Lecture Note of Naval Architectural Calculation

Ship Stability

Ch. 8 Curves of Stability and Stability Criteria

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Contents

Ch. 8 Curves of Stability and Stability Criteria

Introduction

In general, the document which contains the following list is submitted to ship owner and classification society, and get approval from them 9 months before steel cutting.

- Principle particulars
- General arrangement
- Midship section plan
- Lines plan
- Hydrostatic table
- Bonjean table
- Tank capacity table
- Light weight summary
- Allowable Minimum GM Curve
- Trim & stability calculation (Intact stability)
- Damage stability calculation
- Freeboard Calculation
- Visibility Check
- Equipment number calculation

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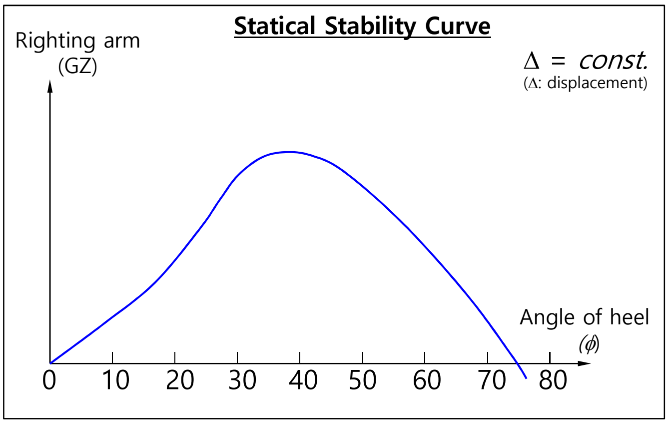
Today's main subject!

1. Statical Stability Curve

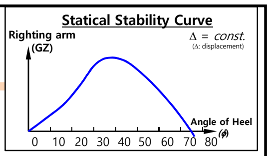
Definition and Purpose of the Statical Stability Curve



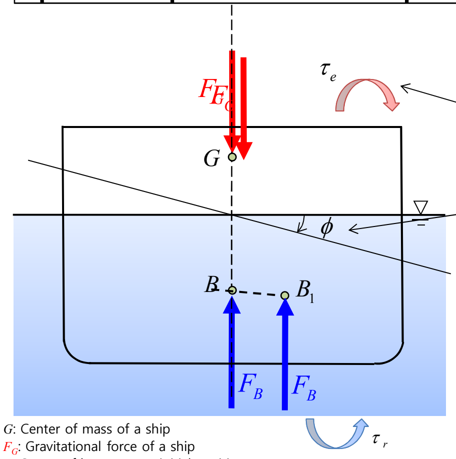
The statical stability curve is a plot of righting arm or righting moment against angle of heel for a given loading condition.

So far as the intact ship is concerned, the statical stability curve provides useful data for judging the adequacy of the ship's stability for the given loading condition.

Definition and Purpose of the Statical Stability Curve - Calculation Method of "GZ" (1/2)



1. At a certain angle of heel, calculate the static equilibrium position of the ship.



The fixed angle of heel means that there is an external moment that causes the ship to heel.

G: Center of mass of a ship
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined

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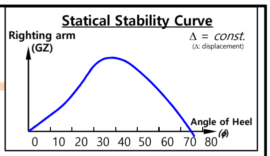
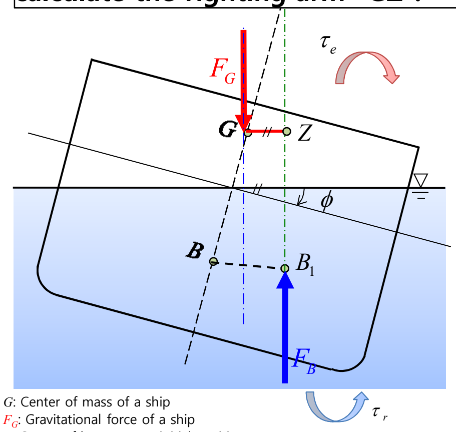
Definition and Purpose of the Statical Stability Curve - Calculation Method of "GZ" (2/2)



2. By using the center of mass (G) and new position of the center of buoyancy (B_1), calculate the righting arm "GZ".



G: Center of mass of a ship
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection point of a vertical line through the new position of the center of buoyancy (B_1) with the transversely parallel line to a waterline through the center of mass (G)

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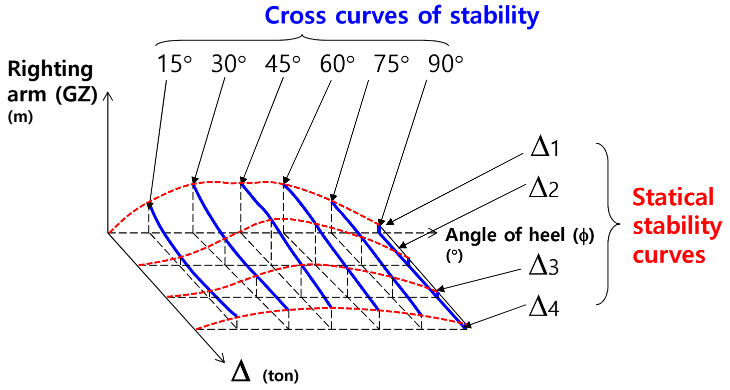
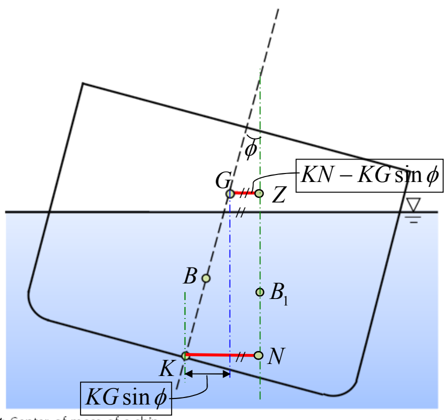
Definition and Purpose of Cross Curves of Stability



The cross curve of stability is a plot of righting arm against displacement for a given angle of heel.

A statical stability curve for a certain value of displacement (loading condition) can be obtained from the cross curves of stability.

Method for Obtaining Cross Curves of Stability



G: Center of mass of a ship
B: Center of buoyancy at initial position
B₁: New position of center of buoyancy after the ship has been inclined
Z: The intersection point of a vertical line through the new position of the center of buoyancy (B_1) with the longitudinally parallel line to a waterline through the center of mass (G)
K: Keel
N: The intersection point of a vertical line through the new position of the center of buoyancy (B_1) with the transversely parallel line to a waterline through the point K

Suppose that the center of mass is located at K.

Then, the KN represents the righting arm.

The KN depends only on the geometry of the submerged volume. But KN is not dependent on the location of the center of mass G.

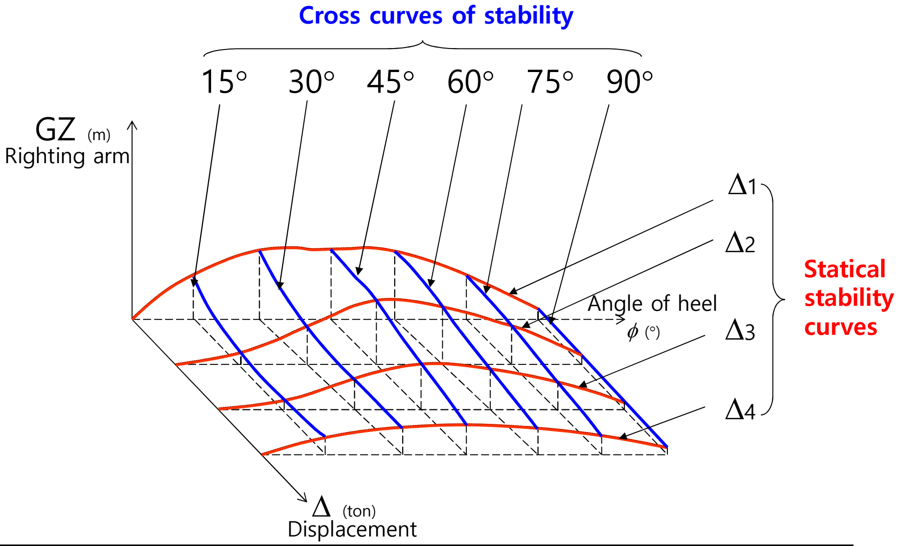
$$KN = KN(\phi, \Delta)$$

If the center of mass moves to vertical direction, so it is located at G, the values of righting arm (GZ) is

$$GZ = KN - KG\sin\phi$$

Obtaining Statical Stability Curves from Cross Curves of Stability

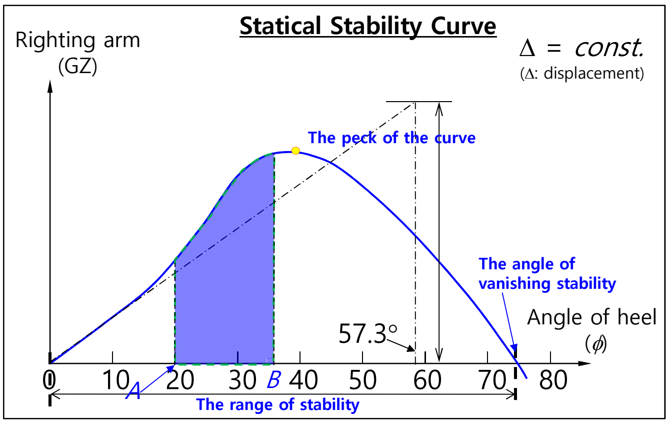
- **Three-dimensional cross curves of stability**



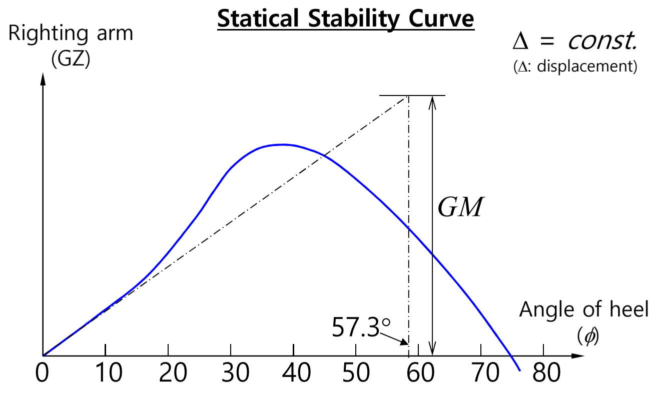
Significance of the Statical Stability Curve (1/5)

The statical stability curve has a number of features that are significant in the analysis of the ship's stability.

- The slope of the curve at zero degree, the peak of the curve, the range of stability, the angle of vanishing stability, and the area under the curve



Significance of the Statical Stability Curve (2/5)
(1) The Slope of the Curve at Zero Degrees



The slope of the curve at zero degree is **the metacentric height (GM)**.

This is a convenient **check for major error in the initial portion of the statical stability curve**.

✓**Derivation**

At small angles of heel

$$GZ \approx GM \sin|\phi|$$

Slop of the curve at ϕ = 0

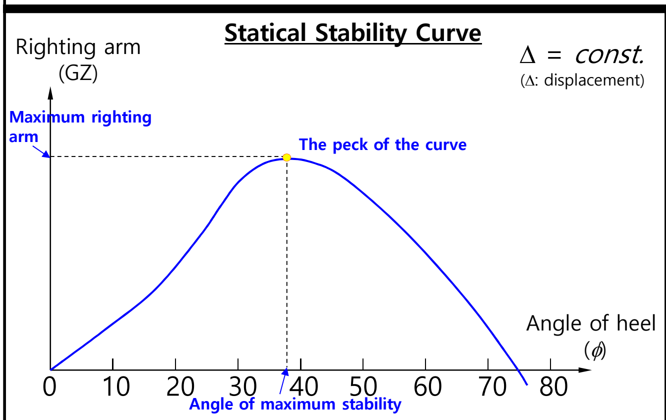
$$\lim_{\phi \to 0} \frac{GZ}{\phi} = \lim_{\phi \to 0} \frac{GM \sin\phi}{\phi}$$

$$= \frac{GM}{1(rad)} \quad , \left(\lim_{\phi \to 0} \frac{\sin\phi}{\phi} = 1 \right)$$

$$= \frac{GM}{57.3^\circ}$$

If the righting arm continues to increase at the same rate as at the origin, it would be equal to GM at an inclination of 1 radian or 57.3°.

Significance of the Statical Stability Curve (3/5)
(2) The Peak of the Curve

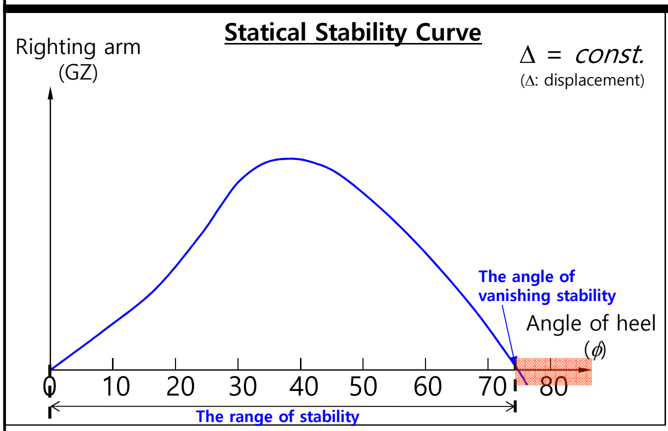


The peak of a statical stability curve identifies two quantities that are the **maximum righting arm** and the **angle of maximum stability**.

The product of the displacement and the maximum righting arm is **the maximum heeling moment** that the ship can experience **without capsizing**.

In other words, **if the ship is forced over to the angle of maximum stability** by an **externally applied constant heeling moment**, the ship will **capsize**.

Significance of the Statical Stability Curve (4/5)
(3) The Range of Stability and the Angle of Vanishing Stability

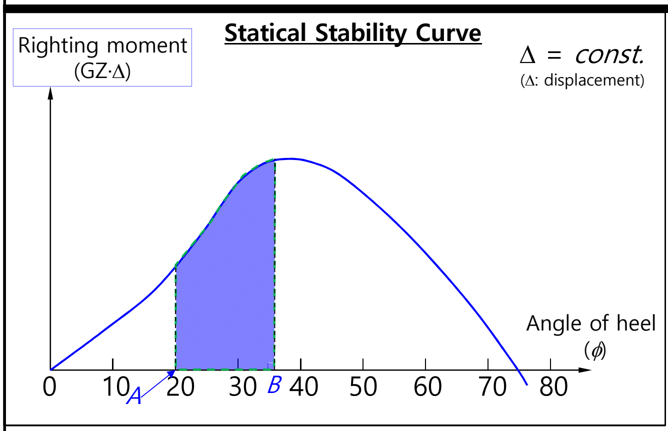


The range of stability is **the range over which the ship has positive righting arms.**

The angle of vanishing stability is **the angle of heel at which the righting arm returns to zero.**

If the ship **heels beyond this angle**, **the moment caused by gravitational force and buoyant force will act to capsize, rather than to right, the ship**.

Significance of the Statical Stability Curve (5/5)
(4) The Area under the Curve



The statical stability curve can be the plot of righting moment against angle of heel for a given condition by the product of the displacement and the righting arm.

The area under the curve, such as between angle A and angle B, represents the **work required to heel the ship from angle A to angle B.**

✓Derivation

When M is the moment at any angle of heel (ϕ),

the work required to rotate the ship against this moment (M) through an angle ($d\phi$)

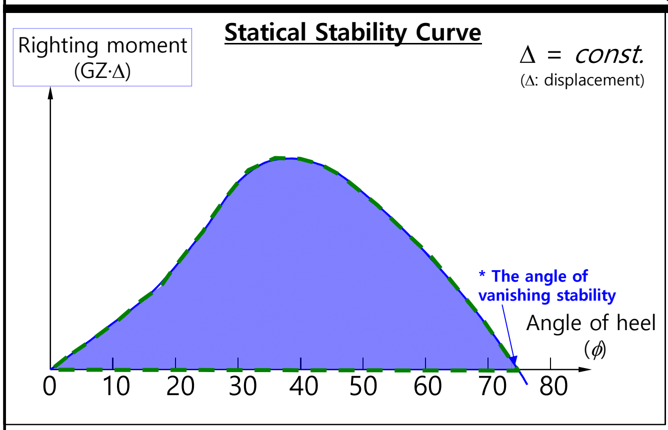
$= M\,d\phi$

The work (W) required to rotate from angle A to angle B: $W = \int_A^B M\,d\phi$, (ϕ in radinas)

Significance of the Statical Stability Curve (5/5)
(4) The Area under the Curve – Total Area

The work (W) required to rotate from A to B:

$$W = \int_A^B M \, d\phi, \quad (\phi \text{ in radinas})$$



* The angle of vanishing stability is the angle of heel at which the righting arm returns to zero.

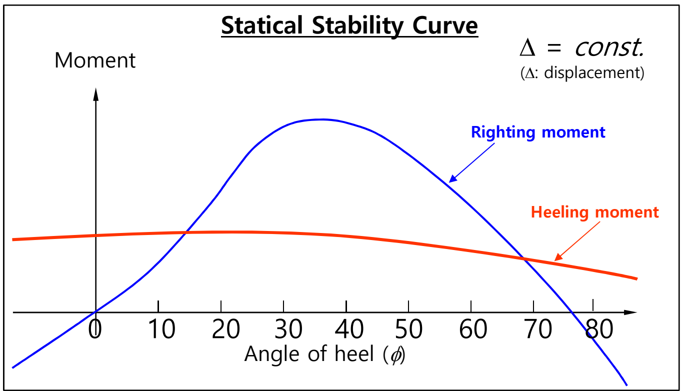
The total area between the statical stability curve (at zero degree to the angle of vanishing stability) and the horizontal axis represents **the total work to capsize the ship from the upright position.**

This is often referred to as **dynamic stability**.

Representation of Heeling Moments on the Statical Stability Curve

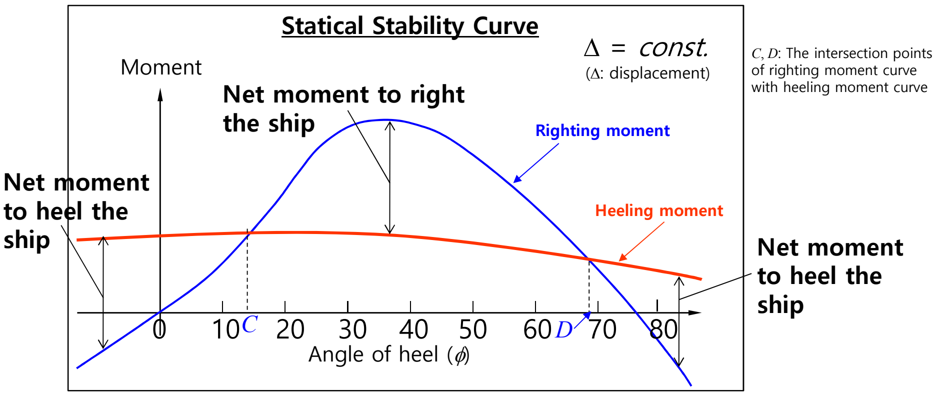
There are **the nature of certain heeling forces acting on the ship**.

These forces may be caused by action of the beam winds, action of the waves in rolling the ship, lifting of heavy weights over the side, high-speed turn, etc.



By **superimposing various heeling moments** caused by these forces on a curve of righting moment, **statical stability of the ship in any operating (loading) condition can be evaluated.**

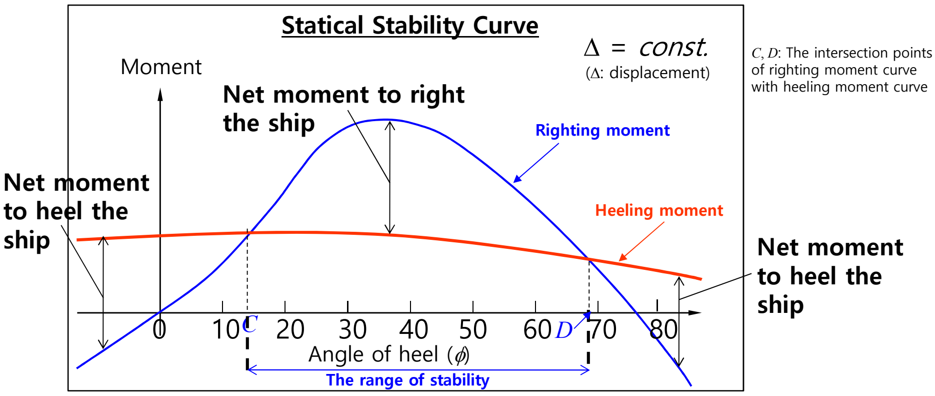
Representation of Heeling Moments on the Statical Stability Curve
- Significance between the Statical Stability Curve and Heeling Moments Curve (1/3)



At angle C and D, the heeling moment is equal to the righting moment and the forces are in equilibrium.

The vertical distance between the heeling moment and righting moment curves at any angles represents the net moment acting at that angle either to heel or right the ship, depending on the relative magnitude of the righting and heeling moments.

Representation of Heeling Moments on the Statical Stability Curve
- Significance between the Statical Stability Curve and Heeling Moments Curve (2/3)

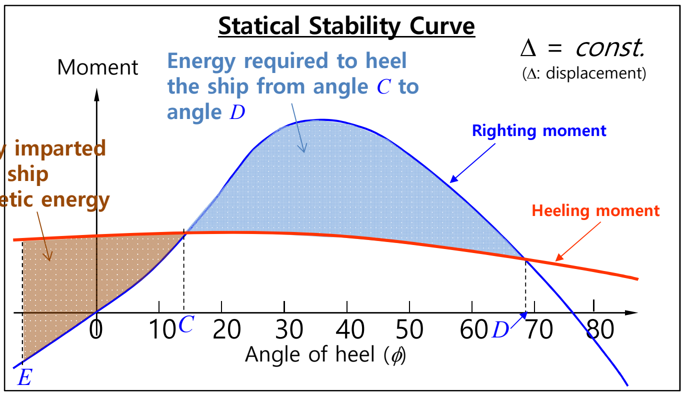


If the ship is heeled to angle C, an inclination in either direction will generate a moment tending to restore the ship to angle C.

If the ship is heeled to beyond angle D, the ship will capsize.

The range of stability is decreased by the effect of the heeling moment.

Representation of Heeling Moments on the Statical Stability Curve
- Significance between the Statical Stability Curve and Heeling Moments Curve (3/3)



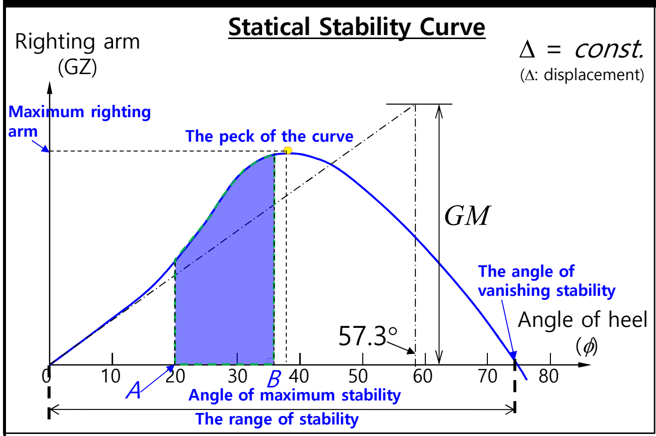
Assumption) The ship was heeled to the left to angle E, has come to rest, and is about to be hilled in the opposite direction.

If the energy imparted to the ship as kinetic energy is larger than the energy required to heel the ship from angle C to angle D, the ship will capsize.

To reduce the danger of capsizing under these conditions, the area between the heeling and righting moment curves and between angle C and angle D should be greater, by some margin, than that between angle E and angle C.

2. Stability Criteria for Intact State

[Review] Statical Stability Curve



✓Slope of the curve at zero degree: **metacentric height (GM)**

$$GZ \approx GM \sin|\phi|$$

✓The peak of a statical stability curve: **maximum righting arm, angle of maximum stability**

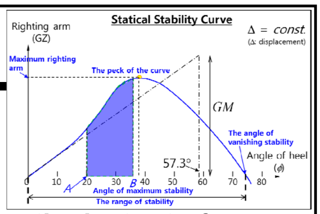
✓The product of the displacement and the maximum righting arm: **the maximum heeling moment** that the ship can experience **without capsizing**

✓The range of stability: **the range over which the ship has positive righting arms**

✓ The area under the curve, such as between angle A and angle B: **the work required to heel the ship from angle A to angle B.**

What criteria is considered to evaluate the ship's stability?
What is satisfactory stability? How much stable a ship must be?

Stability Criteria in General



☑ Various researchers and regulatory bodies prescribed criteria for deciding if the stability is satisfactory.
In this section, we present examples of such criteria based on consideration of actual shape and characteristics of the curves of righting and heeling moment (or arm) for an undamaged ship (intact ship) through large angles of heel.

☑ Features of the curves that warrant consideration from a purely static viewpoint are:

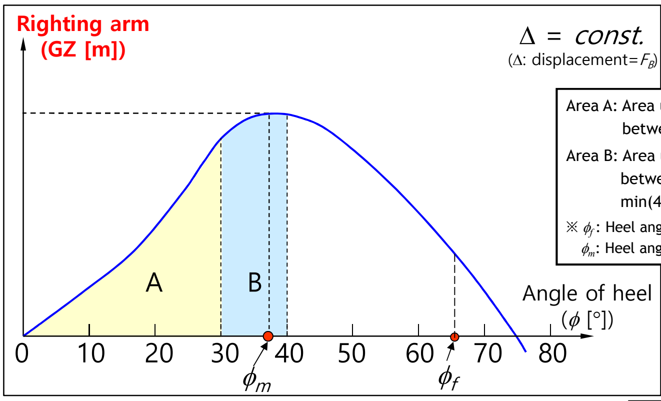
Static considerations
- The angle of steady heel
- The range of positive stability
- The relative magnitudes of the heeling arm and the maximum righting arm.

The work and energy considerations (dynamic stability)

Stability Criteria
- IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.3.1)

$\tau_r = GZ \cdot F_B$



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f: Heel angle at which openings in the hull submerge
ϕ_m: Heel angle of maximum righting arm

IMO Regulations for Intact Stability

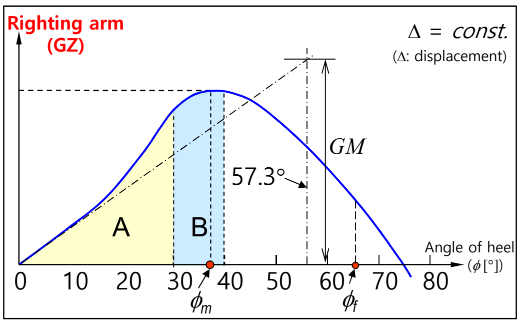
(a) Area A ≥ 0.055 (m·rad)
(b) Area A + B ≥ 0.09 (m·rad)
(c) Area B ≥ 0.030 (m·rad)
(d) GZ ≥ 0.20 (m) at an angle of heel equal to or greater than 30°.
(e) GZ_{max} should occur at an angle of heel equal to or greater than 25°.
(f) The initial metacentric height GM_o should not be less than 0.15 (m).

※ After receiving the approval for the intact and damage stability of IMO regulation from owner and classification society, ship construction can be proceed.

Merchant Ship Stability Criteria
- IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.3.1)

☑ **IMO recommendation on intact stability for passenger and cargo ships**



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f: Heel angle at which openings in the hull
ϕ_m: Heel angle of maximum righting arm

※ After receiving approval of calculation of IMO regulation from Owner and Classification Society, ship construction can proceed.

IMO Regulations for Intact Stability

(a) Area A ≥ 0.055 (m·rad)
(b) Area A + B ≥ 0.09 (m·rad)
(c) Area B ≥ 0.030 (m·rad)

→ **The work and energy considerations (dynamic stability)**

(d) GZ ≥ 0.20 (m) at an angle of heel equal to or greater than 30°
(e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25°.
(f) The initial metacentric height GM_o should not be less than 0.15 (m).

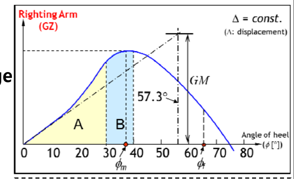
→ **Static considerations**

Merchant Ship Stability Criteria
- IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.4.9)

☑ Special Criteria for Certain Types of Ships
- Containerships greater than 100 m

These requirements apply to containerships greater than 100 m. They may also be applied to other cargo ships with considerable flare or large water plane areas. The administration may apply the following criteria instead of those in paragraphs of previous slide.



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f: Heel angle at which openings in the hull submerge
ϕ_m: Heel angle of maximum righting arm

IMO Regulations for containerships greater than 100 m

(a) Area A ≥ 0.009/C (m-rad)

(b) Area A + B ≥ 0.016/C (m-rad)

(c) Area B ≥ 0.006/C (m-rad)

(d) GZ ≥ 0.033/C (m) at an angle of heel equal to or greater than 30°

(e) GZ_{max} ≥ 0.042/C (m)

(f) The total area under the righting arm curve (GZ curve) up to the angle of flooding ϕ_f should not be less than 0.029/C (m-rad)

In the above criteria **the form factor C** should be calculated using the formula and figure on the right-hand side.

$$C = \frac{dD'}{B_m^2}\sqrt{\frac{d}{KG}}\left(\frac{C_B}{C_W}\right)^2\sqrt{\frac{100}{L}}$$

where, d: Mean draught (m)

$D' = D + h\frac{2b - B_D}{B_D}\left(\frac{2\sum l_H}{L}\right)$ as defined in figure on the right-hand side.

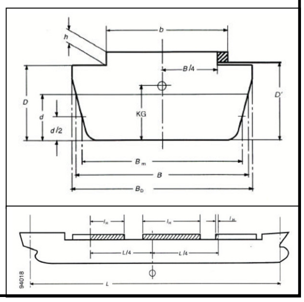
D: Moulded depth of the ship (m)

B: Moulded breadth of the ship (m)

KG: Height of the centre of gravity in (m) above the keel not to be taken as less than d

C_B: Block coefficient

C_W: Water plane coefficient

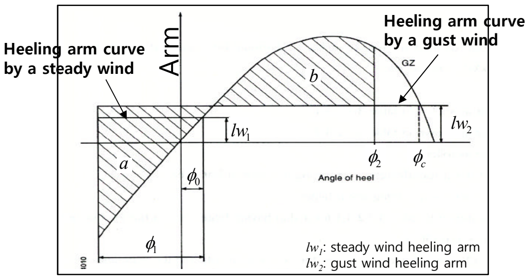


Design Criteria Applicable to All Ships
- IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

(IMO Res.A-749(18) ch.3.2)

☑ Scope

The weather criteria should govern the minimum requirements for passenger or cargo ships of 24 m in length and over.



ϕ_0: Angle of heel under action of steady wind

ϕ_1: Angle of roll to windward due to wave action

ϕ_2: Angle of down flooding (ϕ_f) or 50°, whichever is less

where,

ϕ_f: Angle of heel at which openings in the hull submerge

ϕ_c: Angle of the second intersection between wind heeling arm and GZ(righting arm) curves

Area a: The shaded area between angle ϕ_1 and the first intersection of righting arm curve with heeling arm curve

Area b: The shaded area between the first intersection of righting arm curve with heeling arm curve and angle ϕ_2

IMO Regulations for Severe Wind and Rolling Criteria (Weather Criteria)

(a) ϕ_0 should be limited to 16° or 80% of the angle of deck edge immersion (ϕ_f), whichever is less.

The ship is subjects to a steady wind pressure acting perpendicular to the ship's center line which results in a steady wind heeling arm (lw_1).

$$lw_1 = \frac{P \cdot A \cdot Z}{1000 \cdot g \cdot \nabla} \ (m) \qquad (P = 504\ N/m^2,\ g = 9.81\,m/s^2)$$

A : Lateral projected area above water line.
Z : Vertical distance from the center of wind pressure to the center of water pressure

(b) Under these circumstances, *area b* should be equal to or greater than *area a*. ⇨ **The work and energy considerations (dynamic stability)**

The ship is subjected to a gust wind pressure which results in a gust wind heeling arm (lw_2).

$$lw_2 = 1.5 \cdot lw_1 \ (m)$$

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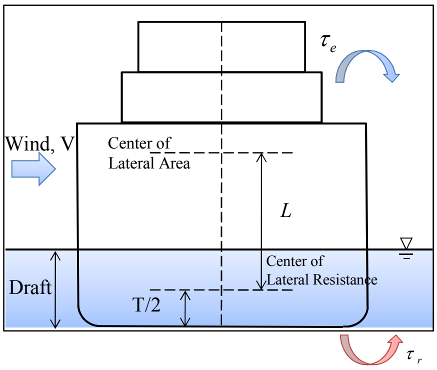
War Ship Stability Criteria
- U.S. Navy Criteria (1/2)

☑ **General U.S. Navy criteria are intended to ensure the adequacy of stability of all types and sizes of naval ships, as evidenced by sufficient righting energy to withstand various types of upsetting of heeling moments.**

(Example) Beam Winds Combined with Rolling



When winds of high velocity exist, beam winds and rolling are considered simultaneously.

If the water were still, the ship would require only sufficient righting moment to overcome the heeling moment produced by the action of the wind on the ship's "sail area".

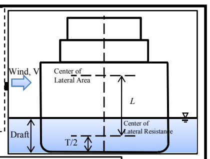
However, when the probability of wave action is taken into account, an additional allowance of dynamic stability is required to absorb the energy imparted to the ship by the rolling motion.

L: Center height of projected sail area above 0.5T
A: Projected sail area (ft^2), V: average wind speed (knots)
ϕ: Angle of heel (degree), Δ: Displacement (LT)
ϕ_m: Angle of maximum righting arm (degree)

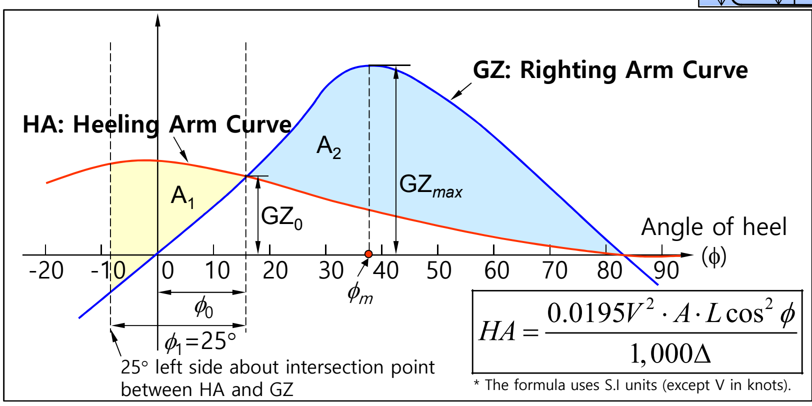
* Brown, A.J., Deybach, F., "Towards A Rational Intact Stability Criteria For Naval Ships", Naval Engineers Journal, pp.65-77, 1998

War Ship Stability Criteria
- U.S. Navy Criteria (2/2)

L: Center height of projected sail area above 0.5T
A: Projected sail area (ft^2)
V: Average wind speed (knots)
ϕ: Angle of heel (degree)
Δ: Displacement (LT)
ϕ_m: Angle of maximum righting arm



☑ **Stability is considered satisfactory if:**



$$HA = \frac{0.0195V^2 \cdot A \cdot L\cos^2\phi}{1{,}000\Delta}$$

* The formula uses S.I units (except V in knots).

- **Regulation**

  (a) $GZ_0 \le 0.6 \cdot GZ_{max}$: **Static considerations**

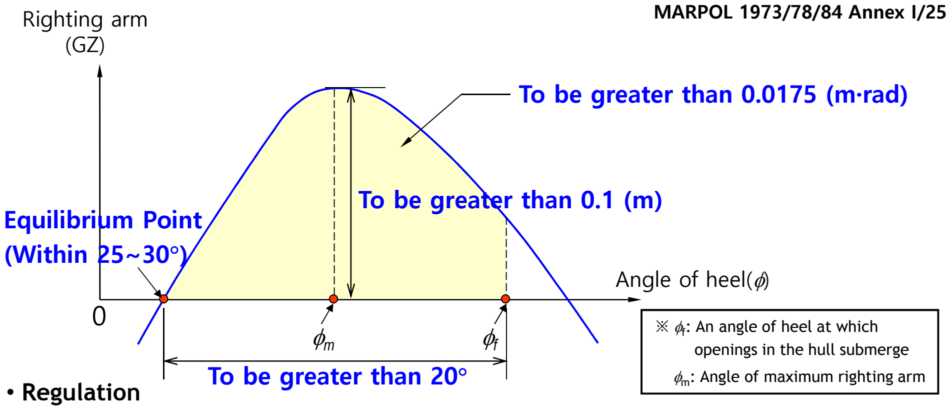
  (b) $A_2 \ge 1.4 \cdot A_1$: **The work and energy considerations (dynamic stability)**

* Brown, A.J., Deybach, F., "Towards A Rational Intact Stability Criteria For Naval Ships", Naval Engineers Journal, pp.65-77, 1998

3. Stability Criteria for Damage State

MARPOL Regulation for Damage Stability - Deterministic Damage Stability

MARPOL 1973/78/84 Annex I/25

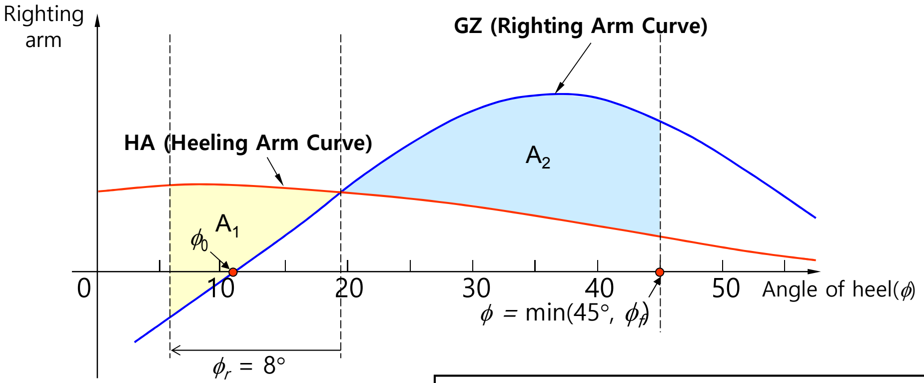


- **Regulation**
  - (a) The final waterline shall be below the lower edge of any opening through which progressive flooding may take place.
  - (b) The angle of heel due to unsymmetrical flooding shall not exceed 25 degrees, provided that this angle may be increased up to 30 degrees if no deck edge immersion occurs.
  - (c) The statical stability curve has at least a range of 20 degrees beyond the position of equilibrium in association with a maximum residual righting arm of at least 0.1 meter within the 20 degrees range
  - (d) The area under the curve within this range shall not be less than 0.0175 meter-radians.

Damage Stability Criteria in Battleship*

- **Regulation**

ϕ_0 (Initial Angle of Heel) $\leq 15°$, $A_2 \geq 1.4 \cdot A_1$



ϕ_r: Angle of heel in transverse wind
(It varies depending on displacement, ϕ_r = 8° in case of battleship with displacement of 9,000 ton.)
ϕ_f: Angle of heel at which openings in the hull submerge

* Surko, S.W., "An Assessment of Current Warship Damaged Stability Criteria", Naval Engineers Journal, 1994