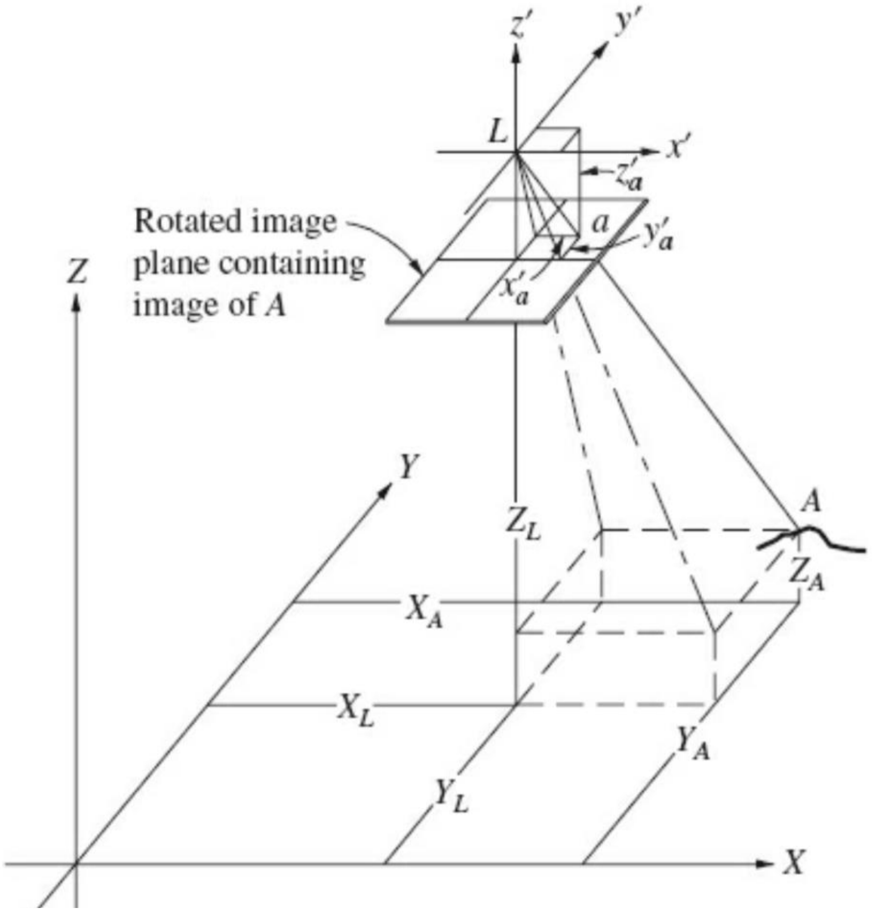

FIGURE D-2 Image coordinate system rotated so that it is parallel to the object space coordinate system.

- Collinearity condition is the most fundamental and useful relationship in A.P.
- The collinearity condition equations are developed from similar triangles of Figure D-2 as follows:

$$\frac{x'_a}{X_A - X_L} = \frac{y'_a}{Y_A - Y_L} = \frac{-z'_a}{Z_L - Z_A}$$

$$\rightarrow x'_a = \left(\frac{X_A - X_L}{Z_A - Z_L}\right) z'_a \,,\; y'_a = \left(\frac{Y_A - Y_L}{Z_A - Z_L}\right) z'_a$$

$$Z'_a = \left(\frac{Z_A - Z_L}{Z_A - Z_L}\right) z'_a$$

D-3 Development of the Collinearity Condition Equations

$$\begin{aligned} x &= m_{11}x' + m_{12}y' + m_{13}z' \\ y &= m_{21}x' + m_{22}y' + m_{23}z' \\ z &= m_{31}x' + m_{32}y' + m_{33}z' \end{aligned} \quad \rightarrow \quad \begin{aligned} x_a &= m_{11}x_a{}' + m_{12}y_a{}' + m_{13}z_a{}' \\ y_a &= m_{21}x_a{}' + m_{22}y_a{}' + m_{23}z_a{}' \\ z_a &= m_{31}x_a{}' + m_{32}y_a{}' + m_{33}z_a{}' \end{aligned}$$

$$x_a = m_{11}\left(\frac{X_A - X_L}{Z_A - Z_L}\right)z'_a + m_{12}\left(\frac{Y_A - Y_L}{Z_A - Z_L}\right)z'_a + m_{13}\left(\frac{Z_A - Z_L}{Z_A - Z_L}\right)z'_a \qquad (1)$$

$$y_a = m_{21}\left(\frac{X_A - X_L}{Z_A - Z_L}\right)z'_a + m_{22}\left(\frac{Y_A - Y_L}{Z_A - Z_L}\right)z'_a + m_{23}\left(\frac{Z_A - Z_L}{Z_A - Z_L}\right)z'_a \qquad (2)$$

$$z_a = m_{31}\left(\frac{X_A - X_L}{Z_A - Z_L}\right)z'_a + m_{32}\left(\frac{Y_A - Y_L}{Z_A - Z_L}\right)z'_a + m_{33}\left(\frac{Z_A - Z_L}{Z_A - Z_L}\right)z'_a \qquad (3)$$

- Dividing (1) and (2) by (3), substituting z_a with $-f$ and adding corrections for offset of the principal point (x_0, y_0) following collinearity equations are obtained:

D-3 Development of the Collinearity Condition Equations

$$x_a = x_0 - f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \qquad \text{(D-5)}$$

$$y_a = y_0 - f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \qquad \text{(D-6)}$$

D-5 Linearization of the Collinearity Equations

$$F = x_a = x_0 - f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] = x_0 - f\,\frac{r}{q} \qquad \text{(D-5')}$$

$$G = y_a = y_0 - f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] = y_0 - f\,\frac{s}{q} \qquad \text{(D-6')}$$

$$x_a = F_0 + \left(\frac{\partial F}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial F}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial F}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial F}{\partial X_L}\right)_0 dX_L + \left(\frac{\partial F}{\partial Y_L}\right)_0 dY_L + \left(\frac{\partial F}{\partial Z_L}\right)_0 dZ_L + \left(\frac{\partial F}{\partial X_A}\right)_0 dX_A + \left(\frac{\partial F}{\partial Y_A}\right)_0 dY_A + \left(\frac{\partial F}{\partial Z_A}\right)_0 dZ_A$$

$$\rightarrow J + v_{x_a} = b_{11}d\omega + b_{12}d\phi + b_{13}d\kappa - b_{14}dX_L - b_{15}dY_L - b_{16}dZ_L + b_{14}dX_A + b_{15}dY_A + b_{16}dZ_A$$

$$y_a = G_0 + \left(\frac{\partial G}{\partial \omega}\right)_0 d\omega + \left(\frac{\partial G}{\partial \phi}\right)_0 d\phi + \left(\frac{\partial G}{\partial \kappa}\right)_0 d\kappa + \left(\frac{\partial G}{\partial X_L}\right)_0 dX_L + \left(\frac{\partial G}{\partial Y_L}\right)_0 dY_L + \left(\frac{\partial G}{\partial Z_L}\right)_0 dZ_L + \left(\frac{\partial G}{\partial X_A}\right)_0 dX_A + \left(\frac{\partial G}{\partial Y_A}\right)_0 dY_A + \left(\frac{\partial G}{\partial Z_A}\right)_0 dZ_A$$

$$\rightarrow K + v_{y_a} = b_{21}d\omega + b_{22}d\phi + b_{23}d\kappa - b_{24}dX_L - b_{25}dY_L - b_{26}dZ_L + b_{24}dX_A + b_{25}dY_A + b_{26}dZ_A$$

D-5 Linearization of the Collinearity Equations

$$b_{11} = \frac{f}{q^2}[r(-m_{33}\,\Delta Y + m_{32}\,\Delta Z) - q(-m_{13}\,\Delta Y + m_{12}\,\Delta Z)]$$

$$b_{12} = \frac{f}{q^2}[r(\cos\phi\,\Delta X + \sin\omega\sin\phi\,\Delta Y - \cos\omega\sin\phi\,\Delta Z)$$
$$-q(-\sin\phi\cos\kappa\,\Delta X + \sin\omega\cos\phi\cos\kappa\,\Delta Y - \cos\omega\cos\phi\cos\kappa\,\Delta Z)]$$

$$b_{13} = \frac{-f}{q}(m_{21}\,\Delta X + m_{22}\,\Delta Y + m_{23}\,\Delta Z)$$

$$b_{14} = \frac{f}{q^2}(rm_{31} - qm_{11})$$

$$b_{15} = \frac{f}{q^2}(rm_{32} - qm_{12})$$

$$b_{16} = \frac{f}{q^2}(rm_{33} - qm_{13})$$

$$J = x_a - x_o + f\frac{r}{q}$$

D-5 Linearization of the Collinearity Equations

$$b_{21} = \frac{f}{q^2}[s(-m_{33}\,\Delta Y + m_{32}\,\Delta Z) - q(-m_{23}\,\Delta Y + m_{22}\,\Delta Z)]$$

$$b_{22} = \frac{f}{q^2}[s(\cos\phi\,\Delta X + \sin\omega\sin\phi\,\Delta Y - \cos\omega\sin\phi\,\Delta Z)$$
$$-q(\sin\phi\sin\kappa\,\Delta X - \sin\omega\cos\phi\sin\kappa\,\Delta Y + \cos\omega\cos\phi\sin\kappa\,\Delta Z)]$$

$$b_{23} = \frac{f}{q}(m_{11}\,\Delta X + m_{12}\,\Delta Y + m_{13}\,\Delta Z)$$

$$b_{24} = \frac{f}{q^2}(sm_{31} - qm_{21})$$

$$b_{25} = \frac{f}{q^2}(sm_{32} - qm_{22})$$

$$b_{26} = \frac{f}{q^2}(sm_{33} - qm_{23})$$

$$K = y_a - y_o + f\frac{s}{q}$$

5. Coplanarity Condition

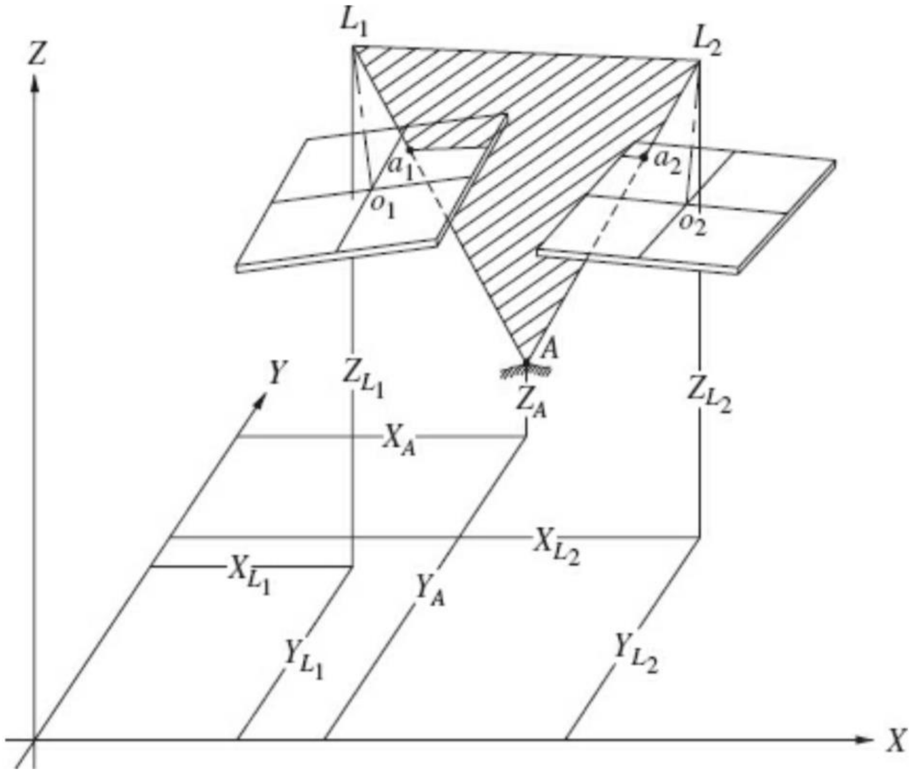

FIGURE 11-2 The *coplanarity* condition.

- Coplanarity, as illustrated in Fig. 11-2, is the condition that the two exposure stations of a stereopair, any object point, and its corresponding image points (L_1, L_2, a_1, a_2, and A) on the two photos all lie in a common plane.
- Coplanarity condition:

$$0 = B_X(E_1F_2 - E_2F_1) + B_Y(F_1D_2 - F_2D_1) + B_Z(D_1E_2 - D_2E_1)$$

Where, $B_X = X_{L_2} - X_{L_1}$, $B_Y = Y_{L_2} - Y_{L_1}$, $B_Z = Z_{L_2} - Z_{L_1}$

$$D = (m_{11})x + (m_{21})y - (m_{31})f$$

$$E = (m_{12})x + (m_{22})y - (m_{32})f$$

$$F = (m_{13})x + (m_{23})y - (m_{33})f$$

D-7 Development of the Coplanarity Condition Equation

$$\begin{bmatrix} D_1 \\ E_1 \\ F_1 \end{bmatrix} = M_1^T \begin{bmatrix} x_1 \\ y_1 \\ -f \end{bmatrix} = \begin{bmatrix} (m_{11})_1 x_1 + (m_{21})_1 y_1 - (m_{31})_1 f \\ (m_{12})_1 x_1 + (m_{22})_1 y_1 - (m_{32})_1 f \\ (m_{13})_1 x_1 + (m_{23})_1 y_1 - (m_{33})_1 f \end{bmatrix}$$

$$\begin{bmatrix} D_2 \\ E_2 \\ F_2 \end{bmatrix} = M_1^T \begin{bmatrix} x_2 \\ y_2 \\ -f \end{bmatrix} = \begin{bmatrix} (m_{11})_2 x_2 + (m_{21})_2 y_2 - (m_{31})_2 f \\ (m_{12})_2 x_2 + (m_{22})_2 y_2 - (m_{32})_2 f \\ (m_{13})_2 x_2 + (m_{23})_2 y_2 - (m_{33})_2 f \end{bmatrix}$$

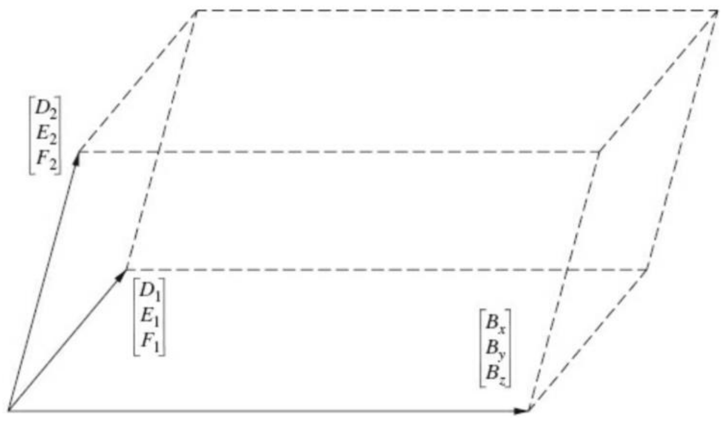

FIGURE D-4 Parallelepiped formed by three vectors used in the coplanarity condition equation.

$$\begin{vmatrix} B_X & B_Y & B_Z \\ D_1 & E_1 & F_1 \\ D_2 & E_2 & F_2 \end{vmatrix} = B_X(E_1F_2 - E_2F_1) + B_Y(F_1D_2 - F_2D_1) + B_Z(D_1E_2 - D_2E_1) = 0$$

6. Space Resection by Collinearity

- Space resection (후방교회법), a method of determining the six elements of exterior orientation (ω, ϕ, κ, X_L, Y_L, and Z_L) of a photograph.

- This method requires a minimum of three control points, with *known XYZ object space coordinates*, to be imaged in the photograph.

- Linearized forms of the space resection collinearity equations are:

$$b_{11}d\omega + b_{12}d\phi + b_{13}d\kappa - b_{14}dX_L - b_{15}dY_L - b_{16}dZ_L = J + v_{x_a}$$

$$b_{21}d\omega + b_{22}d\phi + b_{23}d\kappa - b_{24}dX_L - b_{25}dY_L - b_{26}dZ_L = K + v_{y_a}$$

[Example 11-1] A near-vertical aerial photograph taken with a 152.916-mm-focal-length camera contains images of four ground control points A through D. Refined photo coordinates and ground control coordinates (in a local vertical system) of the four points are listed in the following table. Calculate the exterior orientation parameters ω, ϕ, κ, X_L, Y_L, and Z_L for this photograph.

7. Space Intersection by Collinearity

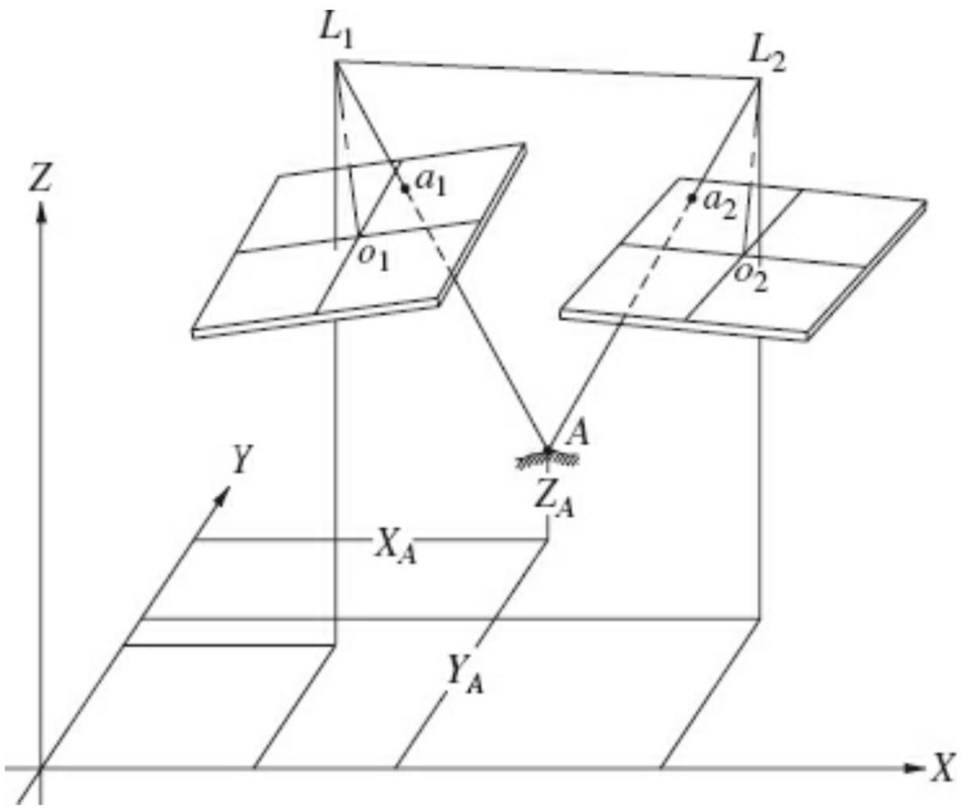

Figure 11-3 Space intersection with a stereopair of aerial photos.

- After the elements of exterior orientation for both photos of a stereopair are determined by the space resection, object point coordinates for points that lie in the stereo overlap area can be calculated using photo coordinates of a_1 and a_2.

$$b_{14}dX_A + b_{15}dY_A + b_{16}dZ_A = J + v_{x_a}$$
$$b_{24}dX_A + b_{25}dY_A + b_{26}dZ_A = K + v_{y_a}$$

$$b'_{14}dX_A + b'_{15}dY_A + b'_{16}dZ_A = J' + v'_{x_a}$$
$$b'_{24}dX_A + b'_{25}dY_A + b'_{26}dZ_A = K' + v'_{y_a}$$

7. Space Intersection by Collinearity

- Because the equations have been linearized using Taylor's theorem, initial approximations are required for each point whose object space coordinates are to be computed.
- For these calculations, vertical photos with normal aerial photography can be assumed, and the initial approximations can be determined by using the parallax equations where H and B can be taken as average of Z_{L_1} and Z_{L_2}, and $\sqrt{\left(X_{L_2} - X_{L_1}\right)^2 + \left(Y_{L_2} - Y_{L_1}\right)^2}$, respectively.

(Parallax equations: $h_A = H - \frac{Bf}{p_a}$, $X_A = B\,\frac{x_a}{p_a}$, $Y_A = B\,\frac{y_a}{p_a}$)

7. Space Intersection by Collinearity

- X and Y coordinates of point A from the parallax equations should be converted to X and Y ground system using such as below coordinate transformation equations:

$$X = ax' - by' + T_x$$
$$Y = ay' + bx' + T_y$$

, where a and b are of rotation and scaling, and T_x and T_y are of translation.

8. Analytical Stereomodel

- Adjacent two photographs having more than 50% of overlapped area form a stereopair where object points in the overlap area constitute a stereomodel.
- Analytical stereomodel are formed with 3D ground coordinates of points in the stereomodel mathematically calculated by analytical photogrammetric techniques.
- The analytical stereomodel is formed by three primary steps: *interior orientation*, *relative orientation*, and *absolute orientation*.
- The points in the stereomodel having object coordinates in the ground coordinate system can be used for many purposes such as digital mapping

9. Analytical Interior Orientation

- Interior orientation for analytical photogrammetry, commonly called photo coordinate refinement, is the step which mathematically recreates the geometry in the photograph based upon camera calibration information as well as atmospheric refraction effects.

- For both film and digital photography the lens distortion and principal-point information from camera calibration are then used to refine the coordinates so that they are correctly related to the principal point and free from lens distortion.

10. Analytical Relative Orientation

- Analytical relative orientation is the process of determining the relative angular attitude and positional displacement between the photographs that existed when the photos were taken.
- This involves defining certain elements of exterior orientation and calculating the remaining ones.
- It is common practice to fix the exterior orientation elements ω, ϕ, κ, X_L, and Y_L of the left photo of the stereopair to zero values. Also for convenience, Z_L of the left photo (Z_{L_1}) is set equal to *f*, and X_L of the right photo (X_{L_2}) is set equal to the photo base *b*.

10. Analytical Relative Orientation

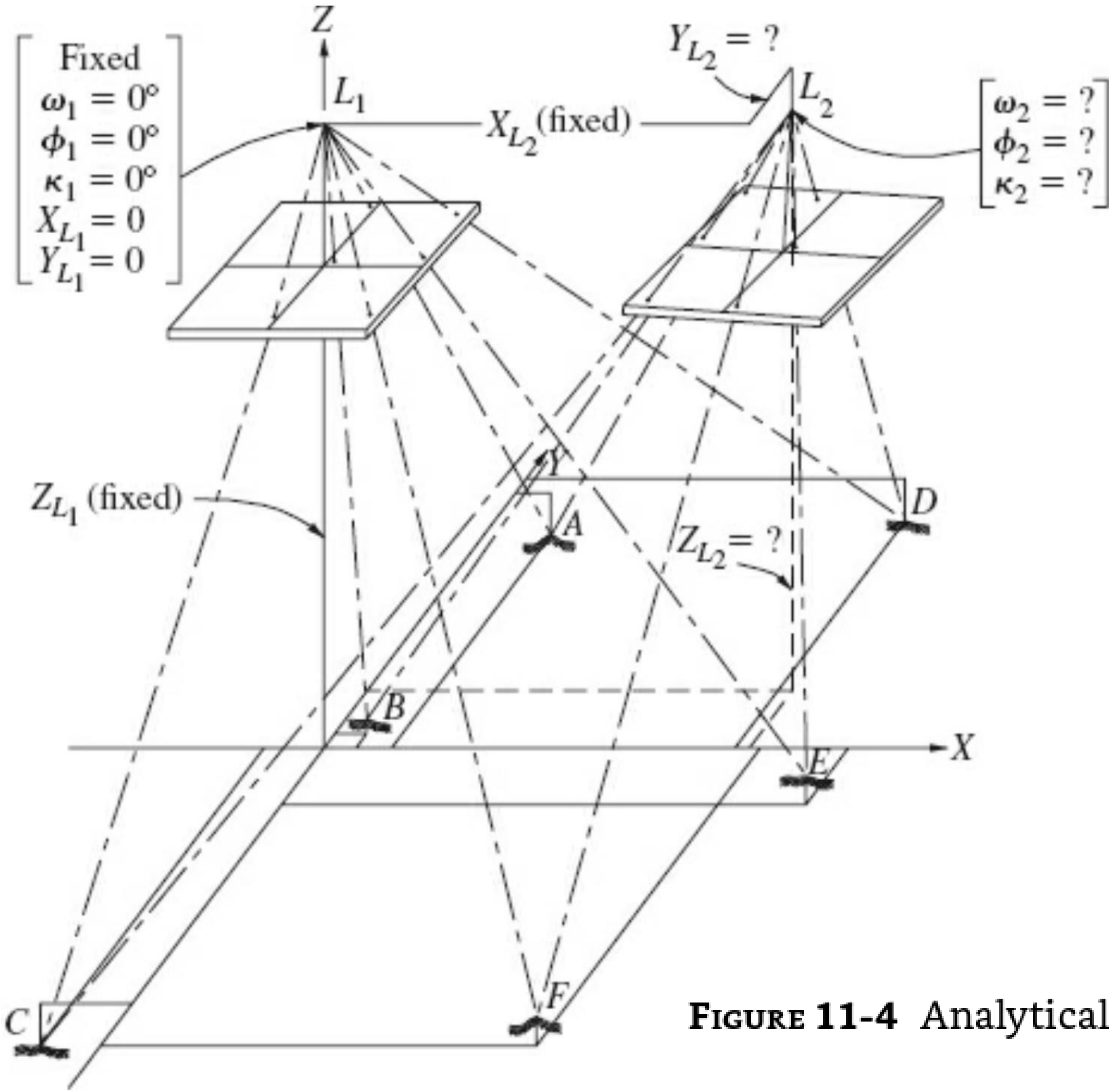

FIGURE 11-4 Analytical relative orientation of a stereopair.

11. Analytical Absolute Orientation

- After analytical relative orientation the relative coordinates of model points can be converted into absolute ones by 3D conformal coordinates transformation in a ground-based system
- Once the transformation parameters have been computed using control points, they can be applied to the remaining stereomodel points, including the X_L, Y_L, and Z_L coordinates of the left and right photographs. This gives the coordinates of all stereomodel points in the ground system.