Topics in Ship Structures

09 Application to Structure

Reference :

Fracture Mechanics by T.L. Anderson Ch. 9

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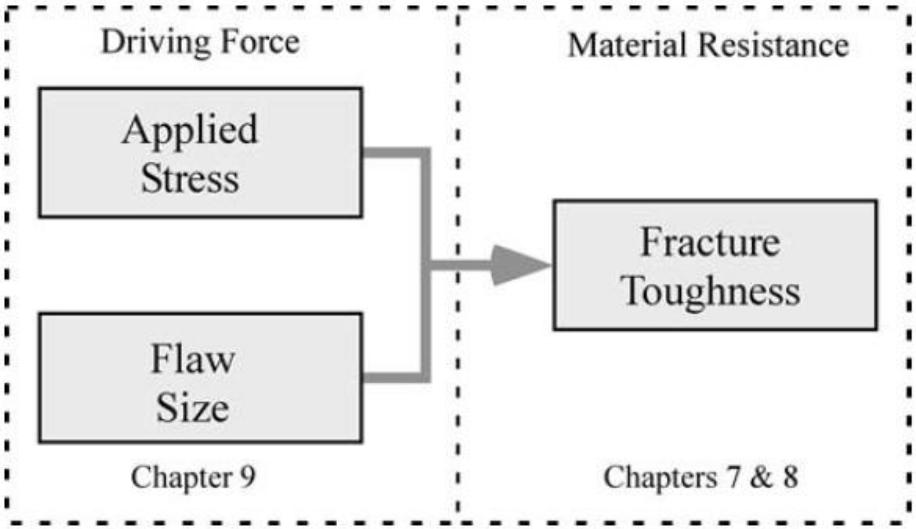
Contents

General

- Three critical variables to be considered in structural design : *stress, flaw size, and toughness.*
- Several parameters for characterizing the fracture driving force.
  - ✓ *Elastic regime :* the stress-intensity factor K and the energy release rate $\mathcal{G}$.
  - ✓ *Elastic-plastic regime :* The J integral and crack-tip-opening displacement (CTO.D)
- This chapter focuses on fracture initiation and instability in structures made from linear elastic and elastic-plastic materials.
- A number of engineering approaches are discussed; the basis of these approaches and their limitations.
- Only quasistatic methodologies.



Relationship between the three critical variables in fracture mechanics.

0. Introduction

- The fracture behavior of a linear elastic structure can be inferred by comparing the applied *K* (the driving force) to a critical *K* (K_{IC}) or a *K-R* curve (the fracture toughness).
- For Mode I loading,

$$K_I = Y\sigma\sqrt{\pi a}$$

where

Y = dimensionless geometry correction factor

σ = characteristic stress

a = characteristic crack dimension

- A large number of stress-intensity solutions have been published over the past 50 years.
- When a published *K* solution is not available, one can obtain the solution experimentally or numerically. Nearly all new K solutions are obtained numerically.
- Deriving a closed-form solution is probably not a viable alternative, since this is possible only with simple geometries and loading, and nearly all such solutions have already been published.

K_I for Part-Though Cracks

❖ K_I for Part-Though Cracks by Newman and Raju.

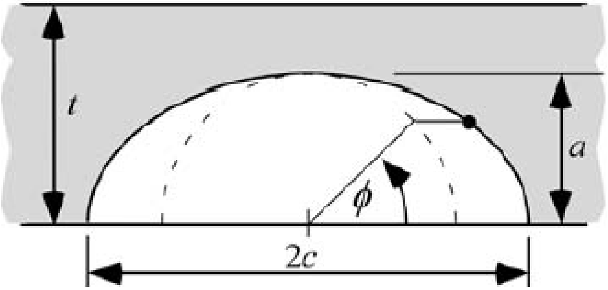
- the stress normal to the flaw = bending and membrane components

$$K_I = (\sigma_m + H\sigma_b)F\sqrt{\frac{\pi a}{Q}}$$

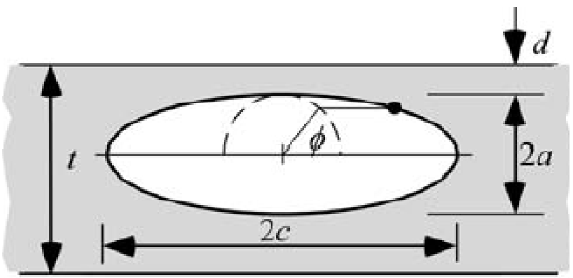
- *F* and *H* are geometry factors and depend on (a/c, a/t, ϕ) and obtained from finite element analysis.

- *Q :* the flaw-shape parameter, which is based on the solution of an elliptical integral of the second kind

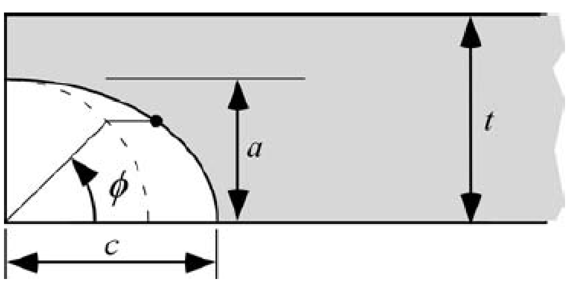
$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad \text{for } a \le c$$



Semielliptical surface crack



Elliptical buried flaw



Quarter-elliptical corner crack

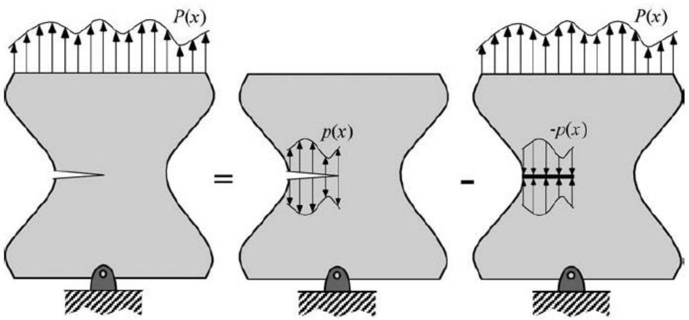
Influence Coefficients for Polynomial Stress Distributions

❖The principle of superposition

- The same K_I for remote boundary traction P(x) and a crack-face pressure $p(x)$.
- Consider a surface crack of depth a with power-law crack-face pressure.

$$p(x) = p_n\left(\frac{x}{a}\right)^n \qquad K_I = G_n p_n \sqrt{\frac{\pi a}{Q}}$$

G_n is an *influence coefficient*



$$K_I^{(a)} = K_I^{(b)} - K_I^{(c)} = K_I^{(b)} \qquad \left(\text{since } K_I^{(c)} = 0\right)$$

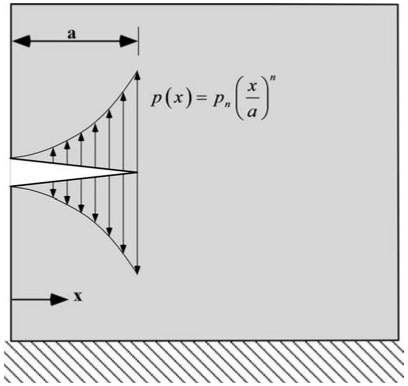
"$K_I^{(c)}$= 0 because the crack faces close, and the plate behaves as if the crack were not present"

- Consider a nonuniform normal stress distribution.

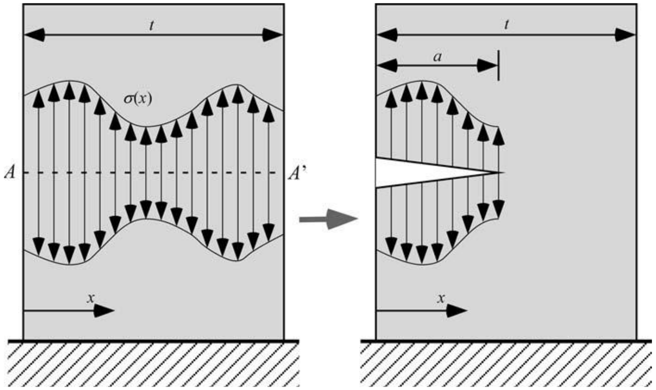
$$\sigma(x) = \sigma_o + \sigma_1\left(\frac{x}{t}\right) + \sigma_2\left(\frac{x}{t}\right)^2 + \sigma_3\left(\frac{x}{t}\right)^3 + \sigma_4\left(\frac{x}{t}\right)^4$$

- The principle of superposition

$$\boxed{K_I = \left[\sigma_o G_o + \sigma_1 G_1\left(\frac{a}{t}\right) + \sigma_2 G_2\left(\frac{a}{t}\right)^2 + \sigma_3 G_3\left(\frac{a}{t}\right)^3 + \sigma_4 G_4\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{\pi a}{Q}}}$$



Power-law pressure distribution along the crack face



Nonuniform stress distribution that can be fit to a four-term polynomial

Influence Coefficients for Polynomial Stress Distributions

❖EX) a pressurized cylinder with an internal axial surface flaw.

- A Taylor series expansion about $x = 0$, $x=r-R_i$.

$$\sigma_{\theta\theta} = \frac{pR_i^2}{R_o^2 - R_i^2}\left[1 + \left(\frac{R_o}{r}\right)^2\right]$$

$$\Rightarrow \quad \sigma_{\theta\theta} = \frac{pR_o^2}{R_o^2 - R_i^2}\left[1 + \left(\frac{R_i}{R_o}\right)^2 - 2\left(\frac{x}{R_i}\right) + 3\left(\frac{x}{R_i}\right)^2 - 4\left(\frac{x}{R_i}\right)^3 + 5\left(\frac{x}{R_i}\right)^4 + \cdots\right] \quad (0 \le x/R_i \le 1)$$

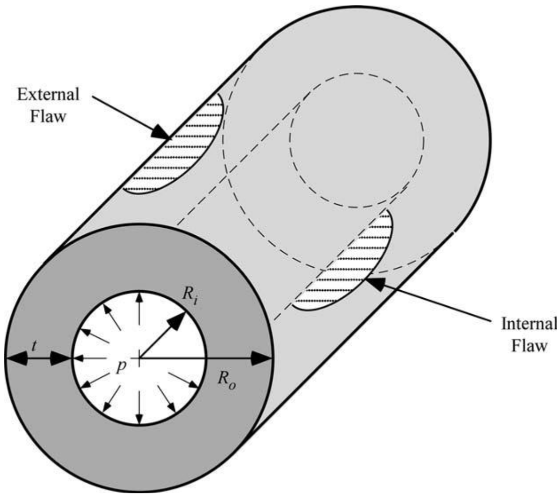
- Superimposing the effect of internal pressure *p*.

$$\frac{pR_o^2}{R_o^2 - R_i^2}\left[1 + \left(\frac{R_i}{R_o}\right)^2\right] + p = \frac{pR_o^2}{R_o^2 - R_i^2}\left[\frac{R_0^2 + R_i^2}{R_0^2} + \frac{R_0^2 - R_i^2}{R_0^2}\right] = 2\frac{pR_o^2}{R_o^2 - R_i^2}$$

$$K_I = \frac{pR_o^2}{R_o^2 - R_i^2}\left[\boxed{2G_o} - 2\left(\frac{a}{R_i}\right)G_1 + 3\left(\frac{a}{R_i}\right)^2 G_2 - 4\left(\frac{a}{R_i}\right)^3 G_3 + 5\left(\frac{a}{R_i}\right)^4 G_4\right]\sqrt{\frac{\pi a}{Q}}$$

- A similar approach to an external surface flaw.

$$K_I = \frac{pR_i^2}{R_o^2 - R_i^2}\left[2G_o + 2\left(\frac{a}{R_o}\right)G_1 + 3\left(\frac{a}{R_o}\right)^2 G_2 + 4\left(\frac{a}{R_o}\right)^3 G_3 + 5\left(\frac{a}{R_o}\right)^4 G_4\right]\sqrt{\frac{\pi a}{Q}}$$



Internal and external axial surface flaws in a pressurized cylinder

Influence Coefficients for Polynomial Stress Distributions

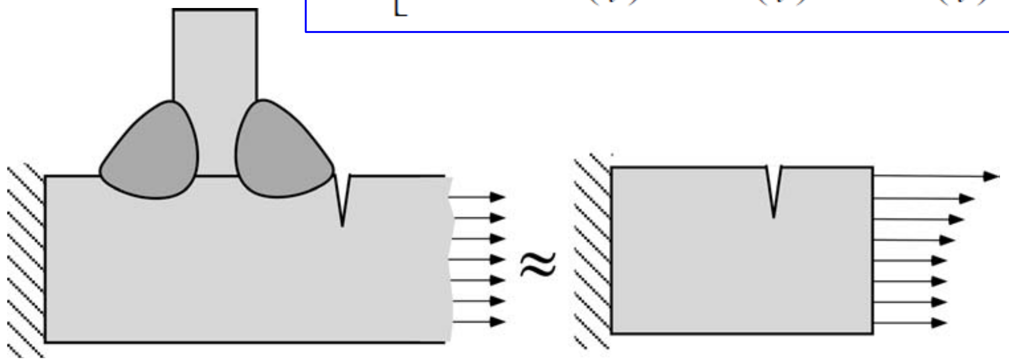
❖Crack at welded joint

- The influence coefficient approach is useful for estimating K_I values for cracks that emanate from stress concentrations.
- If the stress distribution at the weld toe for the uncracked case can be fit to a polynomial,

$$\sigma(x) = \sigma_o + \sigma_1\left(\frac{x}{t}\right) + \sigma_2\left(\frac{x}{t}\right)^2 + \sigma_3\left(\frac{x}{t}\right)^3 + \sigma_4\left(\frac{x}{t}\right)^4$$

- K_I can be estimated by substituting the influence coefficients and polynomial coefficients.

$$K_I = \left[\sigma_o G_o + \sigma_1 G_1\left(\frac{a}{t}\right) + \sigma_2 G_2\left(\frac{a}{t}\right)^2 + \sigma_3 G_3\left(\frac{a}{t}\right)^3 + \sigma_4 G_4\left(\frac{a}{t}\right)^4\right]\sqrt{\frac{\pi a}{Q}}$$



Application of the influence coefficient approach to a complex structural detail such as a fillet weld

Influence Coefficients for Polynomial Stress Distributions

❖Limitations in the application to welded joint.

- The methodology in the previous example is only approximate.
- If the influence coefficients were obtained from an analysis of a flat plate, there may be slight errors if these G_n values are applied to the fillet weld geometry. The actual weld geometry has a relatively modest effect on the G_n values.
- As long as the stress gradient emanating from the weld toe is taken into account, computed K_I values will usually be within 10% of values obtained from a more rigorous analysis.
- Since the flaw is near a weld, there is a possibility that weld residual stresses will be present. These stresses must be taken into account in order to obtain an accurate estimate of K_I.

Primary, Secondary, and Residual Stress

- There are very few practical situations in which a cracked body is subject to pure displacement control.
- Some design codes for structures such as pressure vessels and piping refer to load-controlled stresses as *primary* and displacement-controlled stresses as *secondary.*
- Hoop stress due to internal pressure in a pipe or pressure vessel is an example of a primary stress. Thermal expansion leads to imposed displacements, so thermal stresses are usually considered secondary.
- When plastic deformation occurs, however, secondary stresses redistribute and may relax from their initial values.
- In linear elastic analyses, primary, secondary, and residual stresses are treated in an identical fashion. The total stress intensity is simply the sum of the primary and secondary components:

$$K_I^{total} = K_I^P + K_I^S + K_I^R$$

A Warning about LEFM

- Performing a purely linear elastic fracture analysis and *assuming* that LEFM is valid is potentially dangerous.
- The user must rely on experience to know whether or not plasticity effects need to be considered.
- The safest approach is to adopt an analysis that spans the entire range from linear elastic to fully plastic behavior. Such an analysis accounts for the two extremes of brittle fracture and plastic collapse.
- At low stresses, the analysis reduces to LEFM, but predicts collapse if the stresses are sufficiently high.
- At intermediate stresses, the analysis automatically applies a plasticity correction when necessary; the user does not have to decide whether or not such a correction is needed.
- The failure assessment diagram (FAD) approach, described in Section 9.4, is an example of a general methodology that spans the range from linear elastic to fully plastic material behavior.

The CTOD Design Curve

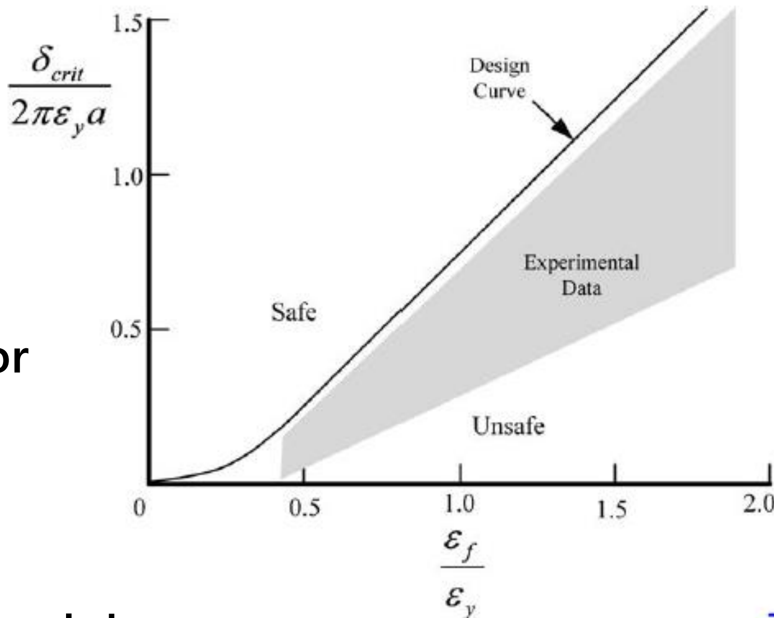
- In 1971, Burdekin and Dawes developed the CTOD design curve, a semiempirical driving force relationship based on elastic-plastic driving force relationship and an empirical correlation between small-scale CTOD tests and wide double-edge-notched tension panels.
- The wide plate specimens were loaded to failure, and the failure strain (ε_f) and crack size (a) of a given large-scale specimen were correlated with the critical CTOD in the corresponding small-scale test.
- Two-part relationship:

$$\frac{\delta_{crit}}{2\pi\varepsilon_y a} = \left(\frac{\varepsilon_f}{\varepsilon_y}\right)^2 \quad \text{for } \frac{\varepsilon_f}{\varepsilon_y} \le 0.5$$

ε_f : failure strain, ε_y : yield strain

$\Rightarrow$ derived from LEFM theory with a safety factor of 2.0 on crack size.

$$\frac{\delta_{crit}}{2\pi\varepsilon_y a} = \frac{\varepsilon_f}{\varepsilon_y} - 0.25 \quad \text{for } \frac{\varepsilon_f}{\varepsilon_y} \le 0.5$$



$\Rightarrow$ represents an upper envelope of experimental data.

The CTOD Design Curve

- British Standards document (PD 6493, 1980), the maximum strain can be estimated from the following equation

$$\varepsilon_1 = \frac{1}{E}[k_t(P_m + P_b) + (S + R)]$$

where

k_t = elastic stress concentration factor
P_m = primary membrane stress
P_b = primary bending stress
S = secondary stress
R = residual stresses

- Since the precise distribution of residual stresses was usually unknown, *R* was typically assumed to equal the yield strength in an as-welded weldment.
- Kamath estimated that the CTOD design curve method corresponds to a 97.5% confidence of survival.
- Direct evaluation of the *J* integral and the FAD approach have replaced CTOD approach.

General

- The most rigorous method to compute J is to perform an elastic-plastic finite element analysis on the structural component that contains a crack. (Ch.12)

- There are a number of simplified methods for estimating J in lieu of elastic-plastic finite element analysis.

- The Electric Power Research Institute(EPRI) J estimation scheme and the reference stress approach,

The EPRI J-Estimation Procedure

- The J integral was first used as a fracture toughness parameter in the early 1970s.
- At that time, there was no convenient way to compute the applied J in a structural component. Stress-intensity factor handbooks were available, but a corresponding handbook for elastic-plastic analysis did not exist.
- A series of finite element analyses were performed at General Electric Corporation in Schenectady, New York, and the first J handbook was an engineering handbook by EPRI in 1981.
- Most of the solutions are for simple two-dimensional geometries such as flat plates with through cracks and edge cracks. Because of these limitations, the EPRI J handbooks are of little value for most real-world problems.
- However, the research funded by EPRI in the late 70s and early 80s did contribute to our understanding of elastic-plastic fracture mechanics.
- The elastic J is actually the elastic energy release rate $\mathcal{G}$, which can be computed from K_I. Fully plastic J solutions were inferred from finite element analysis and were tabulated in a dimensionless form.

$$J_{tot} = J_{el} + J_{pl}$$

The theoretical Background

- Assume a power-law stress-strain curve (the second term in the Ramberg-Osgood model)

$$\frac{\varepsilon_{pl}}{\varepsilon_o} = \alpha\left(\frac{\sigma}{\sigma_o}\right)^n$$

- Close to the crack tip, under *J*-controlled conditions, the stresses are given by the HRR singularity:

$$\sigma_{ij} = \sigma_o\left(\frac{J}{\alpha\varepsilon_o\sigma_o I_n r}\right)^{\frac{1}{n+1}}\tilde{\sigma}_{ij}(n,\theta)$$

- Solving for *J* in the HRR equation gives

$$J = \alpha\varepsilon_o\sigma_o I_n r\left(\frac{\sigma_{ij}}{\sigma_o}\right)^{n+1}\tilde{\sigma}_{ij}^{\;n+1}$$

- The local stresses must increase in proportion to the remote load *P*

$$J = \alpha\varepsilon_o\sigma_o hL\left(\frac{P}{P_o}\right)^{n+1}$$

where

h = dimensionless function of geometry and n

L = characteristic length dimension for the structure

P_o = reference load

Estimation Equations

- The fully plastic equations for J, crack-mouth-opening displacement V_p, and load line displacement Δ_p have the following form for most geometries:

$$J_{pl} = \alpha \varepsilon_o \sigma_o b h_1(a/W, n)\left(\frac{P}{P_o}\right)^{n+1} \quad V_p = \alpha \varepsilon_o a h_2(a/W, n)\left(\frac{P}{P_o}\right)^{n} \quad \Delta_p = \alpha \varepsilon_o a h_3(a/W, n)\left(\frac{P}{P_o}\right)^{n}$$

where
b = uncracked ligament length
a = crack length
h_1, h_2, and h_3 = dimensionless parameters that depend on geometry and hardening exponent

- The reference load P_o normally corresponds to the load at which the net cross section yields.
- The elastic J is equal to $\mathcal{G}(a_{eff})$, the energy release rate for an effective crack length which is based on modified Irwin plastic zone correction)

$$a_{eff} = a + \frac{1}{1+(P/P_o)^2}\frac{1}{\beta\pi}\left(\frac{n-1}{n+1}\right)\left(\frac{K_I}{\sigma_o}\right)^2$$

- β= 2 for plane stress and β = 6 for plane strain conditions

Estimation Equations

❖ Ex. 9.1) Consider a single-edge-notched tensile panel with W = 1 m, B = 25 mm, and a = 125 mm, Calculate J vs. applied load assuming plane stress conditions. Neglect the plastic zone correction.

- Given: σ_o = 414 MPa, n = 10, α = 1.0, E = 207,000 MPa, $\varepsilon_o = \sigma_o / E$ = 0.002

Sol) From Table A9.13, the referece load for this configuration.

$$P_o = 1.072\eta\sigma_o bB \qquad \eta = \sqrt{1+\left(\frac{a}{b}\right)^2} - \frac{a}{b} = 0.867 \quad \text{for} \quad a/b = 125/875 = 0.143$$

$$J_{pl} = \alpha\varepsilon_o\sigma_o \frac{ba}{W} h_1(a/W,n)\left(\frac{P}{P_o}\right)^{n+1}$$

$$J_{pl} = (1.0)(0.002)(414{,}000\,\text{kPa})\frac{(0.875\text{ m})(0.125\text{m})}{1.0\text{ m}}(4.14)\left(\frac{P}{8.42\text{ MN}}\right)^{11}$$

$$= 2.486\times10^{-8}P^{11}\ (\text{kJ/m}^2),\quad \text{P in MN}$$

From Table 2.4

$$J_{el} = \frac{K_I^2}{E} = \frac{P^2\boxed{f^2(a/W)}}{B^2\,W\,E} \qquad J_{el} = \frac{1000P^2(0.770)^2}{(0.025\,\text{m})^2(1.0\,\text{m})(207{,}000\,\text{MPa})} = 4.584P^2$$

$$J = 4.584P^2 + 2.486\times10^{-8}P^{11}\ (\text{kJ/m}^2),\quad \text{P in MN}$$

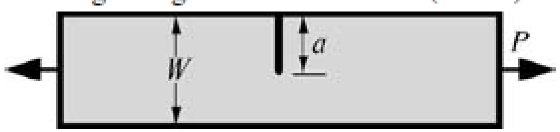
Estimation Equations

❖Table 2.4 K_I Solutions for Common Test Specimens

TABLE 2.4
K_I Solutions for Common Test Specimens[a]

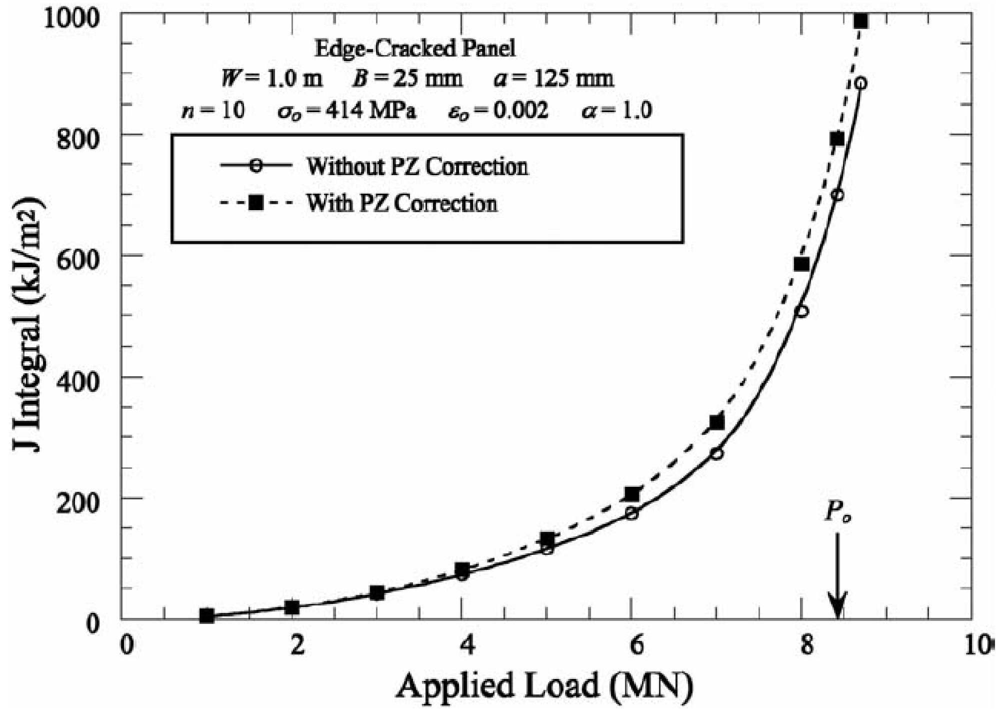
GEOMETRY

Single Edge Notched Tension (SENT)



$$f\left(\frac{a}{W}\right)^{*}$$

$$\frac{\sqrt{2\tan\frac{\pi a}{2W}}}{\cos\frac{\pi a}{2W}}\left[0.752+2.02\left(\frac{a}{W}\right)+0.37\left(1-\sin\frac{\pi a}{2W}\right)^{3}\right]$$



Applied J vs. applied load in an edge-cracked panel

Estimation Equations

❖Table A9.13 Fully Plastic J and Displacement for an Edge-Cracked Tension Specimen in Plane Stress.

TABLE A9.13
Fully Plastic *J* and Displacement for an Edge-Cracked Tension Specimen in Plane Stress [23].

a/W:		$n = 1$	$n = 2$	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 13$	$n = 16$	$n = 20$
	h_1	3.58	4.55	5.06	5.30	4.96	$\boxed{4.14}$	3.29	2.60	1.92
0.125	h_2	5.15	5.43	6.05	6.01	5.47	4.46	3.48	2.74	2.02
	h_3	26.1	21.6	18.0	12.7	9.24	5.98	3.94	2.72	2.0
	h_1	3.14	3.26	2.92	2.12	1.53	0.960	0.615	0.400	0.230
0.250	h_2	4.67	4.30	3.70	2.53	1.76	1.05	0.656	0.419	0.237
	h_3	10.1	6.49	4.36	2.19	1.24	0.630	0.362	0.224	0.123
	h_1	2.88	2.37	1.94	1.37	1.01	0.677	0.474	0.342	0.226
0.375	h_2	4.47	3.43	2.63	1.69	1.18	0.762	0.524	0.372	0.244
	h_3	5.05	2.65	1.60	0.812	0.525	0.328	0.223	0.157	0.102
	h_1	2.46	1.67	1.25	0.776	0.510	0.286	0.164	0.0956	0.0469
0.500	h_2	4.37	2.73	1.91	1.09	0.694	0.380	0.216	0.124	0.0607
	h_3	3.10	1.43	0.871	0.461	0.286	0.155	0.088	0.0506	0.0247
	h_1	2.07	1.41	1.105	0.755	0.551	0.363	0.248	0.172	0.107
0.625	h_2	4.30	2.55	1.84	1.16	0.816	0.523	0.353	2.42	0.150
	h_3	2.27	1.13	0.771	0.478	0.336	0.215	0.146	0.100	0.0616
	h_1	1.70	1.14	0.910	0.624	0.447	0.280	0.181	0.118	0.0670
0.750	h_2	4.24	2.47	1.81	1.15	0.798	0.490	0.314	0.203	0.115
	h_3	1.98	1.09	0.784	0.494	0.344	0.211	0.136	0.0581	0.0496
	h_1	1.38	1.11	0.962	0.792	0.677	0.574			
0.875	h_2	4.22	2.68	2.08	1.54	1.27	1.04			
	h_3	1.97	1.25	0.969	0.716	0.591	0.483			

$$J_{pl} = \alpha \varepsilon_o \sigma_o \frac{ba}{W} h_1(a/W, n)\left(\frac{P}{P_o}\right)^{n+1}$$

$$V_p = \alpha \varepsilon_o a h_2(a/W, n)\left(\frac{P}{P_o}\right)^{n}$$

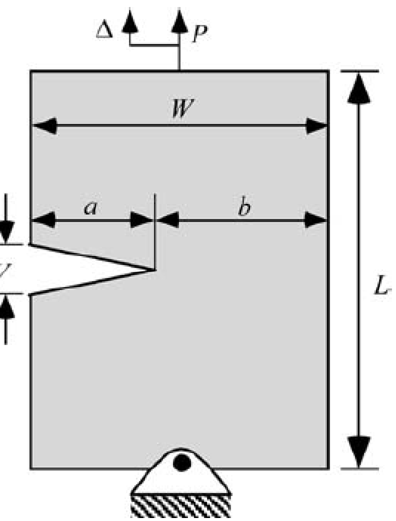
$$\Delta_{p(c)} = \alpha \varepsilon_o a h_3(a/W, n)\left(\frac{P}{P_o}\right)^{n}$$

$$\Delta_{p(nc)} = \alpha \varepsilon_o L\left(\frac{P}{2BW\sigma_o}\right)^{n}$$

$$P_o = 1.072\,\eta B b \sigma_o$$

where

$$\eta = \sqrt{1+\left(\frac{a}{b}\right)^2} - \frac{a}{b}$$



The Reference Stress Approach

- The EPRI equations assume that the material's stress-plastic strain curve follows a simple power law.
- Many materials, however, have flow behavior that deviates considerably from a power law. For example, most low carbon steels exhibit a plateau in the flow curve immediately after yielding.
- Ainsworth [29] modified the EPRI relationships to reflect more closely the flow behavior of real materials.
- Reference stress $$\sigma_{ref} = (P/P_o)\sigma_o$$

  σ_o = reference stress value that is usually equal to the yield strength
- Reference strain (ε_{ref}): the total axial strain when the material is loaded to a uniaxial stress of σ_{ref}

EPRI Equation

$$J_{pl} = \alpha\varepsilon_o\sigma_o b h_1(a/W,n)\left(\frac{P}{P_o}\right)^{n+1} \Rightarrow J_{pl} = \sigma_{ref} b h_1\left(\varepsilon_{ref} - \frac{\sigma_{ref}\varepsilon_o}{\sigma_o}\right)$$

- h_1, the geometry factor that depends on the power-law-hardening exponent n. relatively insensitive to n except high n values (low-hardening materials)

The Reference Stress Approach

- He proposed the following approximation.

$$h_1(n) \approx h_1(1)$$

EPRI Equation

$$J_{pl} = \alpha \varepsilon_o \sigma_o b h_1(a/W, n) \left(\frac{P}{P_o} \right)^{n+1} \quad \Rightarrow \quad J_{pl} = \frac{\mu K_I^2}{E} \left(\frac{E \varepsilon_{ref}}{\sigma_{ref}} - 1 \right)$$

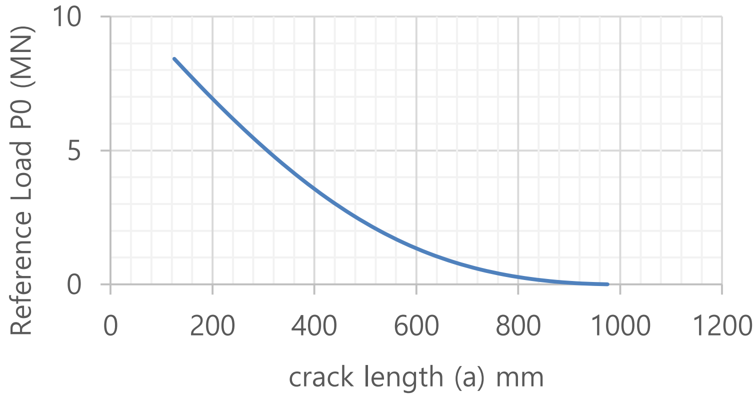
μ = 0.75 for plane strain and μ = 1.0 for plane stress.

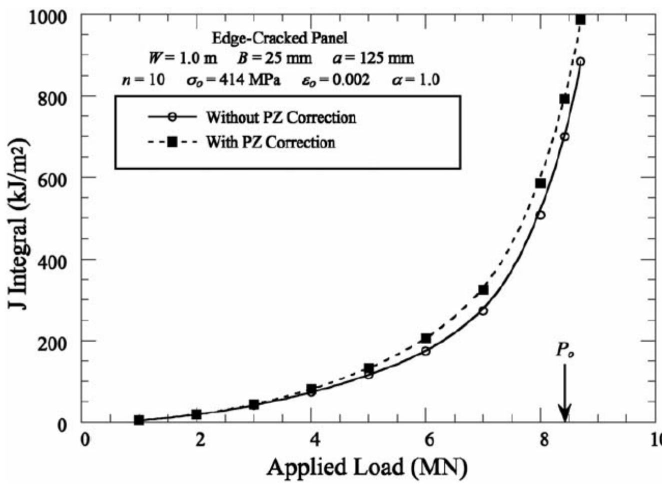
- The above equation is not only simpler than EPRI eq., but also more widely applicable due to thousands of stress-intensity factor solutions in handbooks and the literature.

The Reference Stress Approach

- In Ex. 9.1) P_0 depends on the crack length a.

$$P_o = 1.072\eta\sigma_o bB$$





- *To be independent of Crack length, reference stress is introduced.*

$$\sigma_{ref} = (P/P_o)\sigma_o$$ σ_o = yield strength

EPRI Equation

$$J_{pl} = \alpha\varepsilon_o\sigma_o bh_1(a/W,n)\left(\frac{P}{P_o}\right)^{n+1} \Rightarrow J_{pl} = \sigma_{ref} bh_1\left(\varepsilon_{ref} - \frac{\sigma_{ref}\varepsilon_o}{\sigma_o}\right) \Rightarrow J_{pl} = \frac{\mu K_I^2}{E}\left(\frac{E\varepsilon_{ref}}{\sigma_{ref}} - 1\right)$$

Ductile Instability Analysis

- Crack growth is stable as long as the rate of change in the driving force (J) is less than or equal to the rate of change of the material resistance (J_R)

$$T_{app} = \frac{E}{\sigma_o^2}\left(\frac{dJ}{da}\right)_{\Delta_T} \quad \text{and} \quad T_R = \frac{E}{\sigma_o^2}\frac{dJ_R}{da}$$

Δ_T is the remote displacement:

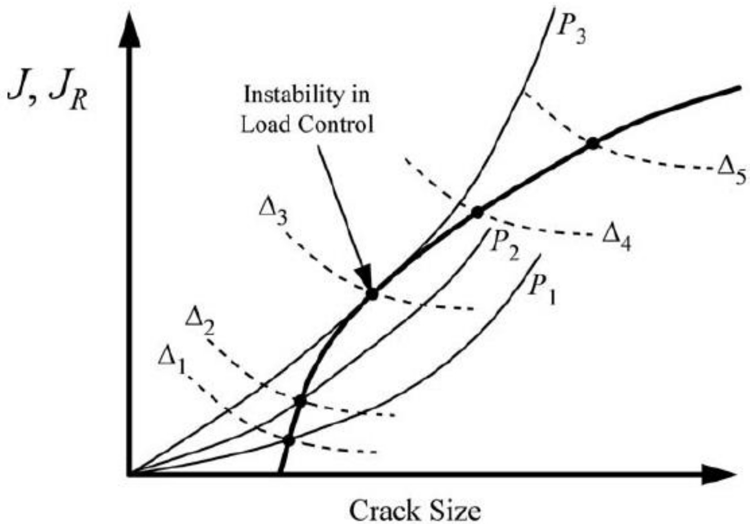
$$\Delta_T = \Delta + C_M P$$

- Crack growth is unstable when

$$T_{app} > T_R$$

- The rate of change in driving force at a fixed remote displacement

$$\left(\frac{dJ}{da}\right)_{\Delta_T} = \left(\frac{\partial J}{\partial a}\right)_P - \left(\frac{\partial J}{\partial P}\right)_a \left(\frac{\partial \Delta}{\partial a}\right)_P \left[C_m + \left(\frac{\partial \Delta}{\partial P}\right)_a\right]^{-1}$$

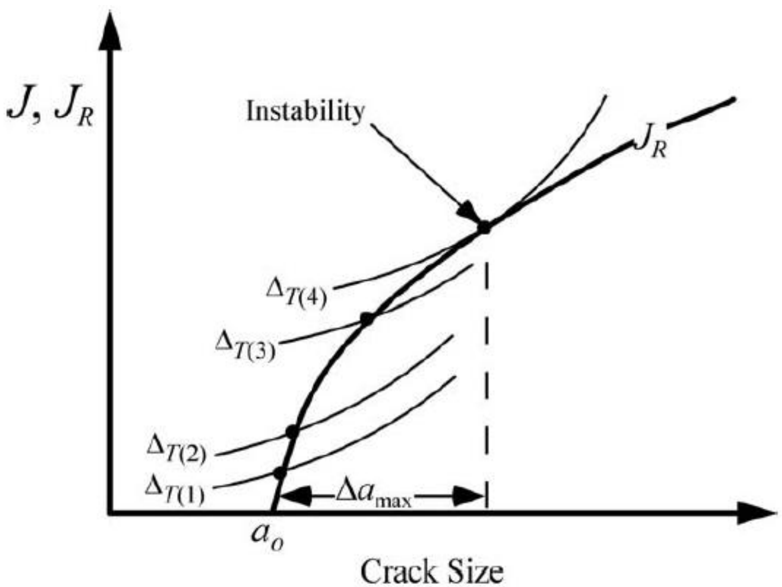


Schematic driving force diagram for a fixed remote displacement.

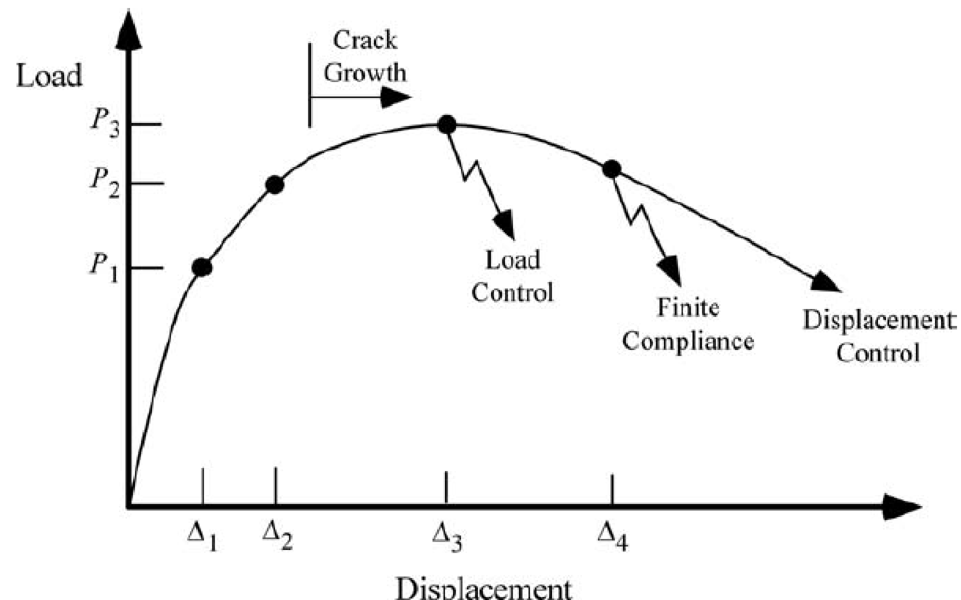
- The structure is unstable at P_3 and Δ_3 in load control, but the structure is stable in displacement control

Ductile Instability Analysis

- Driving force curves for this same structure, but with fixed remote displacement Δ_T and finite system compliance C_M. The structure is unstable at $\Delta_{T(4)}$ in this case ($\Delta_{T(4)} = \Delta_4 + C_M P_4$) .
- A maximum load plateau occurs at P_3 and Δ_3, and the load decreases with further displacement



Schematic driving force diagram for a fixed remote displacement.



Schematic load-displacement curve for the material.

Ductile Instability Analysis

- The applied and material tearing moduli are plotted against J and J_R, respectively. Instability occurs when the T_{app}-J curve crosses the T_R-J_R curve.
- The latter curve is relatively easy to obtain, since J_R depends only on the amount of crack growth.

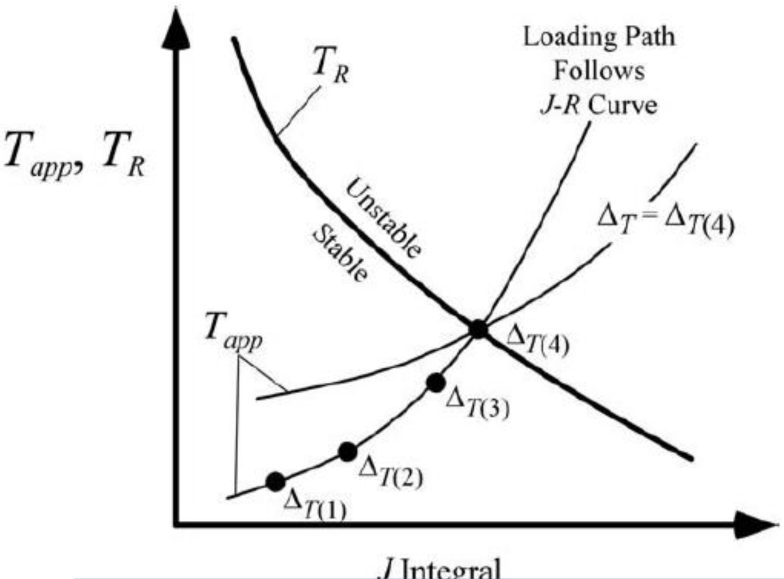
$$J_R = J_R(a - a_o)$$

- There is a unique relationship between T_R and J_R. For example,

$$J_R = C_1(a - a_o)^{C_2}$$

- The material tearing modulus is given by

$$T_R = \frac{E}{\sigma_o^2}\frac{C_2 J_R}{(a-a_o)} = \frac{E}{\sigma_o^2} C_2 C_1^{1/C_2} J_R^{(C_2-1)/C_2}$$



Schematic stability assessment diagram for the material in the three previous figures

Ductile Instability Analysis

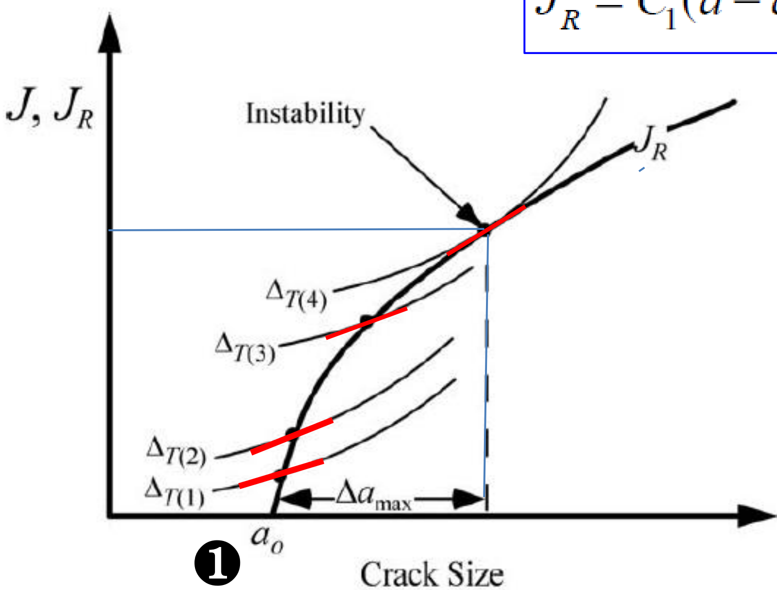
- There are a number of approaches for defining the T_{app} -J curve, depending on the application.
- Method ❶ : Suppose that the initial crack size a_o is known. Since $J = J_R$ during stable crack growth, the applied J at a given crack size can be inferred from the J-R curve.

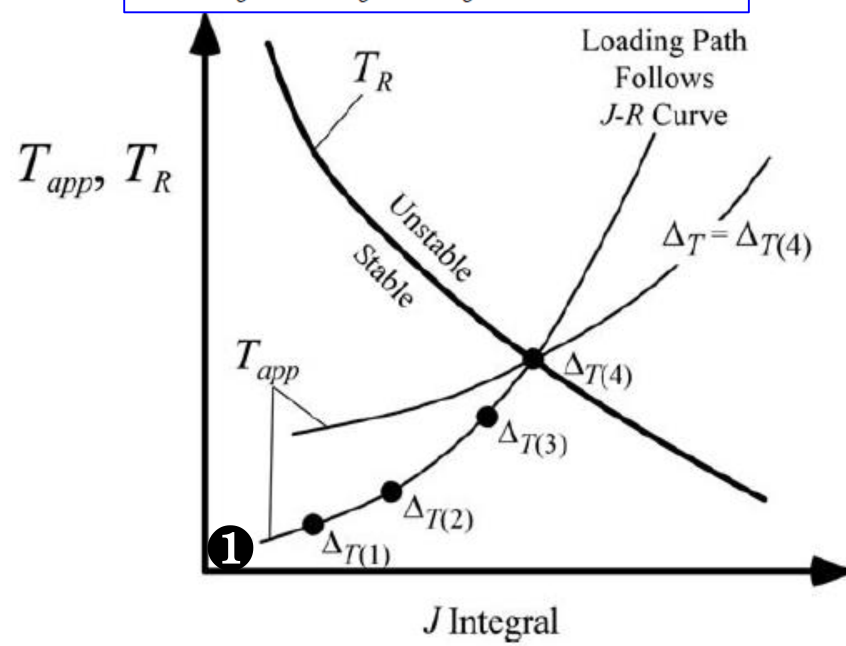
$$T_{app} = \frac{E}{\sigma_o^2}\left(\frac{dJ}{da}\right)_{\Delta_T}$$

- The remote displacement Δ_T increases as the loading progresses up the J-R curve; instability occurs at $\Delta_{T(4)}$.→ $J = J_R$ $T_{app} = T_R$ final load, local displacement, crack size, stable crack extension.

$$J_R = C_1(a - a_o)^{C_2}$$

$$T_R = \frac{E}{\sigma_o^2}\frac{C_2 J_R}{(a-a_o)} = \frac{E}{\sigma_o^2}C_2 C_1^{1/C_2} J_R^{(C_2-1)/C_2}$$





Ductile Instability Analysis

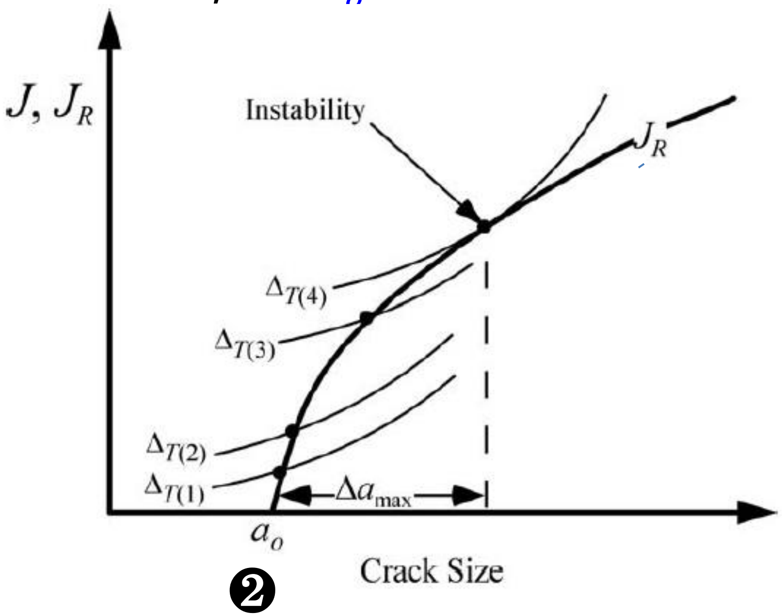
- Method ❷ : by fixing one of the loading conditions (P, Δ, or Δ_T), and determining the critical crack size at failure, as well as a_o.
- For example, if we fix Δ_T at $\Delta_{T(4)}$ in the structure, the same failure point can be predict as the previous analysis but the T_{app}-J curve would follow a different path.

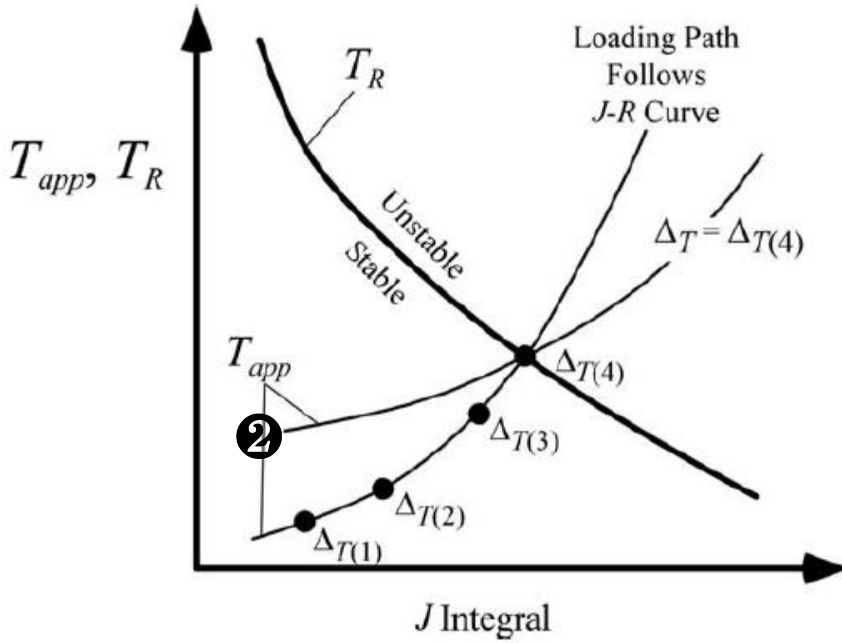
$$\boxed{\Delta_T = \Delta + C_M P}$$

$$\boxed{J_R = C_1 (a - a_o)^{C_2}}$$

$$\boxed{T_R = \frac{E}{\sigma_o^2}\frac{C_2 J_R}{(a-a_o)} = \frac{E}{\sigma_o^2} C_2 C_1^{1/C_2} J_R^{(C_2-1)/C_2}}$$

- If, however, we fix the remote displacement at a different value, we would predict failure at another point on the T_R-J_R curve; the critical crack size, stable crack extension, and a_o would be different from the previous example.





Ductile Instability Analysis

- The Failure Assessment Diagrams (FAD) is probably the most widely used methodology for elastic plastic fracture mechanics analysis of structural components.

- FAD based on the strip-yield plastic zone correction : the strip-yield model has limitations, however. For example, it does not account for strain hardening.

- FAD based on the elastic-plastic *J*-integral solution.

- Simplified FAD that account for strain hardening without a rigorous *J* integral solution

Original Concept based on the strip-yield model

- The first FAD was derived from a modified version of the strip-yield model.
- The effective stress intensity factor for a through crack in an infinite plate.

$$\boxed{K_{eff} = \sigma_{YS}\sqrt{\pi a}\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi\sigma}{2\sigma_{YS}}\right)\right]^{1/2}}$$

- If $\sigma_{YS} \rightarrow \sigma_c$, the strip-yield model predicts failure as the applied stress approaches the collapse stress. For a structure loaded in tension, collapse occurs when the stress on the net cross-section reaches the flow stress (stress required to continue plastically deforming the material) of the material.
- Thus σ_c depends on the tensile properties of the material and the flaw size relative to the total cross section of the structure.

$$\frac{K_{eff}}{K_I} = \frac{\sigma_c}{\sigma}\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_c}\right)\right]^{1/2}$$

$$\boxed{K_I = \sigma\sqrt{\pi a}}$$

Original Concept based on the strip-yield model

$$K_r = \frac{K_I}{K_{eff}} \qquad S_r = \frac{\sigma}{\sigma_c}$$

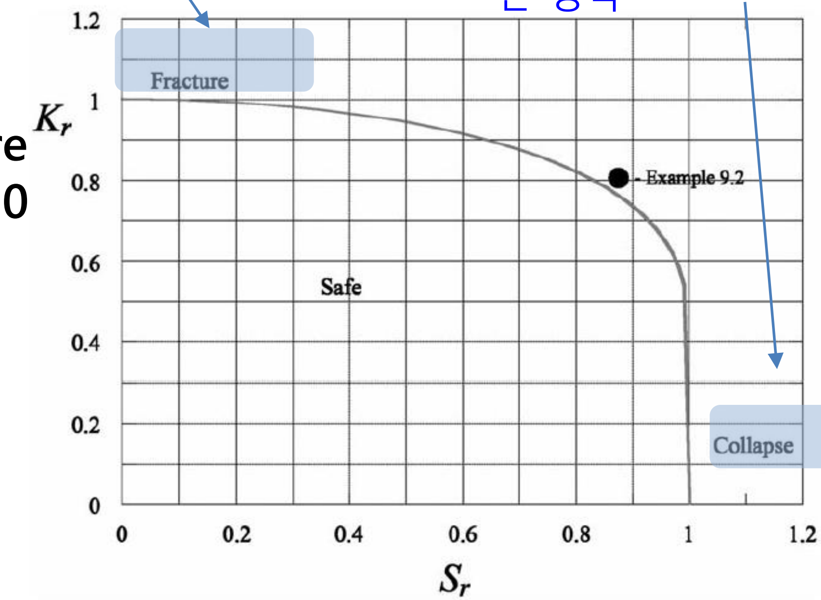
$$\frac{K_{eff}}{K_I} = \frac{\sigma_c}{\sigma}\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_c}\right)\right]^{1/2}$$

$$K_r = S_r\left[\frac{8}{\pi^2}\ln\sec\left(\frac{\pi}{2}S_r\right)\right]^{-1/2}$$

"Applied stress σ 는 작지만 crack length a가 커서 K_I가 K_{mat} 보다 큰 영역"

"Applied stress σ가 collapse strength보다 커서 collapse 되는 영역"

- Fracture is predicted when $K_{eff} = K_{mat}$, where K_{mat} is the fracture toughness in terms of stress intensity units.
- In intermediate cases, collapse and fracture interact, and both K_r and S_r are less than 1.0 at failure.



$$K_r = \frac{K_I}{K_{mat}} \qquad K_I = Y\sigma\sqrt{\pi a}$$

The strip-yield failure assessment diagram

Original Concept based on the strip-yield model

❖ EX 9.2) A middle tension (MT) panel 1 m wide and 25 mm thick with a 200 mm crack must carry a 7.00 MN load. For the material K_{mat} = 200 MPa, σ_{YS} = 350 MPa, and σ_{TS} = 450 MPa.

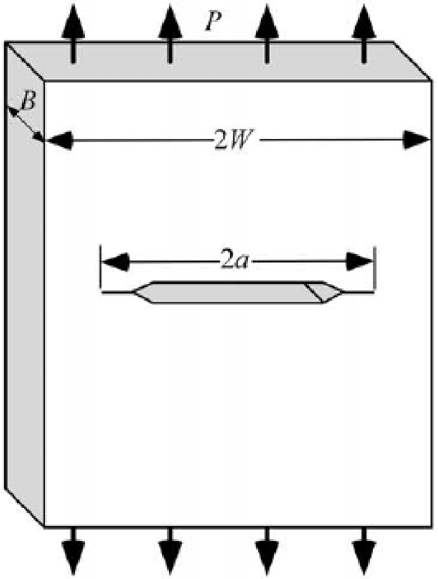
Sol) assuming a flow stress that is the average of yield and tensile strength, $\sigma_{flow} = \sigma_c$ = 400 Mpa.

$$P_c = (400\ \text{MPa})(0.025\ \text{m})(1\ \text{m} - 0.200\ \text{m}) = 8.00\ \text{MN}$$

$$S_r = \frac{7.00\ \text{MN}}{8.00\ \text{MN}} = 0.875$$

$$K_I = \frac{7.00\ \text{MN}}{(0.025\ \text{m})(1.0\ \text{m})}\sqrt{\pi(0.100\ \text{m})\sec\left(\frac{\pi(0.100\ \text{m})}{1.00\ \text{m}}\right)} = 161\ \text{MPa}\sqrt{\text{m}}$$

$$\boxed{K_I = \sigma\sqrt{\pi a}\left[\sec\left(\frac{\pi a}{2W}\right)^{1/2}\right]\underbrace{\left[1 - 0.025\left(\frac{a}{W}\right)^2 + 0.06\left(\frac{a}{w}\right)^4\right]}_{\text{negligible}}} \quad (2.46)$$



middle tension (MT) specimen

$$K_r = \frac{161}{200} = 0.805$$

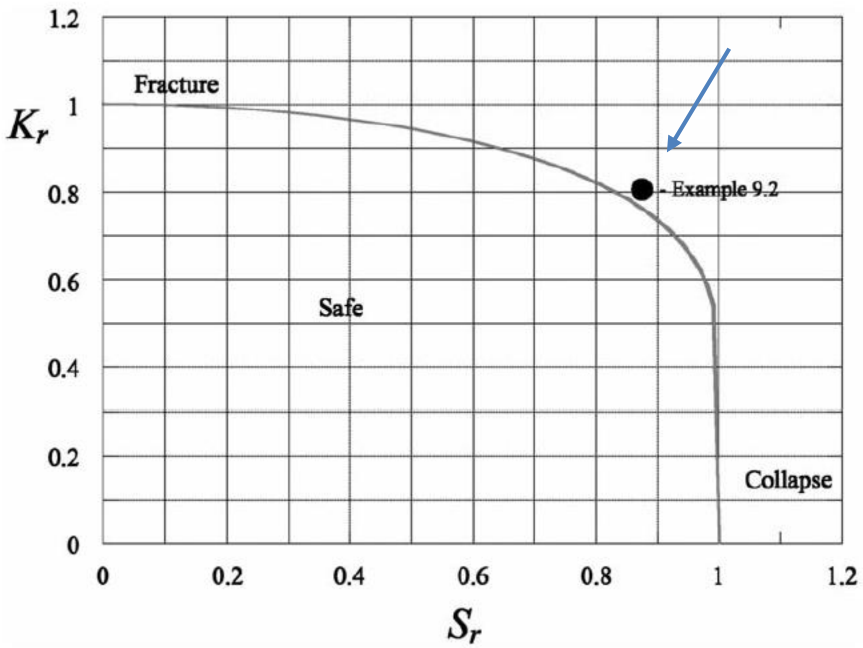
Original Concept based on the strip-yield model

❖EX 9.2) A middle tension (MT) panel 1 m wide and 25 mm thick with a 200 mm crack must carry a 7.00 MN load. For the material K_{mat} = 200 MPa, σ_{YS} = 350 MPa, and σ_{TS} = 450 MPa.

Sol)

$$S_r = \frac{7.00\ \text{MN}}{8.00\ \text{MN}} = 0.875$$

$$K_r = \frac{161}{200} = 0.805$$



- This point falls outside of the failure assessment diagram, the panel will fail before reaching 7 MN.

J-Based FAD

- The shape of the FAD curve is a function of plasticity effects.
- The applied J can be converted to an equivalent K through the following relationship.

$$K_J = \sqrt{\frac{JE}{1-\nu^2}}$$

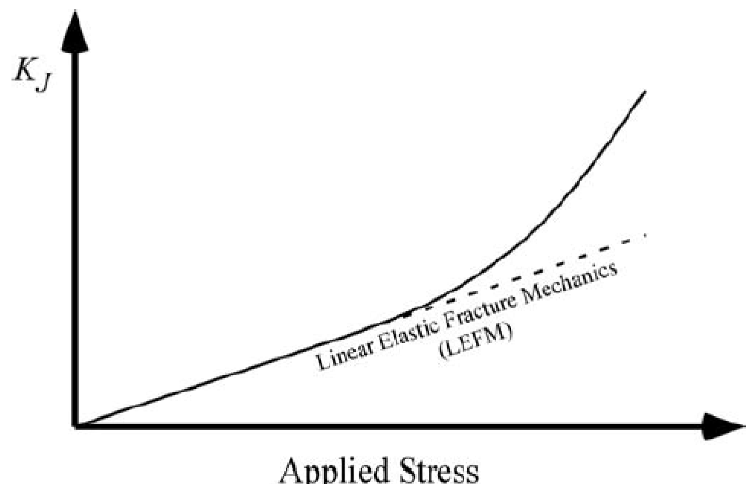
$$J = J_{el} + J_{pl} = \frac{K_I^2}{E'} + \frac{\eta_p U_p}{Bb}, \quad E' = \frac{E}{(1-\nu^2)} \quad in\ 3.2.5$$

- In the linear elastic range, $K_J = K_I$ and stresses near the crack tip are characterized by a singularity.
- In the elastic-plastic range, the plot of K_J vs. stress deviates from linearity and a stress singularity no longer exists. Horizontal axis = L_r

$$K_r = \frac{K_I}{K_J} \qquad L_r = \frac{\sigma_{ref}}{\sigma_{YS}}, \quad \sigma_{ref} = \frac{P}{P_0}\sigma_0$$

where σ_{ref} is the reference stress.

- The reference stress has been based on yield load or limit load solutions for the configuration of interest.



Schematic plot of crack driving force vs. applied stress.

J-Based FAD

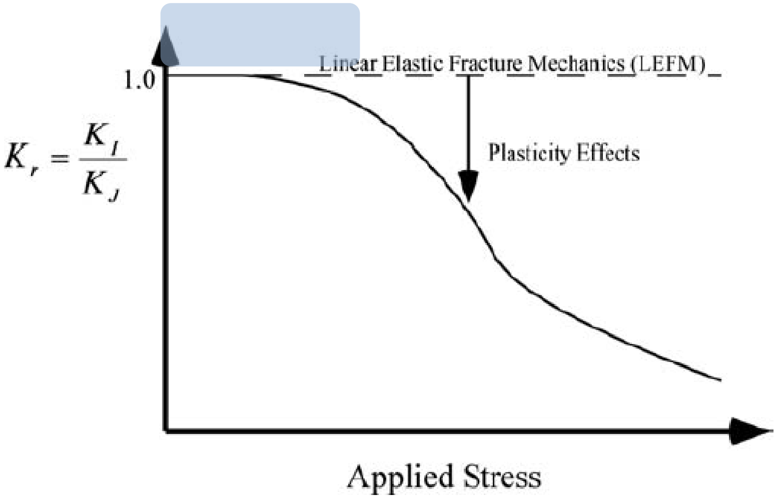
- It needs to incorporate the fracture toughness into the analysis.

$$K_r = \frac{K_I}{K_{mat}}$$

- Fracture toughness is usually characterized by either *J* or CTOD.

$$K_{mat} = \sqrt{\frac{J_{crit}E}{1-\nu^2}} \qquad K_{mat} = \sqrt{\frac{\chi\sigma_{YS}\delta_{crit}E}{1-\nu^2}}$$

- where χ is a constraint factor, which typically ranges from 1.5 to 2 for most geometries and materials.



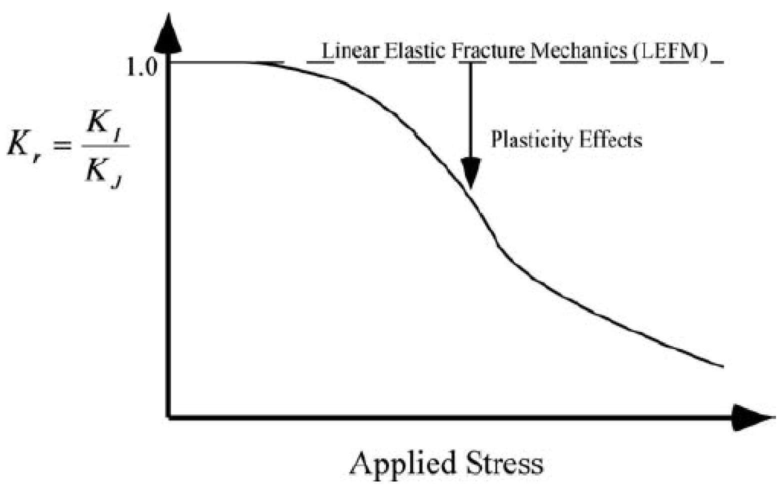
Driving force curve, K_r vs. applied stress

J-Based FAD

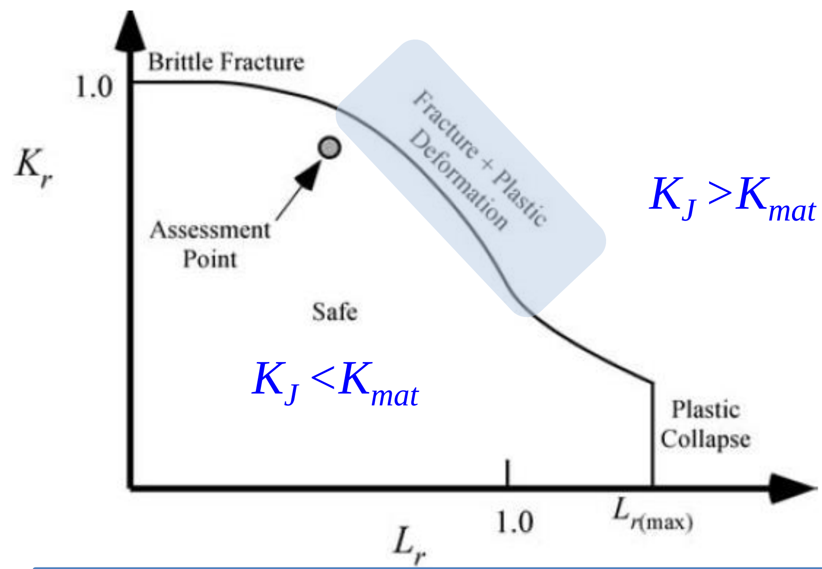
- Provided $L_r < L_{r(\max)}$, the failure criterion in the FAD method can be inferred

$$\boxed{K_r = \frac{K_I}{K_{mat}}} \quad \boxed{K_r = \frac{K_I}{K_J}} \Rightarrow \frac{K_I}{K_{mat}} \geq \frac{K_I}{K_J} \Rightarrow \therefore K_J \leq K_{mat}$$

$$\boxed{J = J_{el} + J_{pl}}$$



Driving force curve, K_r vs. applied stress



Failure assessment diagram (FAD), which spans the range of fully brittle to fully ductile behavior

Approximations of FAD Curve

- The most rigorous method to determine a FAD curve for a particular application is to perform an elastic-plastic *J* integral analysis and define K_r.

$$K_r = \frac{K_I}{K_J}$$

- Simplified approximations of the FAD curve are available.
- Method 1) the material dependent, geometry independent using reference stress.

$$\sigma_{ref} = \frac{P}{P_0}\sigma_0$$

$$K_r = \left(\frac{E\varepsilon_{ref}}{L_r \sigma_{YS}} + \frac{L_r^3 \sigma_{YS}}{2E\varepsilon_{ref}} \right)^{-1/2} \quad \text{for } L_r \le L_{r(\max)} \qquad (9.67)$$

ε_{ref} is inferred from the true stress – true strain curve at $\sigma_{ref.}$

- Method 2) Material & geometry independent

$$K_r = [1 - 0.14(L_r)^2]\{0.3 + 0.7\exp[-0.65(L_r)^6]\} \quad \text{for } L_r \le L_{r(\max)} \qquad (9.68a)$$

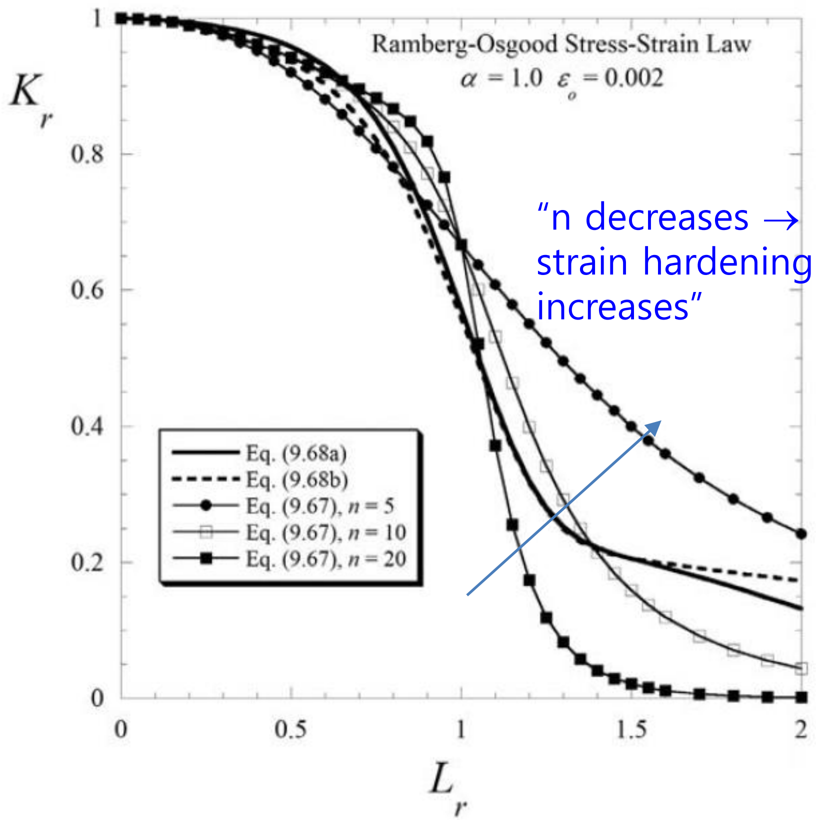
$$K_r = [1 + 0.5(L_r)^2]^{-1/2}\{0.3 + 0.7\exp[-0.6(L_r)^6]\} \quad \text{for } L_r \le L_{r(\max)} \qquad (9.68b)$$

an empirical fit of FAD curves generated with Equation (9.67)

Approximations of FAD Curve

- As strain-hardening increases (i.e., as *n* decreases), there is a more gradual "tail" in the FAD curve. The material dependence in the FAD curve manifests itself primarily in the fully plastic regime (L_r > 1).

$$\frac{\varepsilon_{pl}}{\varepsilon_o} = \alpha\left(\frac{\sigma}{\sigma_o}\right)^n$$



$$K_r = \left(\frac{E\varepsilon_{ref}}{L_r\sigma_{YS}} + \frac{L_r^3\sigma_{YS}}{2E\varepsilon_{ref}}\right)^{-1/2}$$

Comparison of simplified FAD expressions (Eq (9.67)) and Eq (9.68)).

Estimating the Reference Stress

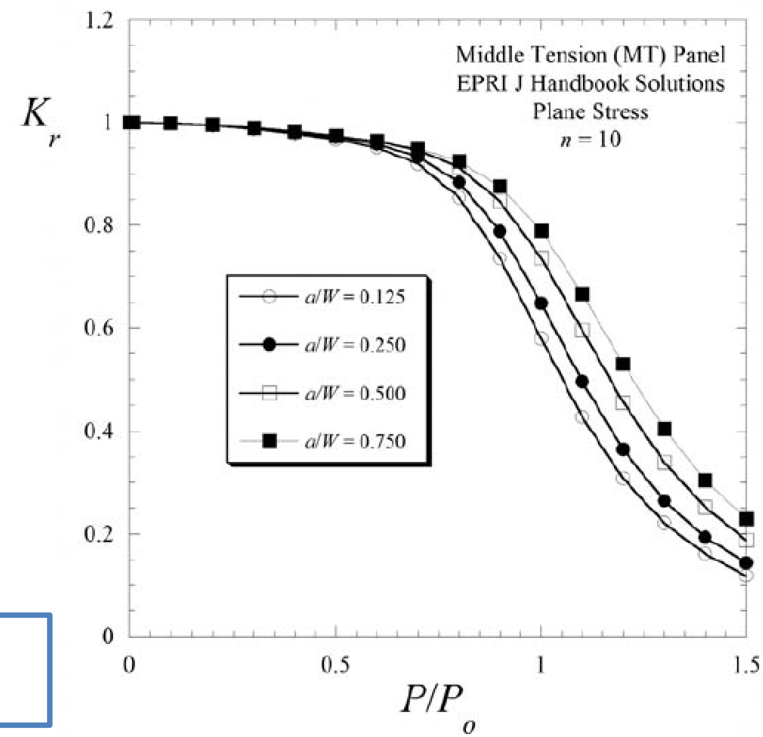
- Most FAD approaches normalize the *x* axis by the limit load or yield load solution. Unfortunately, this practice can lead to apparent geometry dependence in the FAD curve.
- The EPRI *J* handbook procedure was used to generate FAD curves for various normalized crack lengths in a middle tension (MT) specimen, When the applied load is normalized by the yield load P_o on the *x* axis, the resulting FAD curves depend on the relative crack length.

❖An alternative approach for normalizing the *x* axis of the FAD

- Setting $L_r = 1$ in this equation and solving for the ratio of the total *J* to the elastic component.

$$\left.\frac{J}{J_{elastic}}\right|_{L_r=1} = 1 + \frac{0.002E}{\sigma_{YS}} + \frac{1}{2}\left(1 + \frac{0.002E}{\sigma_{YS}}\right)^{-1}$$

$$K_r = \frac{K_I}{K_J} = \sqrt{\frac{J_{elastic}}{J}}$$



EPRI handbook *J*-integral solutions for middle tension panels

Estimating the Reference Stress

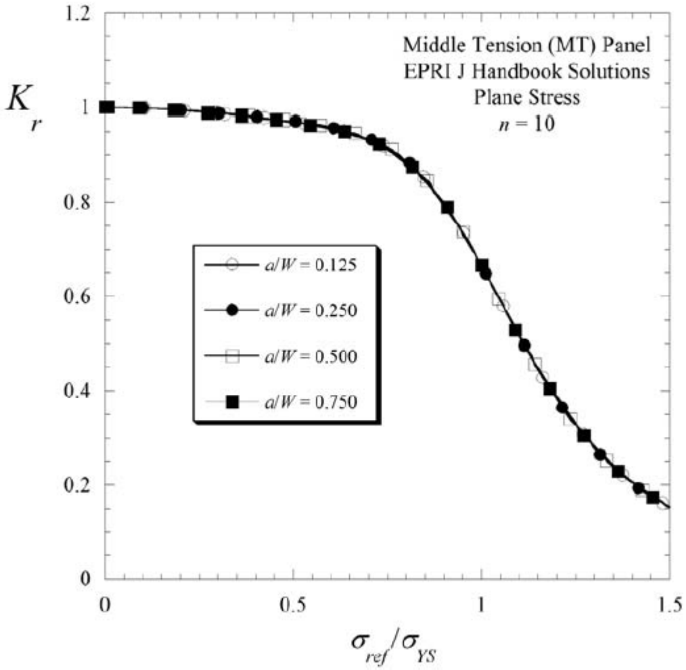
- The reference stress, which is used to compute L_r is proportional to the nominal applied stress: where F is a geometry factor.

$$\sigma_{ref} = \sigma_{nominal} F \qquad L_r = \frac{\sigma_{ref}}{\sigma_{YS}} = \frac{\sigma_{nominal} F}{\sigma_{YS}}$$

- The geometry dependence disappears

$$F = \frac{\sigma_{YS}}{\sigma_{nominal}\big|_{L_r=1}}$$

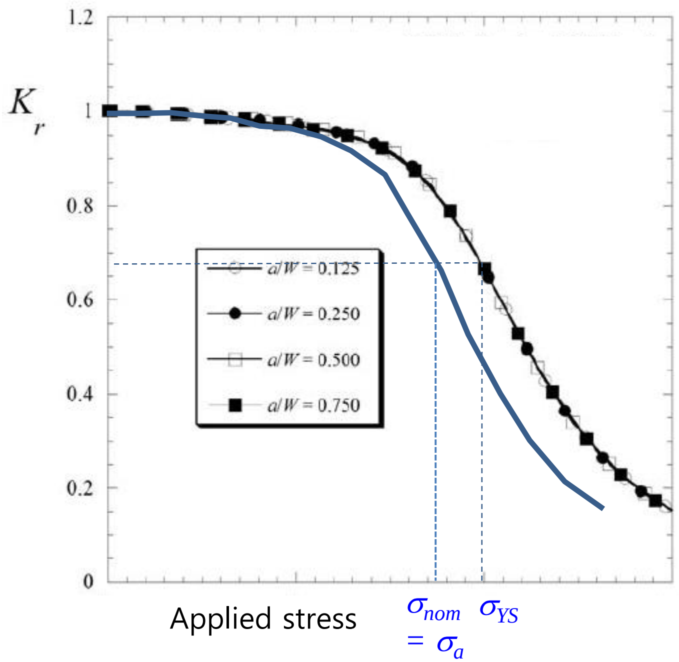
- This method forces all curves to pass through the same point at $L_r = 1$.

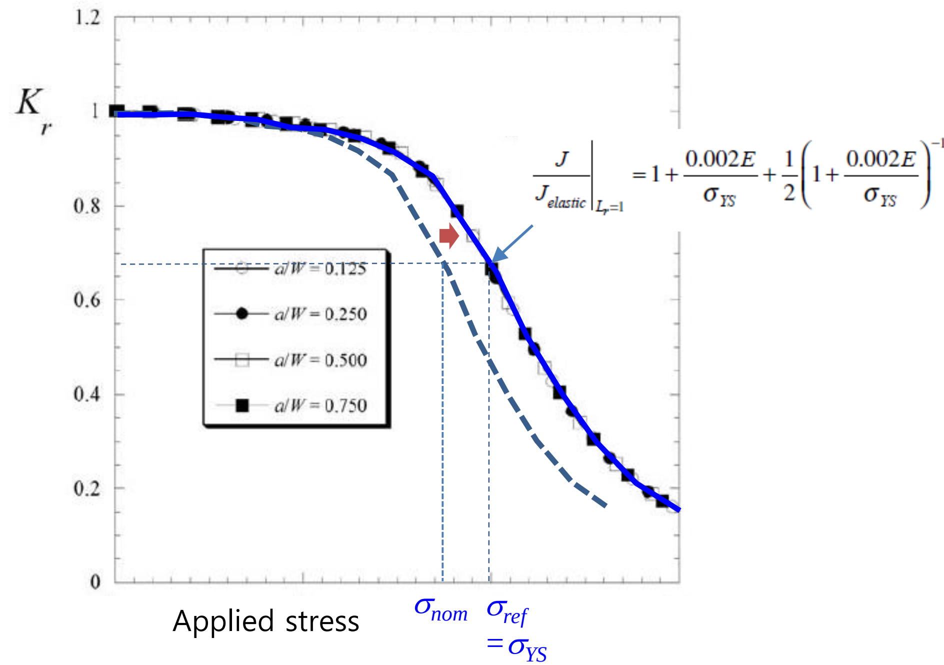


FAD curves with reference stress defined according to the procedure in Equation (9.69) to Equation (9.71).

Estimating the Reference Stress

❖ Illustration of the estimation of reference stress



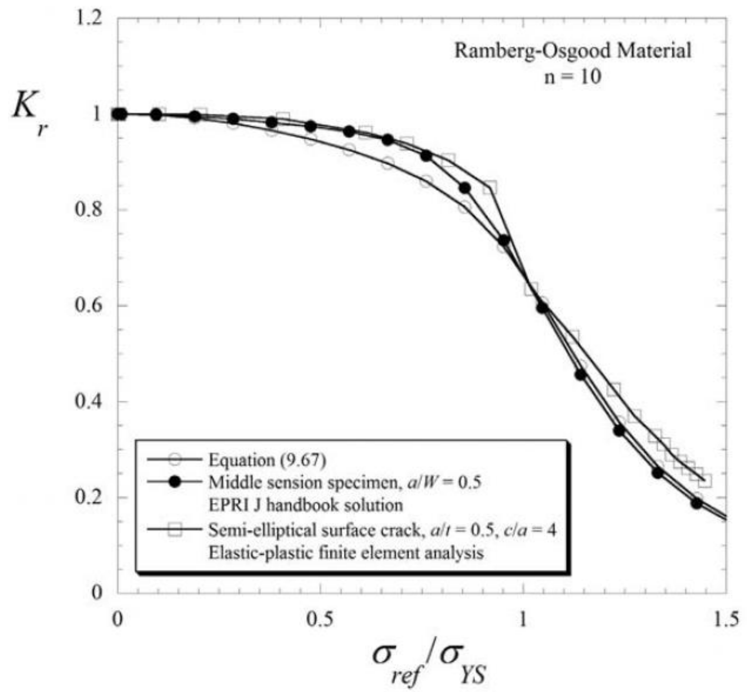


$$F = \frac{\sigma_{YS}}{\left.\sigma_{nominal}\right|_{L_r=1}}$$

Estimating the Reference Stress

- Equation (9.67) : material-specific and geometry-independent FAD expression.
- The three curves are in precise agreement at L_r = 1. At other L_r values, there is good agreement.
- Therefore, the shape of the FAD curve is relatively insensitive to geometry, and the material-specific FAD expression (9.67) agrees reasonably well with a rigorous J solution, provided the reference stress is defined by the procedure in the last slides.

$$\left.\frac{J}{J_{elastic}}\right|_{L_r=1} = 1 + \frac{0.002E}{\sigma_{YS}} + \frac{1}{2}\left(1 + \frac{0.002E}{\sigma_{YS}}\right)^{-1}$$



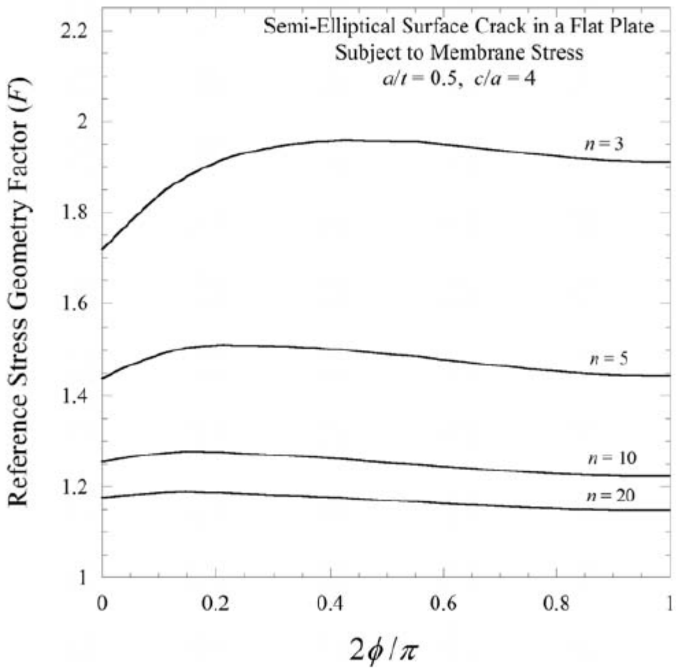
Comparison of the simplified material-dependent FAD (Equation (9.67)) with J-based FAD curves for two geometries.

Estimating the Reference Stress

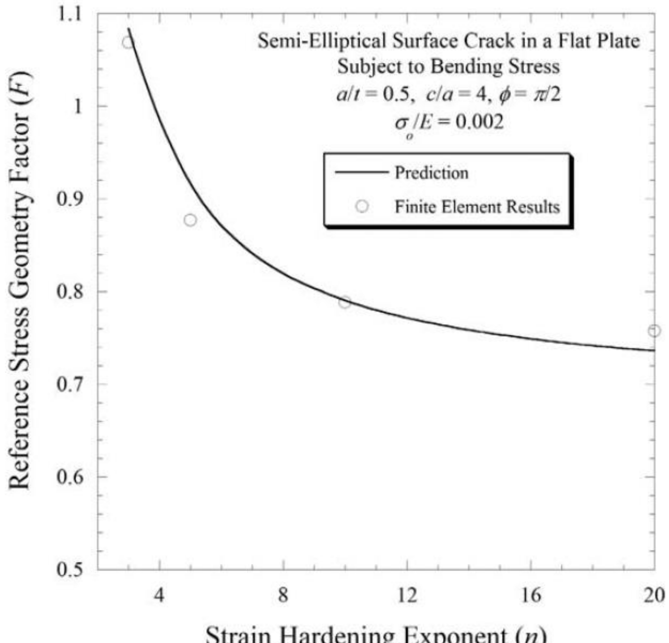
$$\left.\frac{J}{J_{elastic}}\right|_{L_r=1} = 1 + \frac{0.002E}{\sigma_{YS}} + \frac{1}{2}\left(1 + \frac{0.002E}{\sigma_{YS}}\right)^{-1}$$

- The reference stress solution is relatively insensitive to the location on the crack front angle ϕ. but *F* is a strong function of the hardening exponent.

$$F(n_2) = F(n_1)\frac{\left(1 + 2n_2^{1.15}\right)}{\left(1 + 2n_1^{1.15}\right)}$$



Reference stress geometry factor as a function of crack front position and hardening exponent for a semielliptical surface crack in a flat plate.



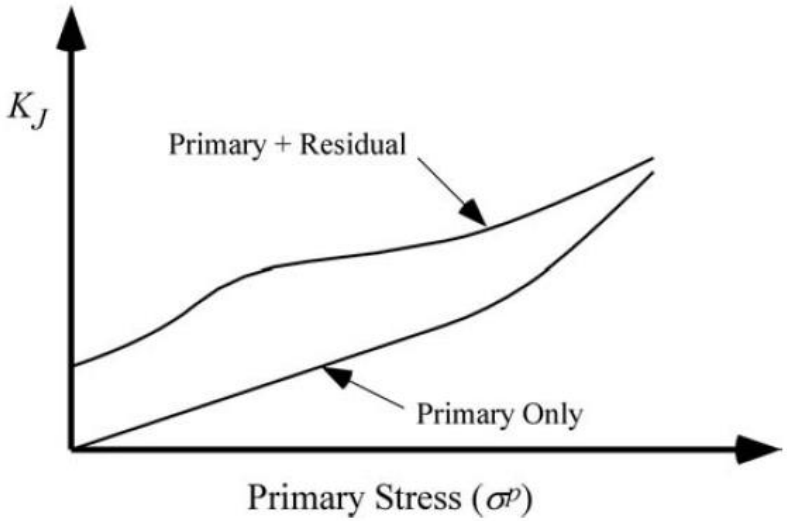
Correlation between the reference stress geometry factor and the hardening exponent for a semielliptical surface crack in a plate subject to a bending stress.

Application to Welded Structures

- The welding process creates residual stresses in and around the weld.
- Geometric anomalies(변칙) such as weld misalignment create additional local stresses.
- The weld metal and heat-affected zone (HAZ) typically have different material properties than the base metal.
- The toughness properties of the weld must, of course, be taken into account in the material resistance.
- The different stress-strain responses of the weld metal and base metal can have a significant effect on the crack driving force.

Incorporating Weld Residual Stresses

- Weld residual stress is usually not considered in most design codes because it does not have a significant effect on the tensile strength of the welded joint, provided the material is ductile.
- When a crack is present, however, residual stresses must be included in the crack driving force. Under linear elastic conditions, residual stresses are treated the same as any other stress.
- At intermediate applied stresses, the K_J vs. stress curve is nonlinear because the combination of primary and residual stresses result in crack-tip plasticity.
- At higher applied stresses, global plasticity results in relaxation of residual stresses.



Schematic plot of crack driving force vs. applied primary stress, with and without an imposed residual stress.

- When the applied primary stress is zero, $K_J > 0$ due to the contribution of the residual stress.
- At higher applied stresses, global plasticity results in relaxation of residual stresses $\rightarrow$ *mechanical stress relief.*
- At intermediate applied stresses, the K_J vs. stress curve is nonlinear because the combination of primary and residual stresses result in crack-tip plasticity.

Incorporating Weld Residual Stresses

- For primary stresses alone, the FAD curve is defined as

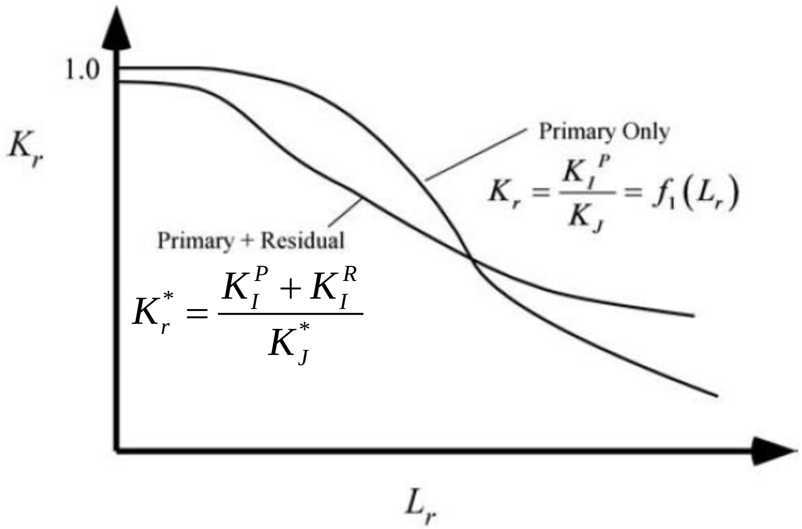
$$K_r = \frac{K_I^P}{K_J} = f_1(L_r)$$

- When residual stresses are present, the shape of the FAD curve is a function of the magnitude of the residual stresses:

$$K_r^* = \frac{K_I^P + K_I^R}{K_J^*} = f_2(L_r, K_I^R)$$

K_J^* accounts for residual stress

- The unusual shape of the FAD curve for the weld with residual stress is due to crack-tip plasticity at intermediate L_r values and mechanical stress relief at high L_r values.



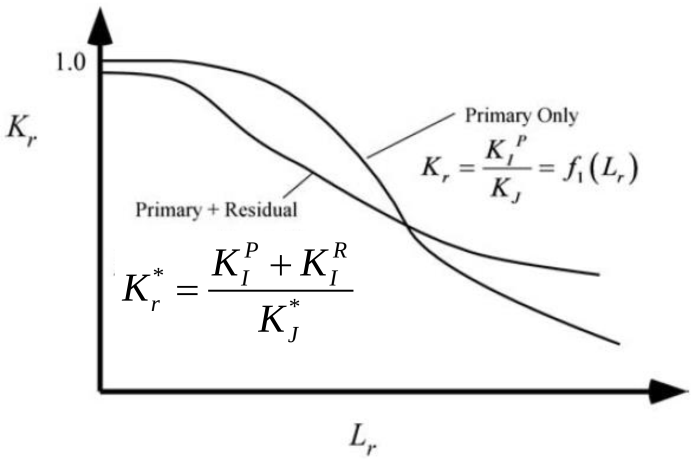
Driving force curves plotted in FAD coordinates

Incorporating Weld Residual Stresses

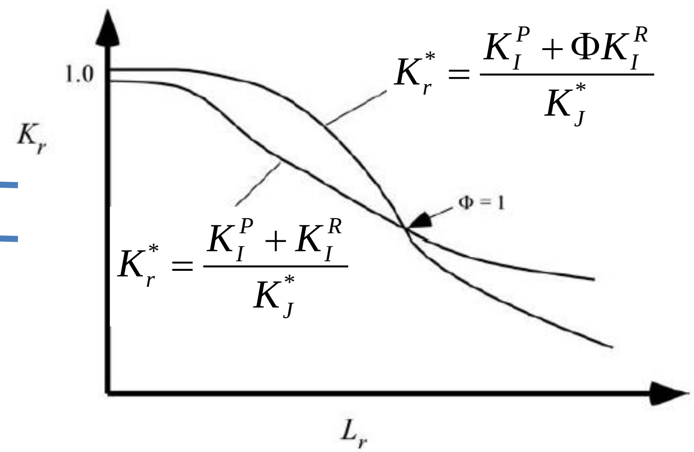
- It is not particularly convenient to apply a FAD curve whose shape is a function of residual stress
- An alternative formulation, where the residual stress effects are decoupled from the FAD curve. Φ : plasticity adjustment.

$$K_r = f_1(L_r) = \frac{K_I^P + \Phi K_I^R}{K_J^*} \qquad \Phi = \frac{f_1(L_r)K_J^* - K_I^P}{K_I^R}$$

- The Φ factor can be derived from elastic-plastic finite element analysis. Various initial residual stress distributions are imposed on a finite element model that contains a crack, and then primary loads are applied.



Driving force curves plotted in FAD coordinates



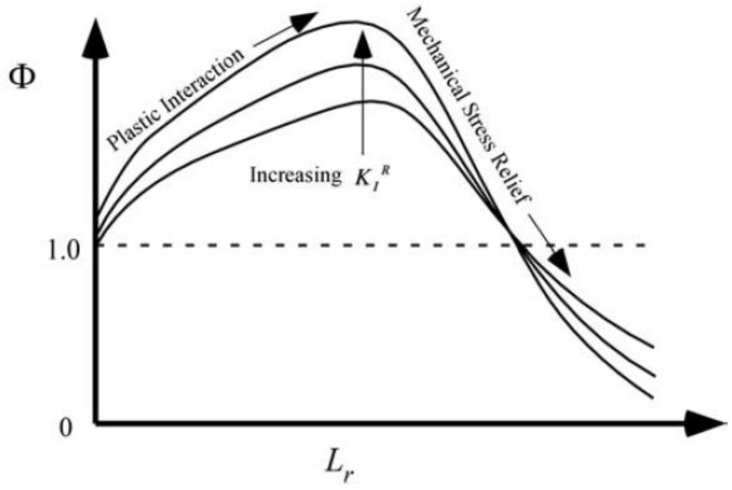
Effect of the plasticity adjustment factor Φ on the FAD curve for a weldment with residual stress.

Incorporating Weld Residual Stresses

- As L_r increases, the crack-tip plasticity magnifies the total driving force, so $\Phi > 1$.

- Eventually Φ reaches a peak and then decreases due to mechanical stress relief.

- The *y* coordinate of the assessment point on the FAD.



Schematic plot of the plasticity adjustment factor on residual stress, Φ, vs. applied primary stress.

$$K_r = \frac{K_I^P + \Phi K_I^R}{K_{mat}}$$

$$\boxed{K_r = f_1(L_r) = \frac{K_I^P + \Phi K_I^R}{K_J^*}}$$

- Plasticity adjustment is made to K_r (without residual stress) using Φ.

$$K_r = f_1(L_r) = \frac{K_I^P + \Phi K_I^R}{K_J^*} \Rightarrow K_J^* = \frac{K_I^P + \Phi K_I^R}{f_1(L_r)}$$

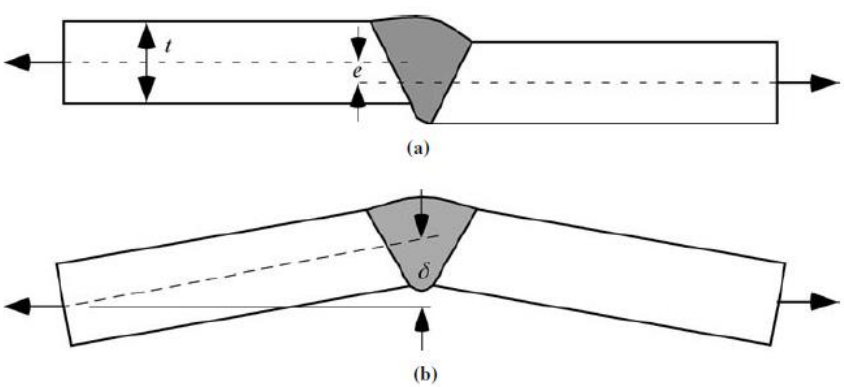
- The failure criterion : $K_J^* \geq K_{mat}$.

Weld Misalignment

- When plates or shells are welded, there is invariably some degree of misalignment. The misalignment creates a local bending stress.

$$\sigma_b^{local} = \sigma_m^{remote}\left(\frac{6e}{t}\right)$$

- This local stress usually does not make a significant contribution to static overload failure, provided the material is ductile.
- Misalignment stresses can, however, increase the risk of brittle fracture and shorten the fatigue life of a welded joint.



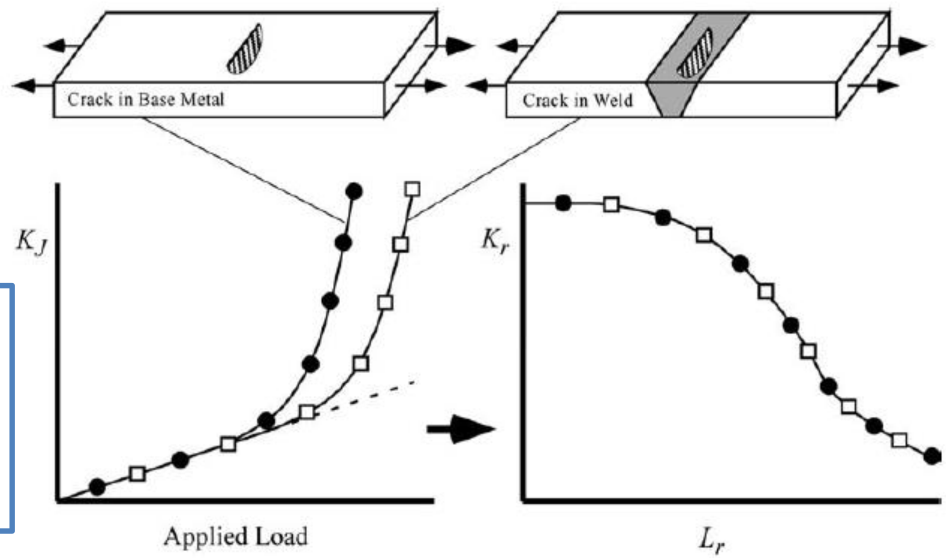
Examples of weld misalignment: (a) centerline offset and (b) angular misalignment

- When applying the FAD method, it is customary to treat misalignment stresses in the same way as weld residual stresses.
- That is, they are not included in the calculation of L_r, and the applied stress-intensity factor due to misalignment stresses is multiplied by Φ.

Weld Strength Mismatch

$$\left.\frac{J}{J_{elastic}}\right|_{L_r=1} = 1 + \frac{0.002E}{\sigma_{YS}} + \frac{1}{2}\left(1 + \frac{0.002E}{\sigma_{YS}}\right)^{-1}$$

- The weld metal is typically stronger than the base metal, but there are instances where the weld metal has lower strength.
- Weldment is said to be *overmatched* when the weld metal has higher strength than the base metal. The reverse situation is known as an *undermatched* weldment.
- Mismatch in strength properties affects the crack driving force in the elastic-plastic and fully plastic regimes.
- Mismatch in properties is normally not a significant issue in the elastic range because the weld metal and base metal typically have similar elastic constants.
- The effect of weld strength mismatch can be taken into account in the FAD method through an appropriate definition of L_r.
- The reference stress for a weldment should be defined from the elastic-plastic J solution using the approach.



Effect of weld strength mismatch on crack driving force. In this schematic, weld residual stress is neglected, and the weld and base metal are assumed to have similar hardening characteristics.