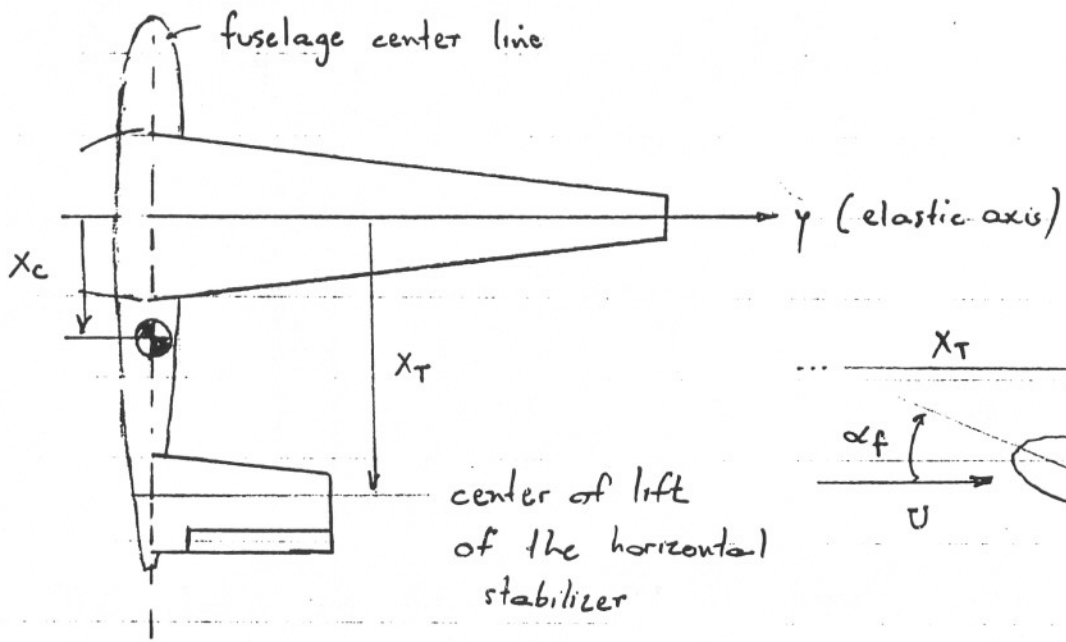
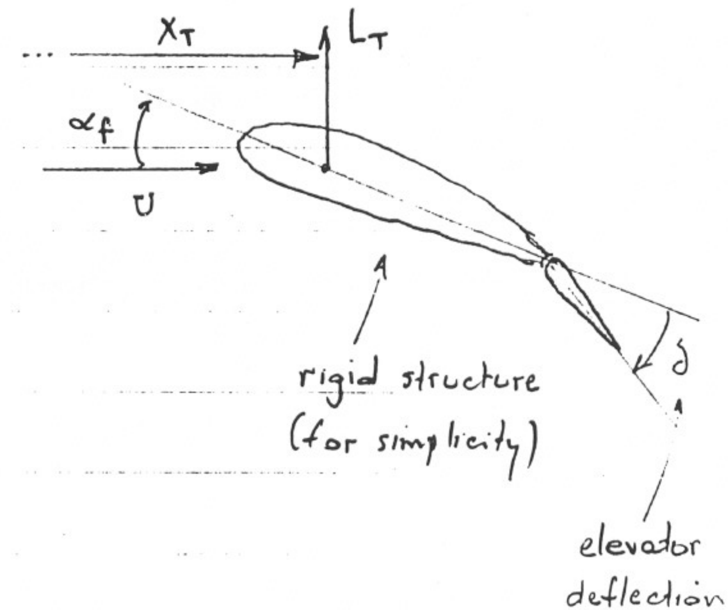
COMPLETE AIRCRAFT

(HANDOUT)





The tail lift :

$$L^T = q S_T \left(C_{L_\alpha}^T \alpha_f + C_{L_\delta}^T \delta \right)$$

where: $(\ )^T$ denotes "tail"

α_f = fuselage angle of attack

The elastic equilibrium about the elastic axis remains the same:

$$\left([A]^{-1} - q[E] \right) \{ c_{e} \} + N g [G] \{ m \} = \alpha_r(0) \{ 1 \} + \{ \alpha_t \} + q [F] \{ c_{mac} \}$$

<u>Note:</u> $\alpha_r(0)$ is related to α_f — for simplicity, we'll assume $\alpha_r(0) = \alpha_f$

If flexible fuselage $\rightarrow$ bending equation will relate α_f and α_t

The total lift equals NW as before :

$$2 \underbrace{\int_0^{\ell} q\, c\, c_\ell \, dy}_{\text{wings}} + \underbrace{q S_T \left(c_{l_\alpha}^T \alpha_f + c_{l_\delta}^T \delta \right)}_{\text{tail}} = NW$$

...or in matrix form :

$$2 q \lfloor 1 \rfloor \left[{}^{\backslash} W_{\backslash} \right] \{ c c_\ell \} + q S_T c_{l_\alpha}^T \alpha_f + q S_T c_{l_\delta}^T \delta = NW$$

The total pitching moment about the center of gravity must be zero for trim:

$$2\int_0^{l} q\left\{ (e+x_c)\, c c_l + c^2 c_{mac} \right\} dy \; +$$

$$-\,(x_T - x_c)\, q S_T \left(c_{l_\alpha}^{T}\, \alpha_f + c_{l_\delta}^{T}\, \delta \right) = 0$$

<u>Note:</u> coefficients of x_c is NW (from total lift relation).

$$\Longrightarrow \quad 2q \lfloor e \rfloor [\,`W_{\backslash}\,] \{ c c_l \} + 2q \lfloor c^2 \rfloor [\,`W_{\backslash}\,] \{ c_{mac} \} \; +$$

$$-\, x_T\, q\, b_\alpha\, \alpha_f - x_T\, q\, b_\delta + x_c\, NW = 0$$

$b_\alpha \equiv S_T\, c_{l_\alpha}^{T}$

$b_\delta \equiv S_T\, c_{l_\delta}^{T}$

The case for prescribed N :

$$\begin{bmatrix} [A]^{-1} - q[E] & -\{1\} & 0 \\ 2q\lfloor 1 \rfloor [\,\grave{}W_\lrcorner\,] & q b_\alpha & q b_\delta \\ 2q\lfloor e \rfloor [\,\grave{}W_\lrcorner\,] & -x_T q b_\alpha & -x_T q b_\delta \end{bmatrix} \begin{Bmatrix} \{cce\} \\ \alpha_f \\ \delta \end{Bmatrix} =$$

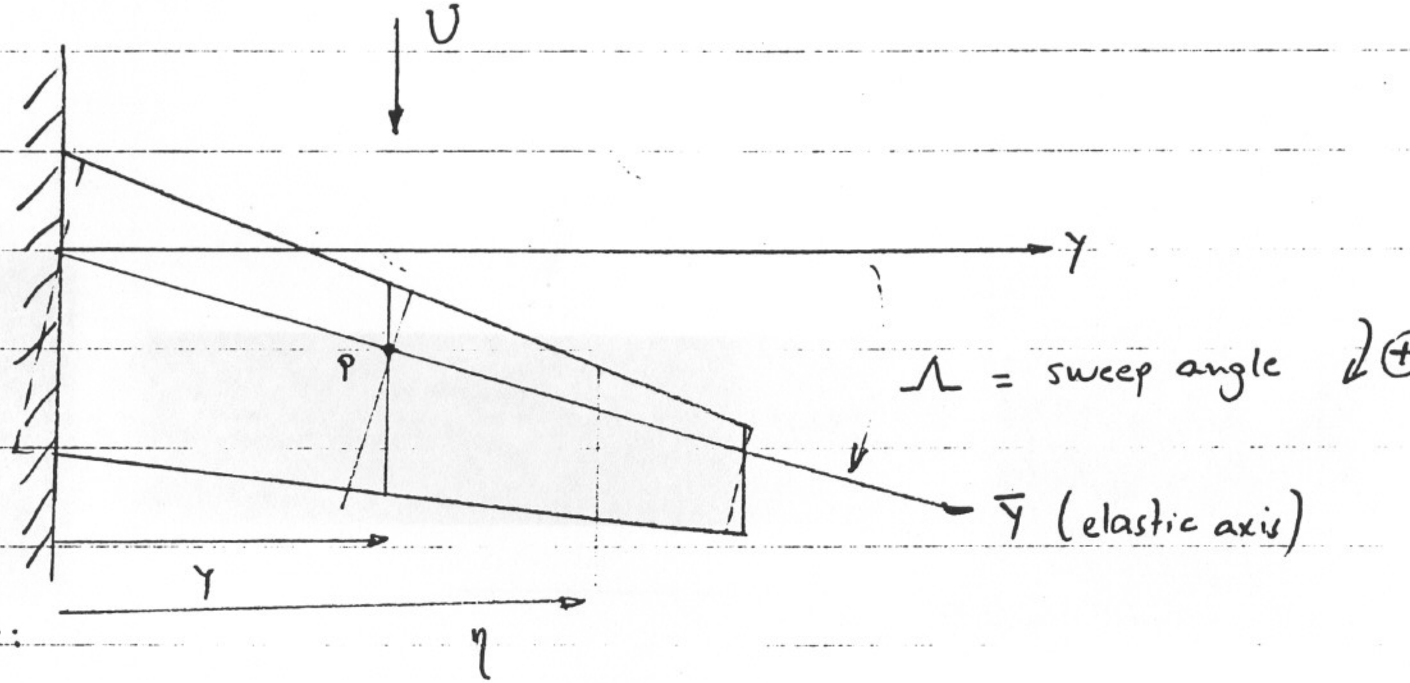
$$= \begin{Bmatrix} \{\alpha_t\} + q[F]\{C_{mac}\} - N_g [G]\{m\} \\ NW \\ -2q\lfloor c^2 \rfloor [\,\grave{}W_\lrcorner\,]\{C_{mac}\} - x_c NW \end{Bmatrix}$$

<u>Note:</u> For a rigid aircraft $\Rightarrow$ $[C^{\theta\theta}] \equiv [0] \rightarrow [E], [F], [G] \equiv [0]$

The condition for divergence comes from the first set of equations for no disturbances (i.e., α_f, $\{\alpha_t\}$, $\{C_{mac}\}$, $[G]$, all zero) :

$$\Longrightarrow \quad \left([A]^{-1} - q[E] \right) \{c_{\ell}\} = \{0\} \quad \longrightarrow \quad q_D = \text{lowest eigenvalue}$$

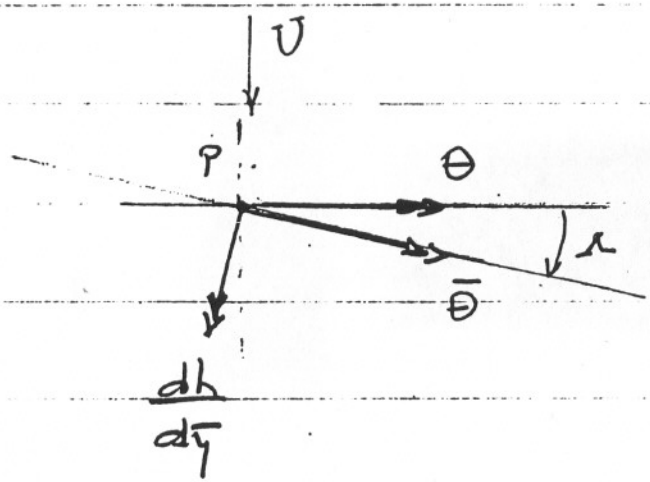
SWEEP EFFECTS ON STATIC AEROELASTIC STABILITY AND AIRLOAD DISTRIBUTION

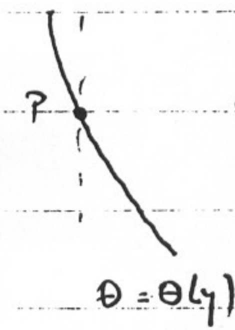


effective root
(remember: it's a <u>beam</u> approx.)

TWO MAIN EFFECTS

1. "Coupling" of torsion and bending of the wing





appears as camber

Note: see BAH § 2.13

BA p.311

$$\Theta = \bar{\Theta}\cos\Lambda - \frac{dh}{d\bar{y}}\sin\Lambda$$

TWO MAIN EFFECTS (cont.)

2. Aerodynamic loading for a given angle of attack distribution depends on the sweep angle.

$\Rightarrow$ $[\bar{A}]$ is different from $[A]$ of unswept case

<u>Note:</u> For strip theory $[\bar{A}] = \left[\ \lceil c\,(C_{L\alpha})_\Lambda \rfloor\ \right]$; $(C_{L\alpha})_\Lambda \cong \cos\Lambda\,(C_{L\alpha})_{\Lambda=0}$

see B.A. p.102

TWIST EQUILIBRIUM

From effect (1), the twist equilibrium

$$\theta(y) = \int_0^{l} C^{\theta\theta}(y,\eta)\, M_{ea}\, d\eta + \int_0^{l} C^{\theta z}(y,\eta)\, F_z\, d\eta$$

where $C^{\theta z}$ represents streamwise elastic twist at y due to transverse force at η.

$$M_{ea} = q\left(e\, c c_l + c^2 c_{mac}\right) - N m g \cdot d$$

$$F_z = q\, c c_l - N m g$$

EXTRA EQUATION : BENDING EQUILIBRIUM

The bending equilibrium :

$$h(y) = \int_0^l C^{zz}(y,\eta)\, F_z \, d\eta + \int_0^l C^{z\theta}(y,\eta)\, Mea \, d\eta$$

Note: Mea causes h because of the component Mea. sin Λ

TWIST EQUILIBRIUM

In matrix form:

$$\{\theta\} = q\,[\bar{E}]\{c\ell\} + q\,[\bar{F}]\{c_{mac}\} - N_g\,[\bar{G}]\{m\}$$

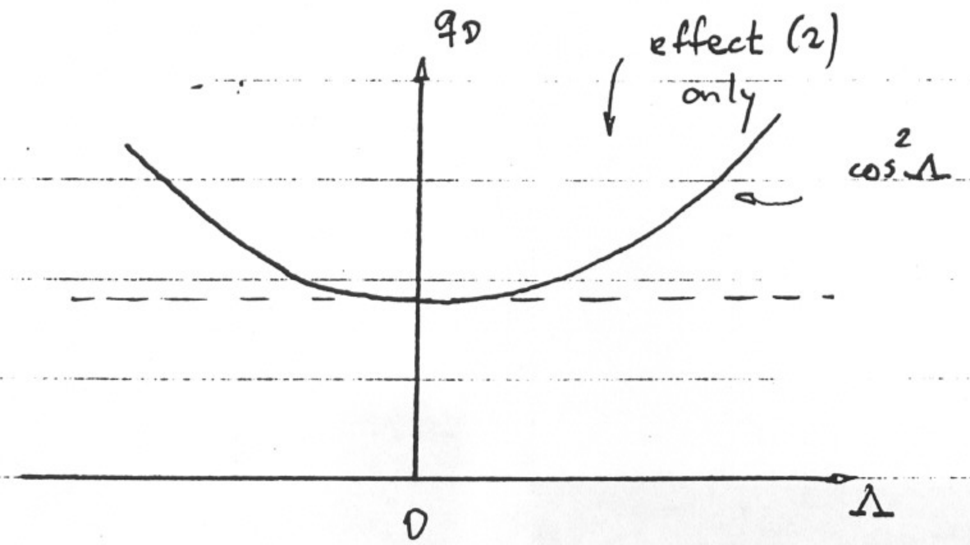
where:

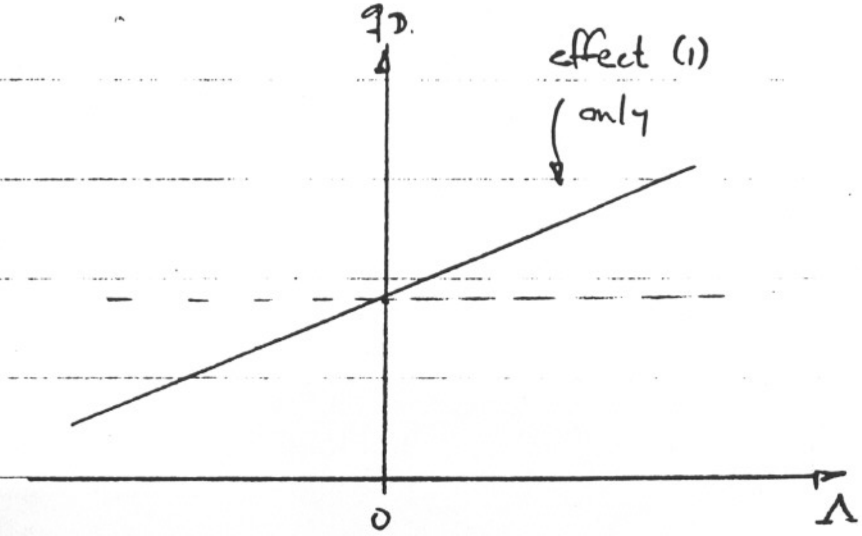
$$[\bar{E}] = \left([C^{\theta\theta}][\diagdown w \diagdown][\diagdown e \diagdown] + [C^{\theta z}][\diagdown w \diagdown] \right)$$

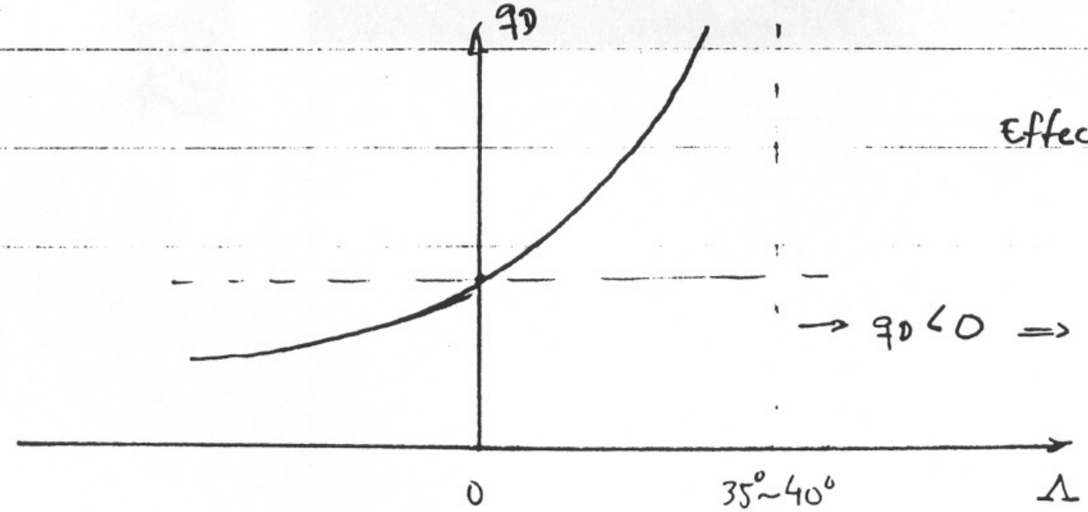
$$[\bar{F}] = [C^{\theta\theta}][\diagdown w \diagdown][\diagdown c^2 \diagdown]$$

$$[\bar{G}] = \left([C^{\theta\theta}][\diagdown w \diagdown][\diagdown d \diagdown] + [C^{\theta z}][\diagdown w \diagdown] \right)$$

TYPICAL DIVERGENCE RESULTS



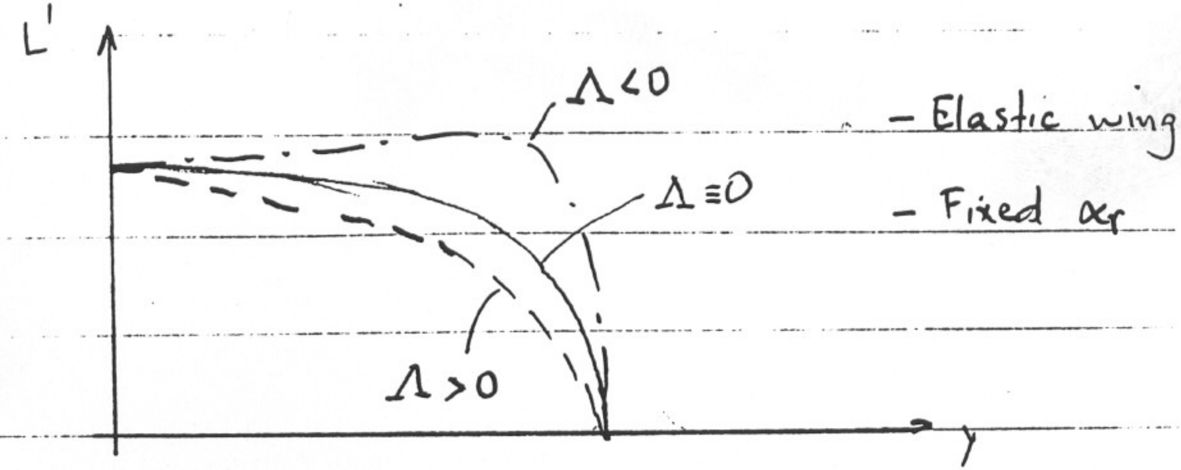


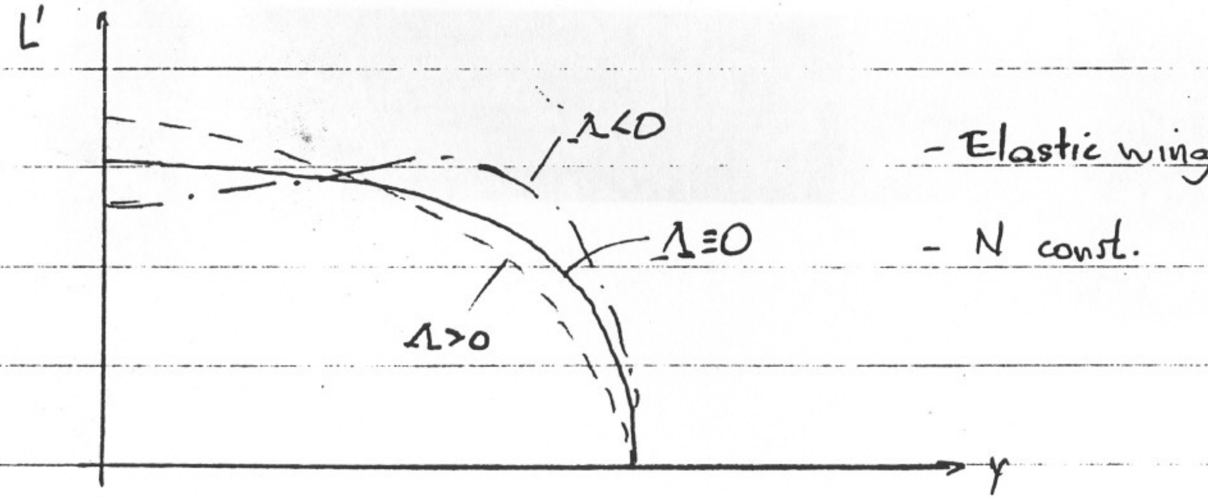


Effect (1) is predominant

→ $q_D < 0$ ⇒ no divergence

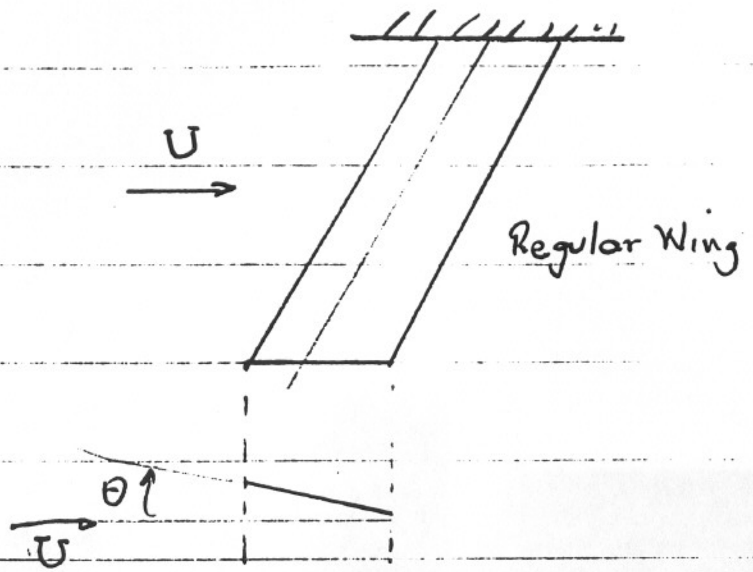
TYPICAL A[illegible]OAD DISTRIBUTION



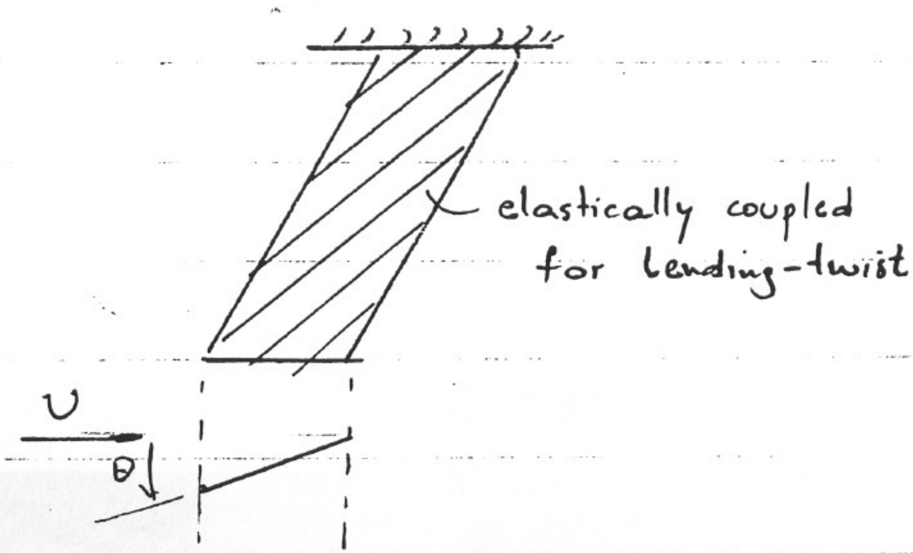


$\therefore$ forward sweep ($\Lambda < 0$) enhances divergence instability and increases structural loads while sweepback can alleviate these concerns

IMPROVING FORWARD SWEEP EFFECTS



$+F \rightarrow +\theta$ BAD!



$+F \rightarrow -\theta$ GOOD!

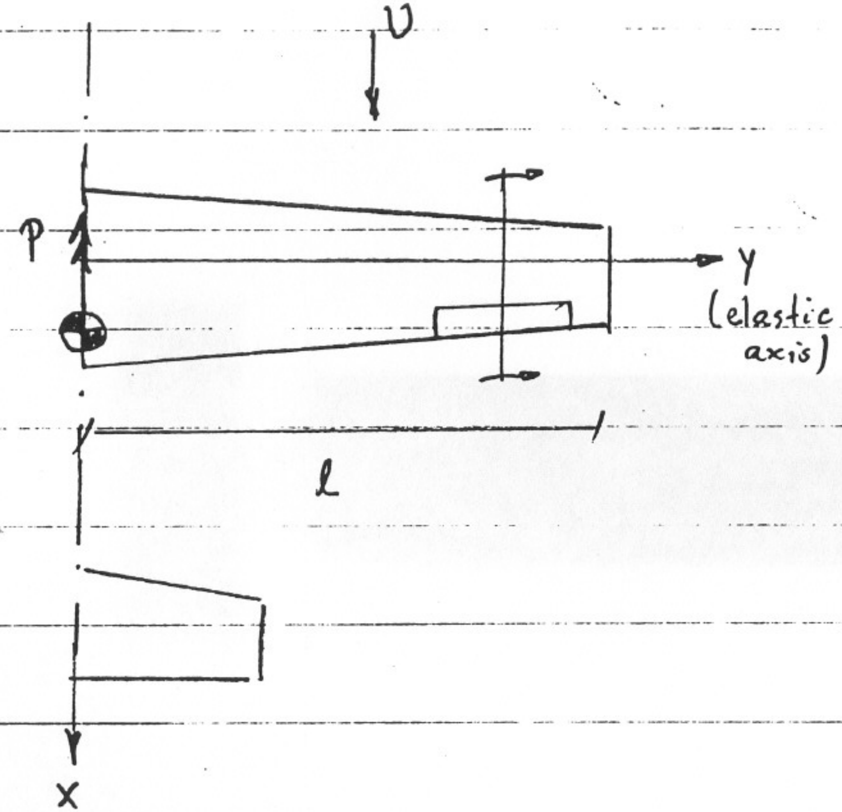
$$\begin{cases} EI\,\bar{h}^{IV} + K\,\bar{\theta}''' = f \\ K\,\bar{h}''' + GJ\,\bar{\theta}'' = -m \end{cases}$$

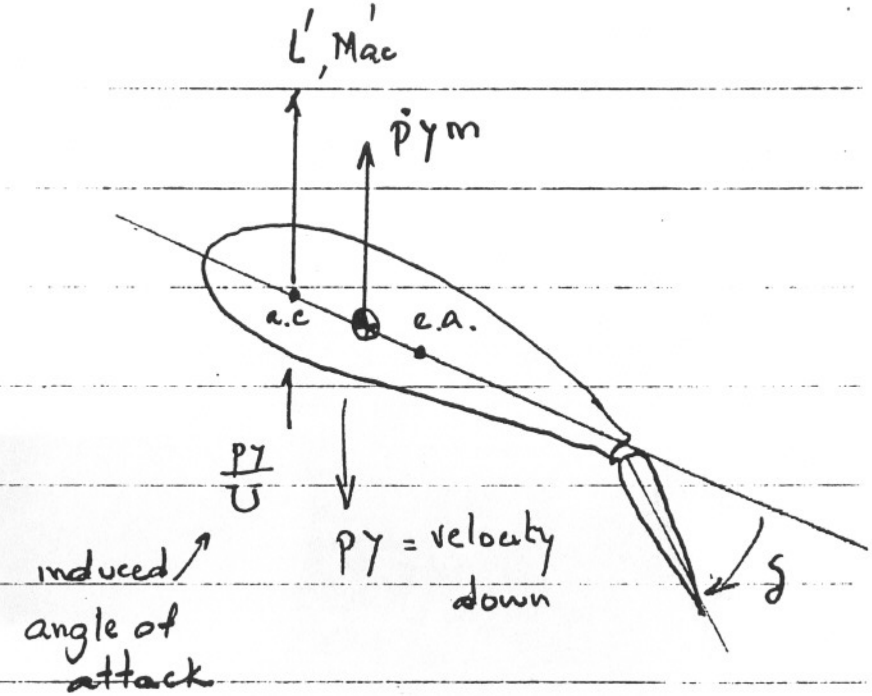
K = bending-twist coupling

↑

can be adjusted to compensate for geometric sweep effects

ROLLING OF A WING





$p \equiv$ roll rate (rad/s) ... positive as indicated (wing down)

ROLLING IS A ANTI-SYMMETRIC MANEUVER

L' due to:

1. anti-symmetric (A.S.) $\Theta \rightarrow c\,c_\ell^{e}$ ← no $\alpha_r(0)$
2. induced α due to $p = \dfrac{p y}{U}$
3. A.S. due to δ

Therefore, the anti-symmetric component of the moment about the elastic axis (M_{ea}) is:

$$M_{ea} = q e \left[c\,c_\ell^{e} + c \frac{\partial c_\ell}{\partial \alpha} \cdot \left(\frac{p y}{U} \right) + \frac{c\, \partial c_\ell}{\partial \delta} \cdot \delta \right] + q c^2 \frac{\partial c_{mac}}{\partial \delta} \delta +$$

$$+ \dot{p} y m d$$

EQUILIBRIUM EQUATIONS

There are two equilibrium equations :

(1) $\theta(y) = \int_0^{\ell} C^{\theta\theta}(y,\eta)\, M_{ea}(\eta)\, d\eta$ ... torsion equation

(2) $2 \int_0^{\ell} y\, F_{AS.}\, dy = I_x\, \dot{p}$ ... rolling moment equation

total roll moment of inertia

or: $-2 \int_0^{\ell} q\, y \left[c\, c_{\ell}^{e} + c \frac{\partial c_{\ell}}{\partial \alpha} \frac{py}{U} + c \frac{\partial c_{\ell}}{\partial \delta} \delta \right] dy = I_x\, \dot{p}$

EQUILIBRIUM EQUATIONS (cont.)

Employing the following change of variables :

$$\frac{\partial c_\ell}{\partial \alpha}\,\frac{p y}{U} = \frac{\partial c_\ell}{\partial\left(\frac{p\ell}{U}\right)} \cdot \underbrace{\frac{\partial\left(\frac{p\ell}{U}\right)}{\partial \alpha}}_{=\frac{\ell}{y}}\,\frac{p y}{U} = c_{\ell_p} \cdot \frac{p\ell}{U}$$

, since $\alpha = \Theta + \frac{p y}{U}$

and writing (1) and (2) in matrix form :

(1) $$\{\Theta\} - q\,[E]\,\{c c_\ell^e\} - q\,[E]\,\{c c_{\ell_p}\}\,\frac{p\ell}{U} - [G]\,[\,`\gamma_\backslash\,]\,\{m\}\,\dot{p} =$$

$$= q\,\left(\,[E]\,\{c c_{\ell_\delta}\} + [F]\,\{c m_{ac_\delta}\}\,\right)\,\delta$$

(2) $$\lfloor H \rfloor\,\{c c_\ell^e\} + \lfloor H \rfloor\,\{c c_{\ell_p}\}\,\frac{p\ell}{U} + \frac{I_x\,\dot{p}}{2q} = -\lfloor H \rfloor\,\{c c_{\ell_\delta}\}\,\delta$$

where : $\lfloor H \rfloor = \lfloor y \rfloor\,[\,`w_\backslash\,]$

EQUILIBRIUM EQUATIONS (cont.)

We also know that :

$$[A]\{\alpha\} = \{c\,c_\ell\} \Longrightarrow \{\theta\} = [A]^{-1}\{c\,c_\ell^e\}$$

that can be used in (1). Solve that now for $\{c\,c_\ell^e\}$ and use the result on (2) yields :

$$\boxed{A_1\,\dot{p} + B_1\,\frac{p\,\ell}{U} = C_1\,\delta}$$

... ordinary diff. equation

where: $A_1 \equiv \dfrac{I_x}{2q} + \lfloor H \rfloor \left([A]^{-1} - q\,[E] \right)^{-1} [G]\,[\,`Y_\backslash]\,\{m\}$

$$B_1 = \lfloor H \rfloor \{c\,c_{\ell_p}\} + q\,\lfloor H \rfloor \left([A]^{-1} - q\,[E] \right)^{-1} [E]\,\{c\,c_{\ell_p}\}$$

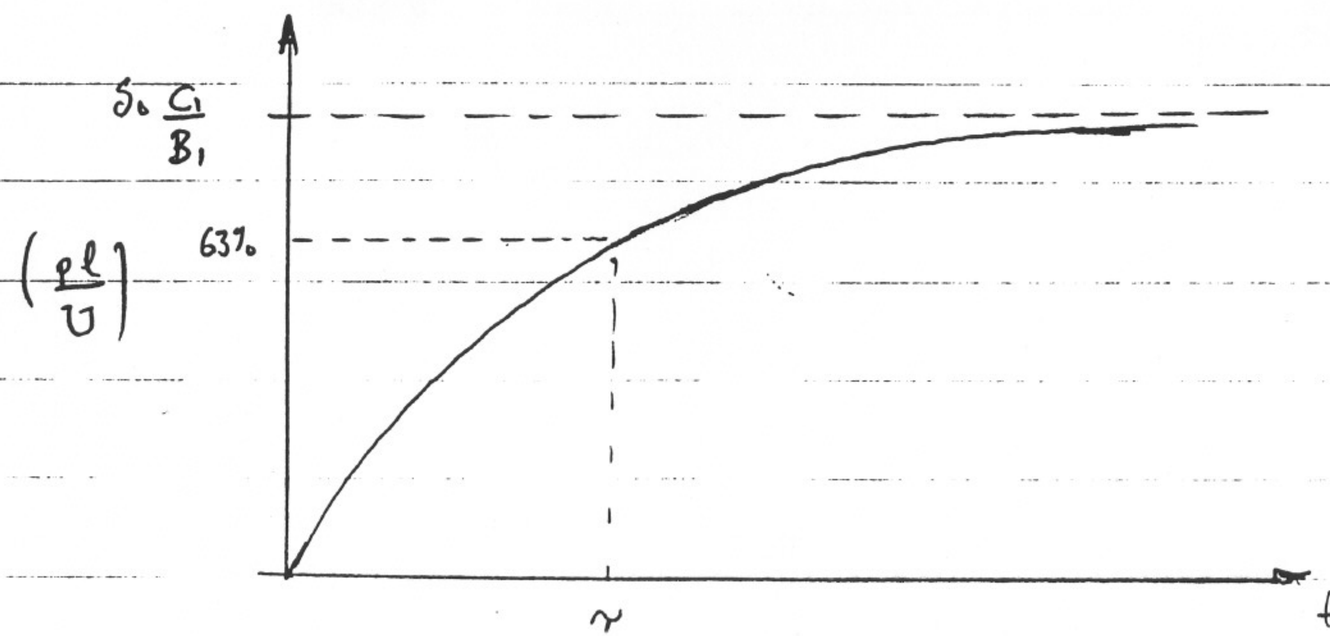
$$C_1 \equiv -\lfloor H \rfloor \{c\,c_{\ell_\delta}\} - q\,\lfloor H \rfloor \left([A]^{-1} - q\,[E] \right)^{-1} \left([E]\,\{c\,c_{\ell_\delta}\} + [F]\,\{c\,m_{ac_\delta}\} \right)$$

Considering a unit control input for the aileron deflection, i.e.,

$$\delta = \delta_0 . 1(t)$$

the solution of the o.d.e. is; for zero initial condition:

$$\boxed{\frac{pl}{U} = \frac{C_1}{B_1}\left(1 - e^{-\frac{l}{U}\frac{B_1}{A_1}t}\right)\delta_0}$$



$$\tau = \frac{A_1}{B_1}\frac{U}{l}$$

time constant

ROLL MOTION

Control Effectiveness (C.E.)

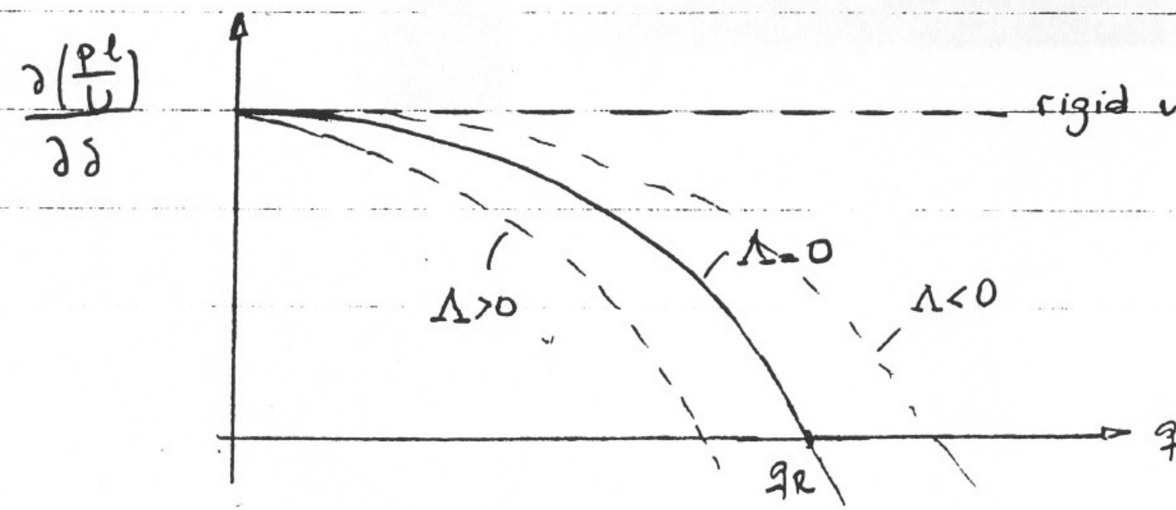
$$C.E. \equiv \frac{\partial\left(\frac{p l}{U}\right)}{\partial \delta}$$ ... change in roll rate due to δ (for $\dot{p} = 0$)

$$\Rightarrow \quad C.E. = \frac{C_1}{B_1}$$

Control Reversal

Loss of C.E. happens when $C_l = 0$. The dynamic pressure corresponding to it can be obtaining from that equation as:

$$q_R = \frac{-\lfloor H \rfloor \{ c\, c_{\ell_\delta} \}}{\lfloor H \rfloor \left([A]^{-1} - q[E] \right)^{-1} \left([E]\{ c\, c_{\ell_\delta} \} + [F]\{ \underset{0}{\hat{c m_{ac_\delta}}} \} \right)}$$



Physical explanation, see:

B.A.H. pp. 507-508