Ship Stability

Ch. 9 Numerical Integration Method in Naval Architecture

Spring 2016

Myung-Il Roh

Department of Naval Architecture and Ocean Engineering
Seoul National University

Contents

☑ Ch. 1 Introduction to Ship Stability
☑ Ch. 2 Review of Fluid Mechanics
☑ Ch. 3 Transverse Stability Due to Cargo Movement
☑ Ch. 4 Initial Transverse Stability
☑ Ch. 5 Initial Longitudinal Stability
☑ Ch. 6 Free Surface Effect
☑ Ch. 7 Inclining Test
☑ Ch. 8 Curves of Stability and Stability Criteria
☑ Ch. 9 Numerical Integration Method in Naval Architecture
☑ Ch. 10 Hydrostatic Values and Curves
☑ Ch. 11 Static Equilibrium State after Flooding Due to Damage
☑ Ch. 12 Deterministic Damage Stability
☑ Ch. 13 Probabilistic Damage Stability

Ch. 9 Numerical Integration Method in Naval Architecture

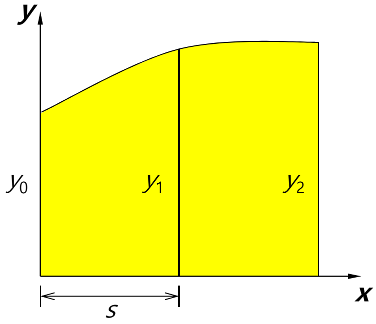
1. Simpson's Rule
2. Gaussian Quadrature
3. Green's Theorem
4. Calculation of Hydrostatic Values by Using Simpson's Rule
5. Calculation of Hydrostatic Values by Using Gaussian Quadrature and Green's Theorem

1. Simpson's Rule

Simpson's 1st and 2nd Rules

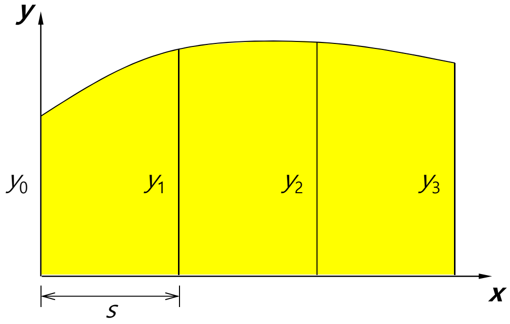
Simpson's 1st and 2nd Rules

Simpson's 1st Rule



$$Area = \frac{1}{3}s(y_0 + 4y_1 + y_2)$$

Simpson's 2nd Rule

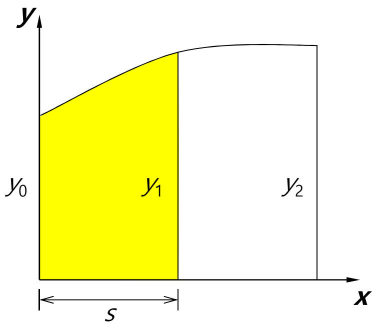


$$Area = \frac{3}{8}s(y_0 + 3y_1 + 3y_2 + y_3)$$

5·8·-1, 3·10·-1, and 7·36·-3 Rules

5·8·-1, 3·10·-1, and 7·36·-3 Rules

5·8·-1 Rule



$$Area = \frac{1}{12}s(5y_0 + 8y_1 - 1y_2)$$

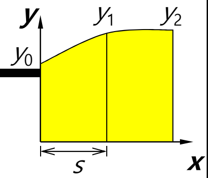
3·10·-1 Rule

$$M_y = \frac{1}{24}s^2(3y_0 + 10y_1 - 1y_2)$$

7·36·-3 Rule

$$I_y = \frac{1}{120}s^3(7y_0 + 36y_1 - 3y_2)$$

Derivation of Simpson's 1st Rule (1/4)



Simpson's 1st Rule:

Approximate the function y by a parabola (quadratic polynomial curve) whose equation has the form

Parabola : $y = a_0 + a_1 x + a_2 x^2$

The parabola is represented by three points defining this curve.

The three points (y_0, y_1, y_2) are obtained by dividing the given interval into equal subintervals "s".

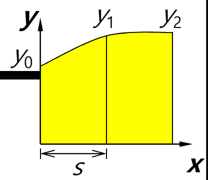
The relation between the coefficients a_0, a_1, a_2 ("Find") and y_0, y_1, and y_2 are

$$x = 0: \quad y_0 = a_0$$

$$x = s: \quad y_1 = a_0 + a_1 s + a_2 s^2$$

$$x = 2s: \quad y_2 = a_0 + 2a_1 s + 4a_2 s^2$$

Derivation of Simpson's 1st Rule (2/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$y_0 = a_0 \quad ①$$

$$y_1 = a_0 + a_1 s + a_2 s^2 \quad\rightarrow\quad a_1 s + a_2 s^2 + y_0 - y_1 = 0 \quad ②$$

$$y_2 = a_0 + 2a_1 s + 4a_2 s^2 \quad\rightarrow\quad 2a_1 s + 4a_2 s^2 + y_0 - y_2 = 0 \quad ③$$

4 x ② - ③:

$$2a_1 s + 3y_0 - 4y_1 + y_2 = 0$$

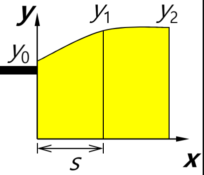
$$\therefore a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2)$$

③ - 2 x ②:

$$2a_2 s^2 - y_0 + 2y_1 - y_2 = 0$$

$$\therefore a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

Derivation of Simpson's 1st Rule (3/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$a_0 = y_0, \quad a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2), \qquad a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

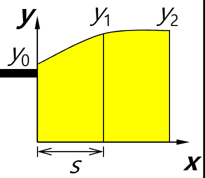
$$y = y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2$$

Integrate the area A from 0 to 2s. (Definite Integral)

$$A = \int_0^{2s} y\,dx$$

$$= \int_0^{2s} y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

Derivation of Simpson's 1st Rule (4/4)



$$A = \int_0^{2s} y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

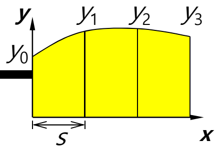
$$= y_0 x + \frac{1}{4s}(-3y_0 + 4y_1 - y_2)x^2 + \frac{1}{6s^2}(y_0 - 2y_1 + y_2)x^3 \Big|_0^{2s}$$

$$= y_0(2s) + \frac{1}{4s}(-3y_0 + 4y_1 - y_2)(2s)^2 + \frac{1}{6s^2}(y_0 - 2y_1 + y_2)(2s)^3$$

$$= 2y_0 s + (-3y_0 + 4y_1 - y_2)s + \frac{4}{3}(y_0 - 2y_1 + y_2)s$$

$$\therefore A = \frac{s}{3}(1y_0 + 4y_1 + 1y_2)$$

Derivation of Simpson's 2nd Rule (1/4)



Simpson's 2nd rule :

Approximate the function by a cubic polynomial curve whose equation has the form

Cubic polynomial curve: $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

The cubic polynomial curve is represented by four points defining this curve.

The four points (y_0, y_1, y_2, y_3) are obtained by dividing the given interval into equal subintervals "s".

The relation between the coefficients a_0, a_1, a_2, a_3 ("Find") and y_0, y_1, y_2, and y_3 are

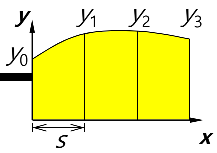
$$x = 0: \quad y_0 = a_0$$

$$x = s: \quad y_1 = a_0 + a_1 s + a_2 s^2 + a_3 s^3$$

$$x = 2s: \quad y_2 = a_0 + 2a_1 s + 4a_2 s^2 + 8s^3$$

$$x = 3s: \quad y_3 = a_0 + 3a_1 s + 9a_2 s^2 + 27s^3$$

Derivation of Simpson's 2nd Rule (2/4)



$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$$y_0 = a_0, \qquad y_1 = a_0 + a_1 s + a_2 s^2 + a_3 s^3,$$

$$y_2 = a_0 + 2a_1 s + 4a_2 s^2 + 8s^3, \qquad y_3 = a_0 + 3a_1 s + 9a_2 s^2 + 27s^3$$

The unknown coefficients, a_0, a_1, a_2, and a_3 lead to

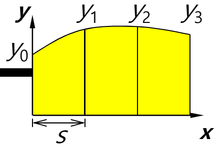
$$a_0 = y_0$$

$$a_1 = \frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3)$$

$$a_2 = \frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3)$$

$$a_3 = \frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)$$

Derivation of Simpson's 2nd Rule (3/4)



$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$a_0 = y_0, \quad a_1 = \frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3),$$

$$a_2 = \frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3), \quad a_3 = \frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)$$

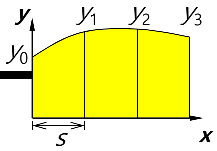
Integrate the area A from 0 to 3s.

$$A = \int_0^{3s} y\,dx = \int_0^{3s} (a_0 + a_1 x + a_2 x^2 + a_3 x^3)\,dx$$

$$= a_0 x + \frac{a_1}{2}x^2 + \frac{a_2}{3}x^3 + \frac{a_3}{4}x^4 \Big|_0^{3s}$$

$$= 3a_0 s + \frac{9}{2}a_1 s^2 + \frac{27}{3}a_2 s^3 + \frac{81}{4}a_3 s^4$$

Derivation of Simpson's 2nd Rule (4/4)



$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$a_0 = y_0, \quad a_1 = \frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3),$$

$$a_2 = \frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3), \quad a_3 = \frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)$$

$$A = 3a_0 s + \frac{9}{2}a_1 s^2 + \frac{27}{3}a_2 s^3 + \frac{81}{4}a_3 s^4$$

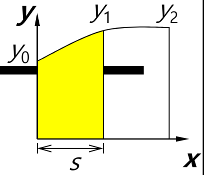
By substituting a_0, a_1, a_2 and a_3 into the equation, the Area "A" leads to

$$A = 3y_0 s + \frac{9}{2}\cdot\frac{1}{6s}(-11y_0 + 18y_1 - 9y_2 + 2y_3)s^2$$

$$+ \frac{27}{3}\cdot\frac{1}{2s^2}(2y_0 - 5y_1 + 4y_2 - y_3)s^3 + \frac{81}{4}\cdot\frac{1}{6s^3}(-y_0 + 3y_1 - 3y_2 + y_3)s^4$$

$$\therefore A = \frac{3}{8}s(y_0 + 3y_1 + 3y_2 + y_3)$$

Derivation of 5·8·-1 Rule (1/4)



5·8·-1 Rule:

Approximate the function y by a parabola whose equation has the form

Parabola : $y = a_0 + a_1 x + a_2 x^2$

The parabola is represented by three points defining this curve.

The three points (y_0, y_1, y_2) are obtained by dividing the given interval into equal subintervals "s".

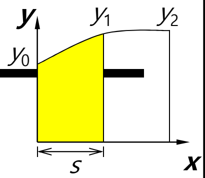
The relation between the coefficients a_0, a_1, a_2 ("Find") and y_0, y_1, and y_2 are

$$x = 0: \quad y_0 = a_0$$

$$x = s: \quad y_1 = a_0 + a_1 s + a_2 s^2$$

$$x = 2s: \quad y_2 = a_0 + 2a_1 s + 4a_2 s^2$$

Derivation of 5·8·-1 Rule (2/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$y_0 = a_0 \quad ①$$

$$y_1 = a_0 + a_1 s + a_2 s^2$$

$$y_2 = a_0 + 2a_1 s + 4a_2 s^2$$

$$a_1 s + a_2 s^2 + y_0 - y_1 = 0 \quad ②$$

$$2a_1 s + 4a_2 s^2 + y_0 - y_2 = 0 \quad ③$$

4 x ② - ③:

$$2a_1 s + 3y_0 - 4y_1 + y_2 = 0$$

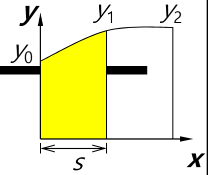
$$\therefore a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2)$$

③ - 2 x ②:

$$2a_2 s^2 - y_0 + 2y_1 - y_2 = 0$$

$$\therefore a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

Derivation of 5·8·-1 Rule (3/4)



$$y = a_0 + a_1 x + a_2 x^2$$

$$a_0 = y_0, \quad a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2), \quad a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$$

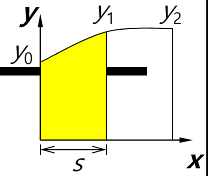
$$y = y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2$$

Integrate the area A from 0 to s.

$$A = \int_0^s y\,dx$$

$$= \int_0^s y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

Derivation of 5·8·-1 Rule (4/4)



$$A = \int_0^s y_0 + \frac{1}{2s}(-3y_0 + 4y_1 - y_2)x + \frac{1}{2s^2}(y_0 - 2y_1 + y_2)x^2 dx$$

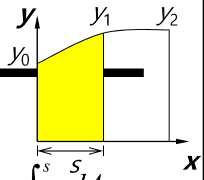
$$= y_0 x + \frac{1}{4s}(-3y_0 + 4y_1 - y_2)x^2 + \frac{1}{6s^2}(y_0 - 2y_1 + y_2)x^3 \Big|_0^s$$

$$= y_0(s) + \frac{1}{4s}(-3y_0 + 4y_1 - y_2)(s)^2 + \frac{1}{6s^2}(y_0 - 2y_1 + y_2)(s)^3$$

$$= y_0 s + \frac{1}{4}(-3y_0 + 4y_1 - y_2)s + \frac{1}{6}(y_0 - 2y_1 + y_2)s$$

$$\therefore A = \frac{s}{12}(5y_0 + 8y_1 - 1y_2)$$

Derivation of 3·10·-1 and 7·36·-3 Rules



3·10·-1 Rule: The first moment of area about y axis $M_y = M_L = \int_0^s x dA$

7·36·-3 Rule: The second moment of area about y axis $I_y = I_L = \int_0^s x^2 dA$

$$M_y = M_L = \int_0^s x dA = \int_0^s xy dx = \int_0^s a_0 x + a_1 x^2 + a_2 x^3 dx$$

← $a_0 = y_0,\ a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2),\ a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$

$$= \boxed{\frac{1}{24}s^2(3y_0 + 10y_1 - y_2)}$$

$$I_y = I_L = \int_0^s x^2 dA = \int_0^s x^2 y dx = \int_0^s a_0 x^3 + a_1 x^4 + a_2 x^5 dx$$

← $a_0 = y_0,\ a_1 = \frac{1}{2s}(-3y_0 + 4y_1 - y_2),\ a_2 = \frac{1}{2s^2}(y_0 - 2y_1 + y_2)$

$$= \boxed{\frac{1}{120}s^3(7y_0 + 36y_1 - 3y_2)}$$

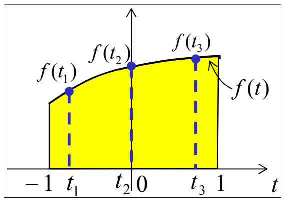
2. Gaussian Quadrature

Calculation of Area by Using Gaussian Quadrature

Gaussian quadrature: $\int_{-1}^{1} f(t)dt \approx \sum_{j=1}^{n} A_j \cdot f(t_j)$

Given: Function $f(t)$

Find: Integration of $f(t)$ at a given interval [-1, 1] $\int_{-1}^{1} f(t)dt$



In the case of cubic Gaussian quadrature,

$$\int_{-1}^{1} f(t)dt \approx A_1 \cdot f(t_1) + A_2 \cdot f(t_2) + A_3 \cdot f(t_3)$$

n	Coefficients A_j	Node t_j
3	A_1 = 0.5555555556	t_1 = -0.7745966692
	A_2 = 0.8888888889	t_2 = 0
	A_3 = 0.5555555556	t_3 = 0.7745966692

n	Coefficients A_j	Node t_j
4 (Quartic)	A_1 = 0.3478548451	t_1 = -0.8611363115
	A_2 = 0.6521451548	t_2 = -0.3399810435
	A_3 = 0.6521451548	t_3 = 0.3399810435
	A_4 = 0.3478548451	t_4 = 0.8611363115
5 (Quintic)	A_1 = 0.2369268850	t_1 = -0.9061798459
	A_2 = 0.4786286704	t_2 = -0.5384693101
	A_3 = 0.6521451548	t_3 = 0.0
	A_4 = 0.4786286704	t_4 = 0.5384693101
	A_5 = 0.2369268850	t_5 = 0.9061798459

In the case of quartic Gaussian quadrature,

$$\int_{-1}^{1} f(t)dt \approx A_1 \cdot f(t_1) + A_2 \cdot f(t_2) + A_3 \cdot f(t_3) + A_4 \cdot f(t_4)$$

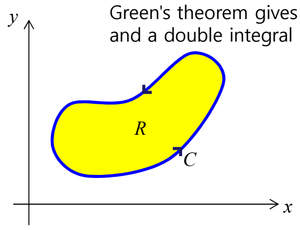
In the case of quintic Gaussian quadrature,

$$\int_{-1}^{1} f(t)dt \approx A_1 \cdot f(t_1) + A_2 \cdot f(t_2) + A_3 \cdot f(t_3) + A_4 \cdot f(t_4) + A_5 \cdot f(t_5)$$

3. Green's Theorem*

* Erwin Kreyszig, Advanced Engineering Mathematics, 9th Edition, pp.439-445, 2006

Calculation of Area by Using Green's Theorem



※ The region should be the left-hand of the curve.

Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Calculation of area $\left(A = \int dA = \iint dxdy\right)$

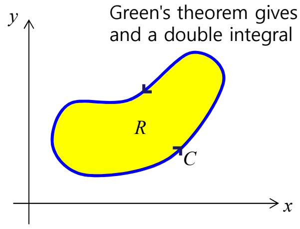
If $M = -y,\ N = x$

L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y)\right) dxdy = \iint_R 2dxdy = 2A$ (A: Area)

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C (-ydx + xdy) = \oint_C (xdy - ydx)$

$\therefore 2A = \oint_C (xdy - ydx)$ $\quad A = \frac{1}{2}\oint_C (xdy - ydx)$

Calculation of First Moment of Area by Using Green's Theorem (1/2)



※ The region should be the left-hand of the curve.

Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓First moment of area about the y-axis in x direction $\left(M_{A,y} = \int xdA = \iint xdxdy\right)$

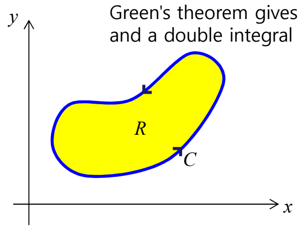
If $M = -xy,\ N = \frac{x^2}{2}$

L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R \left(\frac{\partial}{\partial x}\left(\frac{x^2}{2}\right) - \frac{\partial}{\partial y}(-xy)\right) dxdy = \iint_R 2xdxdy = 2M_{A,y}$

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C \left(-xydx + \frac{x^2}{2}dy\right) = \oint_C \left(\frac{x^2}{2}dy - xydx\right)$

$\therefore 2M_{A,y} = \oint_C \left(\frac{x^2}{2}dy - xydx\right)$ $\quad M_{A,y} = \frac{1}{2}\oint_C \left(\frac{x^2}{2}dy - xydx\right)$

Calculation of First Moment of Area by Using Green's Theorem (2/2)



Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

※ The region should be the left-hand of the curve.

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓First moment of area about the x-axis in y direction $\left(M_{A,x} = \int ydA = \iint ydxdy\right)$

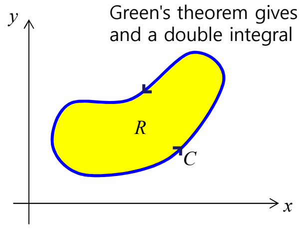
If $M = -\frac{y^2}{2},\ N = xy$

L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}(-\frac{y^2}{2})\right) dxdy = \iint_R 2ydxdy = 2M_{A,x}$

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C \left(-\frac{y^2}{2}dx + xydy\right) = \oint_C \left(xydy - \frac{y^2}{2}dx\right)$

$\therefore 2M_{A,x} = \oint_C \left(xydy - \frac{y^2}{2}dx\right)$ $M_{A,x} = \frac{1}{2}\oint_C \left(xydy - \frac{y^2}{2}dx\right)$

Calculation of Second Moment of Area by Using Green's Theorem (1/2)



Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral **Line Integral**

※ The region should be the left-hand of the curve.

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Second moment of area about the y-axis in x direction $\left(I_{A,y} = \int x^2dA = \iint x^2dxdy\right)$

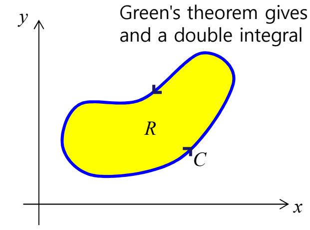
If $M = -x^2y,\ N = \frac{x^3}{3}$

L.H.S = $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(\frac{x^3}{3}) - \frac{\partial}{\partial y}(-x^2y)\right) dxdy = \iint_R 2x^2dxdy = 2I_{A,y}$

R.H.S = $\oint_C (Mdx + Ndy) = \oint_C \left(-x^2ydx + \frac{x^3}{3}dy\right) = \oint_C \left(\frac{x^3}{3}dy - x^2ydx\right)$

$\therefore 2I_{A,y} = \oint_C \left(\frac{x^3}{3}dy - x^2ydx\right)$ $I_{A,y} = \frac{1}{2}\oint_C \left(\frac{x^3}{3}dy - x^2ydx\right)$

Calculation of Second Moment of Area by Using Green's Theorem (2/2)



※ The region should be the left-hand of the curve.

Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy)$$

Surface Integral | **Line Integral**

M, N: The functions of x and y. And M, N, dM/dy, and dN/dx are continuous on R.

✓Second moment of area about the x-axis in y direction $\left(I_{A,x} = \int y^2 dA = \iint y^2 dxdy\right)$

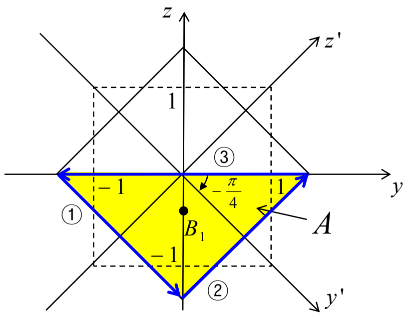
If $M = -\frac{y^3}{3}$, $N = xy^2$

L.H.S $= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \iint_R \left(\frac{\partial}{\partial x}(xy^2) - \frac{\partial}{\partial y}(-\frac{y^3}{3})\right) dxdy = \iint_R 2y^2 dxdy = 2I_{A,x}$

R.H.S $= \oint_C (Mdx + Ndy) = \oint_C \left(-\frac{y^3}{3}dx + xy^2 dy\right) = \oint_C \left(xy^2 dy - \frac{y^3}{3}dx\right)$

$\therefore 2I_{A,x} = \oint_C \left(xy^2 dy - \frac{y^3}{3}dx\right)$ $\quad$ $I_{A,x} = \frac{1}{2}\oint_C \left(xy^2 dy - \frac{y^3}{3}dx\right)$

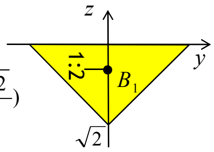
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (1/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2} \cdot 2 \cdot 2 = 2$

$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$



$oy'z'$: Body fixed coordinate

oyz : Water plane fixed coordinate

✓Area A

$$A = \frac{1}{2}\oint_C (xdy - ydx)$$

$$A = \int dA = \iint dydz$$

Green's theorem

$$= \frac{1}{2}\oint_C ydz - zdy$$

"Water plane fixed coordinate"

Segment ①: $y(t) = t$, $z(t) = -t - \sqrt{2}$, $-\sqrt{2} \le t \le 0$

Using the chain rule, convert the line integral for y and z into the integral for only one parameter "t".

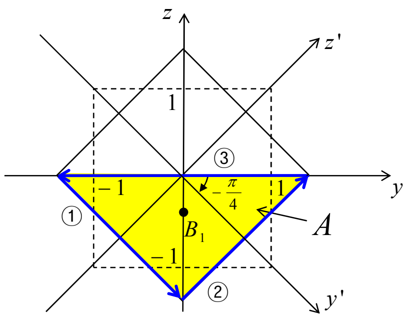
$$\frac{1}{2}\int_{①} ydz - zdy = \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(t(-1) - (-t-\sqrt{2})\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\sqrt{2}dt = \frac{1}{2}\sqrt{2}t\Big|_{-\sqrt{2}}^{0}$$

$$= \frac{1}{2}\sqrt{2}\sqrt{2} = 1$$

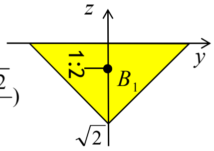
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (2/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$



$oy'z'$: *Body fixed coordinate*

oyz : *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C ydz - zdy$$

Segment ①: $\frac{1}{2}\int_{①} ydz - zdy = 1$

Segment ②: $y(t) = t, \quad z(t) = t - \sqrt{2}, \quad 0 \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{②} ydz - zdy = \frac{1}{2}\int_0^{\sqrt{2}}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)dt$$

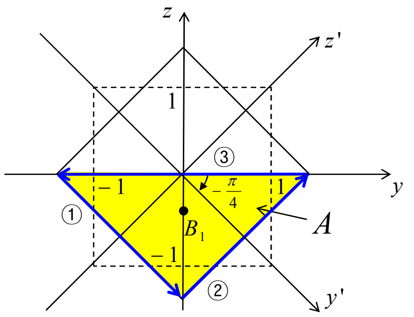
$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(t\cdot 1 - (t-\sqrt{2})\cdot 1\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}}\sqrt{2}dt = \frac{1}{2}\sqrt{2}t\Big|_0^{\sqrt{2}}$$

$$= \frac{1}{2}\sqrt{2}\sqrt{2} = 1$$

29

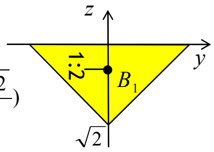
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (3/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$



$oy'z'$: *Body fixed coordinate*

oyz : *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C ydz - zdy$$

Segment ①: $\frac{1}{2}\int_{①} ydz - zdy = 1$

Segment ②: $\frac{1}{2}\int_{②} ydz - zdy = 1$

Segment ③: $y(t) = t, \quad z = 0, \quad -\sqrt{2} \le t \le \sqrt{2}$

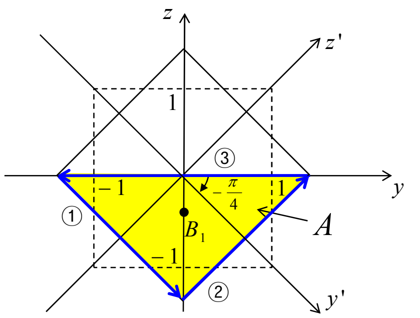
$$\frac{1}{2}\int_{③} ydz - zdy = \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}(t\cdot 0 - 0\cdot 1)dt = 0$$

$$\therefore\ A = \frac{1}{2}\oint_C ydz - zdy = \underset{①}{1} + \underset{②}{1} + \underset{③}{0} = 2$$

30

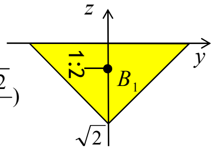
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (4/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z-axis in y direction $M_{A,z}$

$M_{A,y} = \frac{1}{2}\oint_C \left(\frac{x^2}{2}dy - xydx\right)$

$$M_{A,z} = \int y\,dA = \iint y\,dy\,dz$$

Green's theorem

$$= \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy$$

Segment ①: $y(t) = t, \quad z(t) = -t - \sqrt{2}, \quad -\sqrt{2} \le t \le 0$

$$\frac{1}{2}\int_{①} \frac{y^2}{2}dz - yzdy = \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{y^2}{2}\frac{dz}{dt} - yz\frac{dy}{dt}\right)dt$$

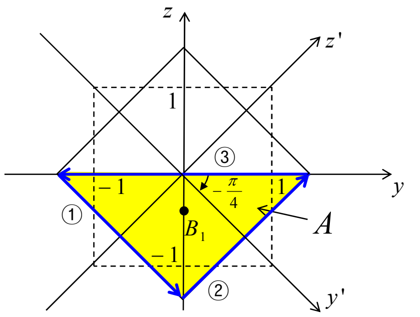
$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{t^2}{2}(-1) - t(-t-\sqrt{2})\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0}\left(\frac{t^2}{2} + \sqrt{2}t\right)dt = \frac{1}{2}\left[\frac{t^3}{6} + \frac{\sqrt{2}}{2}t^2\right]_{-\sqrt{2}}^{0}$$

$$= -\frac{\sqrt{2}}{3}$$

31

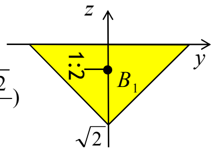
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (5/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z-axis in y direction $M_{A,z}$

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y^2}{2}dz - yzdy = -\frac{\sqrt{2}}{3}$

Segment ②: $y(t) = t, \quad z(t) = t - \sqrt{2}, \quad 0 \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{②} \frac{y^2}{2}dz - yzdy = \frac{1}{2}\int_{0}^{\sqrt{2}}\left(\frac{y^2}{2}\frac{dz}{dt} - yz\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{0}^{\sqrt{2}}\left(\frac{t^2}{2}\cdot 1 - t(t-\sqrt{2})\cdot 1\right)dt$$

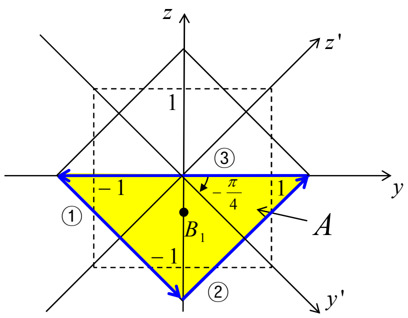
$$= \frac{1}{2}\int_{0}^{\sqrt{2}}\left(-\frac{t^2}{2} + \sqrt{2}t\right)dt$$

$$= \frac{1}{2}\left[-\frac{t^3}{6} + \frac{\sqrt{2}}{2}t^2\right]_{0}^{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{3}$$

32

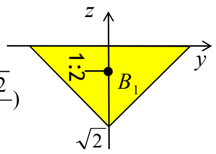
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (6/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z-axis in y direction $M_{A,z}$

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y^2}{2}dz - yzdy = -\frac{\sqrt{2}}{3}$

Segment ②: $\frac{1}{2}\int_{②} \frac{y^2}{2}dz - yzdy = \frac{\sqrt{2}}{3}$

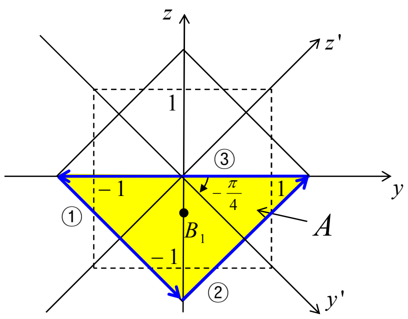
Segment ③: $y(t) = t,\quad z = 0,\quad -\sqrt{2} \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{③} \frac{y^2}{2}dz - yzdy = \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}} \left(\frac{y^2}{2}\frac{dz}{dt} - yz\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}} \left(\frac{t^2}{2}\cdot 0 - t\cdot 0\cdot 1\right)dt = 0$$

$$\therefore M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy = \underset{①}{-\frac{\sqrt{2}}{3}} + \underset{②}{\frac{\sqrt{2}}{3}} + \underset{③}{0} = 0$$

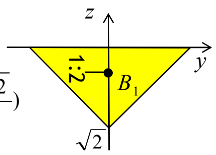
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (7/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the y-axis in z direction $M_{A,y}$

$$M_{A,x} = \frac{1}{2}\oint_C \left(xydy - \frac{y^2}{2}dx\right)$$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy$$

Green's theorem

$$= \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy$$

Segment ①: $y(t) = t,\quad z(t) = -t - \sqrt{2},\quad -\sqrt{2} \le t \le 0$

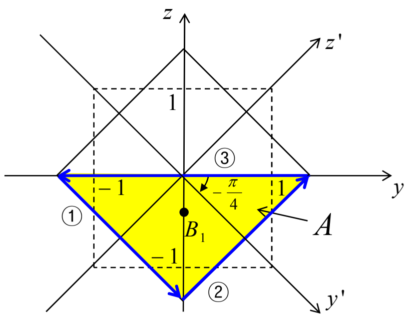
$$\frac{1}{2}\int_{①} yzdz - \frac{z^2}{2}dy = \frac{1}{2}\int_{-\sqrt{2}}^{0} \left(yz\frac{dz}{dt} - \frac{z^2}{2}\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0} \left(t(-t-\sqrt{2})(-1) - \frac{(-t-\sqrt{2})^2}{2}\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0} \left(t^2 + \sqrt{2}t - \frac{t^2 + 2\sqrt{2}t + 2}{2}\right)dt$$

$$= \frac{1}{2}\int_{-\sqrt{2}}^{0} \left(\frac{t^2}{2} - 1\right)dt = \frac{1}{2}\left[\frac{t^3}{6} - t\right]_{-\sqrt{2}}^{0} = -\frac{\sqrt{2}}{3}$$

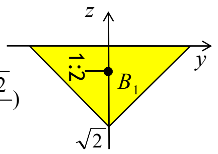
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (8/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



oy'z': Body fixed coordinate

oyz : Water plane fixed coordinate

✓First moment of area about the y-axis in z direction $M_{A,y}$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy$$

Segment ①: $\frac{1}{2}\int_{①} yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3}$

Segment ②: $y(t) = t,\quad z(t) = t - \sqrt{2},\quad 0 \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{②} yzdz - \frac{z^2}{2}dy = \frac{1}{2}\int_0^{\sqrt{2}}\left(yz\frac{dz}{dt} - \frac{z^2}{2}\frac{dy}{dt}\right)dt$$

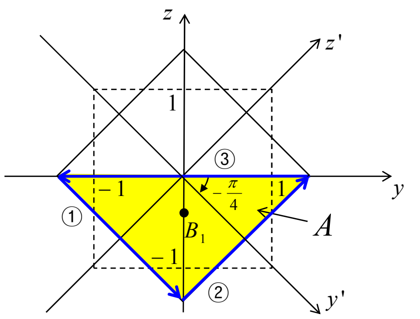
$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(t(t-\sqrt{2})\cdot 1 - \frac{(t-\sqrt{2})^2}{2}\cdot 1\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(t^2 - \sqrt{2}t - \frac{t^2 - 2\sqrt{2}t + 2}{2}\right)dt$$

$$= \frac{1}{2}\int_0^{\sqrt{2}}\left(\frac{t^2}{2} - 1\right)dt = \frac{1}{2}\left[\frac{t^3}{6} - t\right]_0^{\sqrt{2}} = -\frac{\sqrt{2}}{3}$$

35

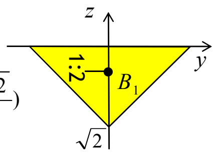
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (9/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



oy'z': Body fixed coordinate

oyz : Water plane fixed coordinate

✓First moment of area about the y-axis in z direction $M_{A,y}$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy$$

Segment ①: $\frac{1}{2}\int_{①} yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3}$

Segment ②: $\frac{1}{2}\int_{②} yzdz - \frac{z^2}{2}dy = -\frac{\sqrt{2}}{3}$

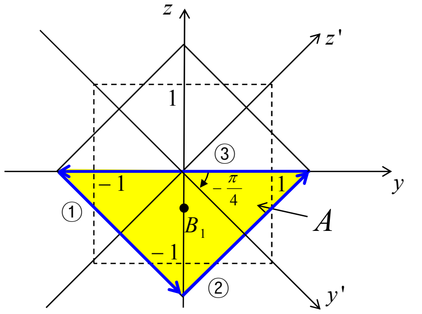
Segment ③: $y(t) = t,\quad z = 0,\quad -\sqrt{2} \le t \le \sqrt{2}$

$$\frac{1}{2}\int_{③} yzdz - \frac{z^2}{2}dy = \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}\left(yz\frac{dz}{dt} - \frac{z^2}{2}\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{\sqrt{2}}^{-\sqrt{2}}\left(t\cdot 0\cdot 1 - \frac{0^2}{2}\cdot 1\right)dt = 0$$

$$\therefore M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy = \underset{①}{-\frac{\sqrt{2}}{3}} \underset{②}{-\frac{\sqrt{2}}{3}} \underset{③}{+0} = -\frac{2\sqrt{2}}{3}$$

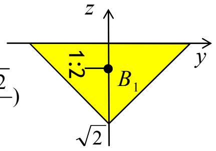
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Inertial Frame (10/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2} \cdot 2 \cdot 2 = 2$

$(y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$



$oy'z'$: *Body fixed coordinate*
oyz: *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C ydz - zdy = 2$$

✓First moment of area about the z-axis in y direction $M_{A,z}$

$$M_{A,z} = \frac{1}{2}\oint_C \frac{y^2}{2}dz - yzdy = 0$$

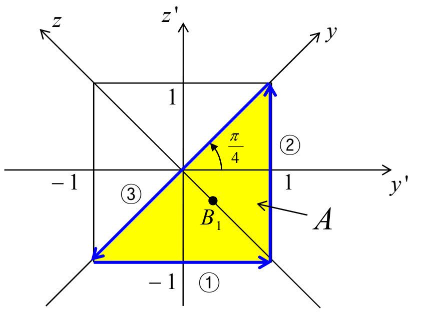
✓First moment of area about the y-axis in z direction $M_{A,y}$

$$M_{A,y} = \frac{1}{2}\oint_C yzdz - \frac{z^2}{2}dy = -\frac{2\sqrt{2}}{3}$$

✓Centroid

$$(y_{B_1}, z_{B_1}) = \left(\frac{M_{A,z}}{A}, \frac{M_{A,y}}{A}\right) = \left(\frac{0}{2}, \frac{1}{2}\cdot\left(-\frac{2\sqrt{2}}{3}\right)\right) = \left(0, -\frac{\sqrt{2}}{3}\right)$$

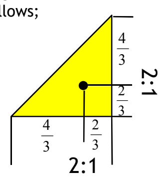
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (1/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2} \cdot 2 \cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: *Body fixed coordinate*
oyz: *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C (xdy - ydx)$$

$$A = \int dA = \iint dy'dz'$$

Green's theorem

$$= \frac{1}{2}\oint_C y'dz' - z'dy'$$

"Body fixed coordinate"

Segment ①: $y'(t) = t, \quad z'(t) = -1, \quad -1 \le t \le 1$

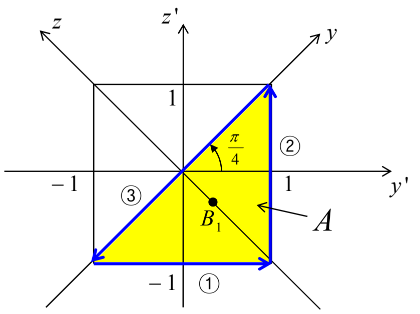
Using the chain rule, convert the line integral for y' and z' into the integral for only one parameter "*t*".

$$\frac{1}{2}\int_{①} y'dz' - z'dy' = \frac{1}{2}\int_{-1}^{1}\left(y'\frac{dz}{dt} - z'\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}(t \cdot 0 - (-1)\cdot 1)dt$$

$$= \frac{1}{2}\int_{-1}^{1} 1dt = \frac{1}{2}t\Big|_{-1}^{1} = 1$$

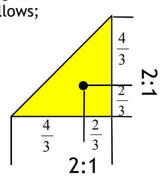
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (2/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = \left(\frac{1}{3}, -\frac{1}{3}\right)$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C y'dz' - z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'dz' - z'dy' = 1$

Segment ②: $y'(t) = 1,\quad z'(t) = t,\quad -1 \le t \le 1$

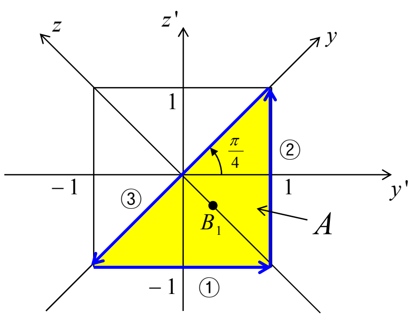
$$\frac{1}{2}\int_{②} y'dz' - z'dy' = \frac{1}{2}\int_{-1}^{1}\left(y'\frac{dz}{dt} - z'\frac{dy}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}(1\cdot 1 - t\cdot 0)dt$$

$$= \frac{1}{2}t\Big|_{-1}^{1} = 1$$

39

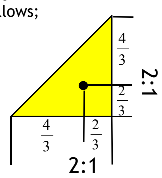
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (3/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = \left(\frac{1}{3}, -\frac{1}{3}\right)$$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓Area A

$$A = \frac{1}{2}\oint_C y'dz' - z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'dz' - z'dy' = 1$

Segment ②: $\frac{1}{2}\int_{②} y'dz' - z'dy' = 1$

Segment ③: $y'(t) = t,\quad z'(t) = t,\quad -1 \le t \le 1$

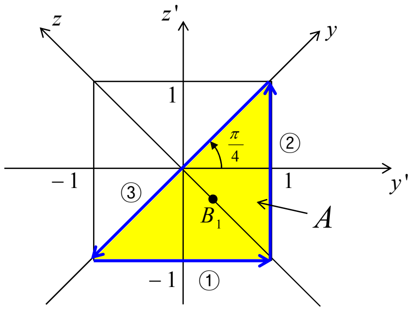
$$\frac{1}{2}\int_{③} y'dz' - z'dy' = \int_{1}^{-1}\left(y'\frac{dz}{dt} - z'\frac{dy}{dt}\right)dt$$

$$= \int_{1}^{-1}(1\cdot 1 - 1\cdot 1)dt = 0$$

$$\therefore\ A = \frac{1}{2}\oint_C y'dz' - z'dy' = \underset{①}{1} + \underset{②}{1} + \underset{③}{0} = 2$$

40

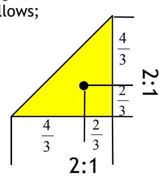
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (4/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \int y' dA = \iint y' dy' dz' \qquad M_{A,y} = \frac{1}{2}\oint_C \left(\frac{x^2}{2}dy - xy dx\right)$$

Green's theorem

$$= \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

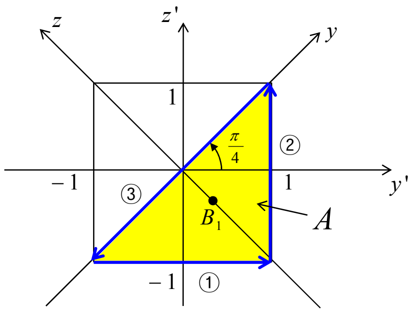
Segment ①: $y'(t) = t, \quad z'(t) = -1, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{①} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}\int_{-1}^{1}\left(\frac{y'^2}{2}\frac{dz'}{dt} - y'z'\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{t^2}{2}\cdot 0 - t(-1)\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1} t dt = \frac{1}{4}t^2\Big|_{-1}^{1} = 0$$

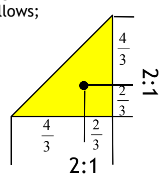
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (5/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y'^2}{2}dz' - y'z'dy' = 0$

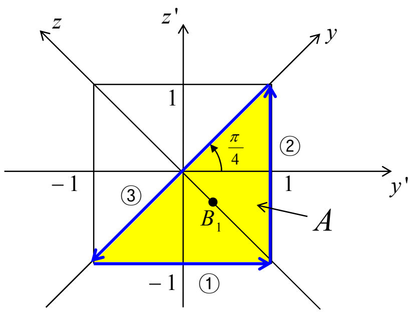
Segment ②: $y'(t) = 1, \quad z'(t) = t, \quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}\int_{-1}^{1}\left(\frac{y'^2}{2}\frac{dz'}{dt} - y'z'\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{1^2}{2}\cdot 1 - 1\cdot t\cdot 0\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\frac{1}{2}dt = \frac{1}{4}t\Big|_{-1}^{1} = \frac{1}{2}$$

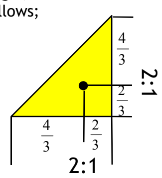
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (6/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

Segment ①: $\frac{1}{2}\int_{①} \frac{y'^2}{2}dz' - y'z'dy' = 0$

Segment ②: $\frac{1}{2}\int_{②} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}$

Segment ③: $y'(t) = t,\quad z'(t) = t,\quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} \frac{y'^2}{2}dz' - y'z'dy' = \frac{1}{2}\int_{-1}^{1}\left(\frac{y'^2}{2}\frac{dz'}{dt} - y'z'\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{t^2}{2}\cdot 1 - t\cdot t\cdot 1\right)dt = \frac{1}{2}\int_{-1}^{1}\left(-\frac{t^2}{2}\right)dt = -\frac{t^3}{12}\Big|_{-1}^{1} = -\frac{1}{6}$$

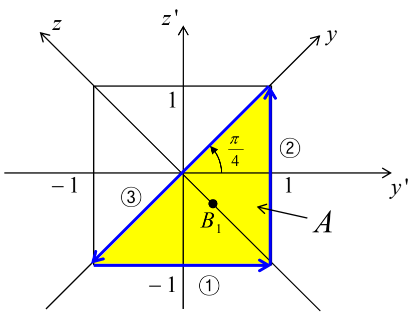
$$\therefore M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy'$$

$$= 0 + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$$

① ② ③

43

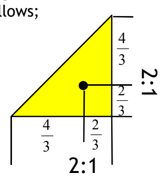
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (7/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: Body fixed coordinate

oyz: Water plane fixed coordinate

✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

$$M'_{A,y'} = \int z'dA = \iint z'dy'dz' \qquad M_{A,x} = \frac{1}{2}\oint_C\left(xydy - \frac{y^2}{2}dx\right)$$

⇩ Green's theorem

$$= \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy'$$

Segment ①: $y'(t) = t,\quad z'(t) = -1,\quad -1 \le t \le 1$

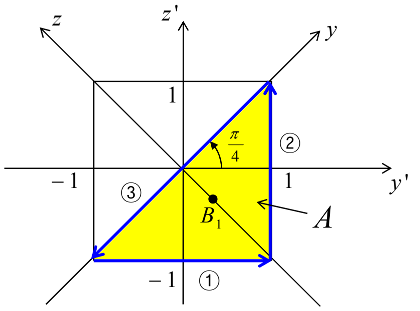
$$\frac{1}{2}\int_{①} y'z'dz' - \frac{z'^2}{2}dy' = \frac{1}{2}\int_{-1}^{1}\left(y'z'\frac{dz'}{dt} - \frac{z'^2}{2}\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(t(-1)\cdot 0 - \frac{(-1)^2}{2}\cdot 1\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(-\frac{1}{2}\right)dt = -\frac{1}{4}t\Big|_{-1}^{1} = -\frac{1}{2}$$

44

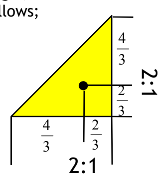
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (8/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

$$M'_{A,y'} = \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'z'dz' - \frac{z'^2}{2}dy' = -\frac{1}{2}$

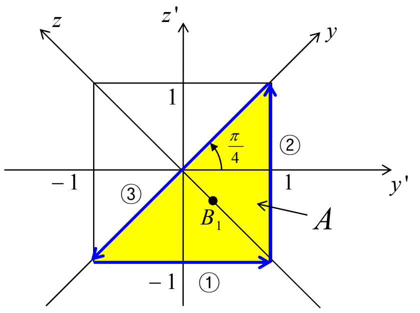
Segment ②: $y'(t) = 1,\quad z'(t) = t,\quad -1 \le t \le 1$

$$\frac{1}{2}\int_{②} y'z'dz' - \frac{z'^2}{2}dy' = \frac{1}{2}\int_{-1}^{1}\left(y'z'\frac{dz'}{dt} - \frac{z'^2}{2}\frac{dy'}{dt}\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1}\left(1\cdot t\cdot 1 - \frac{t^2}{2}\cdot 0\right)dt$$

$$= \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{4}t^2\Big|_{-1}^{1} = 0$$

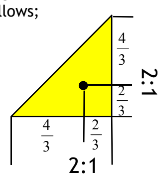
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (9/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$A = \frac{1}{2}\cdot 2\cdot 2 = 2$

$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$



$oy'z'$: *Body fixed coordinate*

oyz: *Water plane fixed coordinate*

✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

$$M'_{A,y'} = \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy'$$

Segment ①: $\frac{1}{2}\int_{①} y'z'dz' - \frac{z'^2}{2}dy' = -\frac{1}{2}$

Segment ②: $\frac{1}{2}\int_{②} y'z'dz' - \frac{z'^2}{2}dy' = 0$

Segment ③: $y'(t) = t,\quad z'(t) = t,\quad -1 \le t \le 1$

$$\frac{1}{2}\int_{③} y'z'dz' - \frac{z'^2}{2}dy' = \frac{1}{2}\int_{-1}^{1}\left(y'z'\frac{dz'}{dt} - \frac{z'^2}{2}\frac{dy'}{dt}\right)dt$$

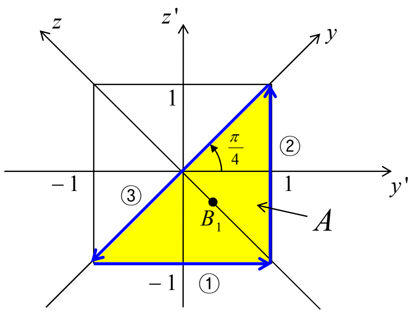
$$= \frac{1}{2}\int_{-1}^{1}\left(t\cdot t\cdot 1 - \frac{t^2}{2}\cdot 1\right)dt = \frac{1}{2}\int_{-1}^{1}\frac{t^2}{2}dt = \frac{t^3}{12}\Big|_{-1}^{1} = \frac{1}{6}$$

$$\therefore M'_{A,y} = \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy'$$

$$= 0 - \frac{1}{2} + \frac{1}{6} = -\frac{2}{3}$$

① ② ③

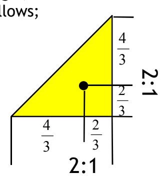
[Example] Calculation of Area, First Moment of Area, and Centroid with Respect to the Body Fixed Frame (10/10)



Cf: From the geometry of the triangle, the area and the centroid can be obtained as follows;

$$A = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$



oy'z': Body fixed coordinate

oyz : Water plane fixed coordinate

✓Area A

$$A = \frac{1}{2}\oint_C y'dz' - z'dy' = 2$$

✓First moment of area about the z′-axis in y′ direction $M_{A,z'}$

$$M'_{A,z'} = \frac{1}{2}\oint_C \frac{y'^2}{2}dz' - y'z'dy' = \frac{2}{3}$$

✓First moment of area about the y′-axis in z′ direction $M_{A,y'}$

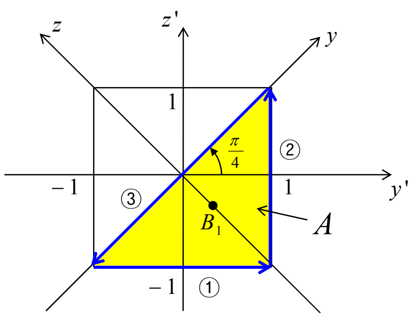
$$M'_{A,y'} = \frac{1}{2}\oint_C y'z'dz' - \frac{z'^2}{2}dy' = -\frac{2}{3}$$

✓Centroid

$$(y'_{B_1}, z'_{B_1}) = \left(\frac{M'_{A,z'}}{A}, \frac{M'_{A,y'}}{A}\right) = \left(\frac{1}{2}\cdot\frac{2}{3}, \frac{1}{2}\cdot\left(-\frac{2}{3}\right)\right) = \left(\frac{1}{3}, -\frac{1}{3}\right)$$

47

[Example] Calculation of Area, First Moment of Area, and Centroid - Transform the Position Vectors with Respect to the Inertial Frame



$$A = 2$$

$$M'_{A,z'} = \frac{2}{3} \quad M'_{A,y'} = -\frac{2}{3}$$

$$(y'_{B_1}, z'_{B_1}) = (\frac{1}{3}, -\frac{1}{3})$$

oy'z': Body fixed coordinate

oyz : Water plane fixed coordinate

✓Calculation of centroid (Center of buoyancy B_1) in the body fixed frame and inertial frame

Body fixed frame	Inertial frame
$\left(\frac{1}{3}, -\frac{1}{3}\right)$	$\left(0, -\frac{\sqrt{2}}{3}\right)$

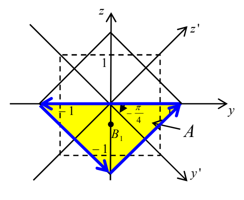
✓ Transform the center of buoyancy in $oy'z'$ frame into oyz frame by rotating the point about the negative x′-axis with an angle of $\frac{\pi}{4}$. Then the result is the same as the calculation result of centroid in the inertial frame.

$$\mathbf{r}_{B_1} = \begin{bmatrix} y_{B_1} \\ z_{B_1} \end{bmatrix} = \begin{bmatrix} \cos\left(-\frac{\pi}{4}\right) & -\sin\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) & \cos\left(-\frac{\pi}{4}\right) \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{3} \end{bmatrix}$$

$$\therefore (y_{B_1}, z_{B_1}) = (0, -\frac{\sqrt{2}}{3})$$



4. Calculation of Hydrostatic Values by Using Simpson's Rule

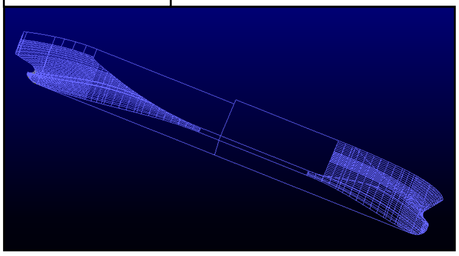
What is a "Hull form"?

☑ **Hull form**
- **Outer shape of the hull** that is streamlined in order to satisfy requirements of a ship owner such as a deadweight, ship speed, and so on
- Like a skin of human

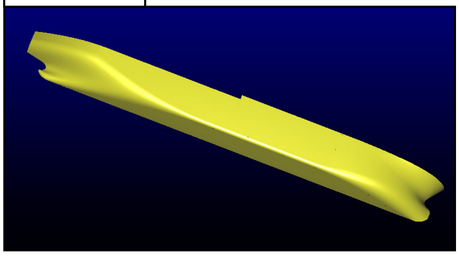
☑ **Hull form design**
- Design task that designs the hull form

Hull form of the VLCC(Very Large Crude oil Carrier)

Wireframe model

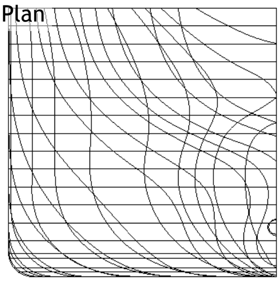


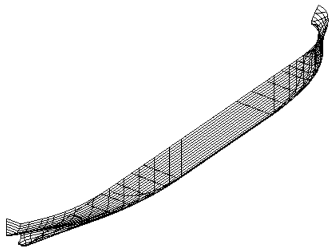
Surface model



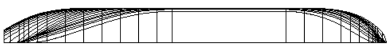
Lines of a 320,000ton VLCC

Body Plan





Water Plan



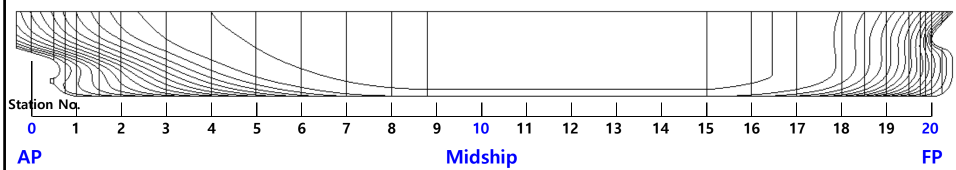
Sheer Plan

Station

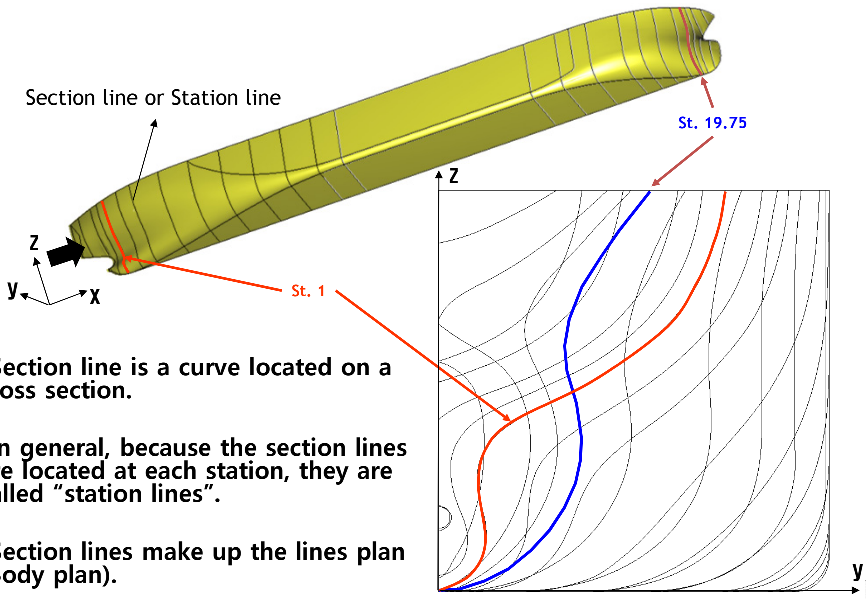
- ☑ **Stations are ship hull cross sections at a spacing of $L_{BP}/20$.**
- ☑ **The station 0 is located at the aft perpendicular and the station 20 is at the forward perpendicular. And the station 10 therefore represents the midship section.**

  - Station spacing = **L_{BP} / 20**
  - X position of the Station "A" = **Station No. of "A" × Station spacing**

Sheer Plan (Elevation View)



Section Line and Body Plan



☑ Section line is a curve located on a cross section.

☑ In general, because the section lines are located at each station, they are called "station lines".

☑ Section lines make up the lines plan (Body plan).

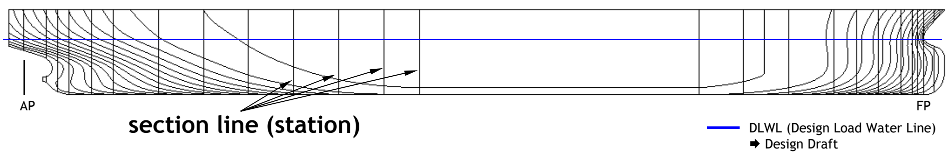
53

Buttock Line and Sheer Plan (Buttock Plan)

☑ Buttock line is a curve located on a profile (lateral) section (x-z plane).

☑ Buttock lines make up the **sheer plan** or **buttock plan** of lines.

Sheer Plan (Elevation View)



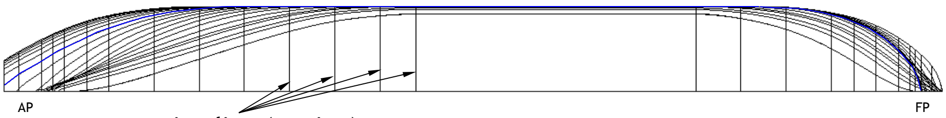
Example of water line of a 320K VLCC

Water Line and Water Plan (Half-Breadth Plan)

☑ Water line is a curve located on a water plane (vertical) section (x-y plane).

☑ Water lines make up the water plan or half-breadth plan of lines.

Water Plan (Plan View)



Example of water line of a 320K VLCC

Example of Offsets Table of a 6,300TEU Container Ship

→ Waterline

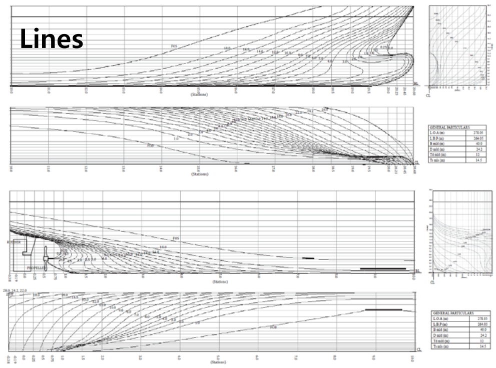
* Unit: mm

↓ Stations

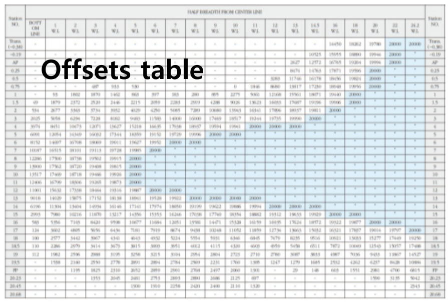
Half-Breadth

Station NO.	HALF BREADTH FROM CENTER LINE																				Station NO.
	BOTTOM LINE	1 W.L	2 W.L	3 W.L	4 W.L	5 W.L	6 W.L	7 W.L	8 W.L	9 W.L	10 W.L	11 W.L	12 W.L	13 W.L	14.5 W.L	16 W.L	18 W.L	20 W.L	22 W.L	24.2 W.L	
Trans. (-0.38)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	14450	18262	19780	20000	20000	Trans. (-0.38)
-0.19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	10525	15955	18890	19944	20000	*	-0.19
AP	-	-	-	-	-	-	-	-	-	-	-	-	-	2627	12572	16765	19204	19994	20000	*	AP
0.25	-	-	-	-	-	-	-	-	-	-	-	-	-	8474	14763	17871	19596	20000	*	*	0.25
0.5	-	-	-	-	-	-	-	-	-	-	-	-	3283	11746	16178	18456	19824	20000	*	*	0.5
0.75	-	-	-	487	933	530	-	-	-	-	0	1846	8680	13817	17230	18948	19956	20000	*	*	0.75
1	-	93	1802	1870	1462	863	397	183	280	895	2275	5061	12168	15561	18071	19440	20000	*	*	*	1
1.5	49	1879	2372	2520	2446	2215	2059	2283	2919	4288	9026	13623	16033	17687	19196	19906	20000	*	*	*	1.5
2	534	2677	3363	3734	3932	4029	4250	5085	7289	10680	13943	16341	17896	18937	19811	20000	*	*	*	*	2
3	2025	5058	6294	7228	8182	9483	11583	14000	16000	17469	18517	19244	19735	19990	20000	*	*	*	*	*	3
4	3974	8451	10473	12071	13627	15218	16635	17938	18937	19594	19941	20000	20000	20000	*	*	*	*	*	*	4
5	6091	12054	14349	16032	17344	18359	19152	19729	19996	20000	20000	*	*	*	*	*	*	*	*	*	5
6	8152	14697	16708	18069	19011	19627	19952	20000	20000	*	*	*	*	*	*	*	*	*	*	*	6
7	10187	16515	18101	19113	19728	19985	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	7
8	12286	17500	18738	19502	19915	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	8
9	13900	17562	18720	19408	19815	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	9
10	13517	17469	18718	19466	19926	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	10
11	12406	16799	18306	19265	19873	20000	*	*	*	*	*	*	*	*	*	*	*	*	*	*	11
12	11001	15632	17338	18464	19316	19887	20000	20000	*	*	*	*	*	*	*	*	*	*	*	*	12
13	9018	14029	15875	17152	18138	18941	19528	19922	20000	20000	20000	20000	*	*	*	*	*	*	*	*	13
14	6196	11304	13404	14934	16146	17141	17974	18650	19199	19622	19886	19994	20000	20000	20000	*	*	*	*	*	14
15	2993	7980	10216	11870	13217	14356	15353	16246	17038	17740	18354	18882	19312	19633	19929	20000	20000	*	*	*	15
16	583	5356	7103	8420	9598	10677	11684	12651	13581	14471	15328	16159	16935	17624	18572	19322	19877	20000	20000	*	16
17	124	3602	4805	5656	6434	7181	7919	8674	9438	10248	11052	11859	12734	13663	15032	16321	17837	19014	19797	20000	17
18	100	2577	3442	3967	4341	4643	4932	5224	5554	5931	6346	6845	7479	8235	9516	10921	13033	15277	17449	19250	18
18.5	110	2286	2979	3414	3673	3815	3893	3951	4012	4115	4320	4603	4959	5458	6511	7872	10049	12543	15057	17488	18.5
19	112	1982	2596	2988	3195	3258	3215	3104	2954	2804	2723	2710	2780	3087	3833	4987	7036	9433	11867	14527	19
19.5	-	1538	2160	2550	2778	2891	2894	2784	2569	2231	1760	1385	1247	1279	1685	2532	4262	6237	8428	10884	19.5
FP	-	-	1195	1825	2310	2652	2859	2901	2768	2497	2060	1301	-	29	148	603	1551	2981	4700	6815	FP
20.23	-	-	-	1353	2045	2481	2753	2893	2890	2686	2125	697	-	-	-	-	-	1590	3135	5042	20.23
20.45	-	-	-	-	-	1300	1910	2258	2420	2400	2110	1320	-	-	-	-	-	-	-	2343	20.45
20.68	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	20.68

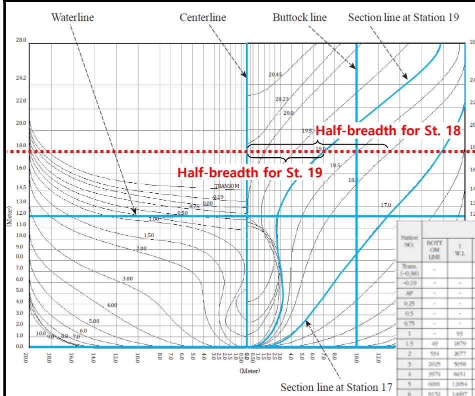
Relationship Between Lines and Offsets Table (1/2)

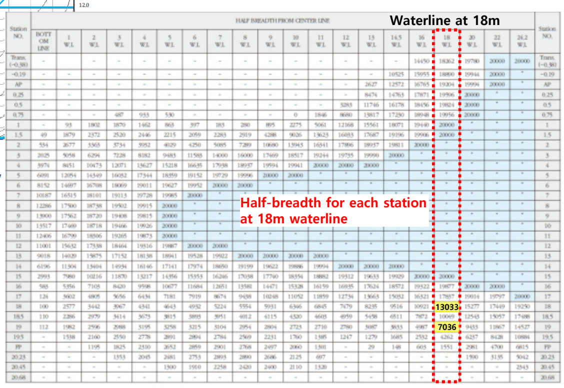


Generation of offsets table from the lines

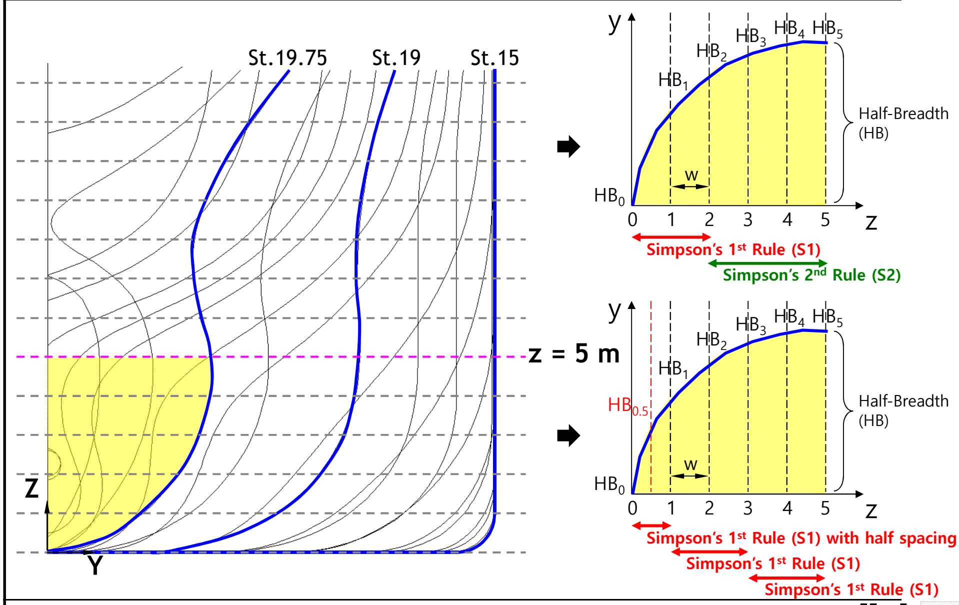


Relationship Between Lines and Offsets Table (2/2)



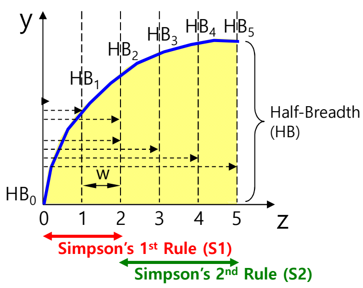


Calculation of Sectional Area



Calculation of the First Moment of Sectional Area

Calculation of Sectional Area



Simpson's 1st Rule

$$Area_1 = \int dA = \frac{1}{3}s(y_0 + 4y_1 + y_2) = \frac{1}{3}w(HB_0 + 4HB_1 + HB_2)$$

Simpson's 2nd Rule

$$Area_2 = \int dA = \frac{3}{8}s(y_0 + 3y_1 + 3y_2 + y_3)$$

$$= \frac{3}{8}w(HB_2 + 3HB_3 + 3HB_4 + HB_5)$$

$$\therefore Area = Area_1 + Area_2$$

Calculation of the First Moment of Sectional Area (about y axis)

Simpson's 1st Rule

Distance of each ordinate from y axis

$$M_{y,1} = \int z dA = \frac{1}{3}s(Y_0 + 4Y_1 + Y_2) = \frac{1}{3}s\left(1\cdot(0\cdot y_0) + 4\cdot(s\cdot y_1) + 1\cdot(2s\cdot y_2)\right) = \frac{1}{3}w\left(1\cdot(0\cdot HB_0) + 4\cdot(w\cdot HB_1) + 1\cdot(2w\cdot HB_2)\right)$$

Simpson's 2nd Rule

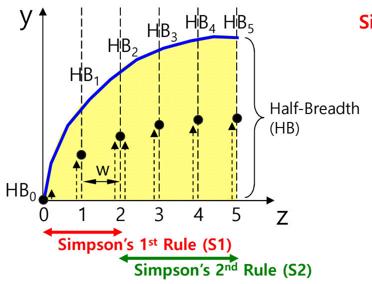
Distance of each ordinate from y axis

$$M_{y,2} = \int z dA = \frac{3}{8}s(y_0 + 3y_1 + 3y_2 + y_3) = \frac{3}{8}s\left(1\cdot(0\cdot y_0) + 3\cdot(s\cdot y_1) + 3\cdot(2s\cdot y_2) + 1\cdot(3s\cdot y_3)\right)$$

$$= \frac{3}{8}w\left(1\cdot(2w\cdot HB_2) + 3\cdot(3w\cdot HB_3) + 3\cdot(4w\cdot HB_4) + 1\cdot(5w\cdot HB_5)\right) \qquad \therefore M_y = M_{y,1} + M_{y,2}$$

Calculation of the First Moment of Sectional Area

Calculation of the First Moment of Sectional Area (about z axis)



Simpson's 1st Rule

$$M_{z,1} = \int z\,dA = \frac{1}{3}s(Y_0+4Y_1+Y_2)$$

Distance of each ordinate from z axis

$$= \frac{1}{3}s\left(1\cdot((y_0/2)\cdot y_0)+4\cdot((y_1/2)\cdot y_1)+1\cdot((y_2/2)\cdot y_2)\right)$$

$$= \frac{1}{3}w\left(1\cdot((HB_0/2)\cdot HB_0)+4\cdot((HB_1/2)\cdot HB_1)+1\cdot((HB_2/2)\cdot HB_2)\right)$$

Simpson's 2nd Rule

$$M_{z,2} = \int z\,dA = \frac{3}{8}s(y_0+3y_1+3y_2+y_3)$$

Distance of each ordinate from z axis

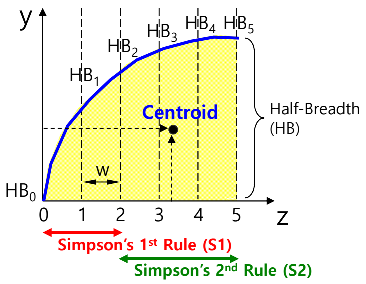
$$= \frac{3}{8}s\left(1\cdot((y_0/2)\cdot y_0)+3\cdot((y_1/2)\cdot y_1)+3\cdot((y_2/2)\cdot y_2)+1\cdot((y_3/2)\cdot y_3)\right)$$

$$= \frac{3}{8}w\left(1\cdot((HB_2/2)\cdot HB_2)+3\cdot((HB_3/2)\cdot HB_3)+3\cdot((HB_4/2)\cdot HB_4)+1\cdot((HB_5/2)\cdot HB_5)\right)$$

$$\therefore M_z = M_{z,1}+M_{z,2}$$

Calculation of the Centroid of Sectional Area

Calculation of the Centroid



$$Area_1 = \frac{1}{3}w(HB_0+4HB_1+HB_2)$$

$$Area_2 = \frac{3}{8}w(HB_2+3HB_3+3HB_4+HB_5)$$

$$\therefore Area = \frac{1}{3}w(HB_0+4HB_1+HB_2)+\frac{3}{8}w(HB_2+3HB_3+3HB_4+HB_5)$$

$$M_{y,1} = \frac{1}{3}w\left(1\cdot(0\cdot HB_0)+4\cdot(w\cdot HB_1)+1\cdot(2w\cdot HB_2)\right)$$

$$M_{y,2} = \frac{3}{8}w\left(1\cdot(2w\cdot HB_2)+3\cdot(3w\cdot HB_3)+3\cdot(4w\cdot HB_4)+1\cdot(5w\cdot HB_5)\right)$$

$$\therefore M_y = \frac{1}{3}w\left(1\cdot(0\cdot HB_0)+4\cdot(w\cdot HB_1)+1\cdot(2w\cdot HB_2)\right)$$

$$+\frac{3}{8}w\left(1\cdot(2w\cdot HB_2)+3\cdot(3w\cdot HB_3)+3\cdot(4w\cdot HB_4)+1\cdot(5w\cdot HB_5)\right)$$

$$M_{z,1} = \frac{1}{3}w\left(1\cdot((HB_0/2)\cdot HB_0)+4\cdot((HB_1/2)\cdot HB_1)+1\cdot((HB_2/2)\cdot HB_2)\right)$$

$$M_{z,2} = \frac{3}{8}w\left(1\cdot((HB_2/2)\cdot HB_2)+3\cdot((HB_3/2)\cdot HB_3)+3\cdot((HB_4/2)\cdot HB_4)+1\cdot((HB_5/2)\cdot HB_5)\right)$$

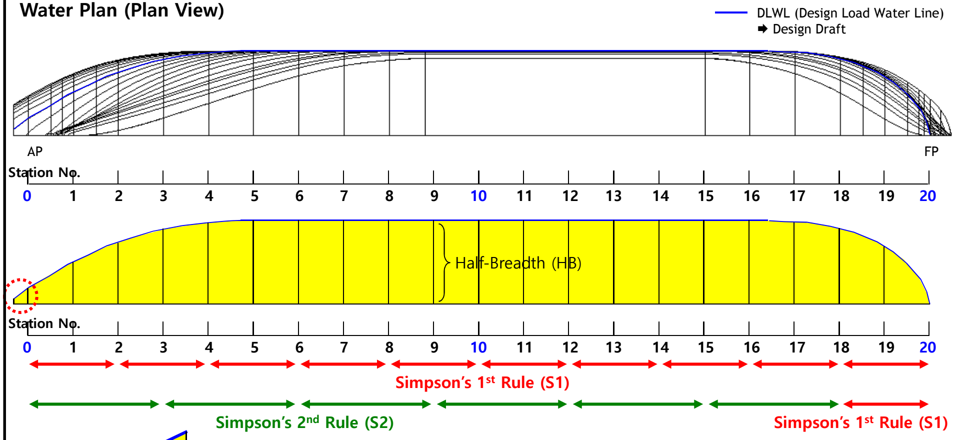
$$\therefore M_z = \frac{1}{3}w\left(1\cdot((HB_0/2)\cdot HB_0)+4\cdot((HB_1/2)\cdot HB_1)+1\cdot((HB_2/2)\cdot HB_2)\right)$$

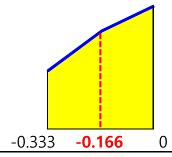
$$+\frac{3}{8}w\left(1\cdot((HB_2/2)\cdot HB_2)+3\cdot((HB_3/2)\cdot HB_3)+3\cdot((HB_4/2)\cdot HB_4)+1\cdot((HB_5/2)\cdot HB_5)\right)$$

$$\therefore Centroid_y = \frac{M_z}{Area},\quad Centroid_z = \frac{M_y}{Area}$$

Calculation of Water Plane Area

Water Plan (Plan View)

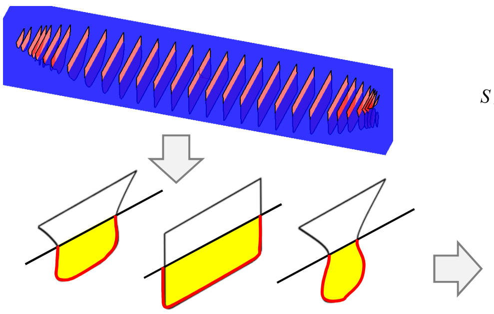




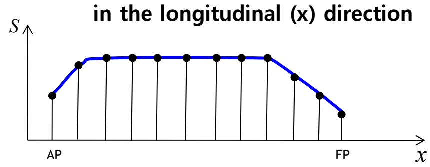
1. Generate a temporary section (e.g., -0.166)
2. Perform Simpson's 1st Rule.

Calculation of Displacement Volume

☑ The displacement volume (underwater volume) at a certain draft can be calculated **by integrating sectional areas in the longitudinal direction**.



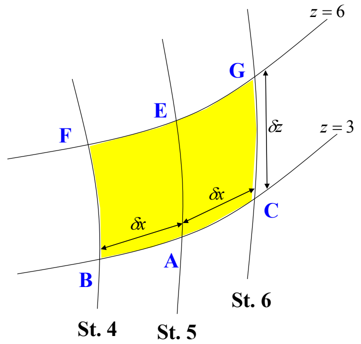
Volume integral
from sectional areas
in the longitudinal (x) direction



☑ In addition, the volume can be calculated **by integrating water plane areas in the vertical direction**. There can be a difference between two volumes due to approximation.

Calculation for Wetted Surface Area

- ☑ The wetted surface area means ship's area which contacts with water.
- ☑ This area can be calculated with the following approximate formula.

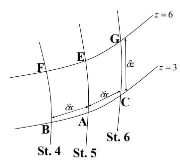


$$S = \delta z \int_{Sta.\,4}^{Sta.\,6} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$$

Example of Calculation for Wetted Surface Area (1/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	−0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	−0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	−0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	−0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	−0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	−0.22*	−0.28*	−0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

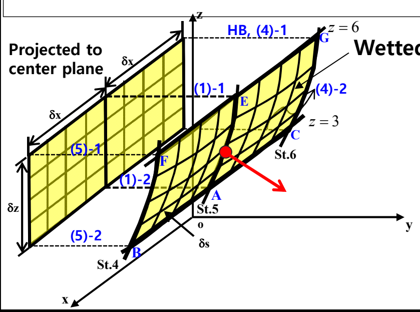
HB_A: Half-breadth afterward

HB_f: Half-breadth forward

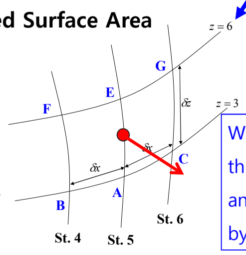
S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$\delta x =$ Station interval $= 13.94\ m$



Wetted Surface Area

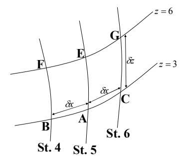


We can find the vertical station shape slope $\frac{dy}{dz}$ and longitudinal water line slope $\frac{dy}{dx}$ by using the central difference.

Example of Calculation for Wetted Surface Area (2/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = \text{Station interval} = 13.94\ m$$

1. Approximated formula for ship's surface area: $S = \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$

1) $\frac{dy}{dz} \approx \frac{\delta y}{\delta z}$

$$\delta z = (6-3) = 3m$$

In the table,

$$\delta y = HB_{W.L.=6m} - HB_{W.L.=3m} \text{ ---------------- } [(1.2)-(1.1)]$$

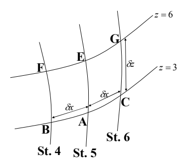
$$\frac{dy}{dz} \approx \frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z} \text{ -------------------- } (2)$$

$$\left(\frac{dy}{dz}\right)^2 \approx \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}\right)^2 \text{ ------------------ } (3)$$

Example of Calculation for Wetted Surface Area (3/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$$\delta x = \text{Station interval} = 13.94\ m$$

1. Approximated formula for ship's surface area: $S = \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$

2) $\frac{dy}{dx} = \frac{1}{2}\left(\left.\frac{dy}{dx}\right|_{W.L.=6m} + \left.\frac{dy}{dx}\right|_{W.L.=3m}\right)$

In the table,

$$\left.\frac{dy}{dx}\right|_{W.L.=6m} \approx \left.\frac{\delta y}{\delta x}\right|_{W.L.=6m} = \frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} \text{ -------------------------------- } [(5.1)-(4.1)]/2\delta x$$

($W.L.$: Waterline)

$$\left.\frac{dy}{dx}\right|_{W.L.=3m} \approx \left.\frac{\delta y}{\delta x}\right|_{W.L.=3m} = \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x} \text{ -------------------------------- } [(5.2)-(4.2)]/2\delta x$$

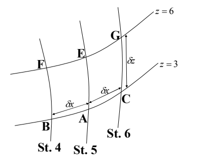
$$\frac{dy}{dx} \approx \frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right) \text{ ------------------------ } (6)$$

$$\left(\frac{dy}{dx}\right)^2 \approx \left[\frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right]^2 \text{ ------------------------ } (7)$$

Example of Calculation for Wetted Surface Area (4/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2$$

$\delta x = \text{Station interval} = 13.94\ m$

1) $\frac{dy}{dz} \approx \frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}$

2) $\frac{dy}{dx} \approx \frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)$

1. Approximated formula for ship's surface area: $S = \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2}\, dx$

$(8) = 1 + (7) + (3)$

2. Substituting 1) and 2) into the formula.

$$S \approx \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2 + \left(\frac{\delta y}{\delta z}\right)^2}\, dx$$

$$= \delta z \int_{Sta.1}^{Sta.5} \sqrt{1 + \left(\frac{1}{2}\left(\frac{HB_{A,W.L.=6m} - HB_{F,W.L.=6m}}{2\cdot\delta x} + \frac{HB_{A,W.L.=3m} - HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right)^2 + \left(\frac{HB_{W.L.=6m} - HB_{W.L.=3m}}{\delta z}\right)^2}\, dx$$

$(9) = \sqrt{(8)}$

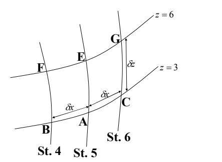
3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area.

69

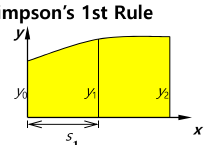
Example of Calculation for Wetted Surface Area (5/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)^1/2	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48

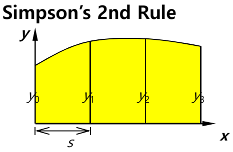


Simpson's 1st Rule



$Area = \frac{1}{3} s (y_0 + 4y_1 + y_2)$

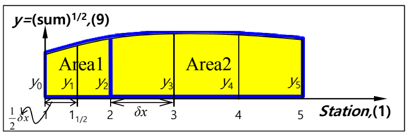
Simpson's 2nd Rule



$Area = \frac{3}{8} s (y_0 + 3y_1 + 3y_2 + y_3)$

3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area.

1) Simpson's multiplier (10)



Simpson's 1st Rule: $Area1 = \frac{1}{3}\cdot\frac{1}{2}\delta x \cdot (y_0 + 4y_1 + y_2)$

Simpson's 2nd Rule: $Area2 = \frac{3}{8}\cdot\delta x \cdot (y_2 + 3y_3 + 3y_4 + y_5)$

Total Area: $Area1 + Area2 = \frac{3}{8}\cdot\delta x \cdot \left(\frac{8}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}y_0 + \frac{8}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot 4y_1 + \frac{8}{3}\cdot\frac{1}{3}\cdot\frac{1}{2}y_2 + y_2 + 3y_3 + 3y_4 + y_5\right)$

$= \frac{3}{8}\cdot\delta x \cdot (\boxed{0.444}y_0 + \boxed{1.778}y_1 + \boxed{1.444}y_2 + \boxed{3}y_3 + \boxed{3}y_4 + \boxed{1}y_5)$ □: S.M, (10)

70

Example of Calculation for Wetted Surface Area (6/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)$^{1/2}$	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84

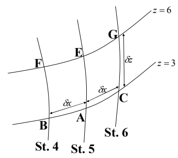
3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area.

$$S \approx \delta z \int_{Sta.1}^{Sta.5} \sqrt{1+\left(\frac{1}{2}\left(\frac{HB_{A,W.L.=6m}-HB_{F,W.L.=6m}}{2\cdot\delta x}+\frac{HB_{A,W.L.=3m}-HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right)^2+\left(\frac{HB_{W.L.=6m}-HB_{W.L.=3m}}{\delta z}\right)^2}\,dx$$

$$=\delta z\cdot\frac{3}{8}\cdot\delta x\cdot\sum \underbrace{S.M}_{(10)}\cdot\underbrace{\sqrt{1+\left(\frac{1}{2}\left(\frac{HB_{A,W.L.=6m}-HB_{F,W.L.=6m}}{2\cdot\delta x}+\frac{HB_{A,W.L.=3m}-HB_{F,W.L.=3m}}{2\cdot\delta x}\right)\right)^2+\left(\frac{HB_{W.L.=6m}-HB_{W.L.=3m}}{\delta z}\right)^2}}_{(9)}$$

$$=\delta z\cdot\frac{3}{8}\cdot\delta x\cdot\sum \underbrace{\text{Prod.}}_{(11)}$$

$$=3\cdot\frac{3}{8}\cdot 13.94\cdot 12.84=201.36\ (m^2)$$



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

$$Sum = 1+\left(\frac{\delta y}{\delta x}\right)^2+\left(\frac{\delta y}{\delta z}\right)^2$$

$\delta x = 13.94\ m, \delta z = 3\ m$

71

Example of Calculation for Wetted Surface Area (7/7)

Calculate the wetted surface area of the ship from St. 1 to St. 5 between 3m and 6m of waterline.

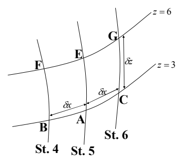
(1) Sta.	(1.1) HB 6m	(1.2) HB 3m	(2) δy/δz	(3) (δy/δz)²	(4) Sta. Ford.	(4.1) HB 6m	(4.2) HB 3m	(5) Sta. Aft.	(5.1) HB 6m	(5.2) HB 3m	(6) Mean δy/δx	(7) (δy/δx)²	(8) Sum	(9) (Sum)$^{1/2}$	(10) S.M	(11) Prod.
5	19.66	18.41	0.42 (1)	0.17	6	20.12	19.84	4	17.56	15.47	-0.12 (2)	0.01	1.18	1.09	1	1.09
4	17.56	15.47	0.70	0.49	5	19.66	18.41	3	13.38	11.16	-0.24	0.06	1.55	1.24	3	3.72
3	13.38	11.16	0.74	0.55	4	17.56	15.47	2	8.14	6.64	-0.33	0.11	1.66	1.29	3	3.87
2	8.14	6.64	0.50	0.25	3	13.38	11.16	1	2.62	2.16	-0.35	0.13	1.38	1.17	1.444	1.69
1 1/2	5.43	4.39	0.35	0.12	2	8.14	6.64	1 (3)	2.62	2.16	-0.36	0.13	1.25	1.12	1.778	1.99
1	2.62	2.16	0.15	0.02	1 1/2	5.43	4.39	1/2	-0.22*	-0.28*	-0.37	0.14	1.16	1.08	0.444	0.48
															Σ	= 12.84

3. By using the Simpson's 1st and 2nd rules, calculate the ship's surface area.

$$S \approx 201.36\ m^2$$

4. Calculate the wetted surface area of both sides of the ship

$$\textit{Wetted Surface Area, Both sides} = 2\cdot S \approx 2\cdot 201.36 = 402.7\ \left(m^2\right)$$



HB: Half-breadth for waterline

HB_A: Half-breadth afterward

HB_f: Half-breadth forward

S: Wetted surface area of the ship

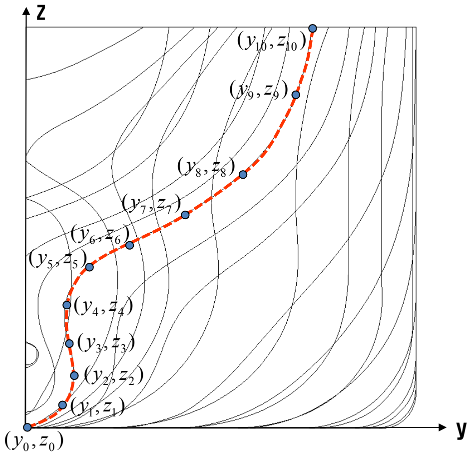
$$Sum = 1+\left(\frac{\delta y}{\delta x}\right)^2+\left(\frac{\delta y}{\delta z}\right)^2$$

5. Calculation of Hydrostatic Values by Using Gaussian Quadrature and Green's Theorem

Description of Section Lines (1/2)

1. Make a text file for describing the body plan of a ship.

Given: Body plan of a ship
Find: Text file describing the body plan of a ship



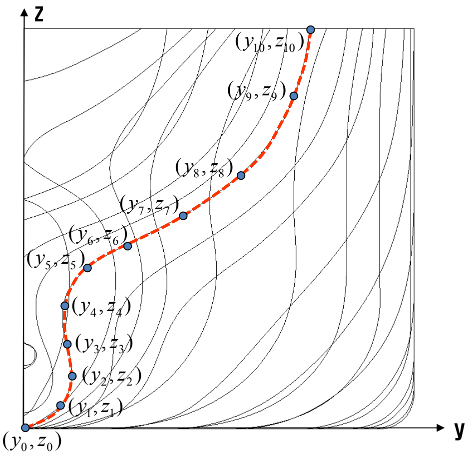
Example of text file for describing the body plan of a ship

300.0 50.0 27.0 18.0	**// *LBP*, B_{mld}, D_{mld}, *T***
27	**// Section Line Num.**
…	
1.0 11	**// Station, Point Num.**
y_0 z_0	**// Y coord., Z coord.**
y_1 z_1	
y_2 z_2	
…	
y_{10} z_{10}	
1.5 10	
…	

Description of Section Lines (2/2)

2. Find cubic B-spline curves passing the points on the section lines.

Given: Data of the points on the section line that describes the body plan of a ship
Find: Cubic B-spline curve which passes the points on the section line



Make cubic B-spline curve which passes through the given points

➡ Refer to the Part "Curve and Surface"
(Ship Hull Form Modeling for 2nd Year Undergraduate Course)

$$\mathbf{r}(u) = \mathbf{d}_0 N_0^3(u) + \mathbf{d}_1 N_1^3(u) + \mathbf{d}_2 N_2^3(u) + \cdots + \mathbf{d}_{D-1} N_{D-1}^3(u)$$

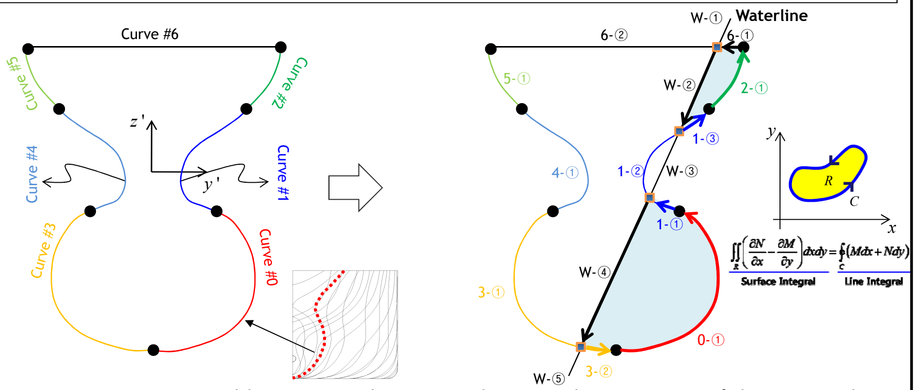
$\mathbf{d}_i$: de Boor points (control points), $i = 0,1,\ldots,D-1$
$N_i^n(u)$: B-splines basis function of degree $n(=3)$
u_j : Knots, $j = 0,1,\ldots,K-1$, where $K = D+n+1$

$$N_i^n(u) = \frac{u - u_{i-1}}{u_{i+n-1} - u_{i-1}} N_i^{n-1}(u) + \frac{u_{i+n} - u}{u_{i+n} - u_i} N_{i+1}^{n-1}(u)$$

$$N_i^0(u) = \begin{cases} 1 & \text{if } u_{i-1} \le u < u_i \\ 0 & \text{else} \end{cases}, \quad \sum_{i=0}^{D-1} N_i^n(u) = 1$$

Calculation of Sectional Area and 1st Moment of Sectional Area Under the Water Plane (1/4)

Given: B-spline curve, the intersection points between the B-spline curves and water plane, and B-spline parameter "u" at each end point of the line segments
Find: Sectional area and 1st moment of sectional area



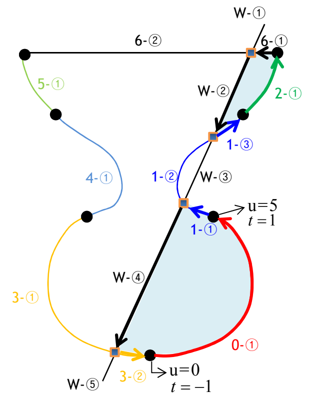
The section is represented by Curve #0 ~ Curve #6

The sectional area and 1st moment of the sectional area under the waterline is calculated by integration of the following line segments.

3-②➡0-①➡1-①➡W-④,
1-③➡2-①➡6-①➡W-②

Calculation of Sectional Area and 1st Moment of Sectional Area Under the Water Plane (2/4)

Given: B-spline curve, the intersection points between the B-spline curve and water plane, and B-spline parameter "u" at each end point of the line segments
Find: Sectional area and 1st moment of section



✓ Relation between the Parameter u and t

$$u = \frac{(t+1)(u_{max}-u_{min})}{2} + u_{min}$$

$$u = \frac{(t+1)(5-0)}{2} + 0$$

<Surface integral> Green's Theorem <Line integral >

$$A = \iint_R dy'dz' \Rightarrow = \frac{1}{2}\oint_C (y'dz' - z'dy')$$

For example, integrate the line segment 0-①

For the line integral of the segment in the y' and z' coordinates, the interval for the integration has to be determined.

- Since the parameter 'u' increases monotone, the interval can be found easily.
- Using the chain rule, convert the line integral for y' and z' into the integral for only one parameter 'u'.

$$\frac{1}{2}\int_0^5 \left(y'(u)\frac{dz'}{du}du - z'(u)\frac{dy'}{du}du \right)$$

$$= \frac{1}{2}\int_0^5 \left(y'(u)\frac{dz'}{du} - z'(u)\frac{dy'}{du} \right) du = \frac{1}{2}\int_0^5 g(u)du$$

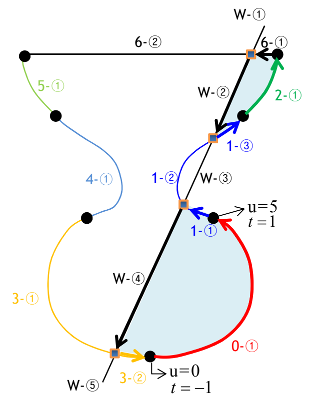
➡ To use Gaussian quadrature, convert the integration parameter 'u' and the interval [0, 5] into 't' and [-1,1]

$$\frac{1}{2}\int_{-1}^1 \left(y'(u(t))\frac{dz'}{du} - z'(u(t))\frac{dy'}{du} \right) \frac{du}{dt} dt = \frac{1}{2}\int_{-1}^1 f(t)dt$$

✓ In the same way, integrate the remained line segments using Green's theorem and Gaussian quadrature.

Calculation of Sectional Area and 1st Moment of Sectional Area Under the Water Plane (3/4)

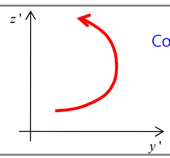
※ **Procedure for calculation of the sectional area and 1st moment of sectional area under the water plane**



✓ Relation between the Parameter u and t

$$u = \frac{(t+1)(u_{max}-u_{min})}{2} + u_{min}$$

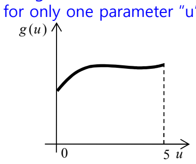
$$u = \frac{(t+1)(5-0)}{2} + 0$$



Convert surface integral into line integral

$$= \frac{1}{2}\oint_C y'dz' - z'dy'$$

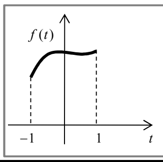
Using the chain rule, convert the line integral for y' and z' into the integral for only one parameter "u".



$$\frac{1}{2}\int_0^5 y'(u)\frac{dz'}{du}du - z'(u)\frac{dy'}{du}du$$

$$= \frac{1}{2}\int_0^5 \left(y'(u)\frac{dz'}{du} - z'(u)\frac{dy'}{du} \right) du$$

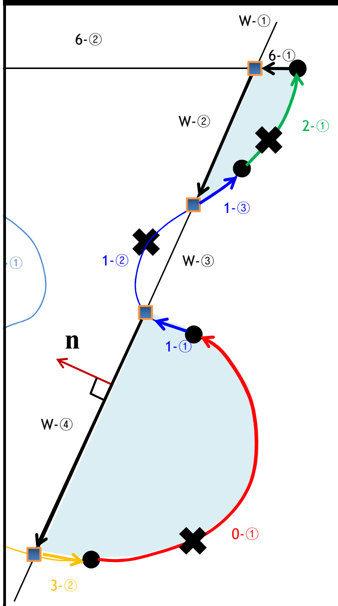
$$= \frac{1}{2}\int_0^5 g(u)du$$



To use Gaussian quadrature, convert the parameter and the interval into "t" and [-1,1].

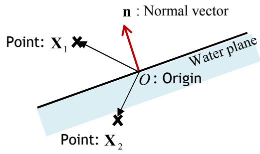
$$\frac{1}{2}\int_{-1}^1 \left(y'(u(t))\frac{dz'}{du} - z'(u(t))\frac{dy'}{du} \right) \frac{du}{dt} dt = \frac{1}{2}\int_{-1}^1 f(t)dt$$

Calculation of Sectional Area and 1st Moment of Sectional Area Under the Water Plane (4/4)



※ Method to check whether the line segments are located under the water plane or not

- To calculate the sectional area under the water plane, it is required to check whether the points on the line segments are located under the water plane or not.



✓Check the location of the point by using the sign of dot product of normal vector of the water plane and position vector of the point

$\mathbf{n} \cdot (\mathbf{X} - \mathbf{O}) > 0$: The point is above the water plane.

$\mathbf{n} \cdot (\mathbf{X} - \mathbf{O}) \le 0$: The point is on or below the water plane.

✓Perform only line integration for the segments which are on or below the water plane.

In this example, the line integration is performed as follows:

The line segment 0-① : $\mathbf{n} \cdot (\mathbf{X} - \mathbf{O}) \le 0$ ➡ Perform integration

The line segment 1-② : $\mathbf{n} \cdot (\mathbf{X} - \mathbf{O}) > 0$ ➡ No integration

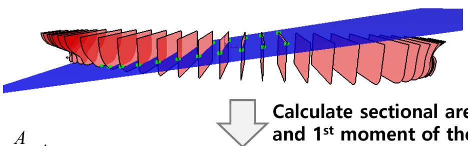
The line segment 2-① : $\mathbf{n} \cdot (\mathbf{X} - \mathbf{O}) \le 0$ ➡ Perform integration

($\mathbf{X}$: the middle point of the each line segment)

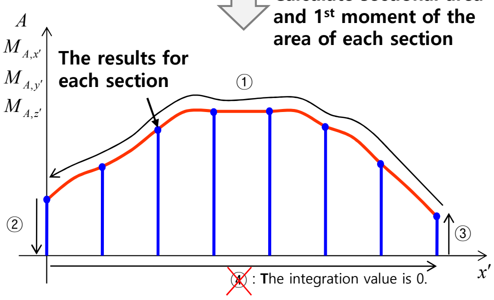
79

Calculation of Ship's Displacement Volume, 1st Moment of Displacement Volume, LCB, TCB, and KB

Given: Sectional areas and 1st moments of the sectional areas under the water plane
Find: Displacement volume, 1st moment of displacement volume, LCB, TCB, and KB



Calculate sectional area and 1st moment of the area of each section



$V = \nabla = \int A(x')dx'$

Displacement Volume

$M_{\nabla,y'z'} = \int M_{A,y'z'}(x')dx'$

$M_{\nabla,x'z'} = \int M_{A,x'z'}(x')dx'$

$M_{\nabla,x'y'} = \int M_{A,x'y'}(x')dx'$

Calculation procedure

✓ **Calculate the displacement volume and 1st moment of the volume by integrating the sectional areas and 1st moments of the sectional areas over ship's length.**

1) **Make the ordinate set along ship's length by using the results for each section.**
2) **Generate B-spline curve which interpolates the ordinates.**
3) **Perform the line integration counter-clockwise using Green's theorem and Gaussian quadrature.**

Displacement: $\Delta = \rho_{sw} \cdot \nabla$

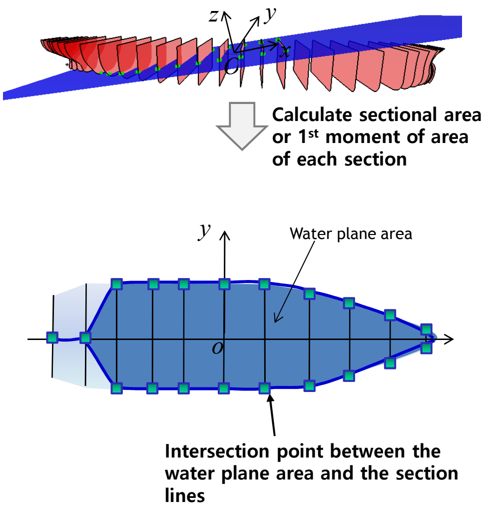
$$LCB = \frac{M_{\nabla,y'z'}}{\nabla}, TCB = \frac{M_{\nabla,x'z'}}{\nabla}, VCB = \frac{M_{\nabla,x'y'}}{\nabla}$$

$$KB = VCB \text{ (from waterline)} + T_d$$

80

Calculation of Water Plane Area, 1st and 2nd Moment of Water Plane Area

Given: Intersection points between the water plane and the section lines
Find: Water plane area, 1st moment and 2nd moment of the water plane area



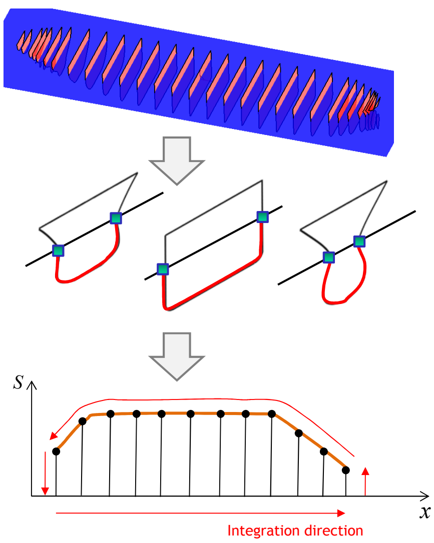
Calculation procedure

- ✓ Transform the intersection points decomposed in body fixed frame into the points decomposed in water plane fixed frame (inertial frame).
- ✓ Generate the curve which interpolates the intersection points. If a section 'x' has no intersection point, input the point as (x, 0, 0).
- ✓ Calculate the area, 1st moment and 2nd moment of area using Green's theorem or Gaussian quadrature.

81

Calculation of Wetted Surface Area

Given: Intersection points between the water plane and the section lines
Find: Wetted surface area



Calculation procedure

1) Calculate the girth length of the section lines under the water plane.

$$s = \int_{t_0}^{t_1} ds = \int_{t_0}^{t_1} \|\dot{\mathbf{r}}(t)\| dt$$

2) Calculate the sectional area surrounded by the girth length and water plane.
3) Make the ordinate set of the sectional area.
4) Generate B-spline curve which interpolates the ordinates.
5) Integrate the area along ship's length using Green's theorem or Gaussian quadrature.

➡ Wetted surface area is calculated.

82