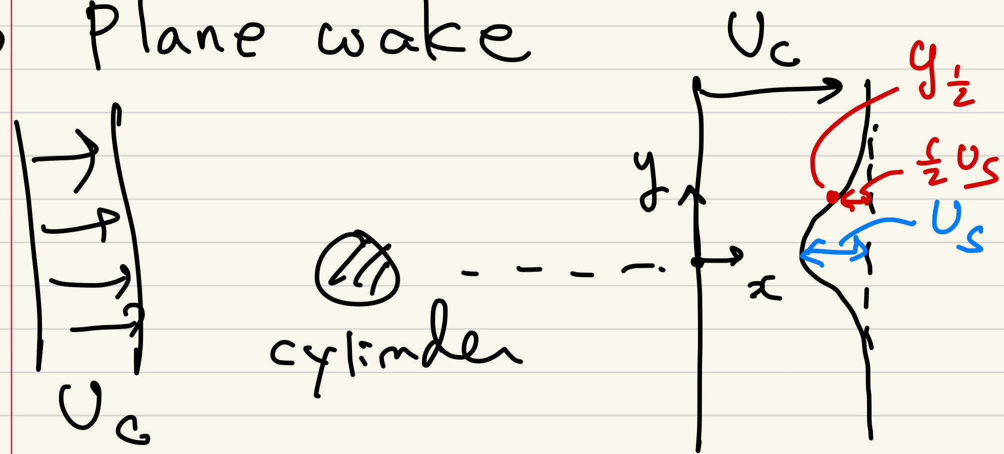
⊙ Plane wake



U_c : char. conv. vel

centerline

$U_s = U_c - \langle U(x,0,0)\rangle$: char. vel. diff.

$y_{\frac{1}{2}}$: $\langle U(x, \pm y_{\frac{1}{2}}, 0)\rangle = U_c - \frac{1}{2}U_s(x)$

$$\xi = y / y_{\frac{1}{2}}$$

$$f(\xi) = \frac{U_c - \langle U(x,y,0)\rangle}{U_s} \quad : \text{self-similar vel. defect}$$

$$\rightarrow f(0) = 1, \quad f(\pm 1) = \frac{1}{2}$$

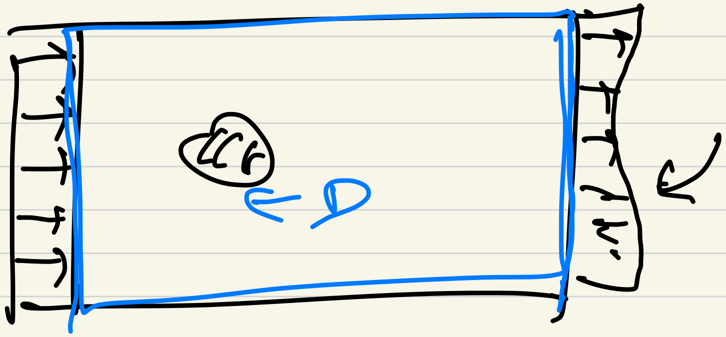
Moment-deficit flow rate

$$\dot{M}(x) = \int_{-\infty}^{\infty} \rho \langle U\rangle (U_c - \langle U\rangle)\, dy$$

ft. of x decays

$$= \rho U_c \underline{U_s(x)\, y_{\frac{1}{2}}(x)} \underbrace{\int_{-\infty}^{\infty} \left(1 - \boxed{\frac{U_s}{U_c}} f(\xi)\right) f(\xi)\, d\xi}$$

not exactly self-similar
but asymptotically self-similar



mtm theorem

$\Downarrow$

$\dot{M}(x) =$ drag force $=$ const

$\therefore$ In the far wake ($U_s/U_c \to 0$),

$U_s(x)\, y_{\frac{1}{2}}(x)$ is indep. of x.

- boundary layer eq. + self similarity

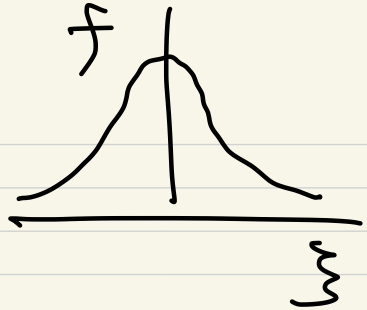
$\rightarrow \underbrace{S}_{\text{ft. } x}\underbrace{(\xi f)'}_{\text{ft. } \xi} = \underbrace{-g'}_{\text{ft. } \xi}$ where $S = \dfrac{U_c}{U_s}\dfrac{dy_{\frac{1}{2}}}{dx}$

$g(\xi) = \dfrac{\langle uv \rangle}{U_s^2}$

$\hookrightarrow$ S should be const.

$U_s \sim x^{-\frac{1}{2}}$, $y_{\frac{1}{2}} \sim x^{\frac{1}{2}}$

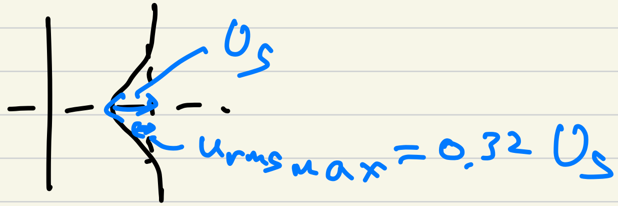
$f(\xi) = \exp(-\alpha \xi^2) \qquad (\alpha = \ln 2 \simeq 0.693)$



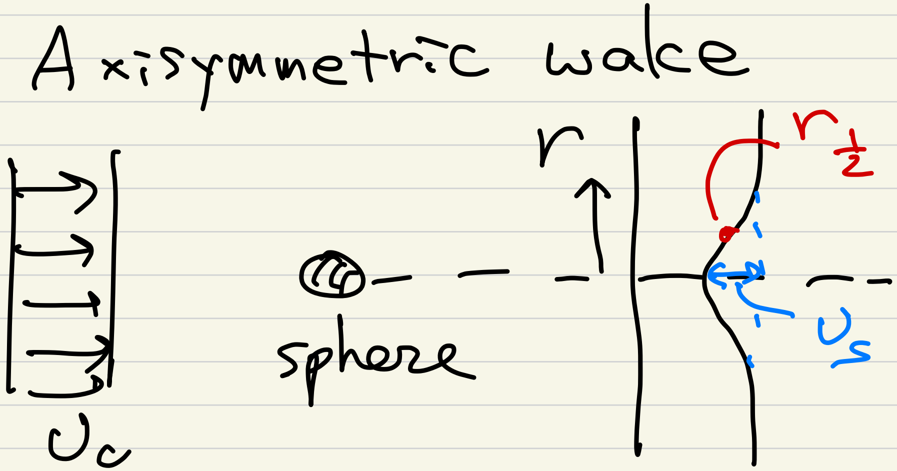
$$Re_0 = \frac{U_s y_{\frac{1}{2}}}{\nu} = \text{const}$$

$$Re_T = \frac{U_s y_{\frac{1}{2}}}{\nu_T} = \frac{1}{\hat{\nu}_T} = \frac{2\ln 2}{S} \simeq 16.7 \qquad (S_{cylinder} = 0.083)$$

$$\frac{\langle u^2 \rangle^{\frac{1}{2}}_{max}}{U_s} \simeq 0.32$$



⊙ Axisymmetric wake



Similar approach as in plane wake

$$\rightarrow S = \frac{U_c}{U_s}\frac{dr_{\frac{1}{2}}}{dx} = \text{const}.$$

mtm deficit flux $= D \sim \rho U_c U_s r_{\frac{1}{2}}^2 = \text{const}$

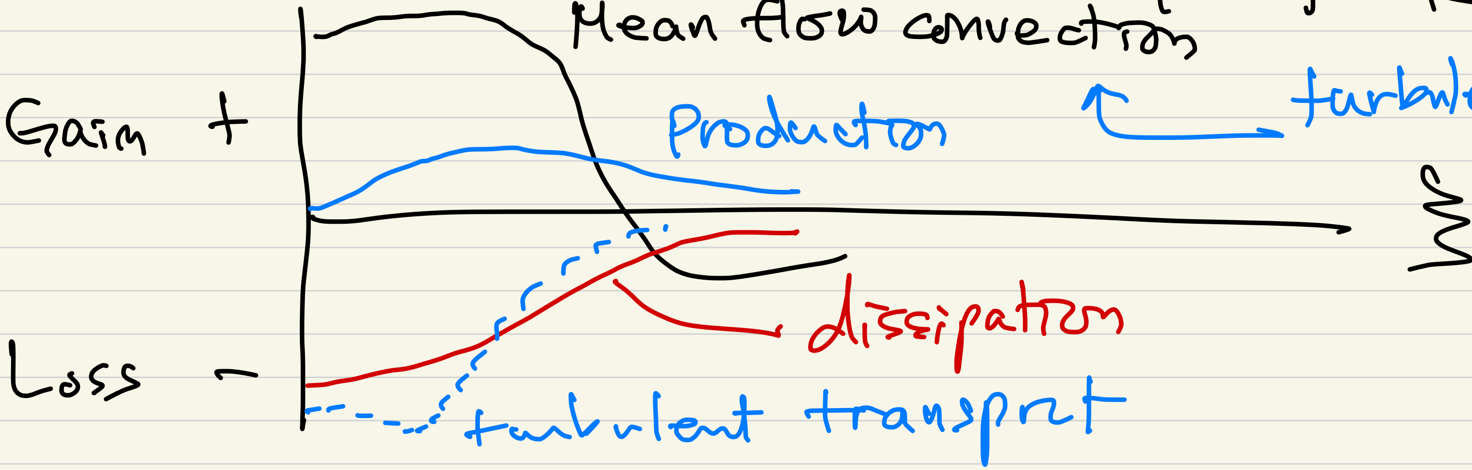
$$\Rightarrow \quad U_s \sim x^{-\frac{2}{3}}, \quad r_{\frac{1}{2}} \sim x^{\frac{1}{3}}$$

$Re_o = \frac{U_s r_{\frac{1}{2}}}{\nu} \sim x^{-\frac{1}{3}}$ decreases! $50 < x/d < 150$

$\rightarrow$ flow can be assumed to relaminarize.

$\langle u^2 \rangle^{\frac{1}{2}}_{max} / U_s \sim 0.9$!

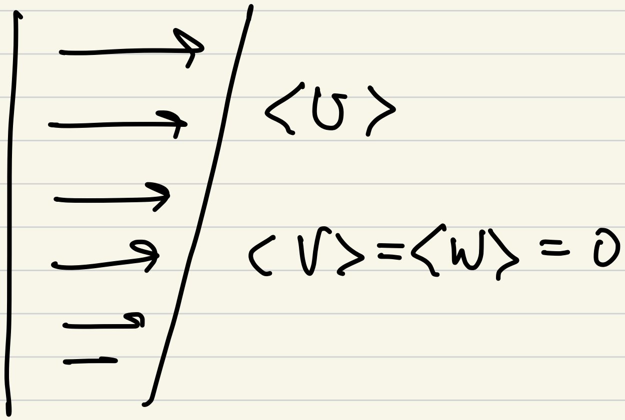
budget for k



turbulence is strongly influenced by upstream conditions.

⊙ Homogeneous shear flows

So far, we have dealt with inhomo. turbulence



"$u_i(\underline{x}, t)$ and $p'(\underline{x}, t)$ are statistically homogeneous"

$\Downarrow$

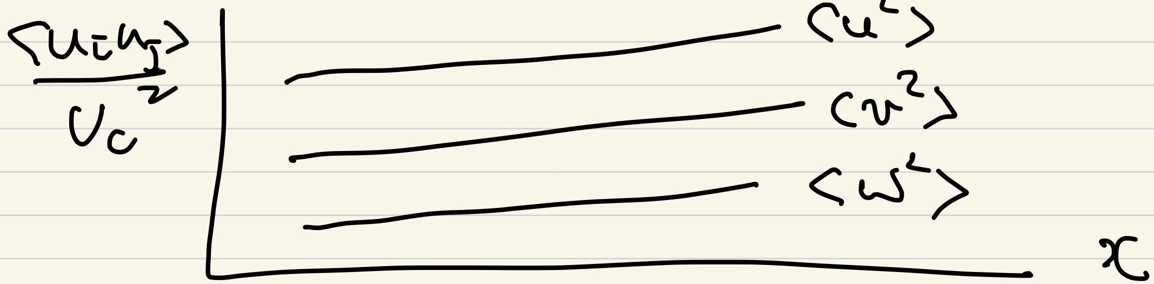
imposed mean vel. grad. $\partial \langle U_i \rangle / \partial x_j$, must be uniform.
(i.e. not ft. of x)

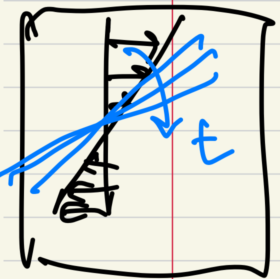
but may vary in time (Prob. 5.41)

Prob. 5.40 $\Rightarrow$

HW3 due by Apr. 30.

In a frame moving with U_c, turb. is approx. homo.





- Direct numerical simulation : Rogallo (1981)
  ↳ no turb. model
  Rogers and Moin (1987)

→ homo. shear flow becomes self-similar after some time.

→ statistics normalized by S $(= \partial \langle U \rangle / \partial y)$ and k become indep. of time.

For evolution of k, $\frac{dk}{dt} = P - \varepsilon$

$$\tau \equiv \frac{k}{\varepsilon} \longrightarrow \frac{\tau}{k}\frac{dk}{dt} = \boxed{\frac{P}{\varepsilon}} - 1$$

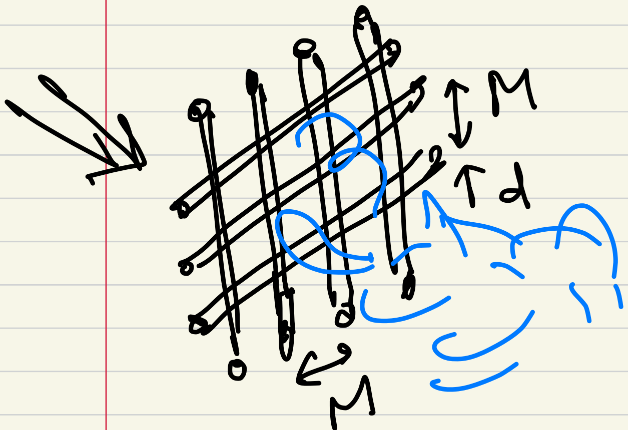
$\simeq 1.7 > 1$

$$P = -\langle u_i u_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j}$$

↳ $k(t) = k(0) \exp\left[\frac{t}{\tau}\underbrace{\left(\frac{P}{\varepsilon} - 1\right)}_{>0}\right]$

→ k grows exponentially in time.

- Grid turbulence



No mean vel. grad

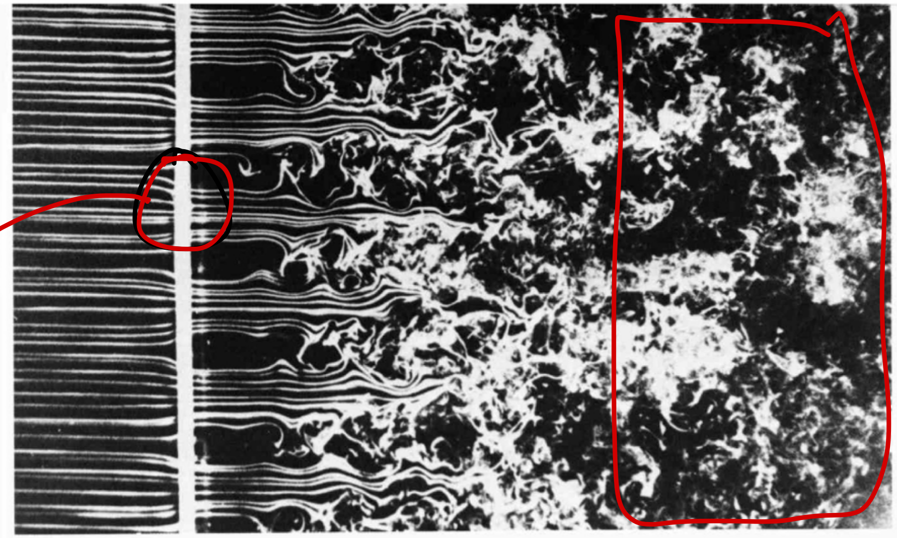
→ $P = 0$, $\varepsilon \neq 0$

→ turbulence decays

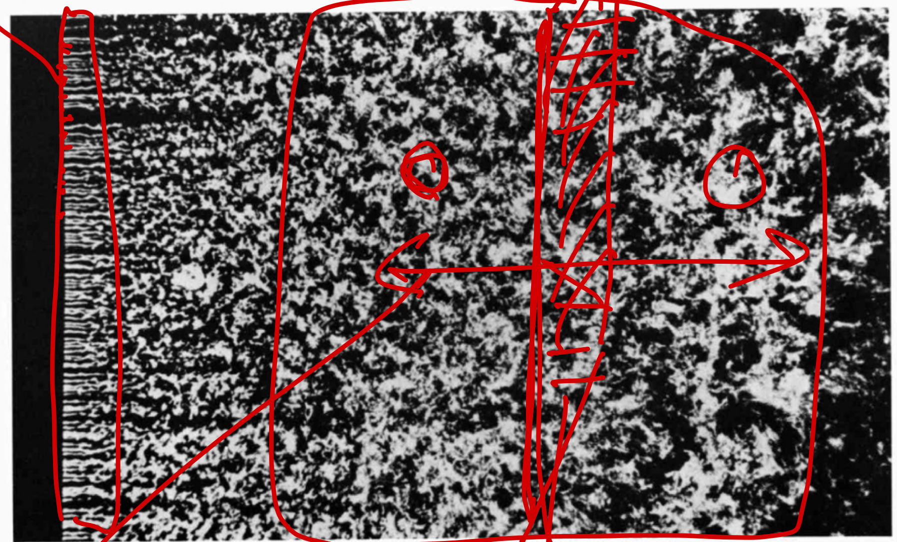
"decaying homo. turbulence

Kang et al. JFM

grid



152. Generation of turbulence by a grid. Smoke wires show a uniform laminar stream passing through a $^1/_{16}$-inch plate with ¾-inch square perforations. The Reynolds number is 1500 based on the 1-inch mesh size. Instability of the shear layers leads to turbulent flow downstream. *Photograph by Thomas Corke and Hassan Nagib*



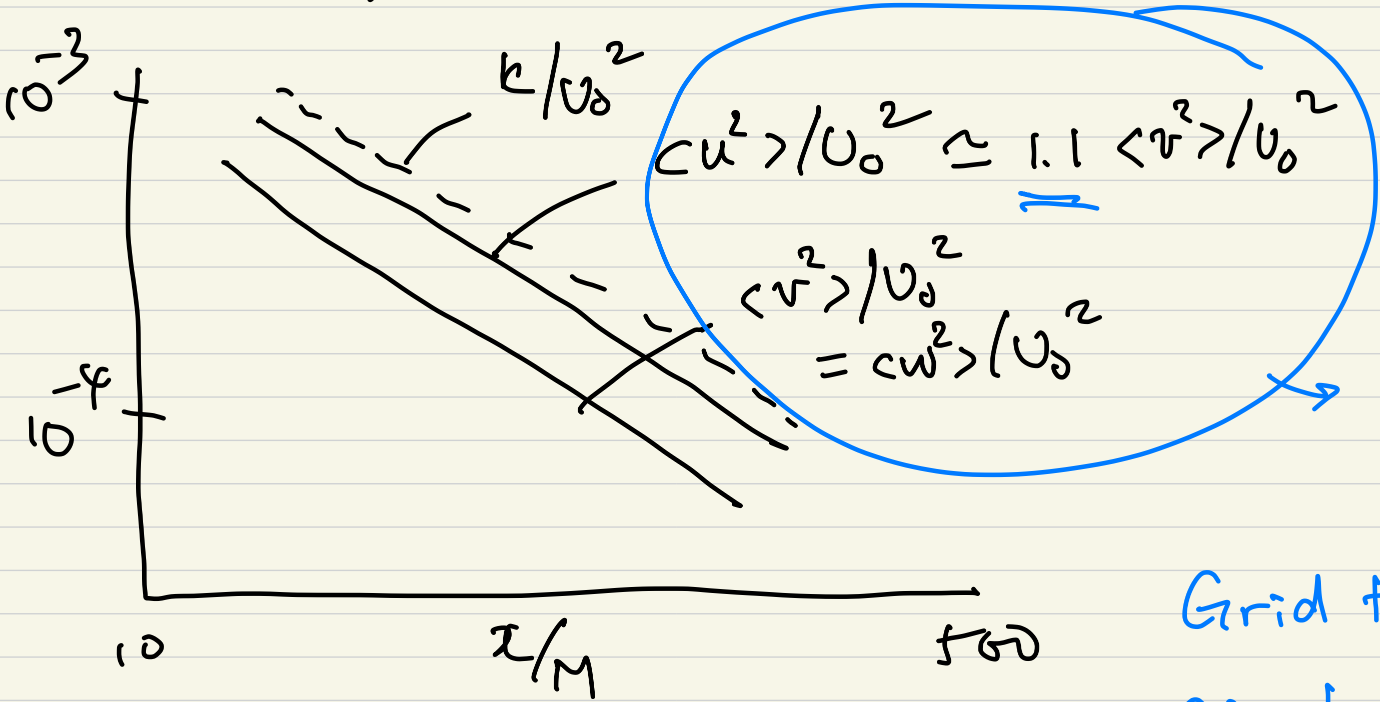
153. Homogeneous turbulence behind a grid. Behind a finer grid than above, the merging unstable wakes quickly form a homogeneous field. As it decays downstream, it provides a useful approximation to the idealization of isotropic turbulence. *Photograph by Thomas Corke and Hassan Nagib*

inhomo.

homo. turb.

In the frame moving with U_0, turb. is homo. and evolves in time ($t = x/U_0$).

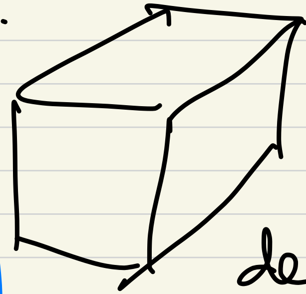
$\rightarrow$ numerical simulation



$\rightarrow$ isotropic

$\Downarrow$

Grid turb. is called as homo. isotropic turb.



periodic in space

$\downarrow$

decays in time

$\langle uv\rangle = \langle uw\rangle = \langle vw\rangle = 0$

all shear stresses are zero.

$$\frac{k}{U_0^2} = A\left(\frac{x - x_0}{M}\right)^{-n}$$

x_0 : virtual origin

n : $1.15 \sim 1.45$

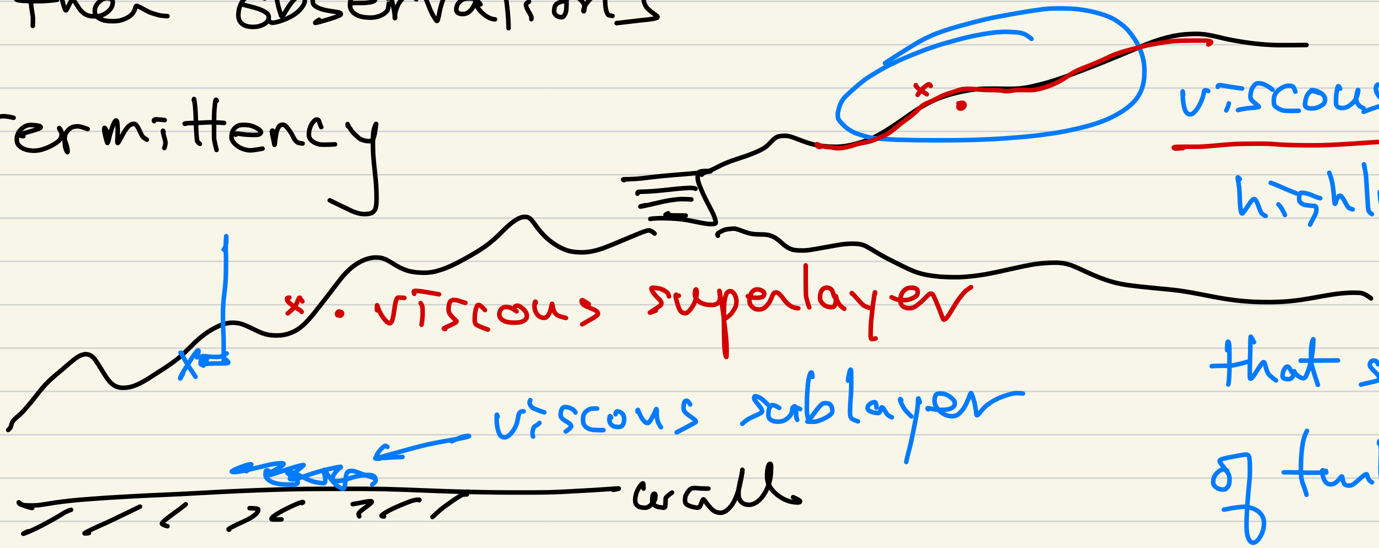
moving frame $\rightarrow k(t) = k_0 (t/t_0)^{-n}$ $\quad (\because P=0)$

$$\frac{dk}{dt} = -\left(\frac{nk_0}{t_0}\right)\left(\frac{t}{t_0}\right)^{-(n+1)} = -\varepsilon$$

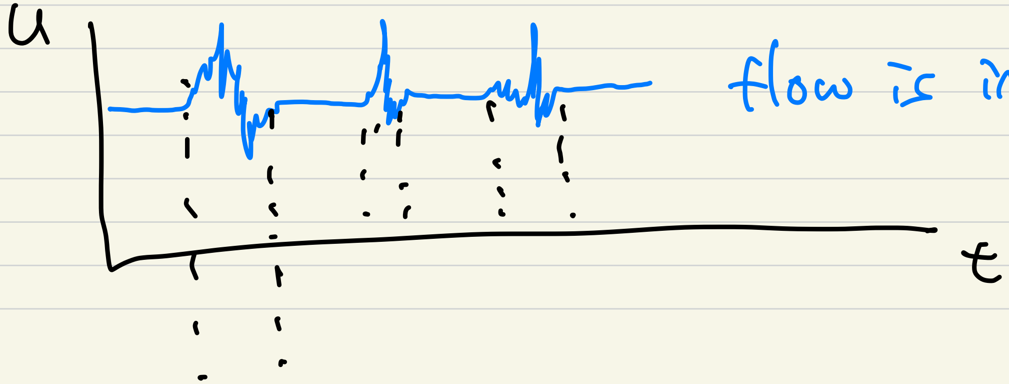
$$\rightarrow \varepsilon = \varepsilon_0 (t/t_0)^{-(n+1)}, \quad \varepsilon_0 = nk_0/t_0$$

5.5 Further observations

· intermittency

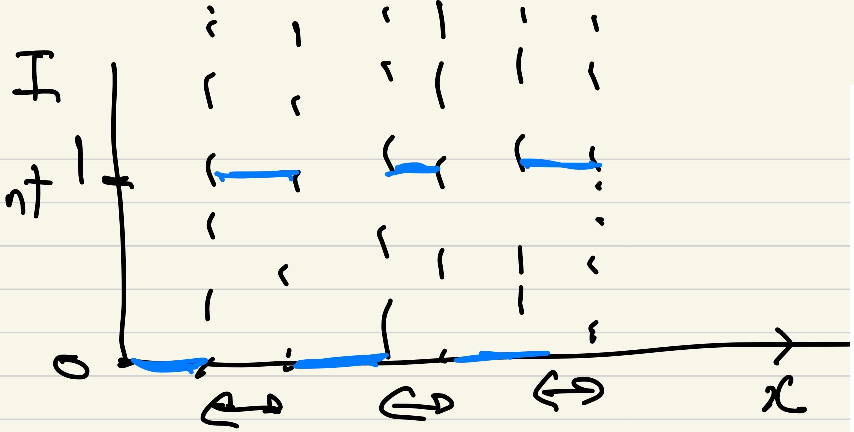


viscous superlayer: highly contorted moving surface that separates regions of turb. & non-turb. flows.



flow is intermittent.

intermittent ff.



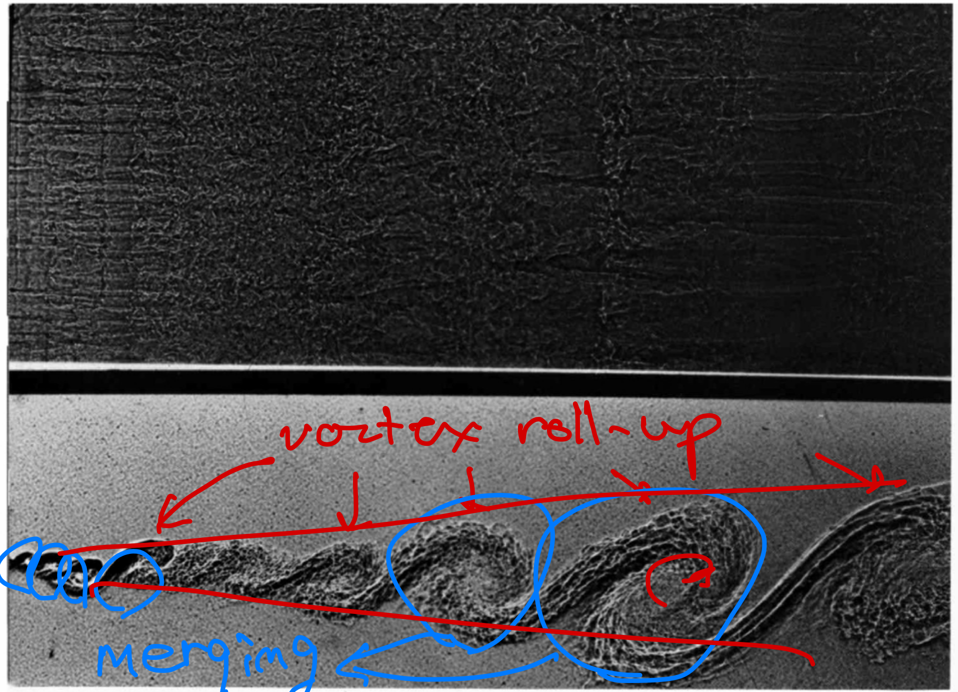
• large-scale turbulent motion

plane mixing layer

vortex roll-up

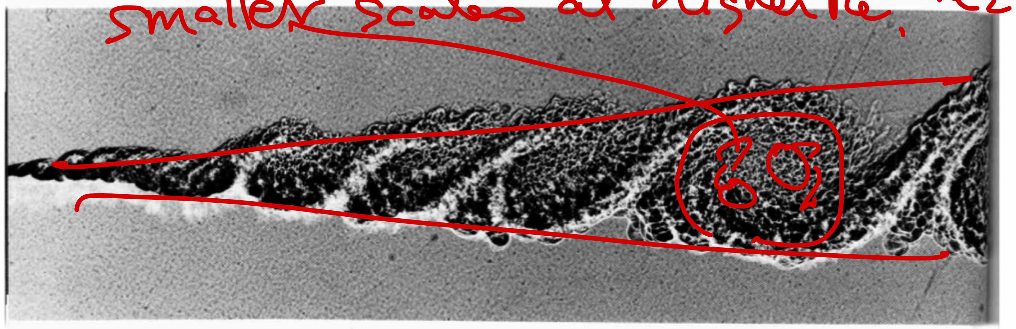
Re_1

merging



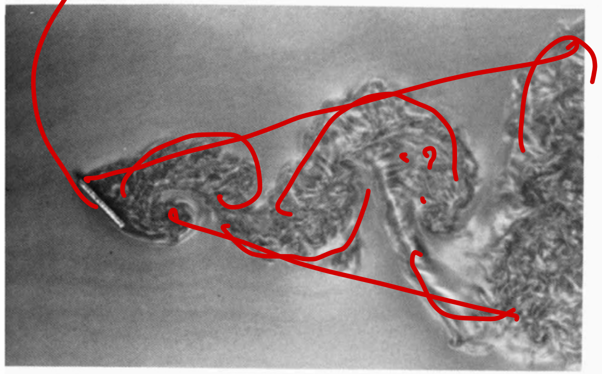
176. Large-scale structure in a turbulent mixing layer. Nitrogen above flowing at 1000 cm/s mixes with a helium-argon mixture below at the same density flowing at 380 cm/s under a pressure of 4 atmospheres. Spark shadow photography shows simultaneous edge and plan views, demonstrating the spanwise organization of the large eddies. The streamwise streaks in the plan view (of which half the span is shown) correspond to a system of secondary vortex pairs oriented in the streamwise direction. Their spacing at the downstream side of the layer is larger than near the beginning. *Photograph by J. H. Konrad, Ph.D. thesis, Calif. Inst. of Tech., 1976.*

smaller scales at higher Re. Re_2



177. Coherent structure at higher Reynolds number. This flow is as above but at twice the pressure. Doubling the Reynolds number has produced more small-scale structure without significantly altering the large-scale structure. *M. R. Rebollo, Ph.D. thesis, Calif. Inst. of Tech., 1976; Brown & Roshko 1974* wake

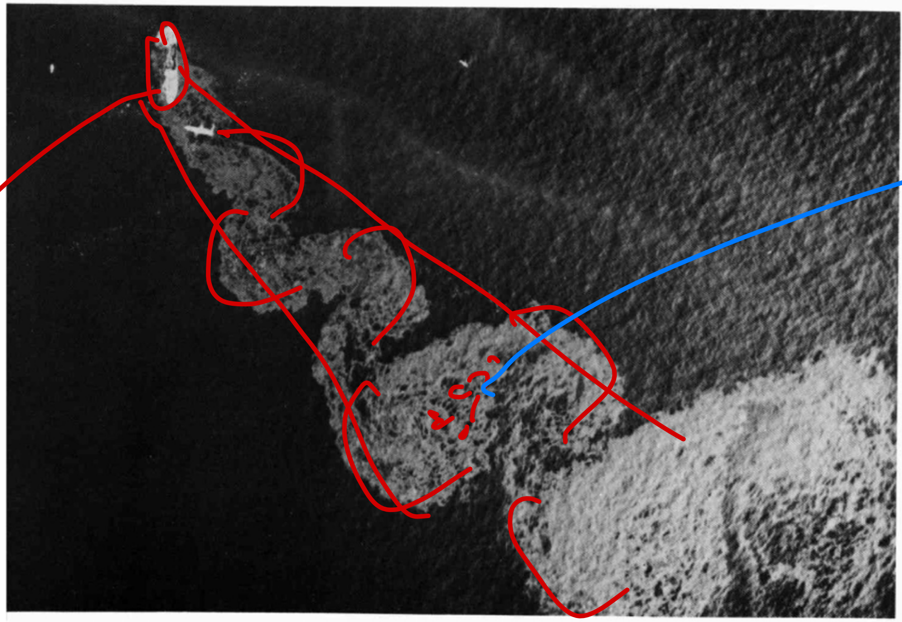
flat plate @ water tunnel



172. Wake of an inclined flat plate. The wake behind a plate at 45° angle of attack is turbulent at a Reynolds number of 4300. Aluminum flakes suspended in water show its characteristic sinuous form. *Cantwell 1981. Reproduced, with permission, from the* Annual Review of Fluid Mechanics, *Volume 13. © 1981 by Annual Reviews Inc.*

large scales are similar

tank ship



173. Wake of a grounded tankship. The tanker *Argo Merchant* went aground on the Nantucket shoals in 1976. Leaking crude oil shows that she happened to be inclined at about 45° to the current. Although the Reynolds number is approximately 10^7, the wake pattern is remarkably similar to that in the photograph at the top of the page. *NASA photograph, courtesy of O. M. Griffin, Naval Research Laboratory.*

small scales are smaller at higher Re.

⇓

turbulent processes are non-local in space & time.