Topics in Ship Structures

10 Fatigue Crack Propagation

Reference :

Fracture Mechanics by T.L. Anderson Ch. 10

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Contents

General

- Most of the material in the preceding chapters dealt with static or monotonic loading of cracked bodies.

- The procedures for analyzing constant amplitude fatigue under small-scale yielding conditions are fairly well established, although a number of uncertainties remain.

- Variable amplitude loading, large-scale plasticity, and short cracks introduce additional complications that are not fully understood.

- This chapter summarizes the fundamental concepts and practical applications of the fracture mechanics approach to fatigue crack propagation

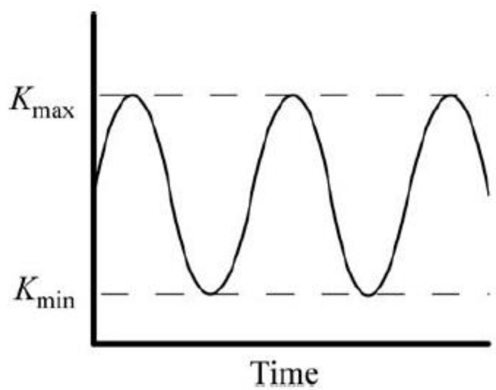
General

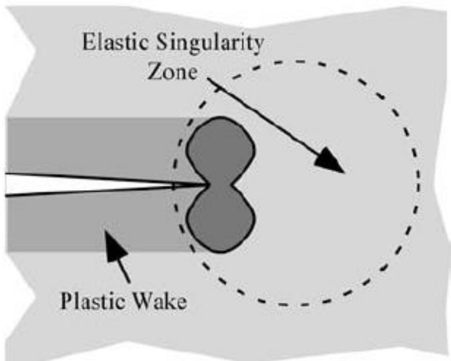
❖ Similitude in Fatigue

- Similitude(유사함) implies that the crack-tip conditions are uniquely defined by a single loading parameter such as the stress-intensity factor.
- In the case of a stationary crack, two configurations will fail at the same critical *K* value, provided an elastic singularity zone exists at the crack tip.
- A cyclic plastic zone forms at the crack tip, and the growing crack leaves behind a plastic wake.
- If the plastic zone is sufficiently small that it is embedded within an elastic singularity zone, the conditions at the crack tip are uniquely defined by the current *K* value.
- Crack growth rate

$$\frac{da}{dN} = f_1(\Delta K, R)$$

$$\Delta K = (K_{max} - K_{min})$$
$$R = K_{min}/K_{max}$$
$$da/dN = \text{crack growth per cycle}$$





Constant amplitude fatigue crack growth under small-scale yielding conditions

General

❖Loading History

- A number of expressions for f_1 have been proposed, most of which are empirical.
- The number of cycles required to propagate a crack from an initial length a_o to a final length a_f is given by

$$N = \int_{a_o}^{a_f} \frac{da}{f_1(\Delta K, R)} \qquad \frac{da}{dN} = f_2(\Delta K, R, \mathcal{H})$$

- If K_{max} or K_{min} varies during cyclic loading, the crack growth in a given cycle may depend on the loading history as well as the current values of K_{min} and K_{max} :
- $\mathcal{H}$ indicates the history dependence $\rightarrow$ violates the similitude assumption; two configurations cyclically loaded at the same ΔK and R will not exhibit the same crack growth rate unless both configurations are subject to the same prior history.

General

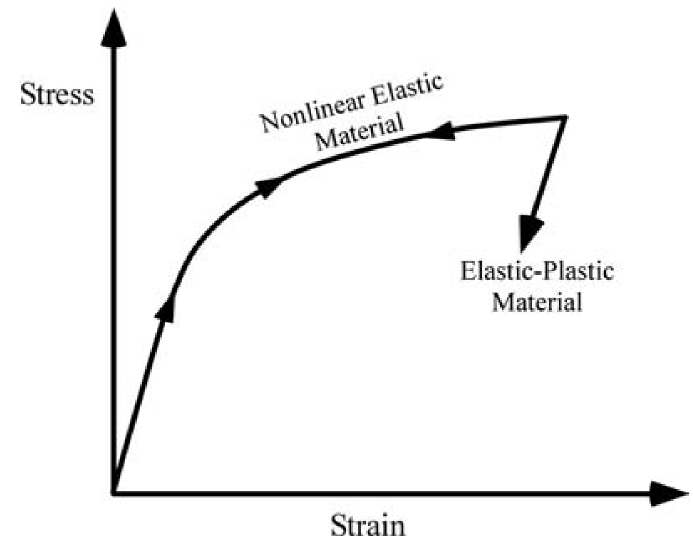
❖Excessive Plasticity

- Excessive plasticity during fatigue can violate similitude, since *K* no longer characterizes the crack-tip conditions in such cases.
- A number of researchers have applied the *J* integral to fatigue accompanied by large-scale yielding; they have assumed a growth law of the form.

$$\frac{da}{dN} = f_3(\Delta J, R)$$

where ΔJ is a contour integral for cyclic loading,

- It is valid in the case of constant amplitude fatigue in small-scale yielding because of the relationship between *J* and *K* under linear elastic conditions.
- When unloading occurs in an elastic-plastic material, deformation plasticity theory no longer models the actual material response.
- The validity has not been proven but this approach appears to be useful for many engineering problems with certain assumptions about the loading and unloading branches.



Schematic comparison of the stress strain behavior of elastic-plastic and nonlinear elastic materials (Ch.4).

Typical fatigue crack growth behavior in metals

❖ Crack Growth Curve

- **Region I** : At the low end, da/dN approaches zero at a threshold ΔK, below which the crack will not grow.

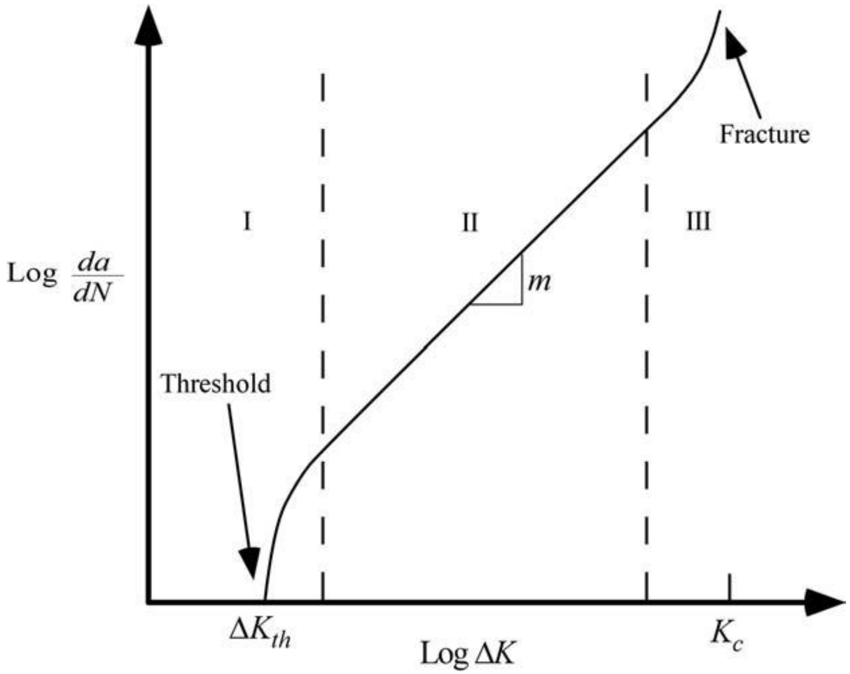
- **Region II** : At intermediate ΔK values, the curve is linear, but the crack growth rate deviates from the linear trend at high and low ΔK levels.

$$\frac{da}{dN} = C\Delta K^{m}$$

- **Region III :**

✓ Hypothesis I : the crack growth rate accelerates as $K_{\max}$ approaches K_c, the fracture toughness of the material

✓ Hypothesis II : influence of crack-tip plasticity on the true driving force for fatigue, ΔJ might be more appropriate.



Typical fatigue crack growth behavior in metals

Typical fatigue crack growth behavior in metals

❖ Paris Law

$$\frac{da}{dN} = C\Delta K^m \qquad m : 2\sim4$$

❖Forman's relationship for Region II and Region III [8]:

$$\frac{da}{dN} = \frac{C\Delta K^m}{(1-R)K_c - \Delta K} \qquad \frac{da}{dN} = \frac{C\Delta K^{m-1}}{\frac{K_c}{K_{\max}} - 1}$$

✓ Crack growth rate becomes infinite as K_{max} approaches K_c

❖Klesnil and Lukas' [10] modification to account for the threshold

$$\frac{da}{dN} = C\left(\Delta K^m - \Delta K_{th}^m\right)$$

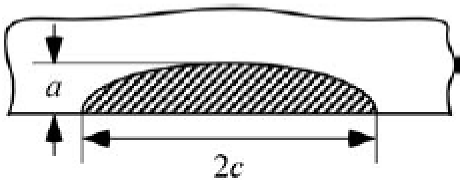
❖NASA : describe fatigue crack growth in all three regions.

$$\frac{da}{dN} = C\Delta K^m \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1 - \frac{K_{\max}}{K_c}\right)^q}$$

Typical fatigue crack growth behavior in metals

❖EX. 10.1) Derive an expression for the number of stress cycles required to grow a semicircular surface crack from an initial radius a_o to a final size a_f assuming the Paris-Erdogan equation describes the growth rate. a_f << plate dimension.

- Sol) The stress-intensity amplitude for a semicircular surface crack in an infinite plate



$$\Delta K \approx \frac{1.04}{\sqrt{2.464}} \Delta\sigma \sqrt{\pi a} = 0.663 \Delta\sigma \sqrt{\pi a}$$

$$K_I = \lambda_s \sigma \sqrt{\frac{\pi a}{Q}} f(\phi)$$

$$\frac{da}{dN} = C(0.663\Delta\sigma)^m (\pi a)^{m/2}$$

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65}$$

$$N = \frac{1}{C(0.663\Delta\sigma\sqrt{\pi})^m} \int_{a_o}^{a_f} a^{-m/2} da$$

$$\lambda_s = \left[1.13 - 0.09\left(\frac{a}{c}\right)\right]\left[1 + 0.1(1-\sin\phi)^2\right]$$

$$f(\phi) = \left[\sin^2(\phi) + \left(\frac{a}{c}\right)^2 \cos^2(\phi)\right]^{1/4}$$

$$= \frac{a_o^{1-m/2} - a_f^{1-m/2}}{C\left(\frac{m}{2}-1\right)(0.663\Delta\sigma\sqrt{\pi})^m} \quad (\text{for } m \neq 2)$$

"Typo in textbook"

General

- At high loads, the compliance ($d\Delta/dP$) agreed with standard formulas, but at low loads, the compliance was close to that of an uncracked specimen due to the contact between crack surfaces (i.e., crack closure) at loads that were low but greater than zero.
- An effective stress-intensity range

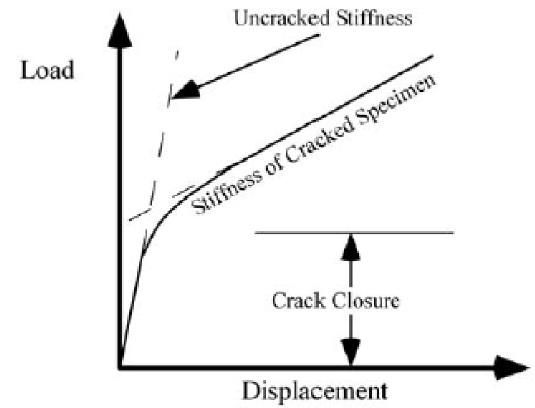
$$\Delta K_{eff} \equiv K_{\max} - K_{op}$$

- An effective stress-intensity ratio:

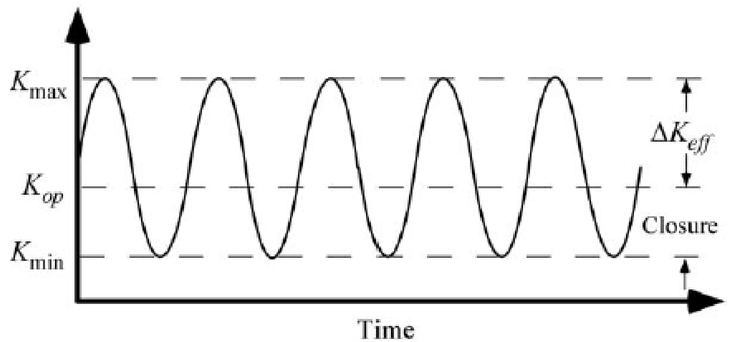
$$U \equiv \frac{\Delta K_{eff}}{\Delta K} = \frac{K_{\max} - K_{op}}{K_{\max} - K_{\min}}$$

- A modified Paris-Erdogan equation

$$\frac{da}{dN} = C\Delta K_{eff}^{m}$$



(a)



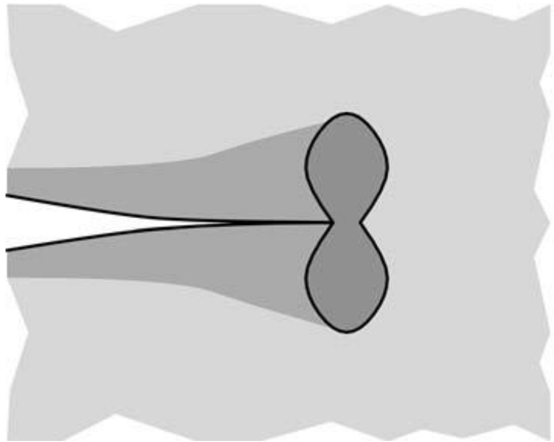
(b)

Crack closure during fatigue crack growth. The crack faces contact at a positive load (a) resulting in a reduced driving force for fatigue, ΔK_{eff} (*b*):

Plasticity-induced closure

❖ Plasticity-induced closure

- Residual stresses in the plastic wake applies a closure force to the crack faces.
- Residual stretch in the plastic wake causes the crack faces to close at a positive remote stress .
- The plastic wake behind the crack causes closure and a reduced crack growth rate.
- The crack growth rate can accelerate because the zone of compressive residual stresses in the plastic wake was removed by machining.



plasticity-induced closure

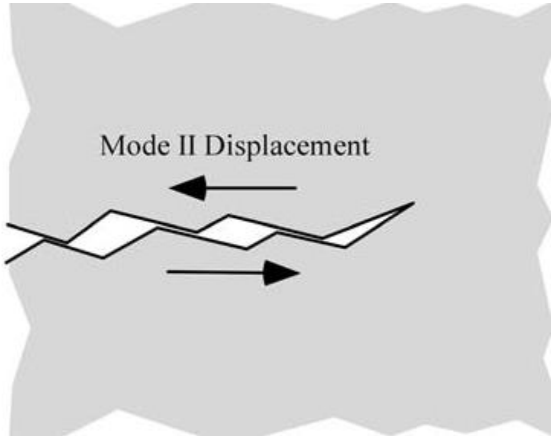
Other closures

❖ Roughness-induced closure

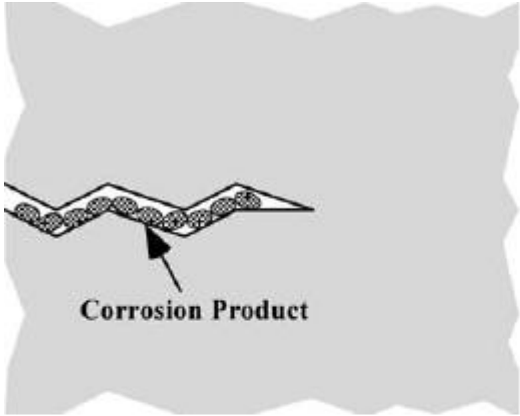
- Although fatigue cracks propagate in pure Mode I conditions, crack deflections due to microstructural heterogeneity(불균질) can lead to mixed mode conditions on the microscopic level.
- These displacements cause mismatch between upper and lower crack faces, which in turn results in contact of crack faces at a positive load.

❖ Oxide-induced closure & closures induced by Oxide or a viscous fluid,

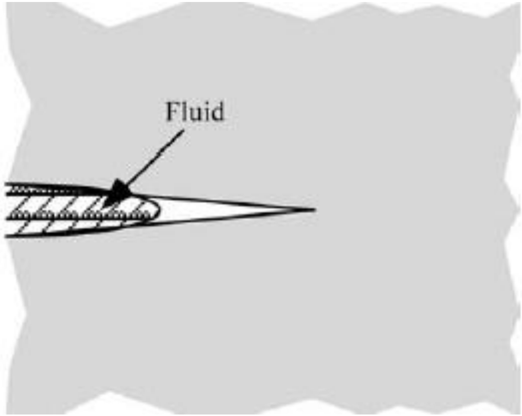
- Oxide debris or other corrosion products become wedged between crack faces. Crack closure can also be introduced by a viscous fluid.



roughness-induced closure



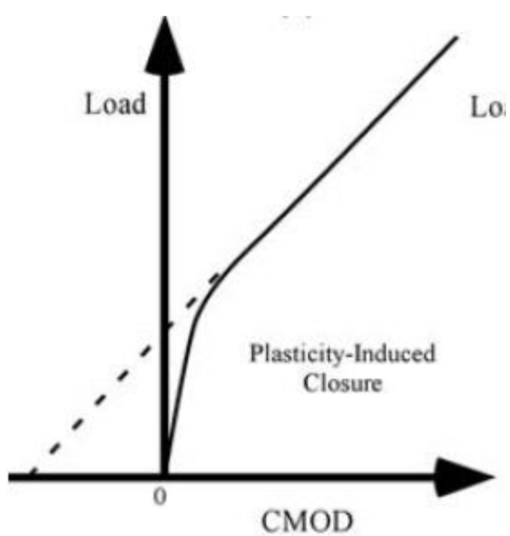
oxide-induced closure



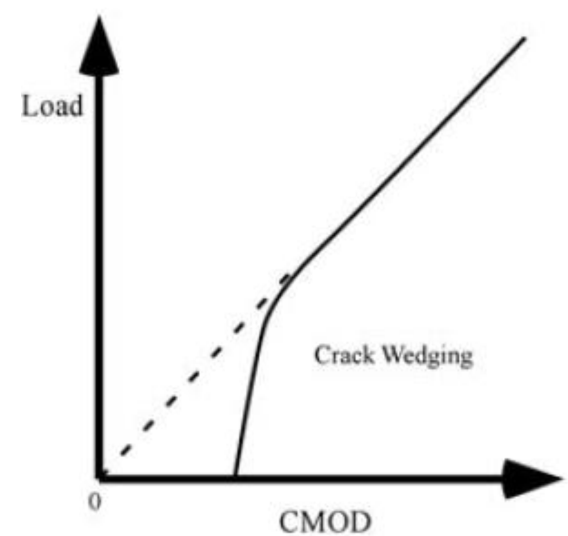
closure induced by a viscous fluid

Crack wedging

- The crack is prevented from closing completely by an obstruction of some type.
- Corrosion product and viscous fluid involve crack wedging.
- Load is plotted vs. crackmouth-opening displacement (CMOD).
- Plasticity-induced closure : the crack is actually closing, the load-CMOD curve should pass through the origin.
- Wedging mechanism, : a residual displacement at zero load, but the load-CMOD curve at high loads should extrapolate to the origin.



Plasticity-induced closure

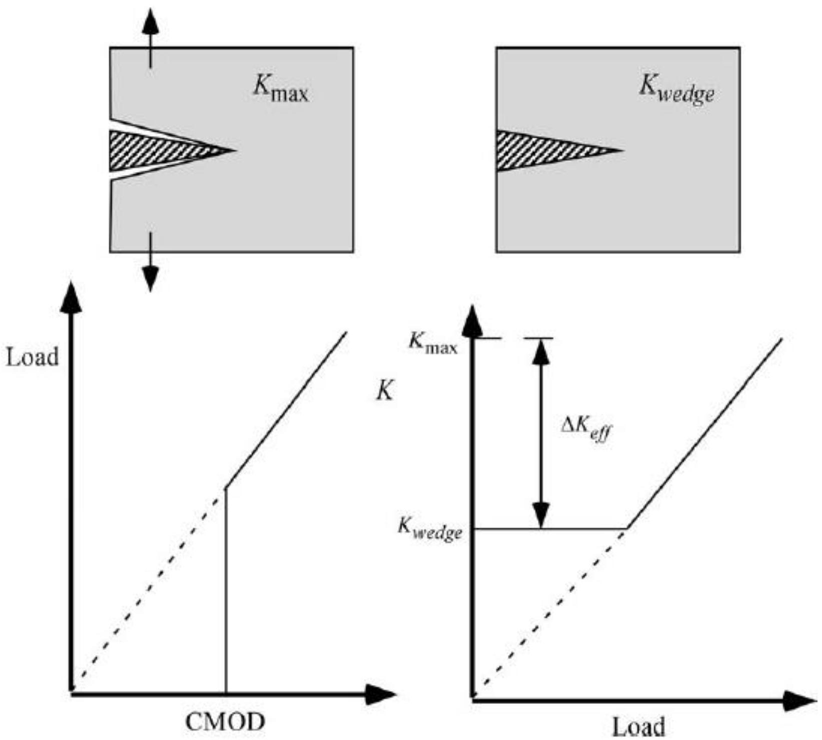


Wedging mechanisms

A Closer Look at Crack-Wedging Mechanisms

❖An idealized scenario of rigid wedge inserted into an open crack

- An applied stress intensity of K_{wedge} exists at the crack tip despite the lack of an externally applied load.



Load-displacement behavior and ΔK_{eff} for an ideal wedge, which is rigid and conforms perfectly to the crack-opening profile.

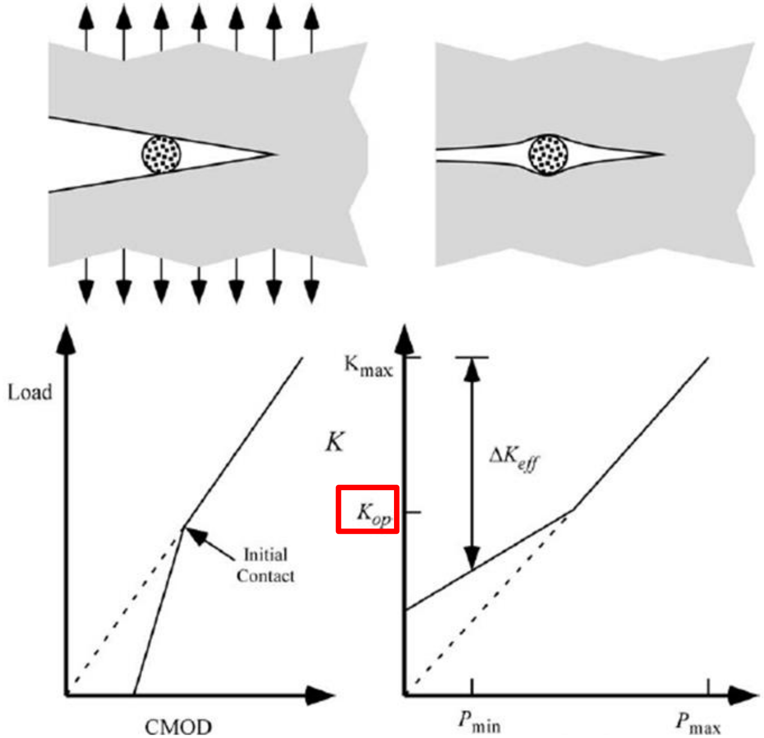
- When load is reapplied, K does not change until load is a sufficient cause for a crack-opening displacement greater than the width of the wedge.
- For cyclic load $\Delta K_{eff} = K_{max} - K_{wedge}$.

A Closer Look at Crack-Wedging Mechanisms

❖A single rigid particle wedge inserted into an open crack

- When this initial contact occurs, the slope of the load-CMOD curve becomes steeper. As load is reduced further, portions of the crack face not in contact with the particle continue to close, resulting in a continual decrease in the applied *K.*

- For cyclic load $\Delta K_{eff} > K_{max} - K_{op}$ (at the point where the compliance changes).
  $\Rightarrow$ closure effects would be overestimated (fatigue driving force is underestimated).

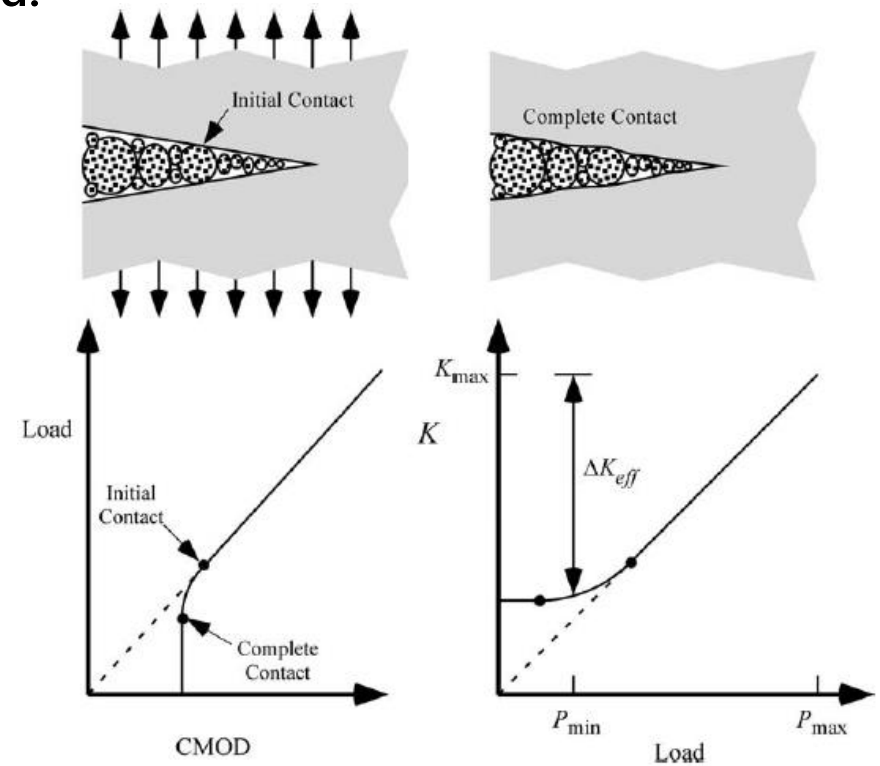


Load-displacement behavior and ΔK_{eff} for a single particle.

A Closer Look at Crack-Wedging Mechanisms

❖the crack is filled with particles of various sizes

- As the load is relaxed, the slope of the load-CMOD curve gradually changes as more particles make contact with the crack.
- Eventually, no further contact occurs and the CMOD attains a constant value, assuming the particles are rigid.
- The measurement of K_{op} has been highly subjective, and different individuals and laboratories have obtained widely varying results for a given material under the same test conditions.
- The new definition, K_{op}, based on the relationship between the applied load and the true applied K removes the subjectivity and provides a more accurate indication.



Load-displacement behavior and ΔK_{eff} for a crack that is filled with multiple particles or asperities (도톨도톨함) of various sizes.

Effects of Loading Variables on Closure

- A number of investigators have attempted to correlate K_{op} to loading variables such as R ratio, but with limited success. ⇐ closure measurements, different closure behavior occurs in different alloys and in different loading regimes for a given alloy.
- Elber [15] measured the closure stress intensity in 2023-T3 aluminum at various load levels and R ratios.

$$U = 0.5 + 0.4R \qquad (-0.1 \le R \le 0.7)$$

$$K_{op} = \Delta K\left(\frac{1}{1-R} - 0.5 - 0.4R\right)$$

$$\boxed{U \equiv \frac{\Delta K_{eff}}{\Delta K} = \frac{K_{\max} - K_{op}}{K_{\max} - K_{\min}}}$$

- Hudak and Davidson performed closure measurements on a 7091 aluminum alloy and 304 stainless steel over a wide range of loading variables. K_o is a material constant.

$$U = 1 - \frac{K_o}{K_{\max}} = 1 - \frac{K_o(1-R)}{\Delta K}$$

$$\Delta K = K_{\max} - K_{\min} = K_{\max} - K_{\max} R$$

$$\Rightarrow K_{\max} = \frac{\Delta K}{1-R}$$

$$\boxed{U \equiv \frac{\Delta K_{eff}}{\Delta K} = \frac{K_{\max} - K_{op}}{K_{\max} - K_{\min}}}$$

$$K_{op} = K_o(1-R) + K_{\max} R$$

$$= K_o(1-R) + \frac{\Delta K R}{1-R}$$

General

- The fatigue threshold ΔK_{th} is the point below which a fatigue crack will not grow.
- Experimental measurements of the threshold are usually inferred from a load-shedding procedure, where ΔK is gradually reduced until the crack growth rate reaches a very small value.
- Most experts believe that the threshold consists of two components: an intrinsic threshold that is a material property, and an extrinsic(외적인) component that is a function of loading variables such as the R ratio.

The Closure Model for The Threshold

$$U \equiv \frac{\Delta K_{eff}}{\Delta K} = \frac{K_{\max} - K_{op}}{K_{\max} - K_{\min}}$$

- Let us assume that a given material has an intrinsic threshold $\Delta K_{th}{}^*$ and that K_{op} is also a material constant that is independent of the R ratio.
- The relationship between the apparent threshold ΔK_{th} and the intrinsic threshold $\Delta K_{th}{}^*$ is given by

$$K_{\max} - K_{\min} = \Delta K_{th},\ K_{\max} - RK_{\max} = \Delta K_{th},\ K_{\max} = \frac{\Delta K_{th}}{1-R}$$

$$U = \frac{K_{\max} - K_{op}}{K_{\max} - K_{\min}} = \frac{\dfrac{\Delta K_{th}}{1-R} - K_{op}}{\Delta K_{th}} = \frac{1}{1-R} - \frac{K_{op}}{\Delta K_{th}}$$

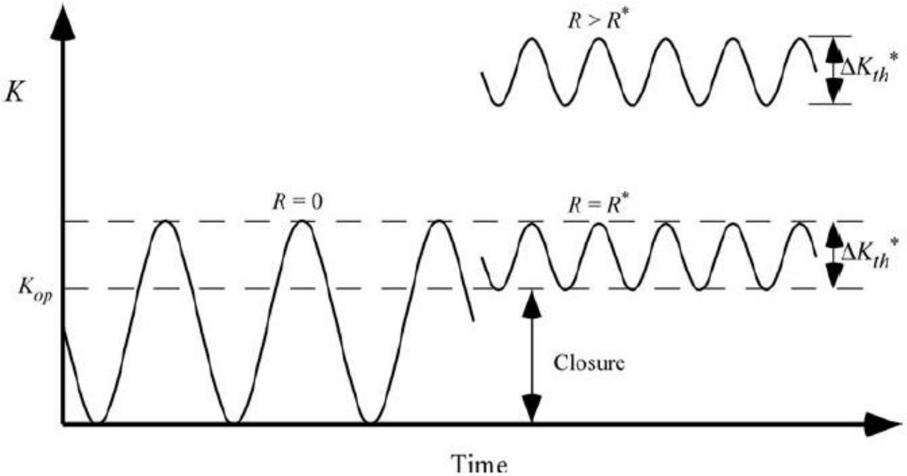
$$U\Delta K_{th} = \Delta K_{th}^*$$

- Rewriting U in terms of ΔK_{th} and R gives

$$U = \min\left[\left(\frac{1}{1-R} - \frac{K_{op}}{\Delta K_{th}}\right), 1\right]$$

$$\Delta K_{th} = \begin{cases} \left(K_{op} + \Delta K_{th}^*\right)(1-R), & R \le R^* \\ \Delta K_{th}^*, & R > R^* \end{cases}$$

$$R^* = 1 - \frac{\Delta K_{th}^*}{K_{op} + \Delta K_{th}^*} \leftarrow \boxed{\frac{1}{1-R^*} - \frac{K_{op}}{\Delta K_{th}} = 1}$$

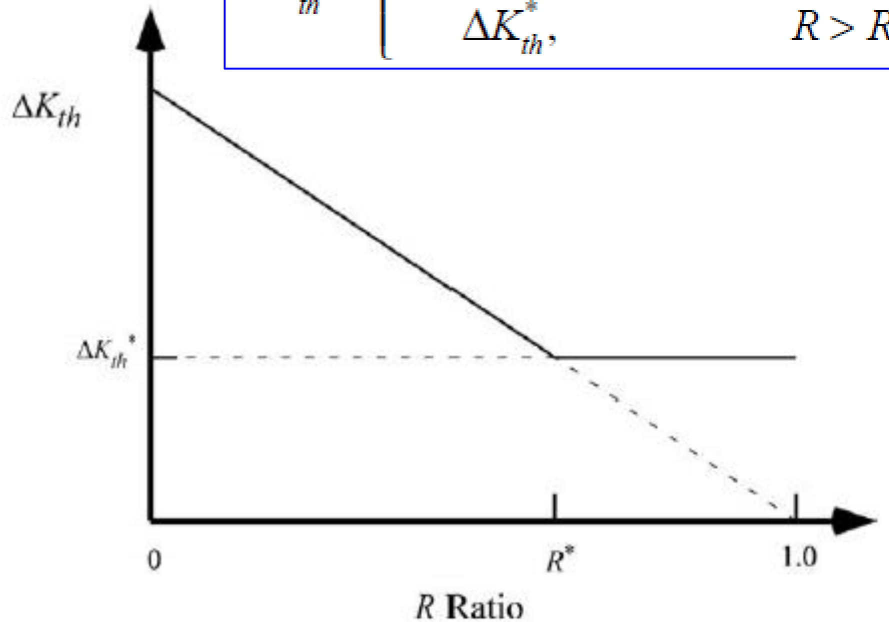


Schematic illustration of the relationship between closure behavior and the R ratio

The Closure Model for The Threshold

- This expression predicts that the threshold stress intensity range varies linearly with *R* below *R** and is constant at higher *R* ratios.

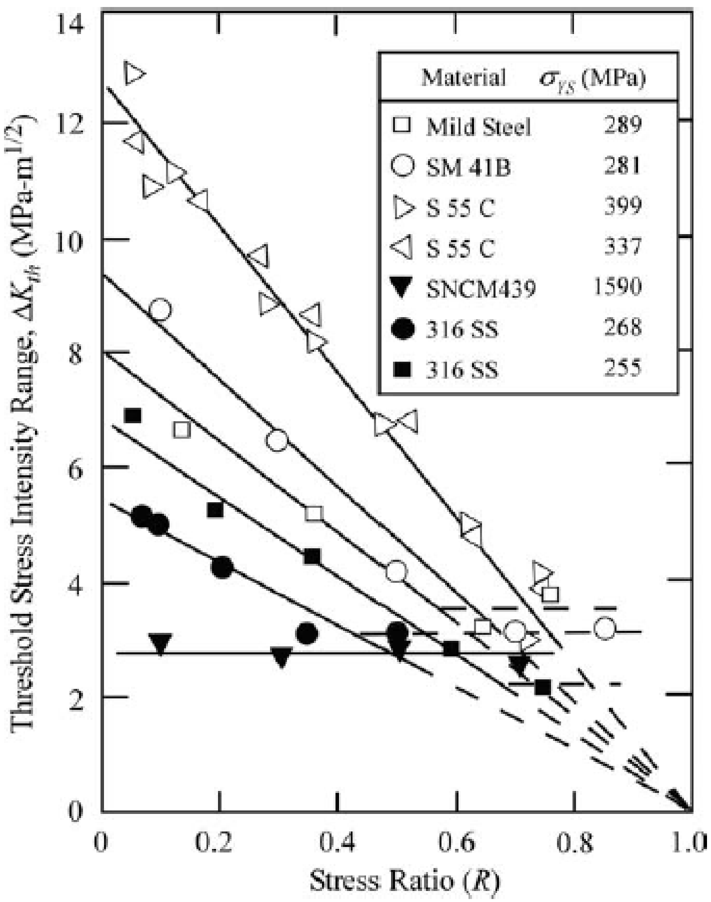
$$\Delta K_{th} = \begin{cases} \left(K_{op} + \Delta K_{th}^{*}\right)(1-R), & R \le R^{*} \\ \Delta K_{th}^{*}, & R > R^{*} \end{cases}$$



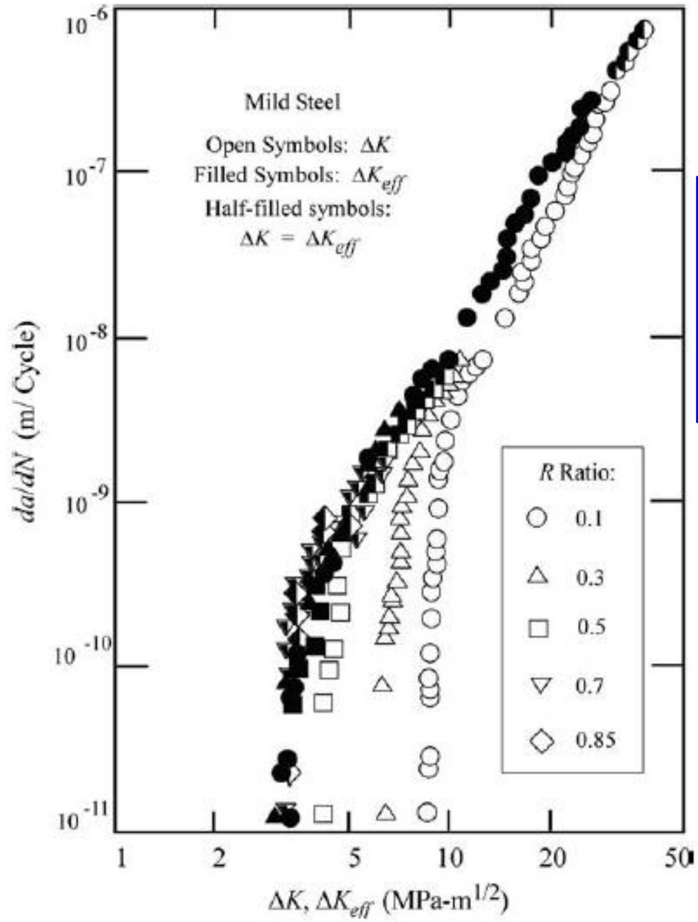
Schematic illustration of the relationship between closure behavior and the *R* ratio on ΔK_{th}

The Closure Model for The Threshold

- For the highest R ratios, closure was not observed, so $\Delta K = \Delta K_{eff}$.
- When data at lower R ratios are corrected for closure, the R ratio effect disappears and all data exhibit the same threshold, which corresponds to ΔK_{th}^* for the material.



Effect of R ratio on the threshold stress-intensity range for various steels



Fatigue crack growth data near the threshold for mild steel at various R ratios

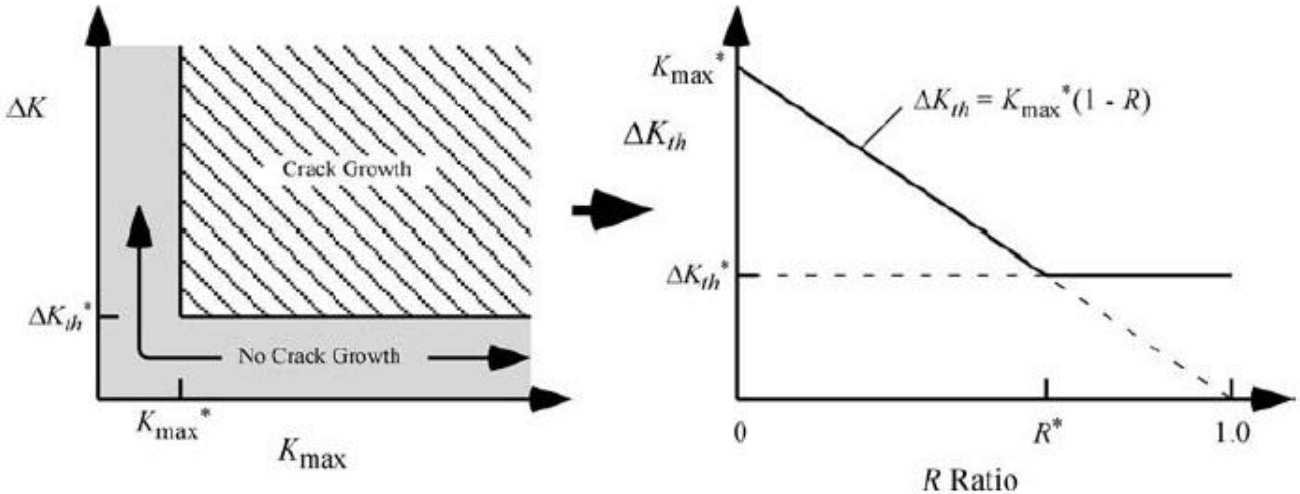
$$\Delta K_{eff} = K_{\max} - K_{op},$$
$$K_{op} \approx 0.6 K^*_{\max},$$
$$K^*_{\max} = K_{op,mat} + \Delta K^*_{th}$$
$$K_{op,mat} : \text{material property}$$

A Two-Criterion Model

- The two thresholds on cyclic stress intensity ΔK_{th}^{*} and maximum stress intensity K_{max}^{*} (or named K_{max} threshold) form an *L*-shaped curve. Both thresholds must be exceeded for crack growth to occur, according to this model.

$$\Delta K_{th} = \begin{cases} K_{\max}^{*}(1-R), & R \le R^{*} \\ \Delta K_{th}^{*} & R > R^{*} \end{cases}$$

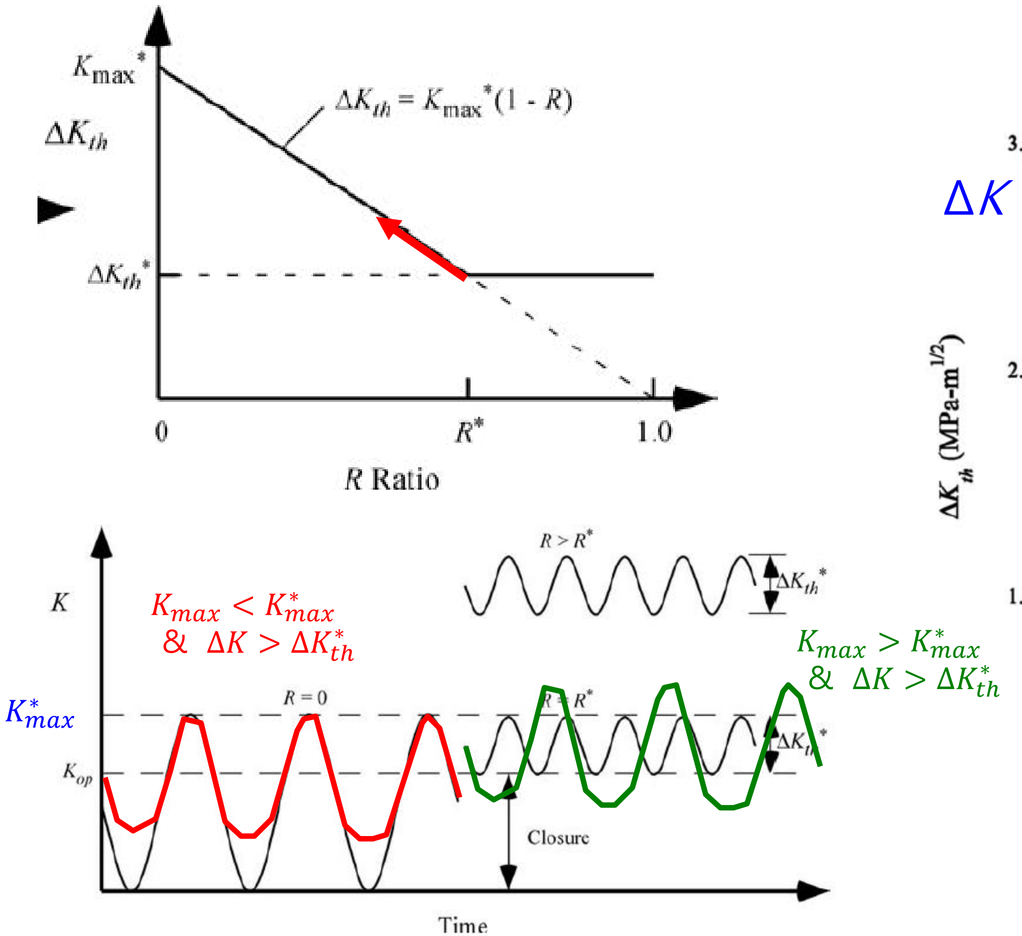
$$K_{\max}^{*} = K_{op} + \Delta K_{th}^{*}$$

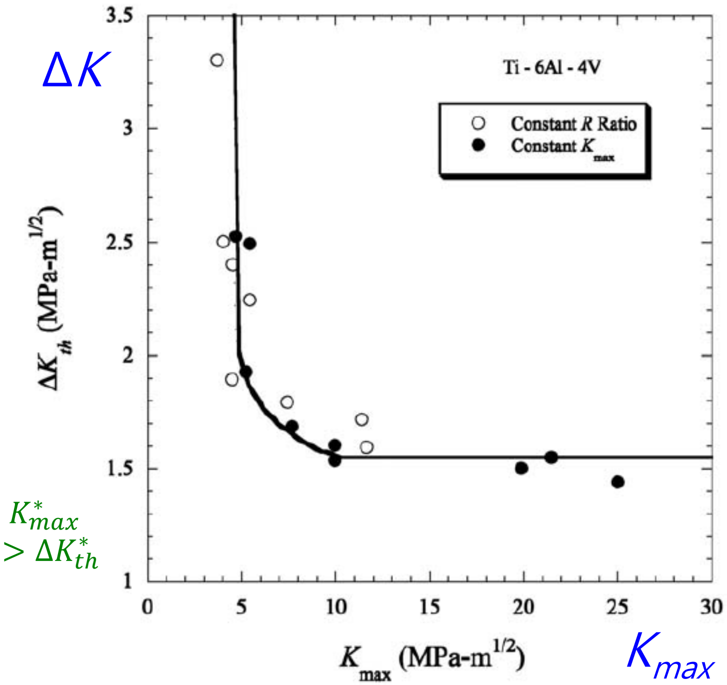


Schematic illustration of the dual threshold model

4. The Fatigue Threshold

A Two-Criterion Model



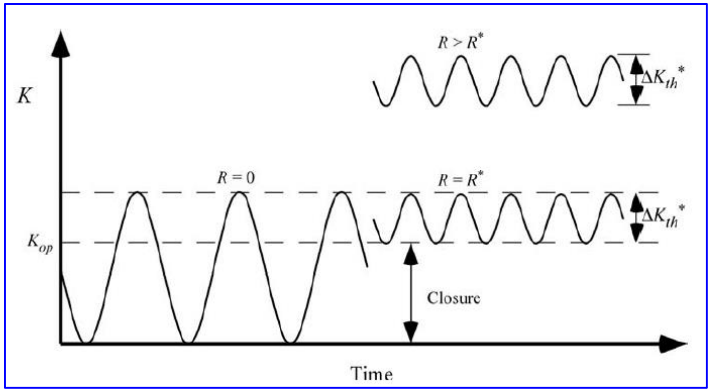


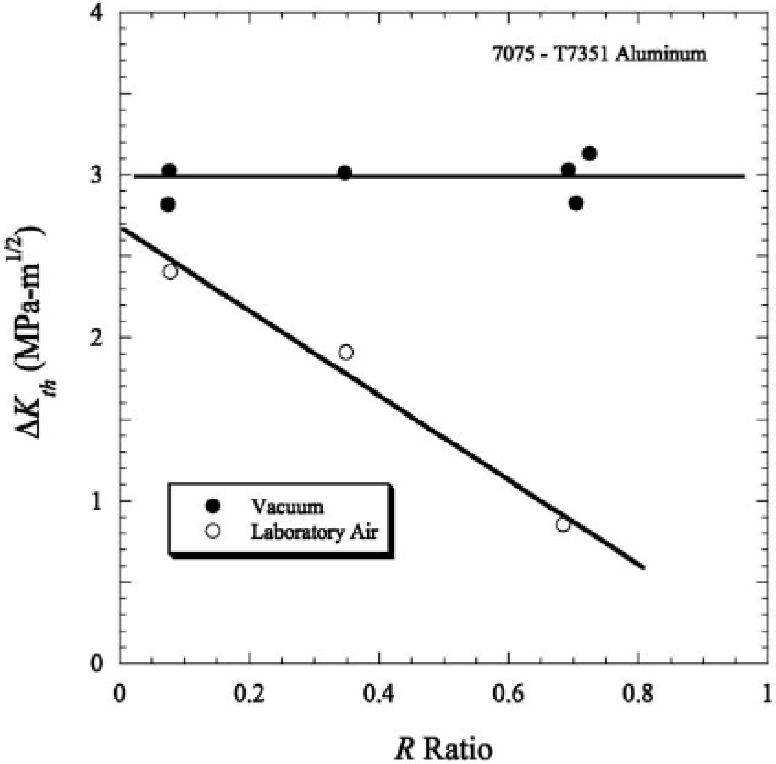
Threshold ΔK vs. K_{max} for Ti – 6Al – 4V

Threshold Behavior in Inert Environments

- It appears that significant crack closure is definitely not occurring in the vacuum.
- The existence of an *R* ratio dependence on ΔK_{th} as an indication of closure effects.
- The *absence* of an *R* ratio dependence on ΔK_{th} in a specific data set would imply the *lack* of significant closure.
- If significant closure is not occurring in the vacuum data, the *R* ratio dependence of ΔK_{th} for the air data cannot be due to plasticity-induced closure. → roughness-induced closure.

$$\Delta K_{th} = \begin{cases} \left(K_{op} + \Delta K_{th}^{*}\right)(1-R), & R \le R^{*} \\ \Delta K_{th}^{*}, & R > R^{*} \end{cases}$$

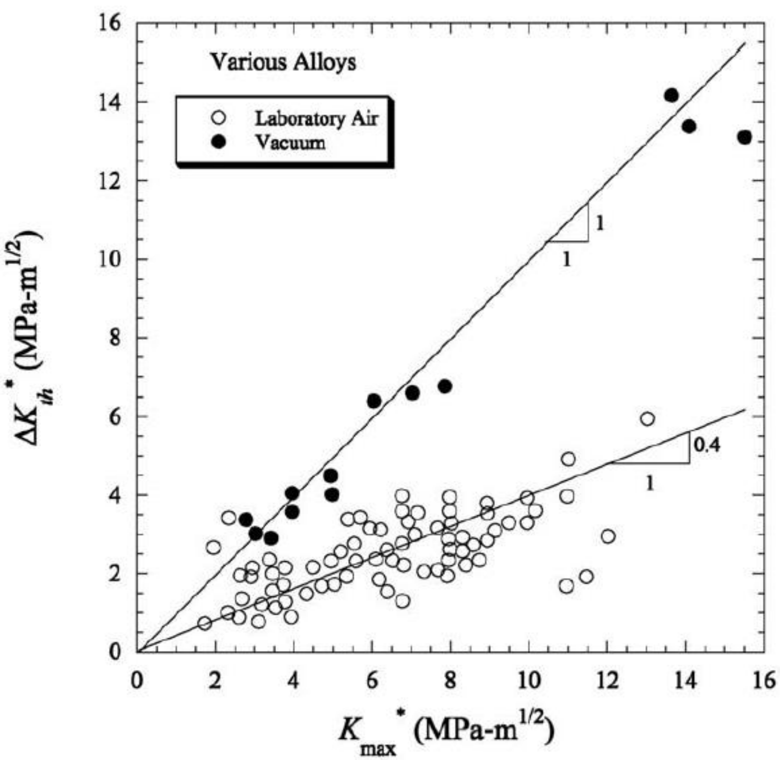




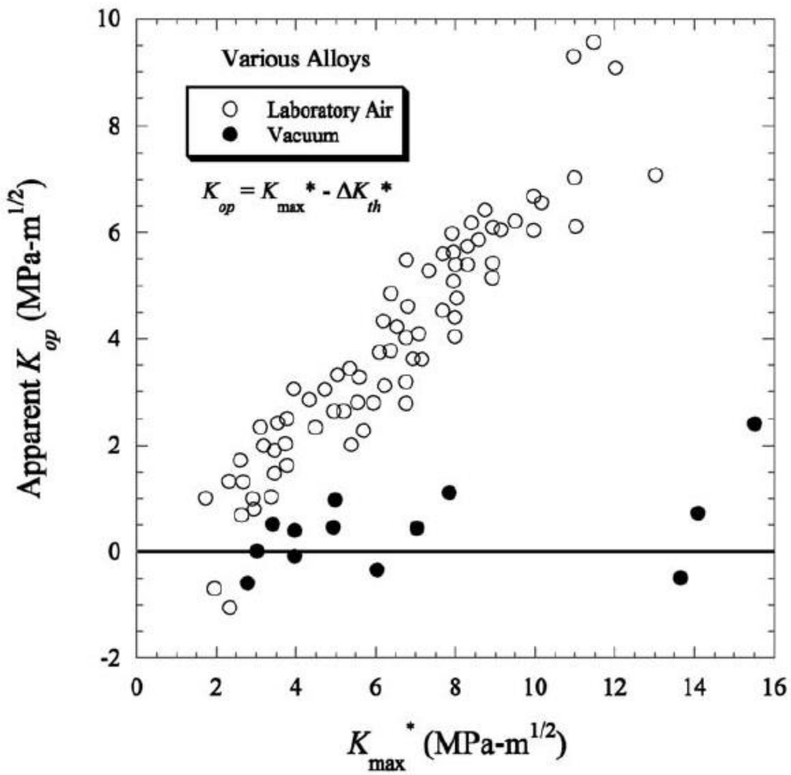
Comparison of ΔK_{th}^{*} for an aluminum alloy in air and vacuum environments

Threshold Behavior in Inert Environments

- In a vacuum environment : $K_{op} \approx 0$
- In air : $K^*_{max} = K_{op} + \Delta K^*_{th} \Rightarrow \Delta K^*_{th} \approx 0.4 K^*_{max} \Rightarrow K_{op} = 0.6 K^*_{max}$.
- The role of plasticity-induced closure in the threshold regime is a controversial topic.

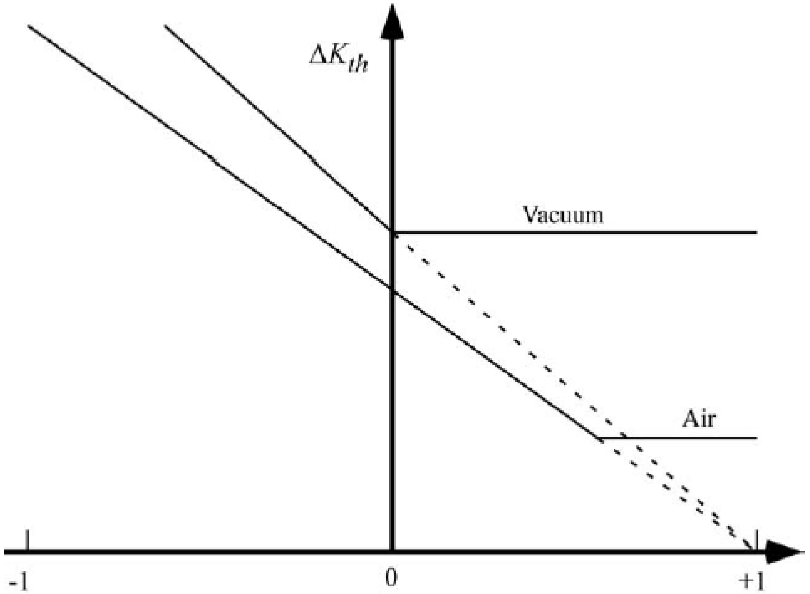


Correlation between ΔK^*_{th} and K^*_{max} in air and vacuum environments



Data from the left figure, replotted in terms of the apparent K_{op}

Threshold Behavior in Inert Environments



Schematic illustration of the threshold behavior in air and vacuum environments, at both positive and negative *R* ratios.

General

- Similitude of crack-tip conditions, which implies a unique relationship between da/dN, ΔK, and R, is rigorously valid only for constant amplitude loading (i.e., $dK/da = 0$).
- Real structures, however, seldom conform to this ideal. A typical structure experiences a spectrum of stresses over its lifetime.
- In such cases, the crack growth rate at any moment in time may depend on the prior history as well as current loading conditions.
- Variable amplitude fatigue analyses that account for prior loading history are considerably more cumbersome than analyses that assume similitude.
- Cyclic loading at high R ratios, where crack closure effects are negligible.
- When similitude applies, at least approximately, the *linear damage model* is suitable for variable amplitude loading.

Linear Damage Model for Variable Amplitude Fatigue

- If we assume that △a<<a, such that *da/dN* for a given cyclic stress does not change significantly during the loading history, the crack growth can be estimated as follows:

$$\Delta a \approx \left(\frac{da}{dN}\right)_1 N_1 + \left(\frac{da}{dN}\right)_2 N_2 + \cdots$$

- If the crack growth is described by the Paris equation

$$\Delta a \approx C[(\Delta K_1)^m N_1 + (\Delta K_2)^m N_2 + \cdots]$$

$$= CY^m (\pi a)^{m/2} \left(\Delta\sigma_1^m N_1 + \Delta\sigma_2^m N_2 + \cdots\right)$$

where *Y* is a geometry factor in the stress-intensity solution

- This weighted average cyclic stress corresponds to an average growth rate for the loading spectrum at a given crack size

$$\Delta\overline{\sigma} = \left(\frac{\sum_{i=1}^{n} \Delta\sigma_i^m N_i}{N_{tot}}\right)^{1/m}$$

- Life prediction is performed as if the loading were constant amplitude with $\bar{\sigma}$

$$\frac{d\overline{a}}{dN} = C(Y\Delta\overline{\sigma}\sqrt{\pi a})^m$$

Linear Damage Model for Variable Amplitude Fatigue

- Life prediction is performed as if the loading were constant amplitude

$$N = \frac{1}{C(\Delta\bar{\sigma}\sqrt{\pi})^m}\int_{a_o}^{a_f}\frac{da}{Ya^{m/2}}$$

- The above procedure for handling variable amplitude loading must be modified when accounting for a threshold.

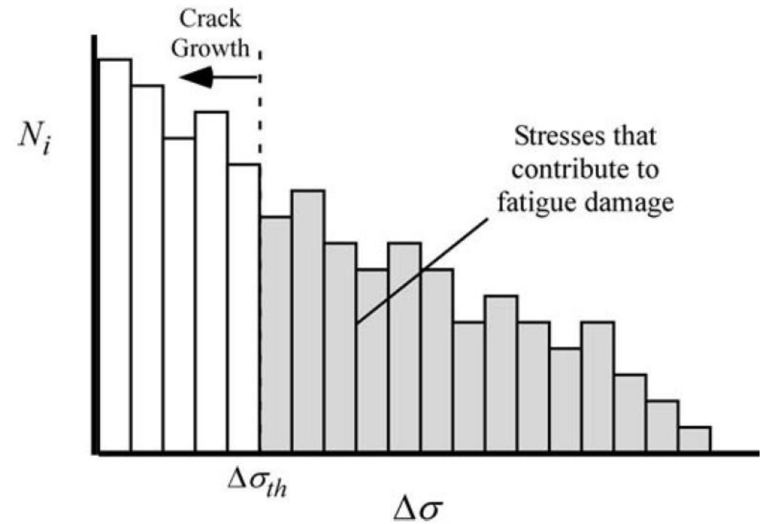
$$\frac{da}{dN} = \begin{cases} C\Delta K^m, & \Delta K > \Delta K_{th} \\ 0, & \Delta K \le \Delta K_{th} \end{cases}$$

- The threshold cyclic stress is given by

$$\Delta\sigma_{th} = \frac{\Delta K_{th}}{Y\sqrt{\pi a}}$$

- This threshold cyclic stress decreases with crack growth.
- The linear damage model for an arbitrary growth law can be generalized by computing an average crack growth rate for the loading spectrum.

$$\frac{d\bar{a}}{dN} = \frac{1}{N_{tot}}\sum_{i=1}^{n}\left(\frac{da}{dN}\right)_i N_i$$

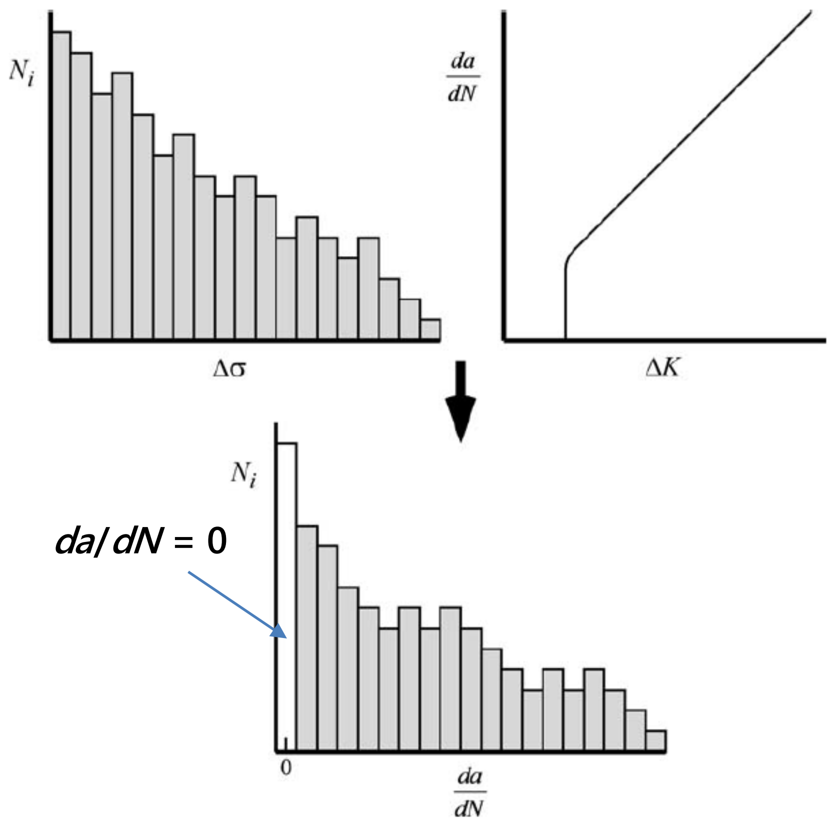


Schematic cyclic stress histogram. Only cycles below $\Delta\sigma_{th}$ contribute to fatigue damage

Linear Damage Model for Variable Amplitude Fatigue

- The construction of a *da/dN* histogram from a cyclic stress histogram and the crack growth law.
- The total number of cycles in the above expression, N_{tot}, must include *all* cycles, including those where *da/dN* = 0.
- Life prediction is achieved through integration of the average growth rate:

$$N = \int_{a_o}^{a_f} \left(\frac{d\bar{a}}{dN} \right)^{-1} da$$



Derivation of a *da/dN* histogram from a cyclic stress histogram and the growth law.

Change of residual stress due to loading

Initial Tensile Residual Stress

Initial state of residual stress …. **A**
(same as tensile yield stress)

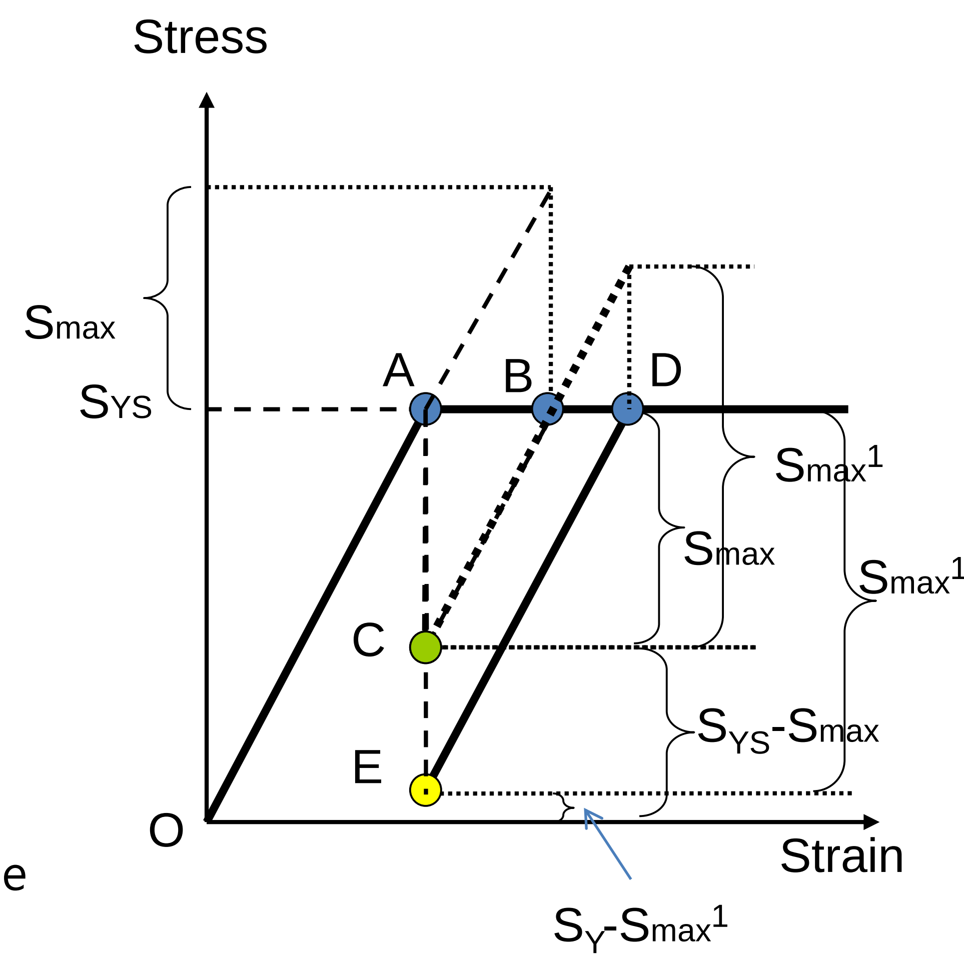
If a stress "S**max**" is applied, the state moves from A to **B**.

If applied stress is released, the state moves from B to **C**.

If another stress "S**max1**" is applied over yielding, the state moves from C to **D** through B.

If applied stress is released, the state moves from D to **E**.

Final residual stress is represented by the location **E**.



Shake down effect under Tensile Load

Change of residual stress due to loading

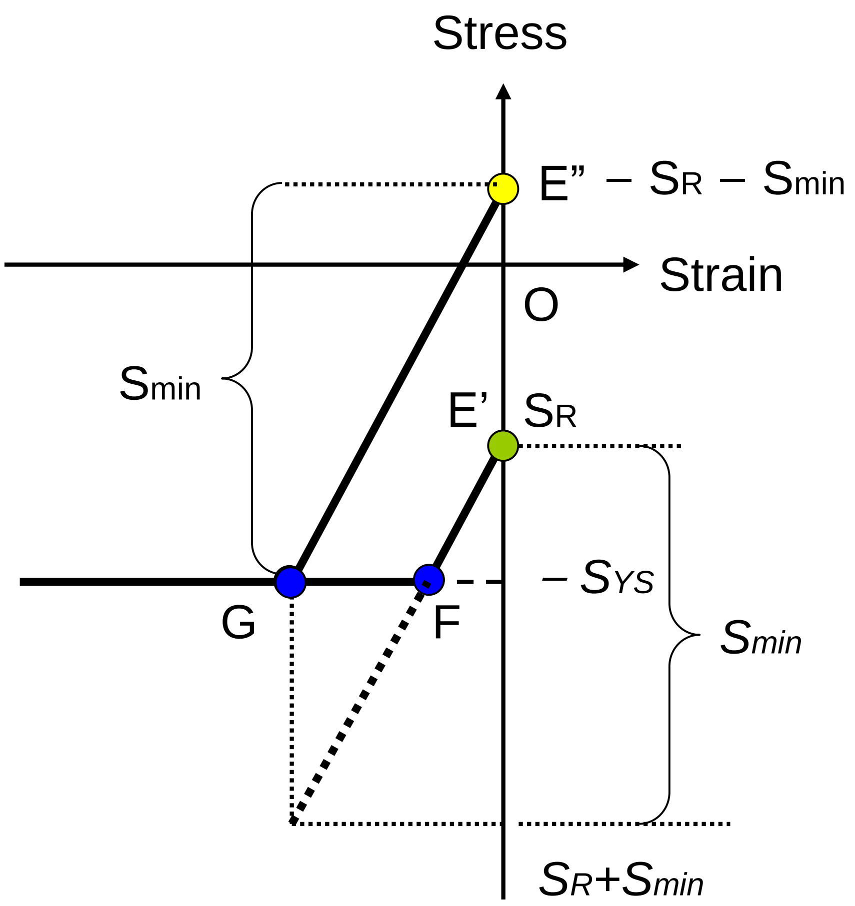
Initial compressive residual stress

Initial state .... **E'**
(compressive initial residual stress; S**R**)

If a stress "S**min**" is applied, the state moves from E' to **G** through F.

If applied stress is released, the state moves from G to **E"**.

Final residual stress is represented by the location **E"**.



Shake down effect under Compressive Load

Summary of shake down effect

- If more higher stress; S_{max} is applied at a certain cycle than before, the residual stress $*S_R$ is renewed as follows.

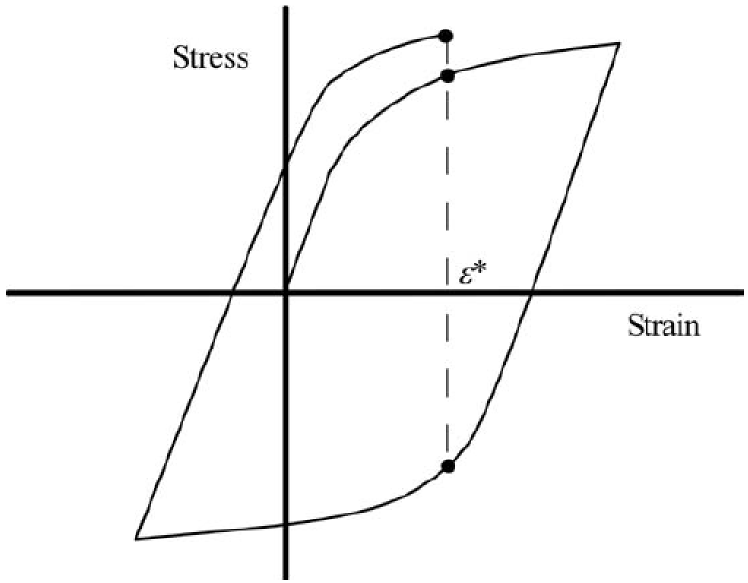
$$*S_R = S_{YS} - S_{max}$$

- If the stress $*S_R + S_{min}$ becomes less than $-S_{YS}$, which means the state becomes plasticity region in compressive side, the residual stress is renewed as follows.

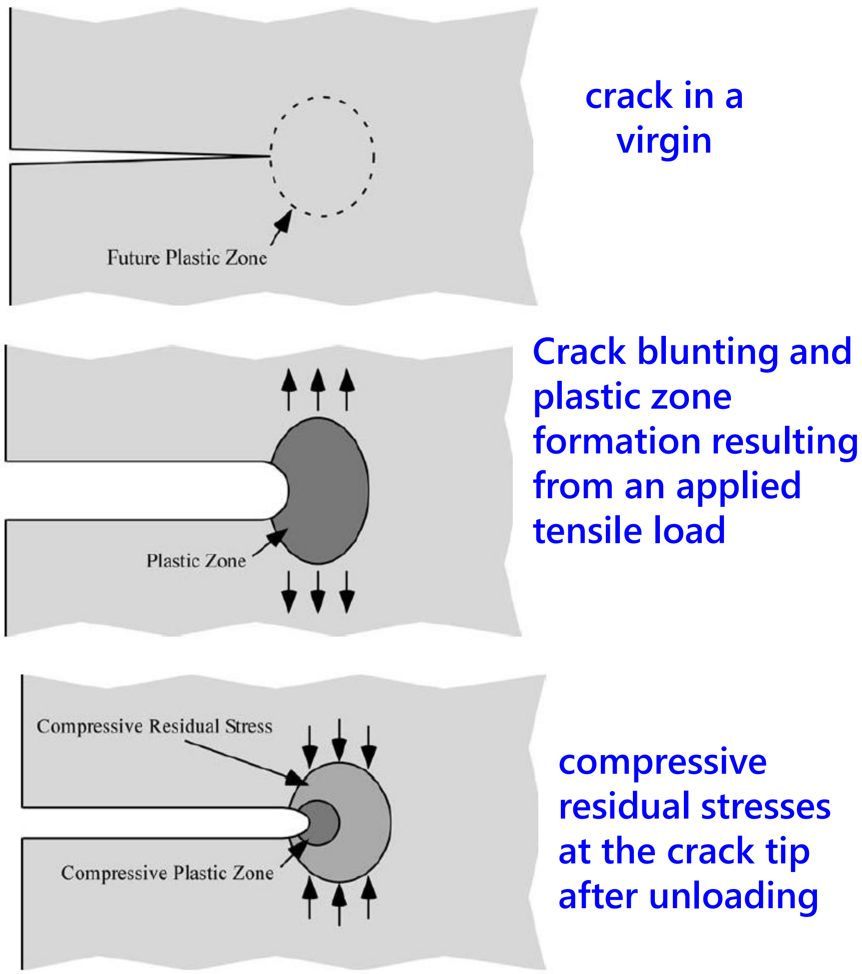
$$*S_R = -S_{YS} - S_{min}$$

Reverse Plasticity at the Crack Tip

- History effects in fatigue are a direct result of the history dependence of plastic deformation

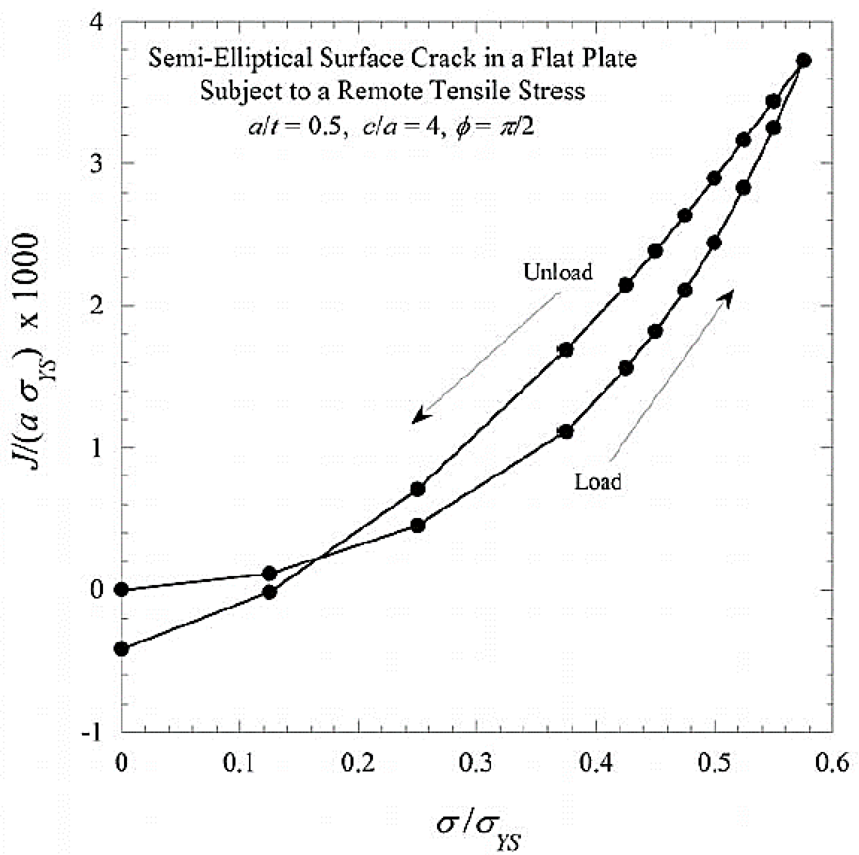


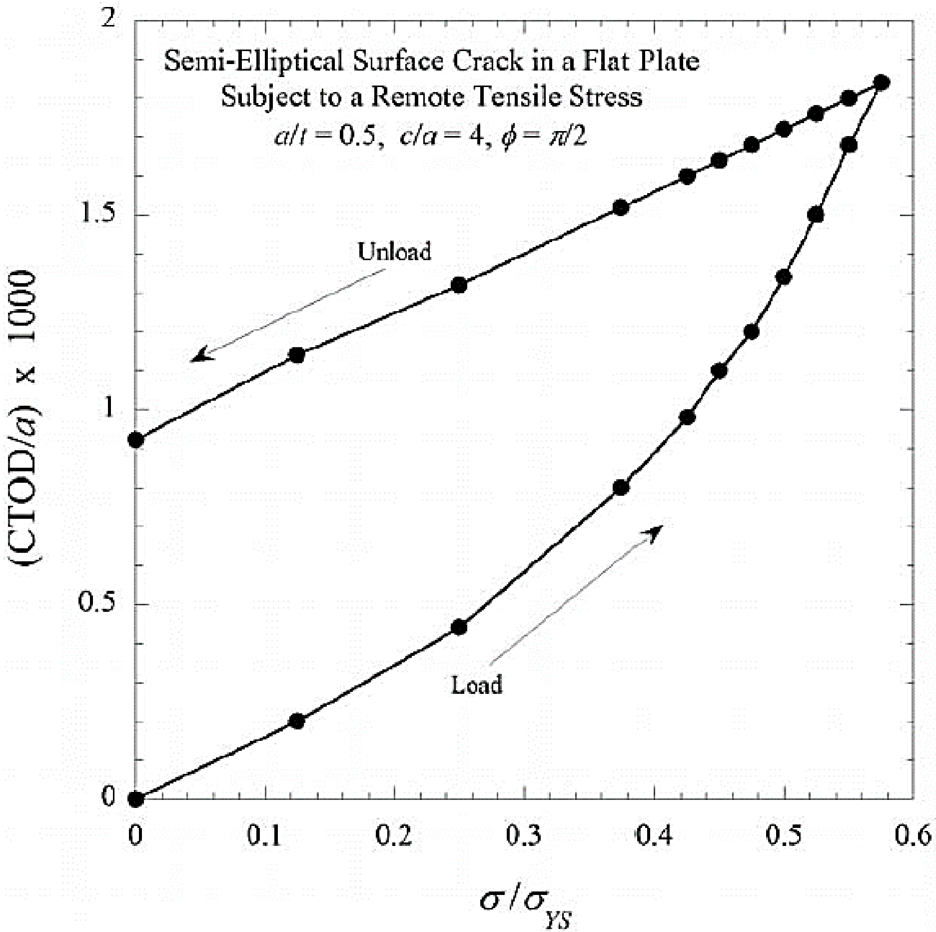
Schematic stress-strain response of a material that is yielded in both tension and compression. The stress at a given strain ε^*, depends on prior loading history.



Deformation at the tip of a crack subject to a single load-unload cycle

Reverse Plasticity at the Crack Tip

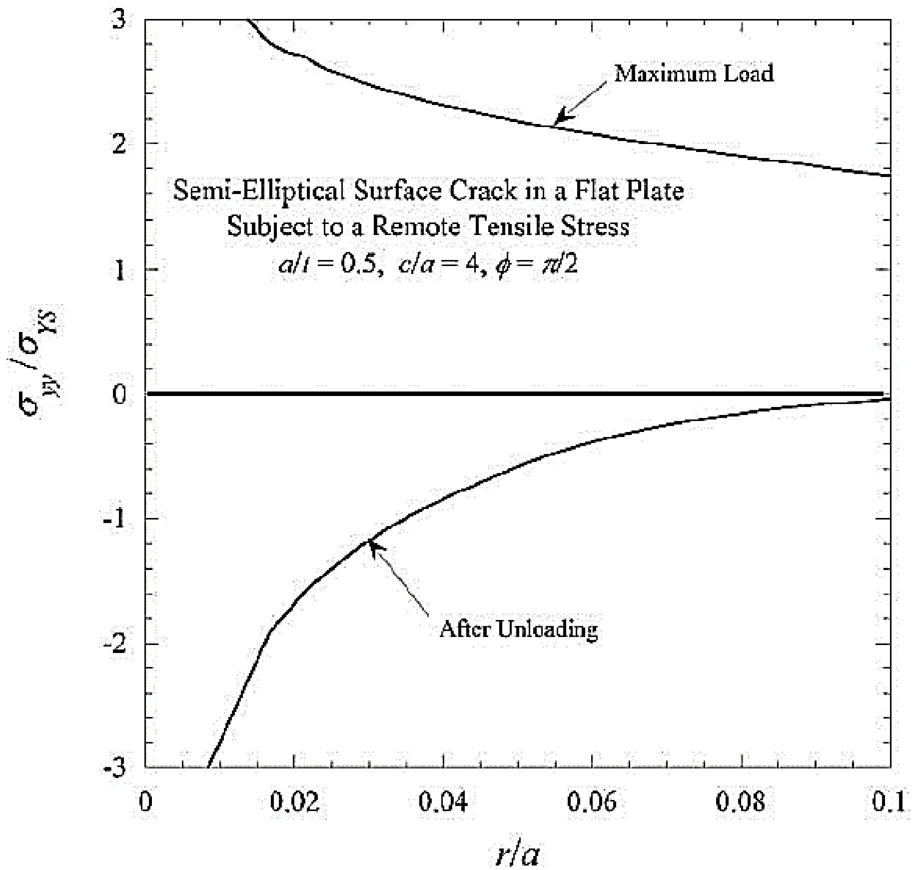


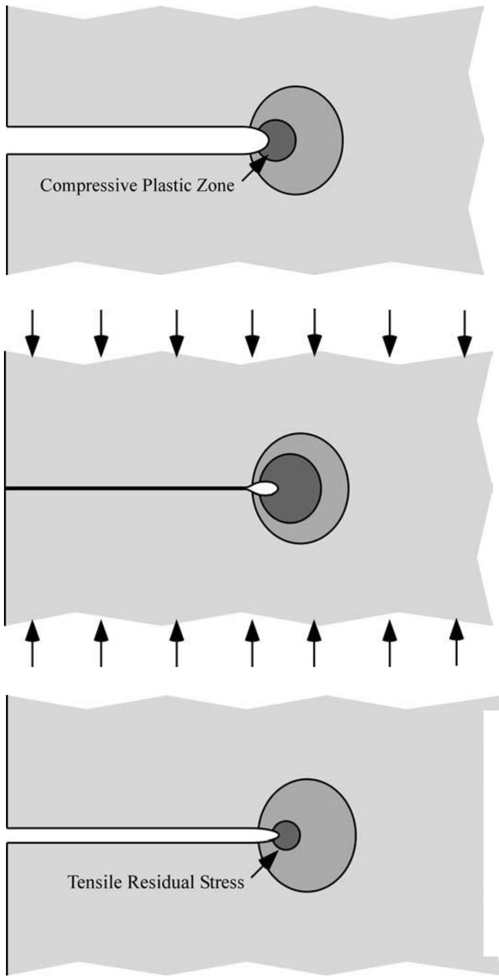


J-integral results obtained from an elastic-plastic finite element analysis of a load-unload cycle in a plate with a semielliptical surface crack.

Crack-tip-opening displacement (CTOD) results obtained from an elastic-plastic finite element analysis of a load-unload cycle in a plate with a semielliptical surface crack.

Reverse Plasticity at the Crack Tip





initial state

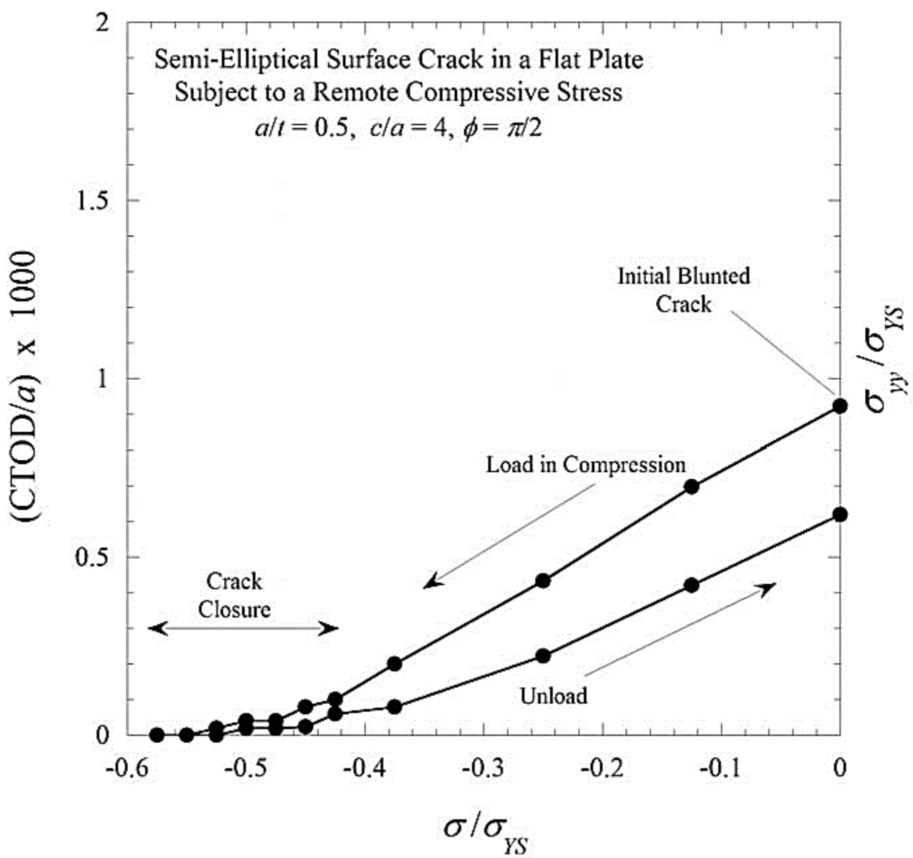
Application of compressive load, and removal of compressive load.

The crack tip resharpens and a tensile residual stress forms at the crack tip.

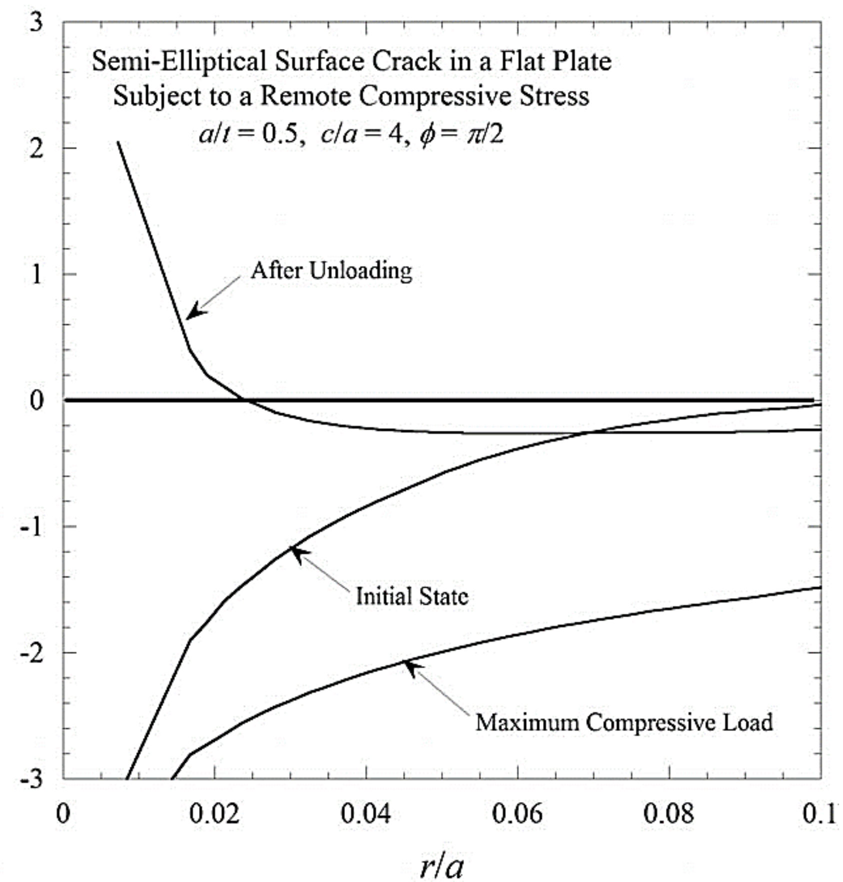
Normal stress vs. distance from the crack tip computed from an elastic-plastic finite element analysis of a load-unload cycle in a plate with a semielliptical surface crack.

Compressive underload of a blunted crack with an initial zone of compressive residual stress at the crack tip

Reverse Plasticity at the Crack Tip



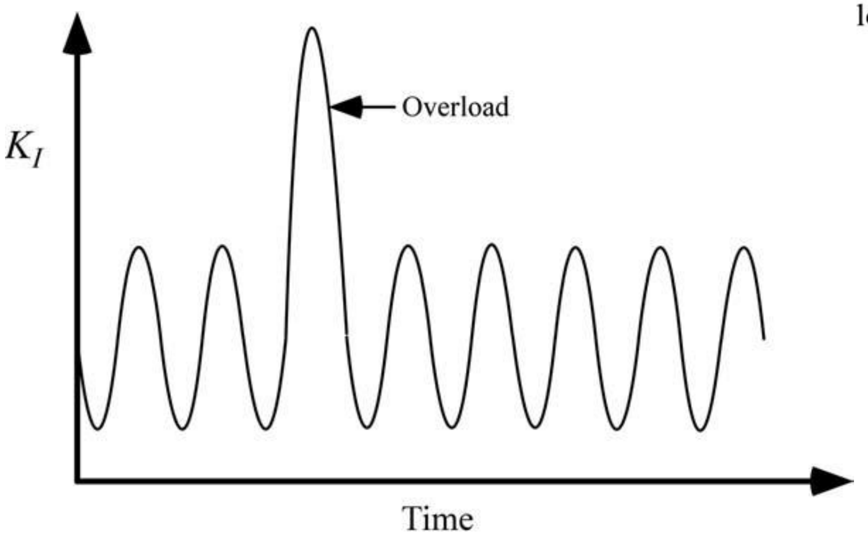
Crack-tip-opening displacement (CTOD) obtained from an elastic-plastic finite element analysis of the compressive underload scenario.



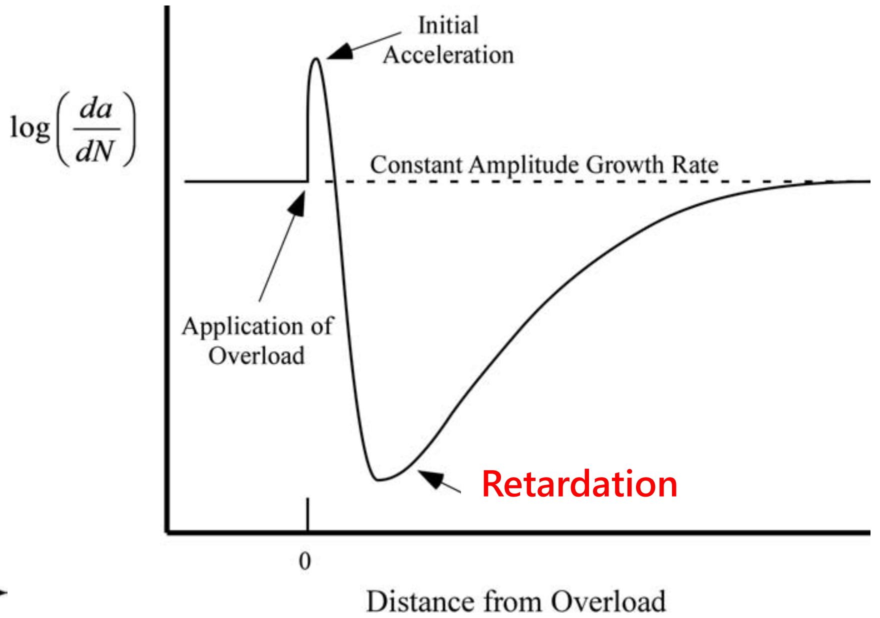
Normal stress vs. distance from the crack tip, obtained from an elastic-plastic finite element analysis of the compressive underload scenario.

The Effect of Overload and Underloads

- Constant amplitude loading is interrupted by a single overload, after which the K amplitude resumes its previous value.
- The overload cycle produces a significantly larger plastic zone → the residual stresses influences subsequent fatigue behavior.
- As the crack grows following the overload, the rate eventually approaches that observed for constant amplitude loading.



A single overload during cyclic loading.



Typical crack growth behavior following the application of a single overload.

The Effect of Overload and Underloads

❖ Three possible mechanisms have been proposed to explain retardation following an overload.

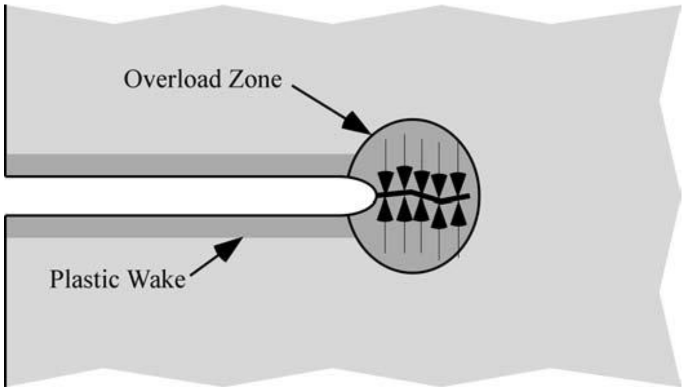
1. The crack blunts following an overload, and crack growth is delayed while the crack tip resharpens ⇒ rejected.
2. The compressive residual stresses in front of the crack tip retard the crack growth rate ⇒ favored.
3. As the crack grows into the overload zone, residual stresses *behind* the crack tip result in plasticity-induced closure ⇒ the majority opinion.

- The most compelling evidence in favor of the closure argument is the phenomenon of *delayed retardation*. retardation generally does not occur immediately following the application of an overload.
- If retardation were driven by a reduced K_{max} due to compressive stresses in front of the crack tip, one would expect the effect to be immediate.

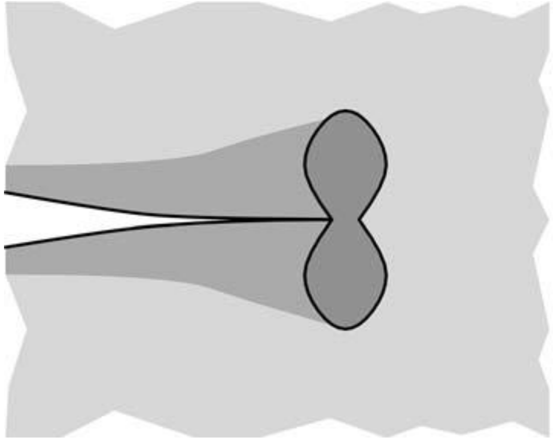
❖ The closure mechanism provides a plausible explanation for the momentary acceleration of crack growth rate following an overload.

The Effect of Overload and Underloads

- When an overload is applied, the resulting crack blunting causes the crack faces to move apart. Closure does not occur in the cycles immediately following the overload, so the crack growth rate is momentarily higher than it was prior to the overload.
- Once the crack grows a short distance into the overload zone, compressive residual stresses result in plasticity-induced closure, which in turn results in retardation.
- The blunting following the overload may result in a momentary acceleration due to the absence of closure, but retardation occurs once the crack grows into the overload zone.



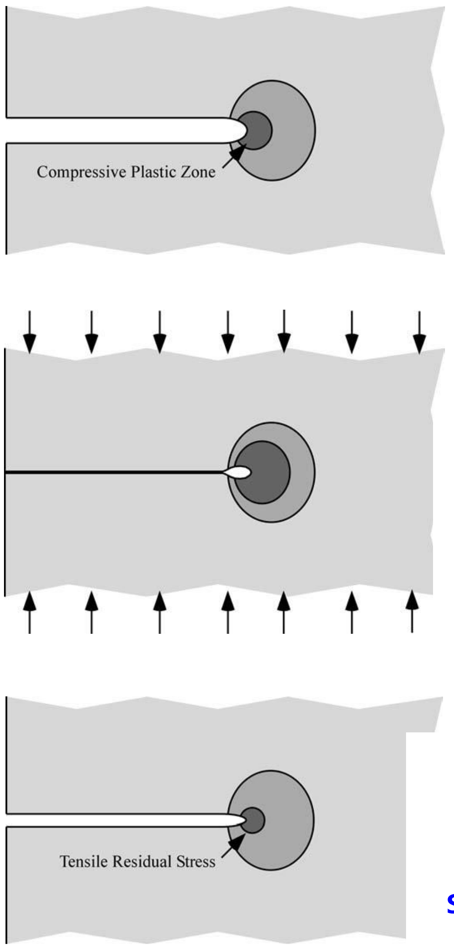
Retardation caused by plasticity-induced closure.



plasticity-induced closure

The Effect of Overload and Underloads

- *Underloads* (i.e., a compressive load or a tensile load that is well below previous minimum loads) can result in an acceleration of crack growth or a reduction in the level of retardation.
- A blunted crack is forced closed by a compressive underload. The crack resharpens and a tensile residual stress forms ahead of the crack tip.
- If the crack faces are in contact at the time an underload is applied, a tensile residual stress zone will not form at the crack tip, but compressive forces on the crack faces may flatten asperities. This may have the effect of reducing the magnitude of roughness-induced closure in subsequent crack growth.



initial state

Application of compressive load, and removal of compressive load.

The crack tip resharpens and a tensile residual stress forms at the crack tip.

Compressive underload of a blunted crack with an initial zone of compressive residual stress at the crack tip

Models for Retardation and Variable Amplitude Fatigue

- More recent models are based on the assumption that plasticity-induced closure is responsible for load interaction effects.

❖ Wheeler Model :

- It relates the current crack growth rate to the distance the crack has grown into the overload zone.
- A retardation factor ϕ_R
- The crack growth rate

$$\phi_R = \left(\frac{\Delta a + r_{y(c)}}{r_{y(o)}}\right)^{\gamma}$$

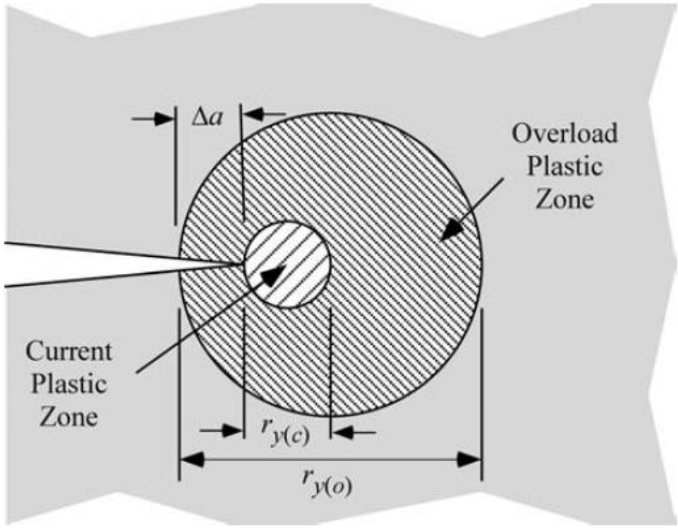
- The overload plastic zo

$$\left(\frac{da}{dN}\right)_R = \phi_R \frac{da}{dN}$$

$$r_{y(o)} = \frac{1}{\beta\pi}\left(\frac{K_o}{\sigma_{YS}}\right)^2$$

where Ko is the stress int ‹ overl

β = 2 for plane stress and β = 6 for plane strain.



Growth of a fatigue crack following an overload.

Models for Retardation and Variable Amplitude Fatigue

- The plastic zone size that corresponds to the current *K*max is given by

$$r_{y(c)} = \frac{1}{\beta\pi}\left(\frac{K_{\max}}{\sigma_{YS}}\right)^2$$

- that retardation effects are assumed to persist as long as the current plastic zone is contained within the overload zone, but the effects disappear when the current plastic zone touches the outer boundary of the overload zone.

Models for Retardation and Variable Amplitude Fatigue

❖ Willenborg model

- A residual stress-intensity factor

$$K_R = \begin{cases} K_o\left(1 - \dfrac{\Delta a}{r_{y(o)}}\right)^{1/2} - K_{\max}, & \Delta a \le r_{y(o)} \\ 0, \quad \Delta a > r_{y(o)} \end{cases}$$

$$r_{y(c)} = \frac{1}{\beta\pi}\left(\frac{K_{\max}}{\sigma_{YS}}\right)^2$$

- An effective *R* ratio

$$R_{eff} = \frac{K_{\min} - K_R}{K_{\max} - K_R}$$

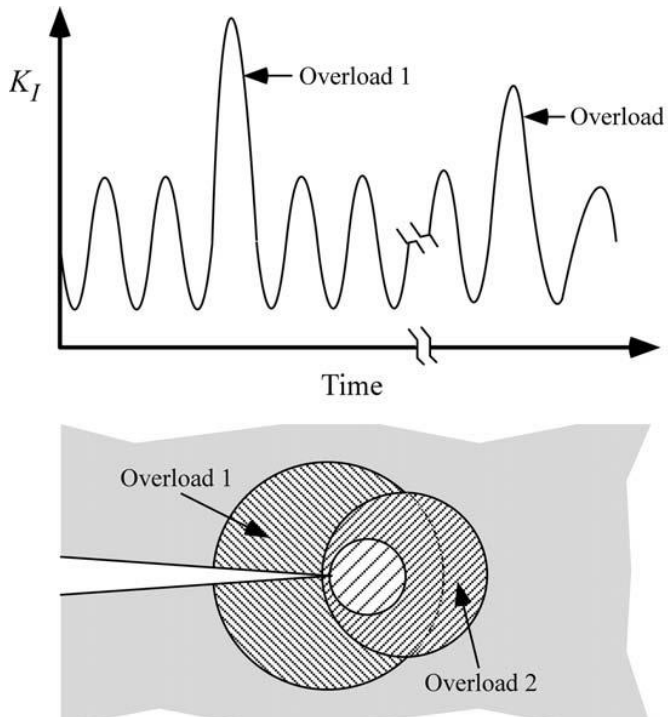
- Walker expression [13] is a modification of the Paris power law.

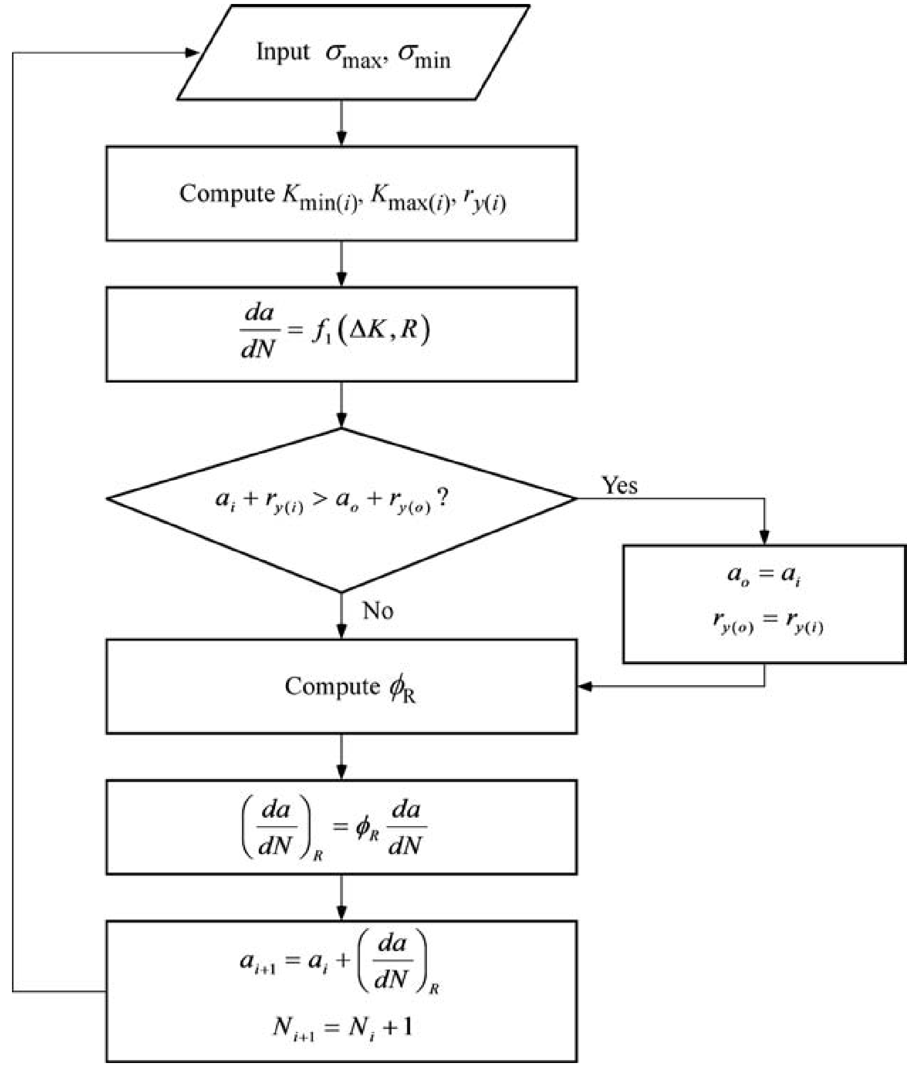
$$\frac{da}{dN} = C\left[\frac{\Delta K}{(1-R)^n}\right]^m$$

where n is a material constant. When applying the Willenborg model, R_{eff} is substituted in place of R.

Models for Retardation and Variable Amplitude Fatigue

- ❖ Retardation models such as the Wheeler and Willenborg approaches can be applied to variable amplitude loading.
- the second overload is used to compute $r_{y(o)}$ because this overload zone extends further ahead of the current crack tip.





Simple example of variable amplitude loading.

Flow chart for variable amplitude fatigue analysis with the Wheeler model.

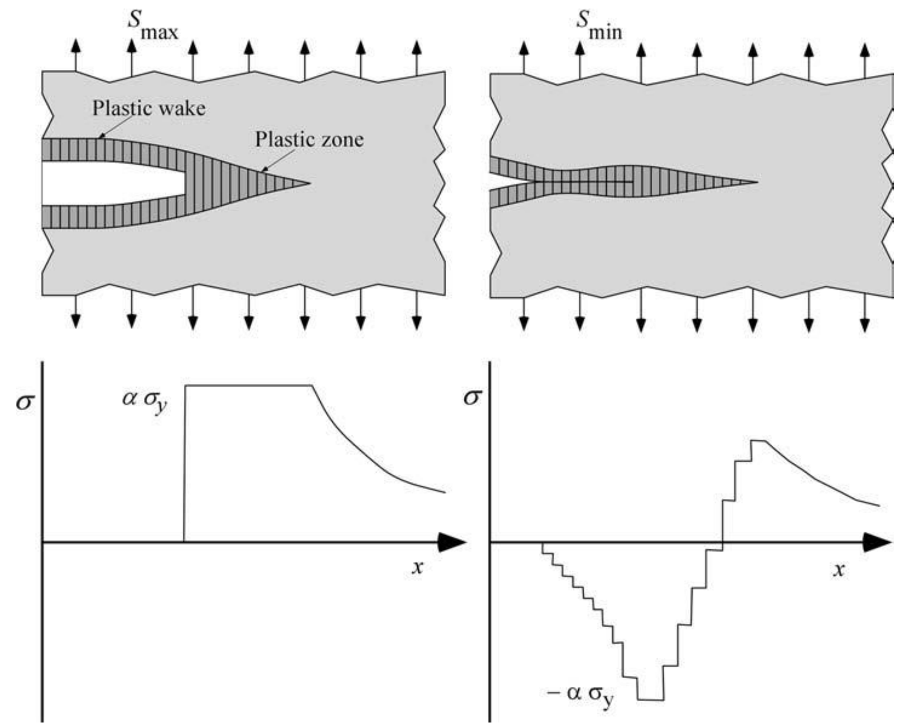
Models for Retardation and Variable Amplitude Fatigue

❖ A closure-based retardation model

- The plastic zone is divided into discrete segments. As the crack advances, segments are broken and become part of the plastic wake. The residual stress in each broken segment is computed from the maximum stretch the segment was subjected to when it was intact.

- At the maximum far-field stress Smax, the crack is fully open, and plastic zone is stressed to αs_y, where α = 1 for plane stress and α = 3 for plane strain.
- At the minimum stress S_{min}, the crack is closed. The residual stress distribution in the plastic wake determines the far-field opening stress S_{σ}.
- The effective stress intensity

$$K_{eff} = S_{max} - S_{\sigma}$$



The Newman closure model