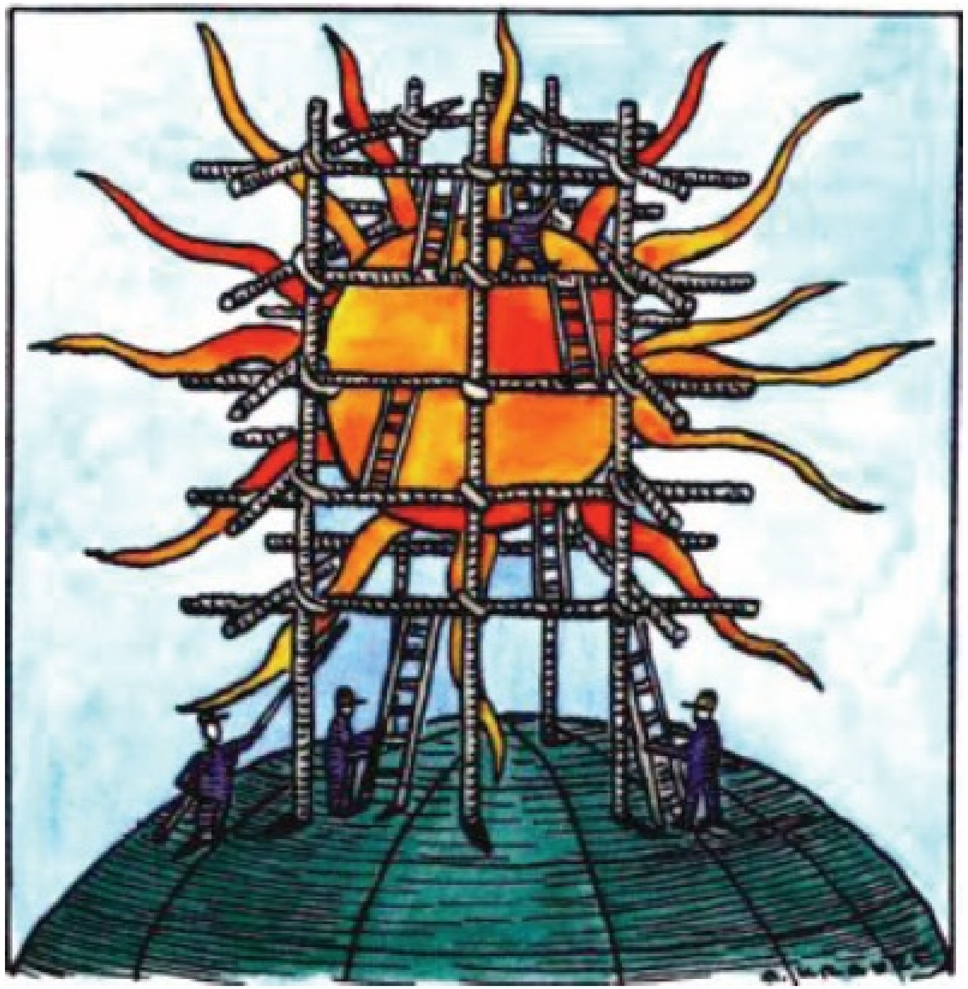
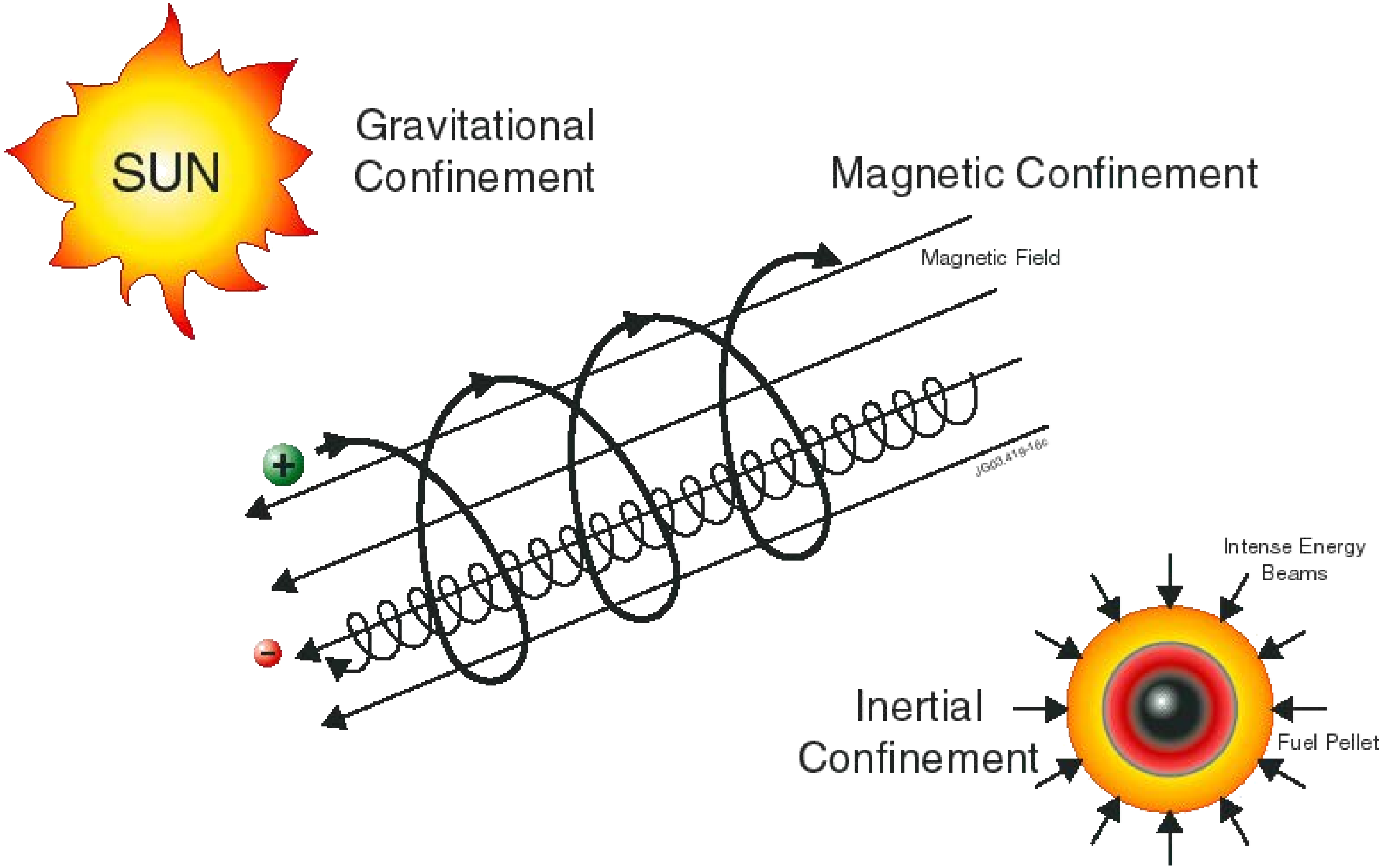
Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

To build a sun on earth



To build a sun on earth



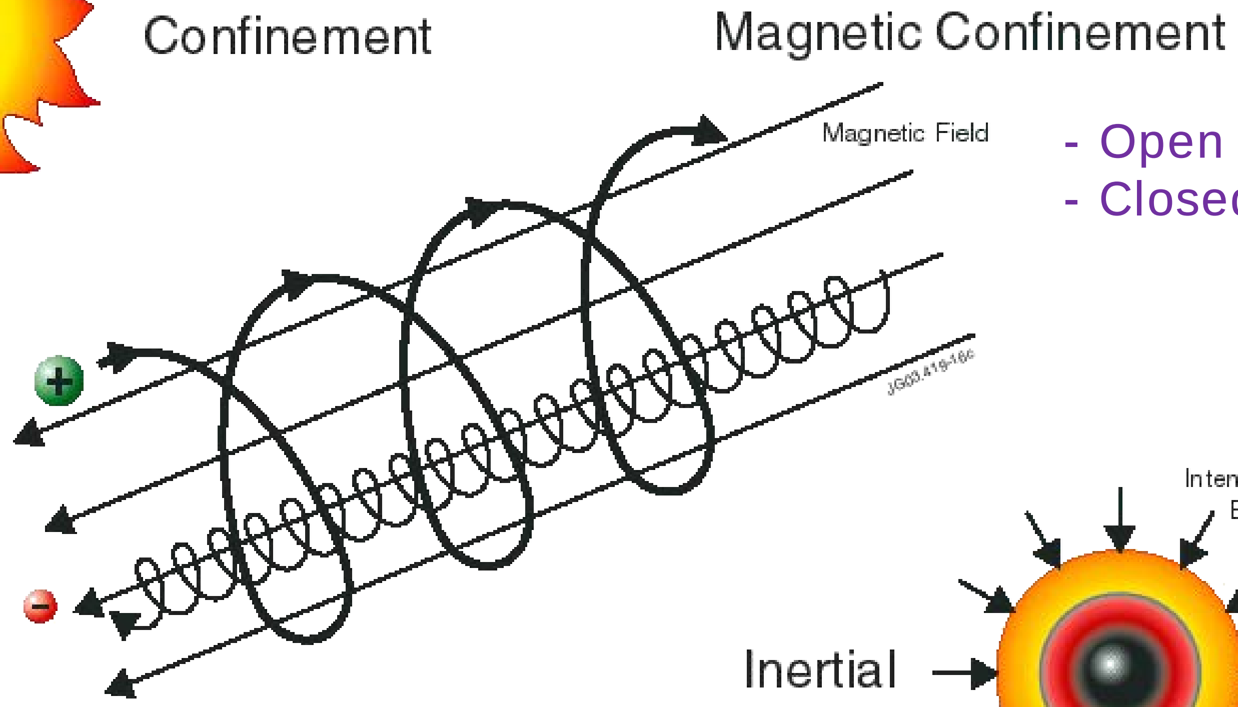
Magnetic Confinement

- **Bring the Sun on the Earth**

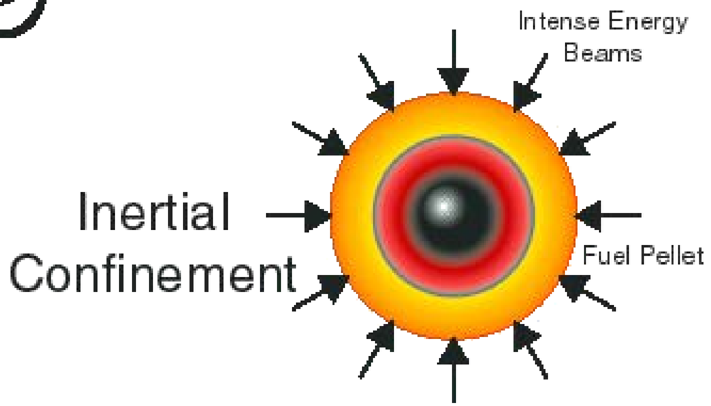
Quantity	ITER	Sun	Ratio
Diameter	16.4 m	140×10^{4} km	~ $1/10^{8}$
Central temp.	200 Mdeg	15 Mdeg	10
Central density	~$10^{20}/m^{3}$	~$10^{32}/m^{3}$	~$1/10^{12}$
Central press.	~5 atm	~10^{12} atm	~$1/10^{11}$
Power density	~0.6 MW/m^{3}	~0.3 W/m^{3}	~2×10^{6}
Reaction	DT	pp	
Plasma mass	0.35 g	2×10^{30} kg	~$1/10^{34}$
Burn time const.	200 s	10^{10} years	~$1/10^{15}$

M. Kikuch, "steady state tokamak reactor and its physics issues", Talk at SNU, Korea, September 30, 2011

To build a sun on earth



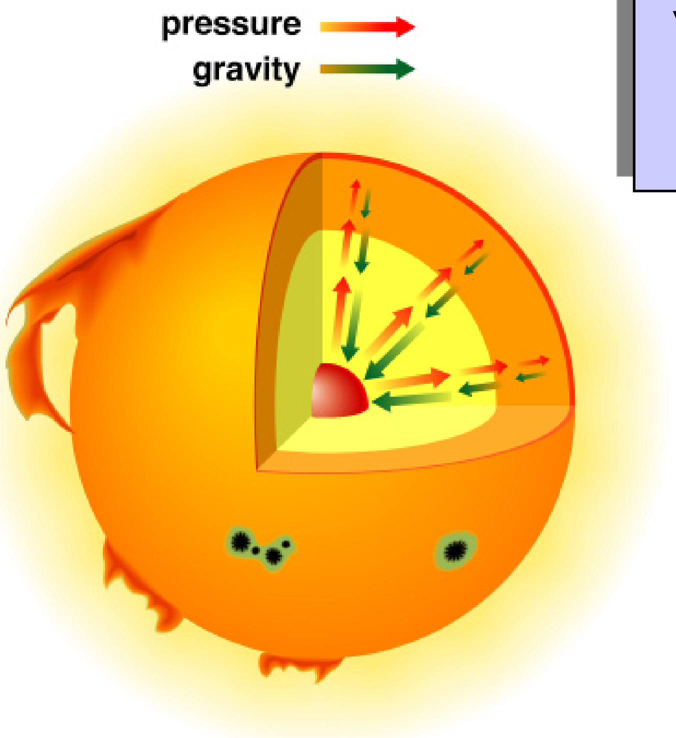
- Open magnetic confinement
- Closed magnetic confinement



What is open magnetic confinement?

Open Magnetic Confinement

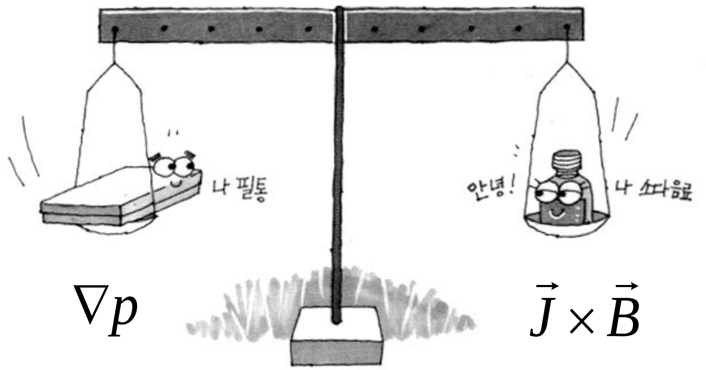
- **Equilibrium – Radial Force Balance**

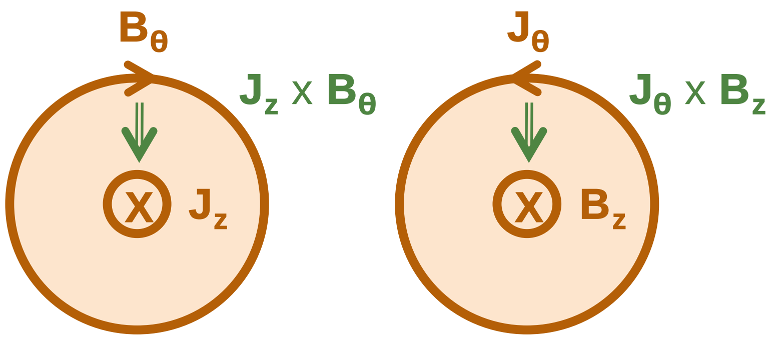


$$\nabla p = \vec{J} \times \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

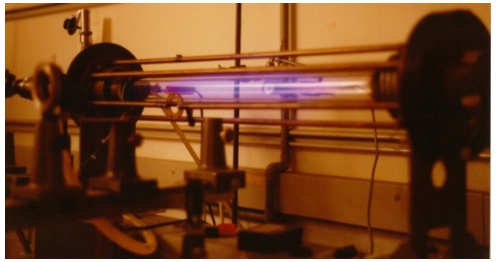
$$\nabla \cdot \vec{B} = 0$$





http://lasp.colorado.edu/education/outerplanets/solsys_star.php

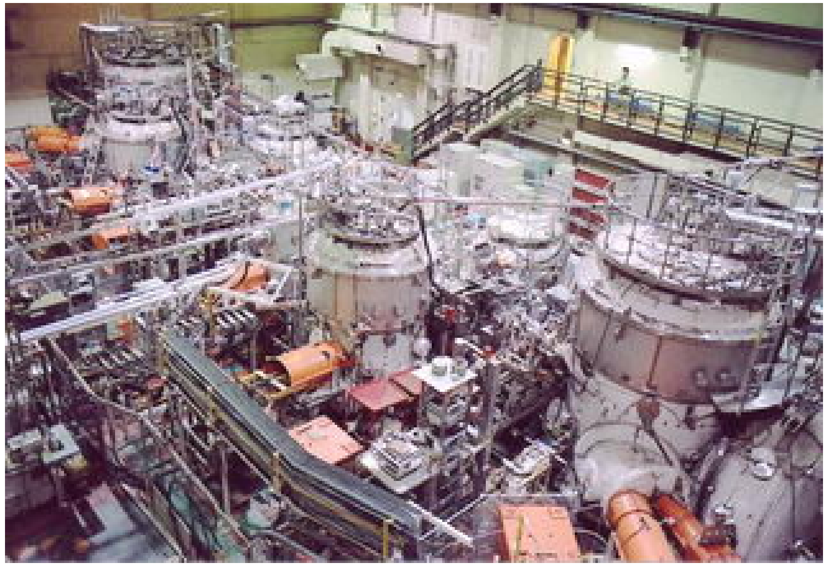
Open Magnetic Confinement



Z pinch

θ pinch

screw pinch

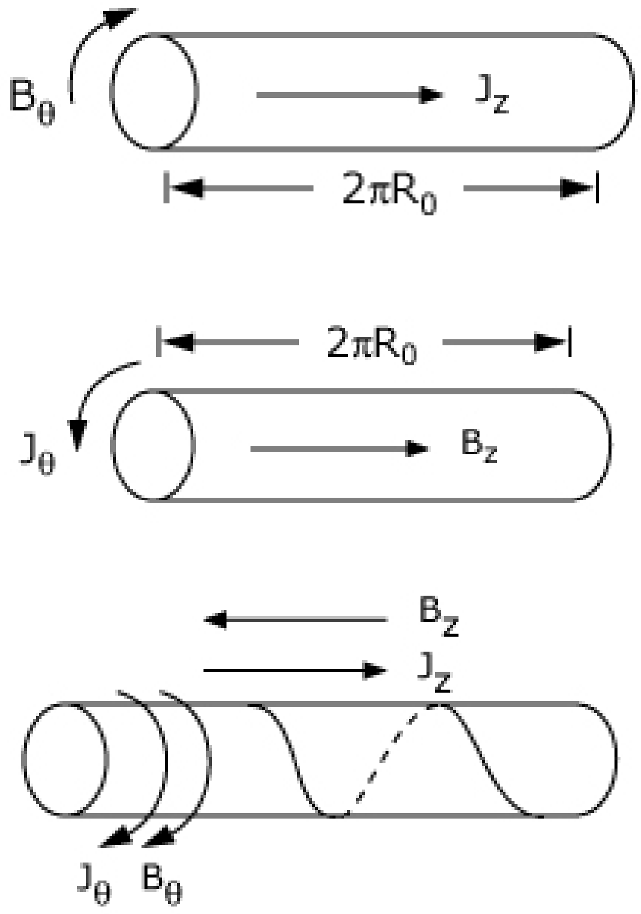


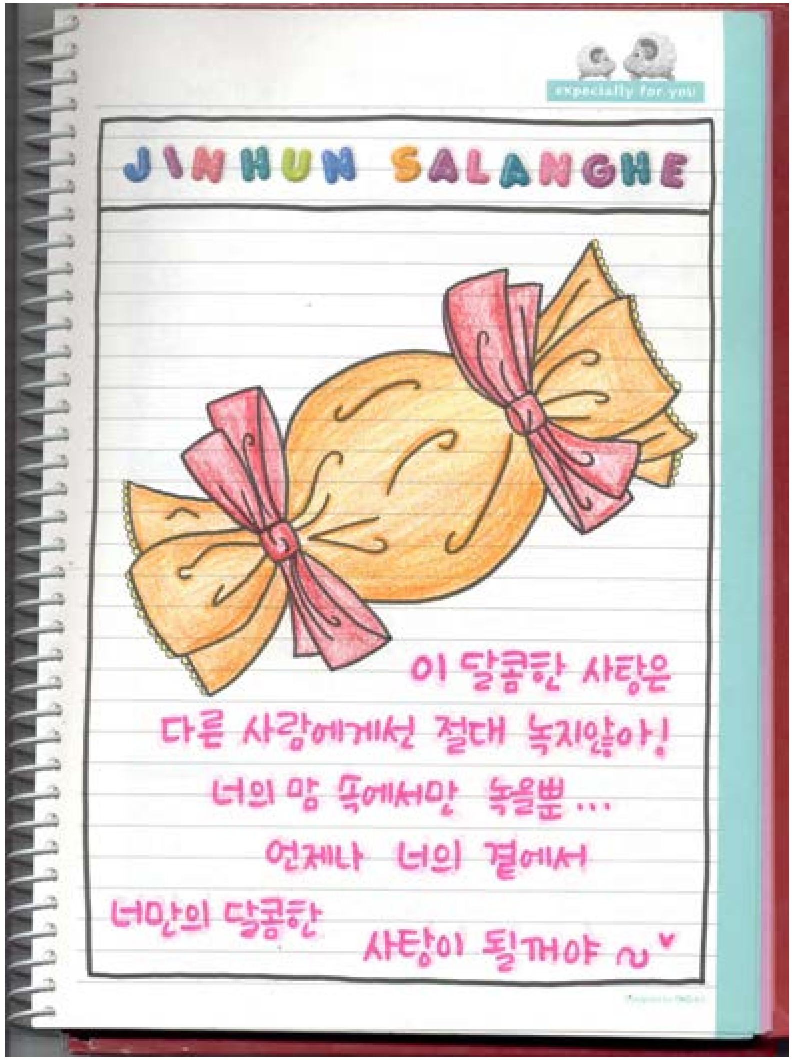
Magnetic mirror

http://phys.strath.ac.uk/information/history/photos.php
http://www.frascati.enea.it/ProtoSphera/ProtoSphera%202001/6.%20Electrode%20experiment.htm

What is magnetic mirror?

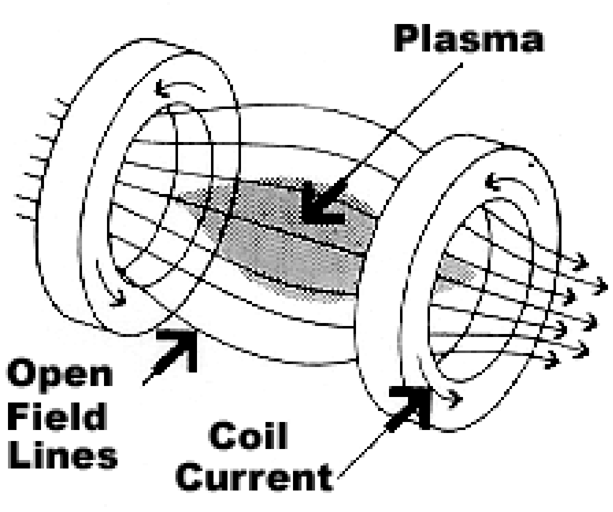
Magnetic Mirror

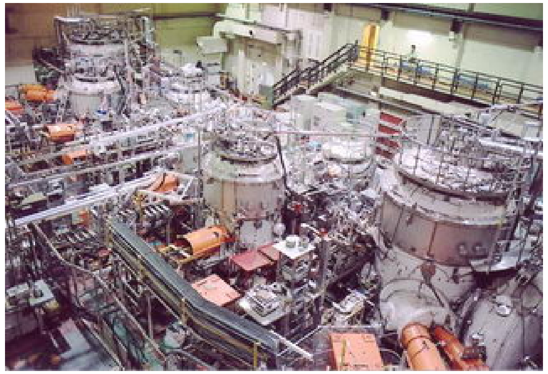


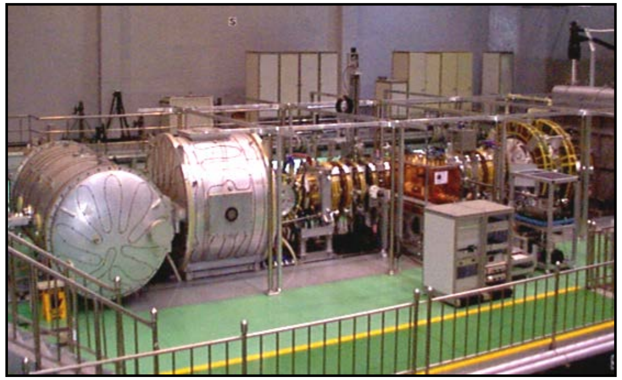


- Suffering from end losses

Magnetic Mirror



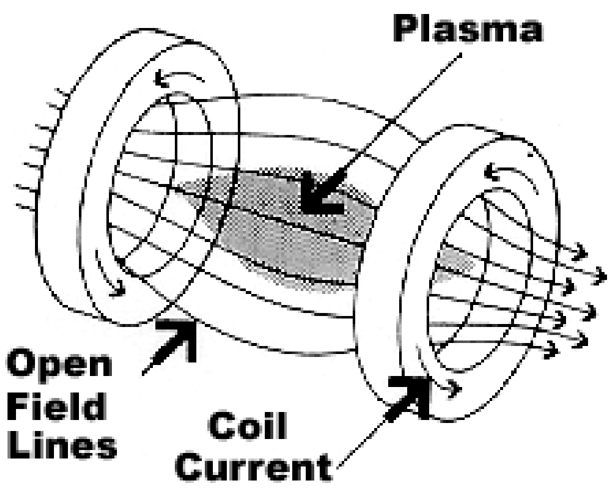




Gamma 10 and Hanbit

- To reduce end-leakage by establishing an increasing magnetic field at the two ends
- Many of charged particles are trapped due to the imposed constraints on particle motion with regards to conservation of energy and the magnetic moment.
- First proposed by Enrico Fermi as a mechanism for the acceleration of cosmic rays

Magnetic Mirror

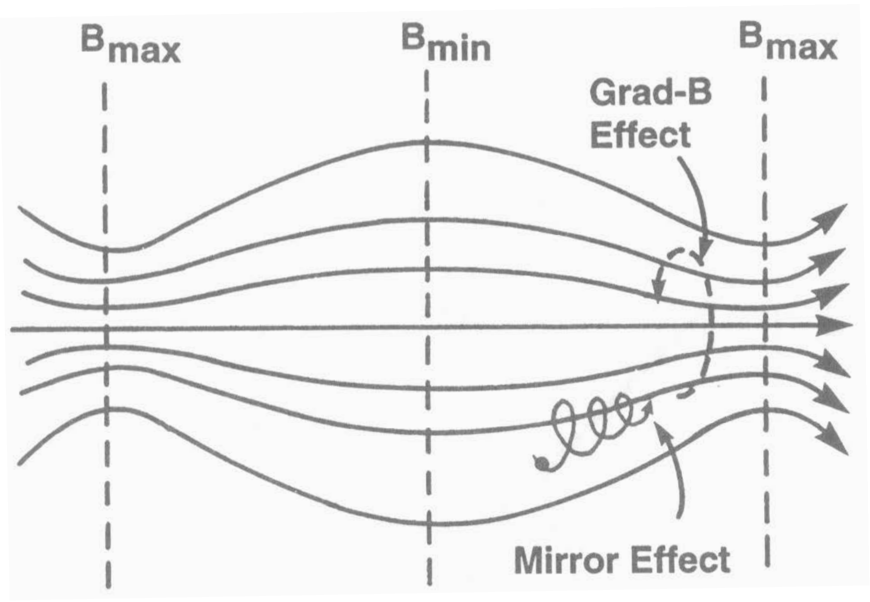


What is the motion of particles?

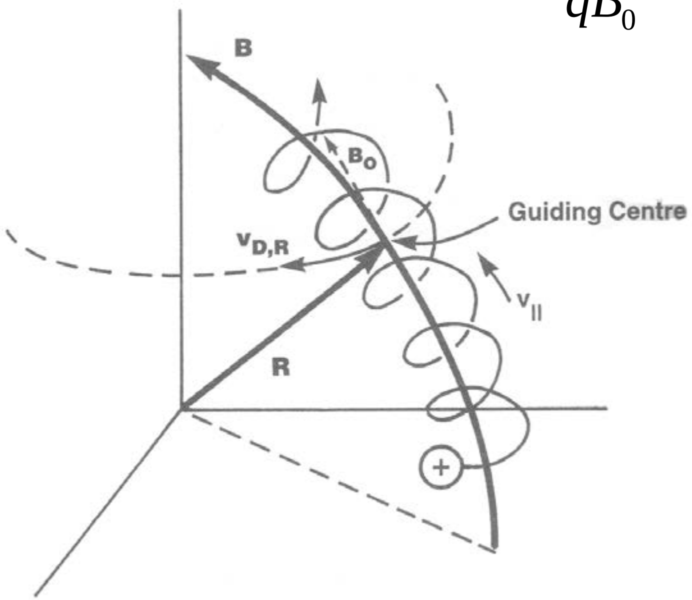
Magnetic Mirror

- **Single particle picture**
  - As the ion approaches the ends, it is increasingly subjected to drifts due to the inhomogeneity of the mirror field.
  - Drifts occur in azimuthal directions and the particles are still bound to their magnetic surfaces.

$$\mathbf{v}_{D,R} = \overline{\mathbf{v}}_{gc,x} = \frac{mv_{\parallel}^2}{qB_0^2}\frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$



$$r_L = \frac{v_{\perp} m}{|q| B_z}$$



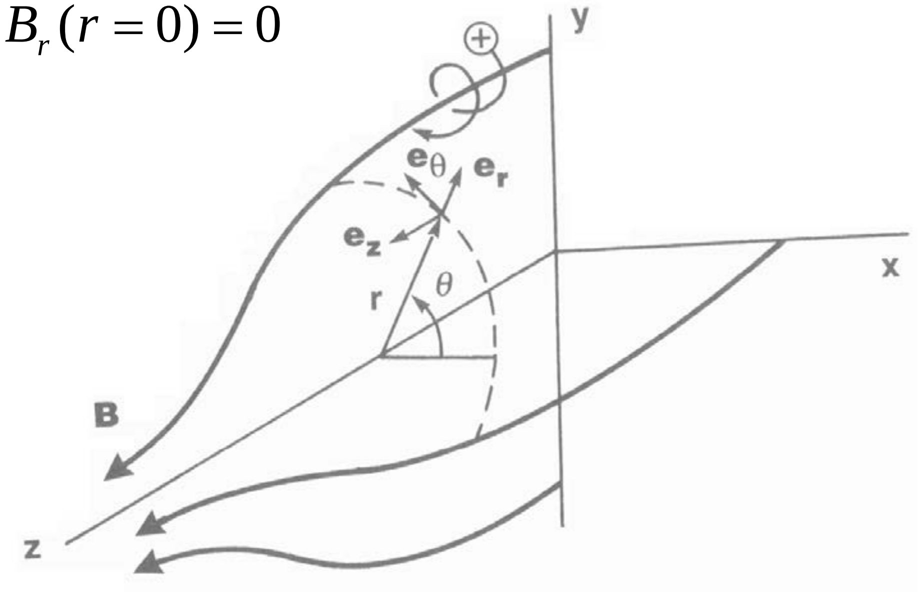
A. A. Harms et al, “Principles of Fusion Energy”, World Scientific (2000)

Individual Charge Trajectories

- **Axial field variation**

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times B_z\mathbf{e}_z\right) + q\left(\mathbf{v}\times B_r\mathbf{e}_r\right)$$

$$\nabla\cdot\mathbf{B} = 0 \rightarrow B_r \approx -\frac{r}{2}\frac{\partial B_z}{\partial z} \qquad B_\theta = 0,\quad B_r(r=0) = 0$$



$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$

μ : magnetic moment of the gyrating particle

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Magnetic Mirror

- **Conservation of kinetic energy and magnetic moment**

$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0$$

$$\frac{d}{dt}(\mu) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B}$$

Individual Charge Trajectories

- **Invariant of motion**

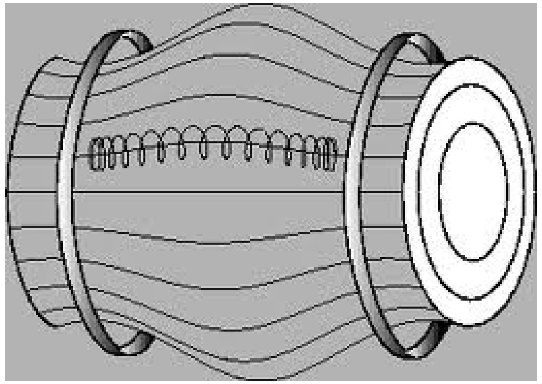
$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B} \qquad m\frac{dv_\parallel}{dt} = -\frac{\mu}{v_\parallel}\frac{dB}{dt} \qquad mv_\parallel\frac{dv_\parallel}{dt} = -\mu\frac{dB}{dt}$$

$$\mathbf{F}_\parallel = m\frac{d\mathbf{v}_\parallel}{dt} = -\mu\nabla_\parallel \mathbf{B} = -\mu\frac{\partial \mathbf{B}}{\partial s} = -\mu\frac{\partial \mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{dt}{ds} = -\mu\frac{\partial \mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt}$$

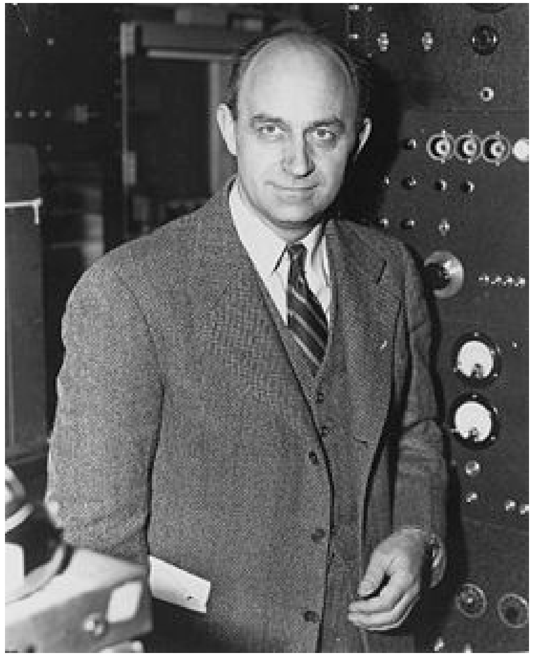
$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = \frac{d}{dt}(\mu B) + \left(-\mu\frac{dB}{dt}\right) = 0$$

$$\frac{d}{dt}(\mu B) = \mu\frac{dB}{dt} + B\frac{d\mu}{dt}$$

$\longrightarrow \dfrac{d\mu}{dt} = 0$: adiabatic invariant

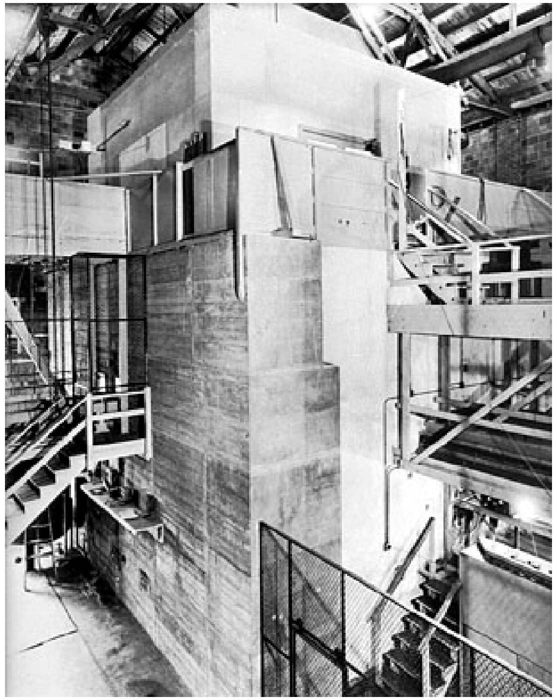


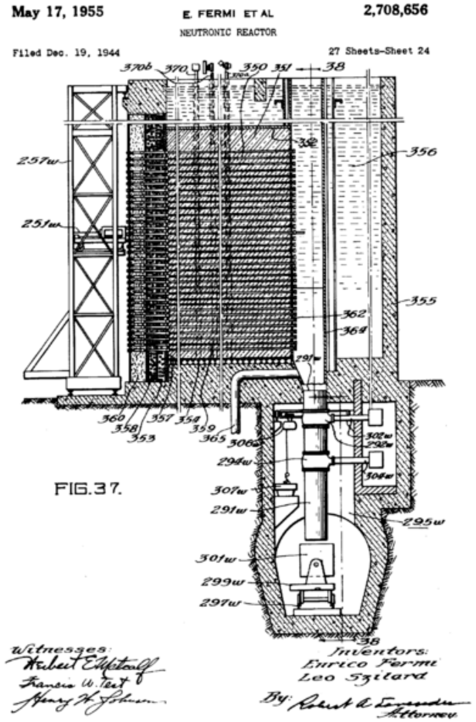
Magnetic Mirror



Enrico Fermi (1901-1954)

Nobel Laureate in physics in 1938
Cf. Marshall Rosenbluth (Doctoral student)





CP-1 (Chicago Pile-1, the world's first human-made nuclear reactor) and Drawings from the Fermi–Szilárd "neutronic reactor" patent

Magnetic Mirror

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

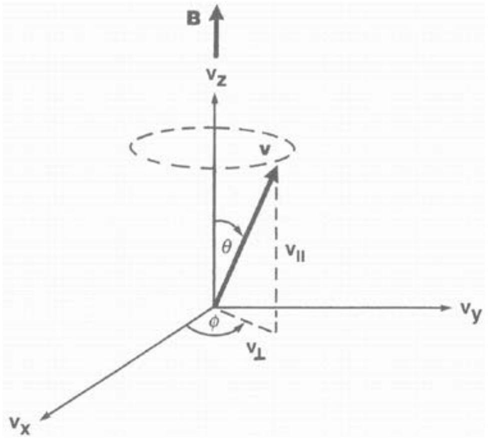
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Magnetic Mirror

$$E_0 = \frac{1}{2}mv^2 = \frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2 = const.$$

$$\boldsymbol{\mu} = \frac{\frac{1}{2}mv_\perp^2}{B} = \frac{\frac{1}{2}mv^2\sin^2\boldsymbol{\theta}}{B} = const.$$



The path of a fast proton in an irregular magnetic field of the type that we have assumed will be represented very closely by a spiraling motion around a line of force. Since the radius of this spiral may be of the order of 10^{12} cm, and the irregularities in the field have dimensions of the order of 10^{18} cm, the cosmic ray will perform many turns on its spiraling path before encountering an appreciably different field intensity. One finds by an elementary discussion that as the particle approaches a region where the field intensity increases, the pitch of the spiral will decrease. One finds precisely that

$$\boxed{\sin^2\vartheta/H \approx \text{constant},} \qquad (12)$$

where ϑ is the angle between the direction of the line of force and the direction of the velocity of the particle, and H is the local field intensity. As the particle approaches a region where the field intensity is larger, one will expect, therefore, that the angle ϑ increases until $\sin\vartheta$ attains the maximum possible value of one. At this point the particle is reflected back along the same line of force and spirals backwards until the next region of high field intensity is encountered. This process will be called a "Type A" reflection. If the magnetic field were static, such a reflection would not produce any change in the kinetic energy of the particle. This is not so, however, if the magnetic field is slowly variable. It may happen that a region of high field intensity moves toward the cosmic-ray particle which collides against it. In this case, the particle will gain energy in the collision. Conversely, it may happen that the region of high field intensity moves away from the particle. Since the particle is much faster, it will overtake the irregularity of the field and be reflected backwards, in this case with loss of energy. The net result will be an average gain, primarily for the reason that head-on collisions are more frequent than overtaking collisions because the relative velocity is larger in the former case.

$$\frac{w'}{w} = \frac{1+2B\beta\cos\vartheta+B^2}{1-B^2}, \qquad (13)$$

where βc is the velocity of the particle, ϑ is the angle of inclination of the spiral, and Bc is the velocity of the perturbation. It is assumed that the

Magnetic Mirror

- **Fermi as a genuine scientist**

spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

some equivalent mechanism. With respect to the injection of heavy nuclei I do not know a plausible answer to this point.

Magnetic Mirror

- **Condition for trapping of particles**

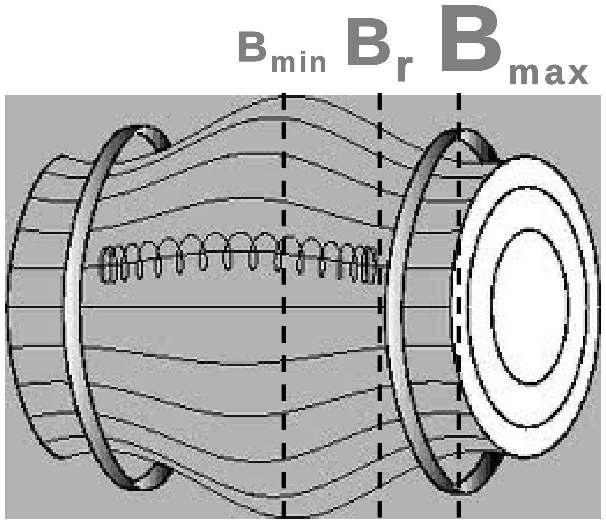
$$v_{\|}\Big|_{B_r \le B_{\max}} = 0$$

$$\mathbf{F}_{\|} = -\frac{1}{2}\frac{mv_{\perp}^2}{B}\nabla_{\|}B = -\mu\nabla_{\|}B$$

$$\mu = \frac{\frac{1}{2}mv_{\perp\min}^2}{B_{\min}} = \frac{\frac{1}{2}mv_{\perp r}^2}{B_r}$$

$$E = \frac{1}{2}mv_{\|\min}^2 + \frac{1}{2}mv_{\perp\min}^2 = \frac{1}{2}mv_{\|r}^2 + \frac{1}{2}mv_{\perp r}^2 = \frac{1}{2}mv_{\perp r}^2$$

$$\frac{v_{\perp\min}^2}{v_{\perp r}^2} = \frac{B_{\min}}{B_r} \longrightarrow \frac{v_{\perp\min}^2}{v_{\|\min}^2 + v_{\perp\min}^2} = \frac{B_{\min}}{B_r}$$



Magnetic Mirror

- **Condition for trapping of particles**

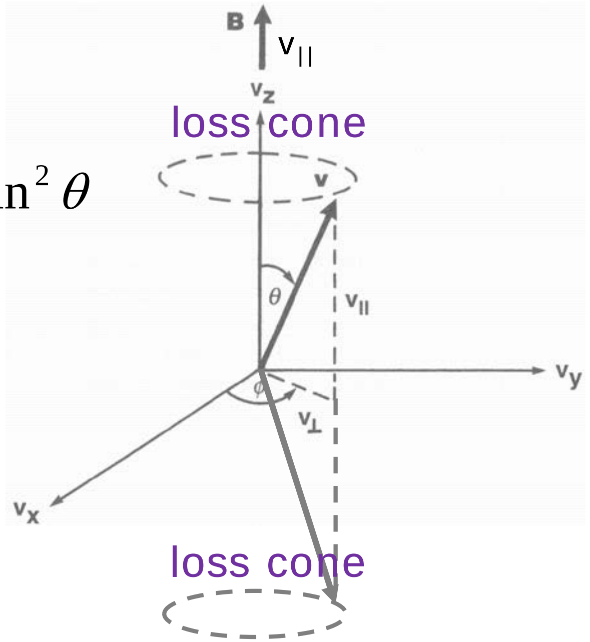
$$v_{\parallel}\Big|_{B_r \le B_{\max}} = 0$$

$$\mathbf{F}_{\parallel} = -\frac{1}{2}\frac{mv_{\perp}^2}{B}\nabla_{\parallel}B = -\mu\nabla_{\parallel}B$$

$$\mu = \frac{\frac{1}{2}mv_{\perp\min}^2}{B_{\min}} = \frac{\frac{1}{2}mv_{\perp r}^2}{B_r}$$

$$E = \frac{1}{2}mv_{\parallel\min}^2 + \frac{1}{2}mv_{\perp\min}^2 = \frac{1}{2}mv_{\parallel r}^2 + \frac{1}{2}mv_{\perp r}^2 = \frac{1}{2}mv_{\perp r}^2$$

$$\frac{v_{\perp\min}^2}{v_{\perp r}^2} = \frac{B_{\min}}{B_r} \longrightarrow \frac{v_{\perp\min}^2}{v_{\parallel\min}^2 + v_{\perp\min}^2} = \frac{B_{\min}}{B_r} = \frac{v_{\perp\min}^2}{v_{\min}^2} = \sin^2\theta$$



Condition for trapping of particles

$$\frac{B_{\min}}{B_r} = \sin^2\theta \ge \frac{B_{\min}}{B_{\max}}$$

Magnetic Mirror

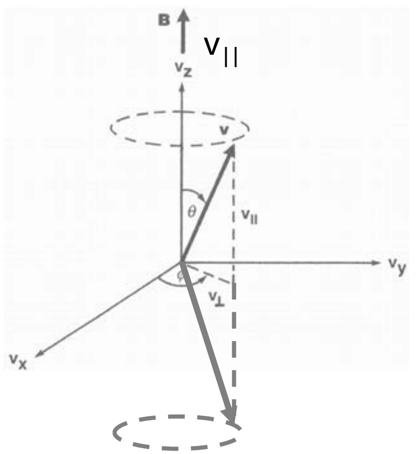
$$\frac{B_{\min}}{B_r} = \sin^2\theta \geq \frac{B_{\min}}{B_{\max}}$$

$$\theta_0 = \arcsin\sqrt{\frac{B_{\min}}{B_{\max}}}$$

- **Mirror ratio**

$$f_{loss} = \frac{\int\limits_{\text{double cone}} f(\vec{v})d^3v}{\int\limits_0^{\infty} f(\vec{v})d^3v} = \frac{\int\limits_0^{2\pi} d\phi\left[\int\limits_0^{\theta_0}\sin\theta d\theta + \int\limits_{\pi-\theta_0}^{\pi}\sin\theta d\theta\right]\int\limits_0^{\infty}\frac{f(v)}{4\pi v^2}v^2dv}{\int\limits_0^{2\pi} d\phi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^{\infty}\frac{f(v)}{4\pi v^2}v^2dv}$$

$$= 1 - \cos\theta_0$$



$$f_{trap} = 1 - f_{loss} = \cos\theta_0 = \sqrt{1 - \frac{B_{\min}}{B_{\max}}}$$

$$\frac{B_{\max}}{B_{\min}} \equiv R_m$$

mirror ratio:
Determining the effectiveness of confinement

$$f_{trap} = \sqrt{1 - \frac{1}{R_m}}$$

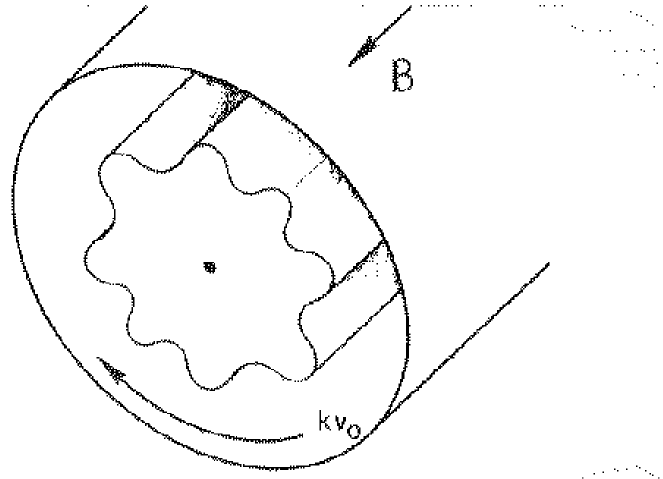
A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Stability in mirror?

Magnetic Mirror

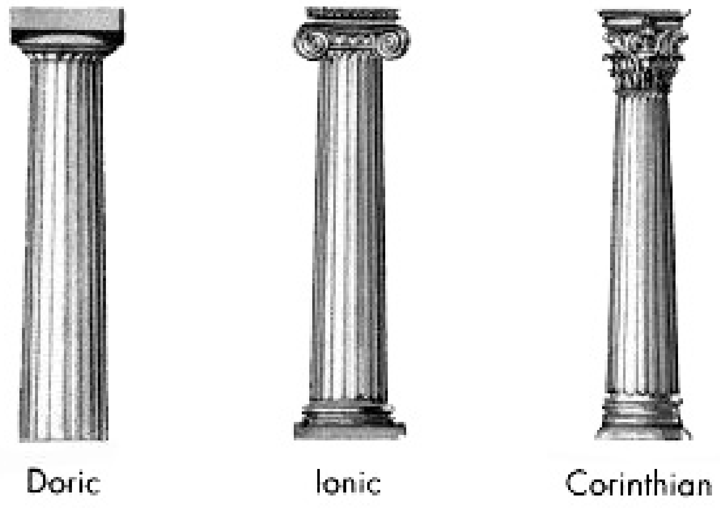
- **Instabilities**

- Flute instability: convex curvature of the magnetic field



A "flute" instability.

Figure 1: Flute instability — From F.F.Chen, 1974



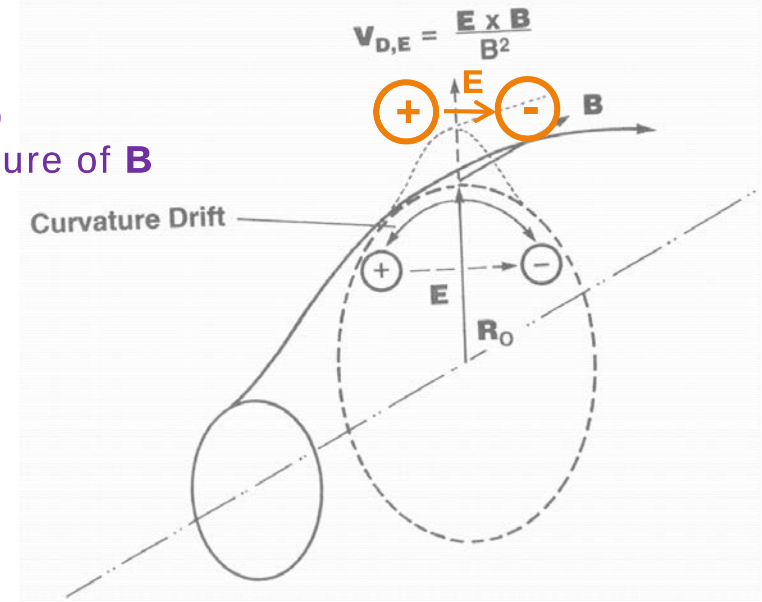
http://www.threes.com/index.php?view=article&id=20:greek-columns&option=com_content&Itemid=39

Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field

due to curvature of **B** in *Z*



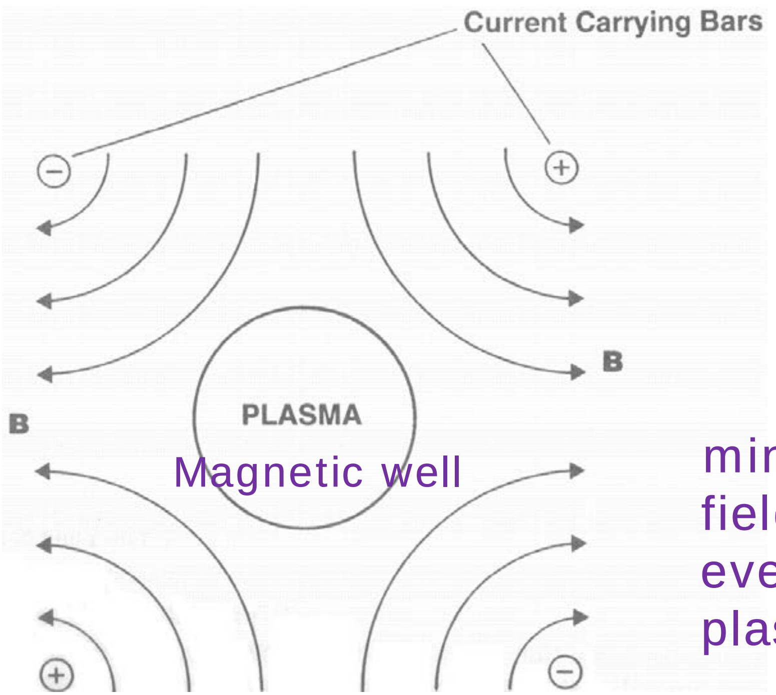
- Particle picture:
  The curvature drift leading to azimuthal polarisation to create the **E**-field resulting in the **E**x**B** drift displacing the plasma particles radially outward

$$\mathbf{v}_{D,\nabla B} = \pm \frac{1}{2} v_\perp r_L \frac{\mathbf{B} \times \nabla B}{B^2} \qquad \mathbf{v}_{D,R} = \frac{m v_\parallel^2}{q B_0^2} \frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$

Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field

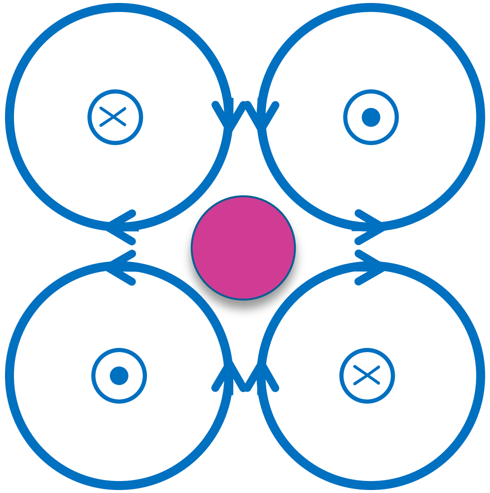


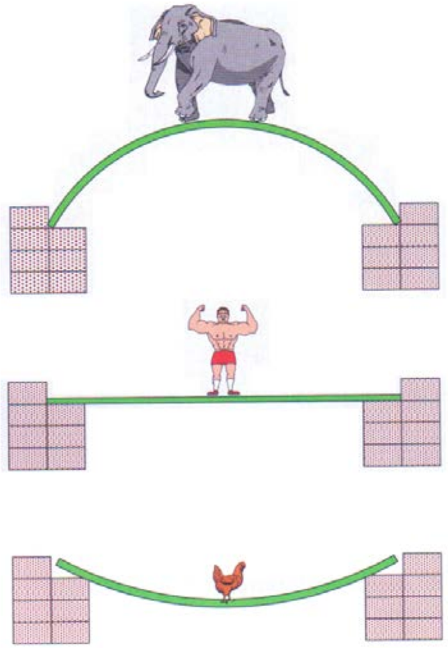
minimum-*B* field configuration: field lines are (almost) everywhere concave into the plasma

Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field





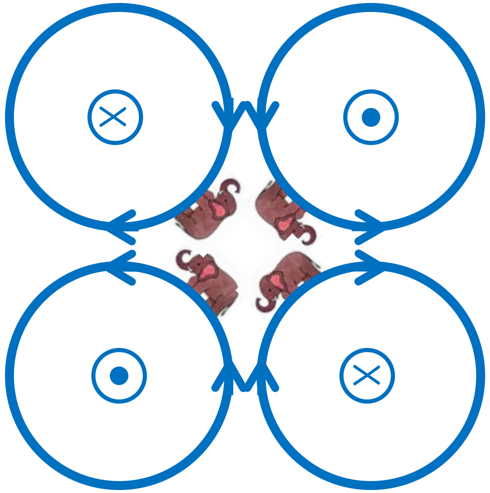
F. F. Chen, "An Indispensable Truth", Springer (2011)

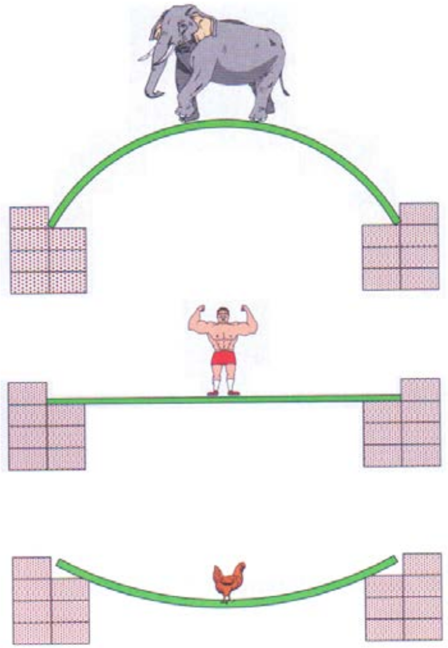
http://blog.naver.com/PostView.nhn?blogId=ray0620&logNo=150112423635&parentCategoryNo=1&viewDate=&currentPage=1&listtype=0
http://en.wikipedia.org/wiki/File:St_Louis_Gateway_Arch.jpg

Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field





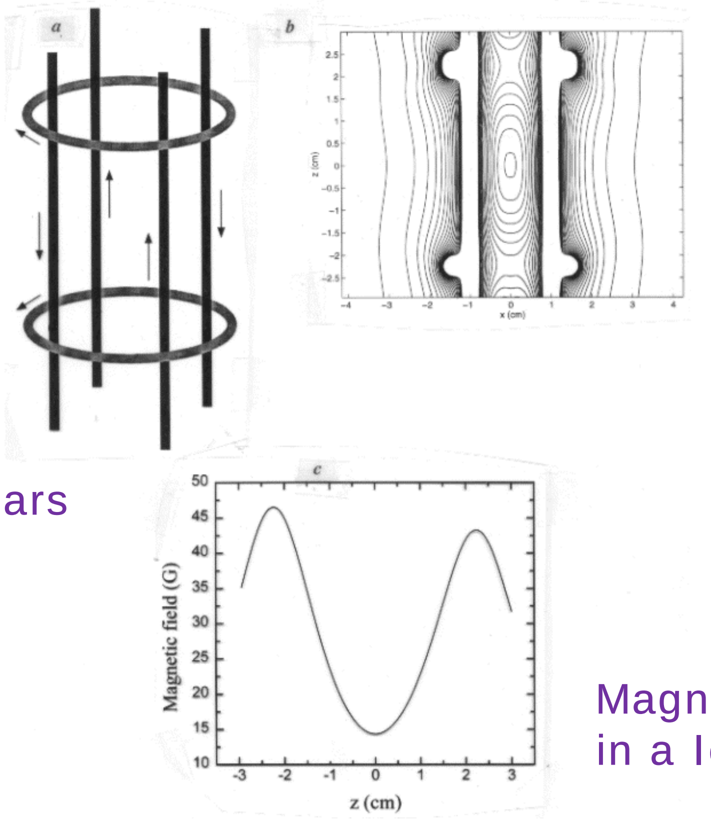
F. F. Chen, "An Indispensable Truth", Springer (2011)

http://blog.naver.com/PostView.nhn?blogId=ray0620&logNo=150112423635&parentCategoryNo=1&viewDate=&currentPage=1&listtype=0
http://en.wikipedia.org/wiki/File:St_Louis_Gateway_Arch.jpg

Magnetic Mirror

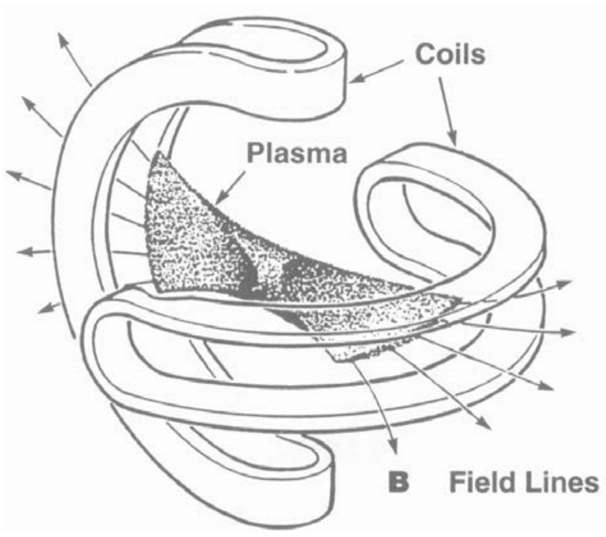
- **Instabilities**

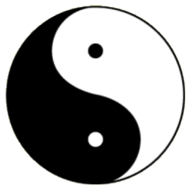
- Flute instability: convex curvature of the magnetic field



Ioffe bars

Magnetic field profiles
in a Ioffe-Pritchard trap





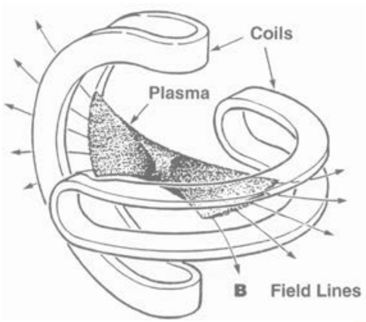
Yin-Yang coil

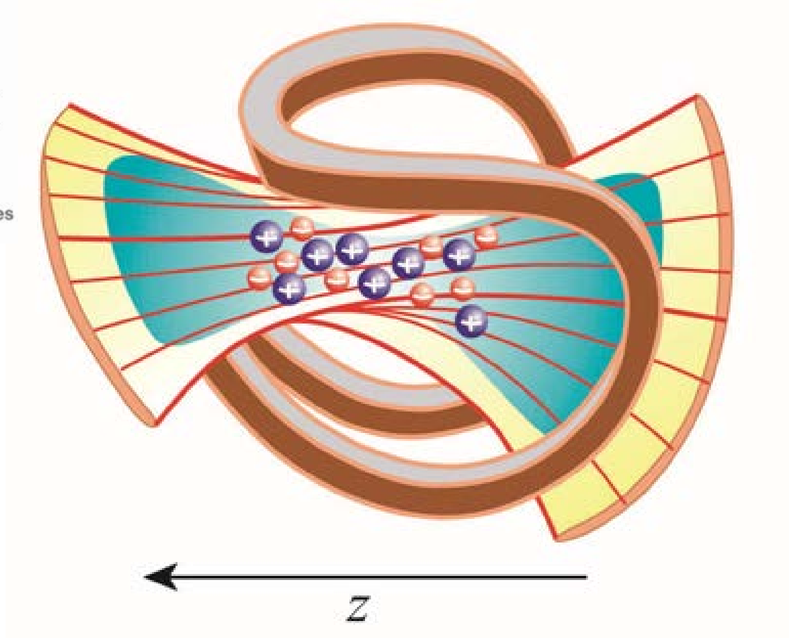
A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field





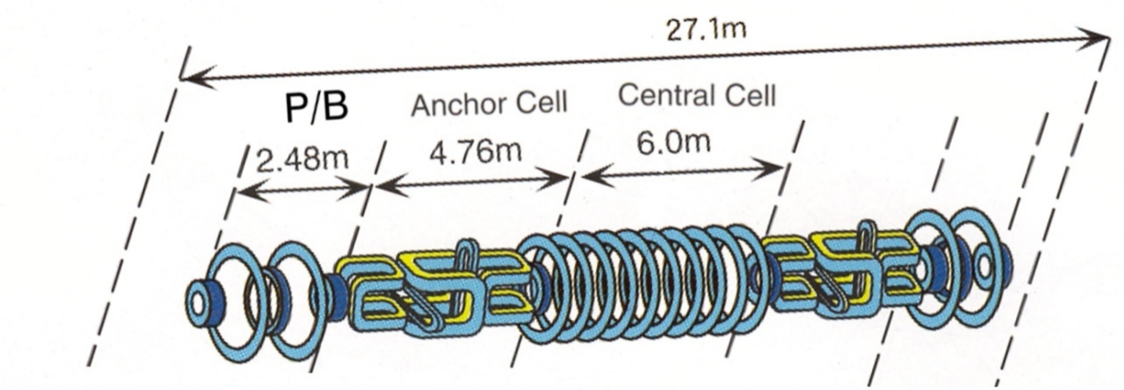
Baseball coil

http://www.wangnmr.com/Copper_magnet_catolog.htm

Magnetic Mirror

- **Instabilities**

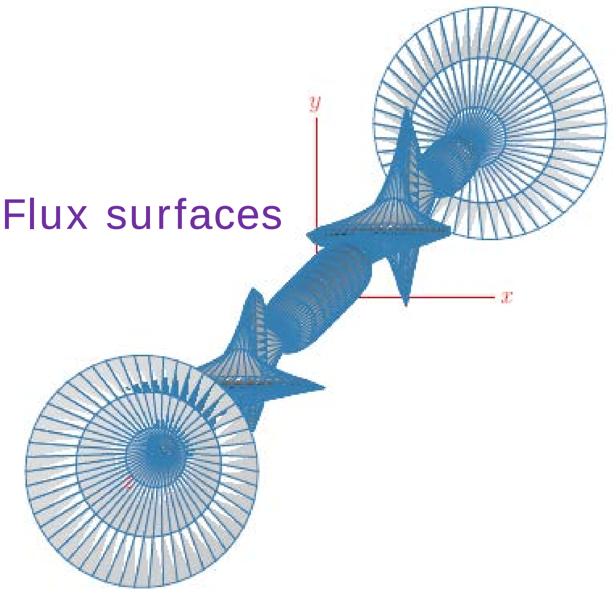
- Flute instability: convex curvature of the magnetic field



Plug/Barrier
(potential
plugging)
ECRH

Baseball coils
(MHD stabilising)
ICRF

Flux surfaces

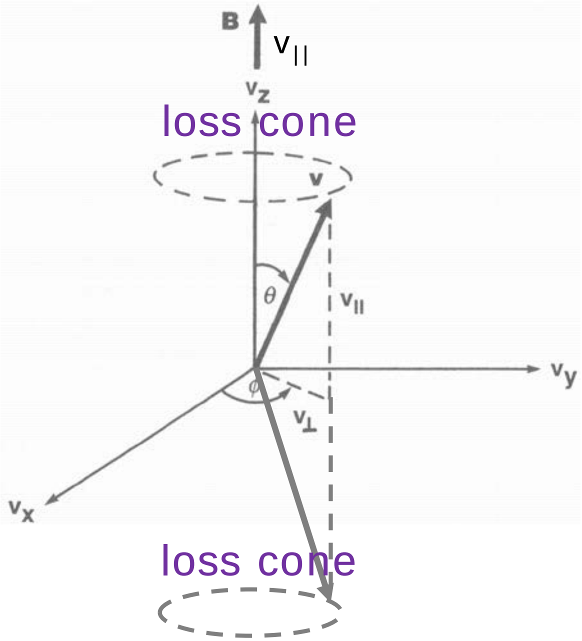


Gamma 10 Tandem mirror
(Univ. of Tsukuba, Japan)

Magnetic Mirror

- **Instabilities**

- Velocity-space instability: driven by the non-Maxwellian velocity distribution due to the preferred loss of particles with large $v_{||}/v_{\perp}$.
- Enhancing the velocity-space diffusion into the loss cone
- Observed that such it is less harmful to plasma confinement when the mirror device is short in dimension



Magnetic Mirror

- **Classical Mirror Confinement**

- If collisions are neglected, particles are trapped in a mirror when they do not appear in the loss cone.
- Collisions can bring them randomly from the confinement region into the loss cone.
- Due to their relatively small mass, electrons diffuse more rapidly.

→ a positive electrostatic potential built up in the confined plasma tending to retain the remaining electrons in the magnetic bottle

- Overall plasma confinement time is governed by the ion escape time.

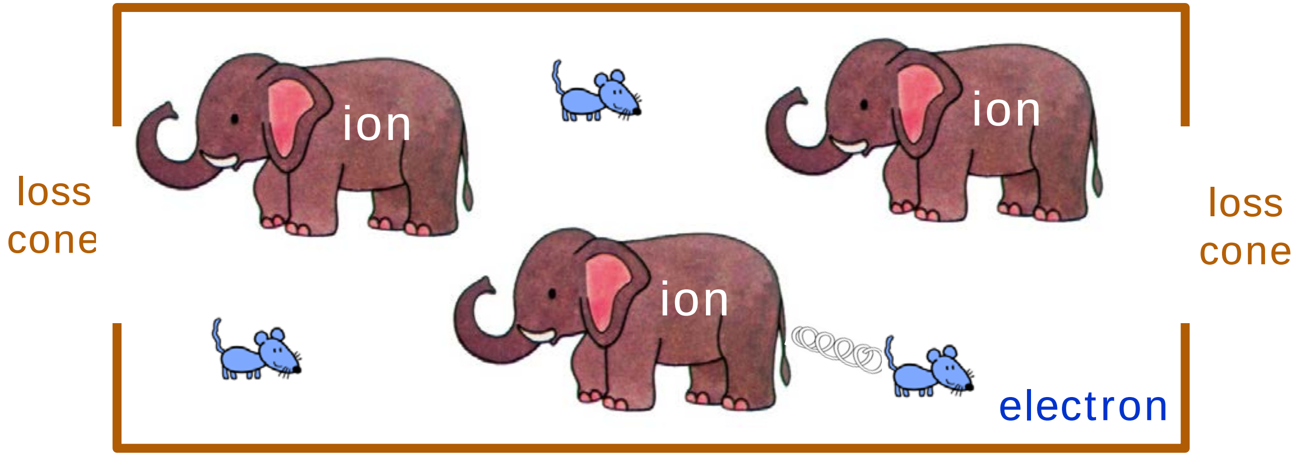
Magnetic Mirror

- **Classical Mirror Confinement**

- If collisions are neglected, particles are trapped in a mirror when they do not appear in the loss cone.
- Collisions can bring them randomly from the confinement region into the loss cone.
- Due to their relatively small mass, electrons diffuse more rapidly.

→ a positive electrostatic potential built up in the confined plasma tending to retain the remaining electrons in the magnetic bottle

- Overall plasma confinement time is governed by the ion escape time.



Magnetic Mirror

- **Classical Mirror Confinement**

- If collisions are neglected, particles are trapped in a mirror when they do not appear in the loss cone.
- Collisions can bring them randomly from the confinement region into the loss cone.
- Due to their relatively small mass, electrons diffuse more rapidly.

→ a positive electrostatic potential built up in the confined plasma tending to retain the remaining electrons in the magnetic bottle

- Overall plasma confinement time is governed by the ion escape time.

$$\tau_M \approx \tau_{ii} \ln\left(\frac{B_{\max}}{B_{\min}}\right) \approx C \frac{A_i^{1/2} T_i^{3/2} \ln\left(\frac{B_{\max}}{B_{\min}}\right)}{n_i Z_i^4}$$: mirror confinement time

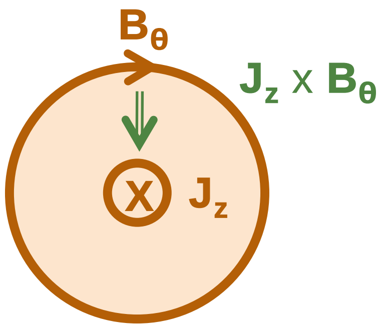
$$\tau_{ii} = \frac{1}{n_i \langle \sigma_s v_r \rangle} \propto \frac{A_i^{1/2} (kT_i)^{3/2}}{n_i q_i^4 \langle \sigma_s v_r \rangle}$$: ion-ion collision time

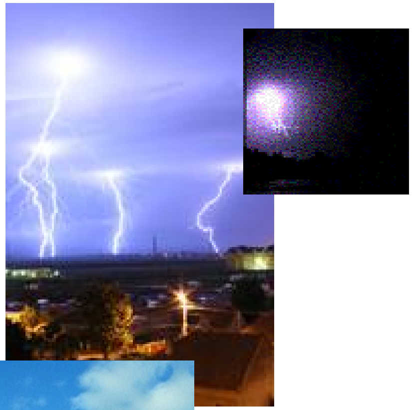
- Mirror confinement time does not depend on the actual magnitude of **B** or the plasma size but on the size of the loss cone.
- Higher density enhances the scattering into the loss cone.

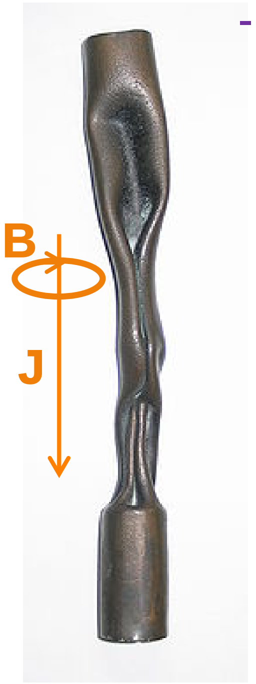
What is Z-pinch?

Magnetic Pinch

- **The *Z* Pinch**







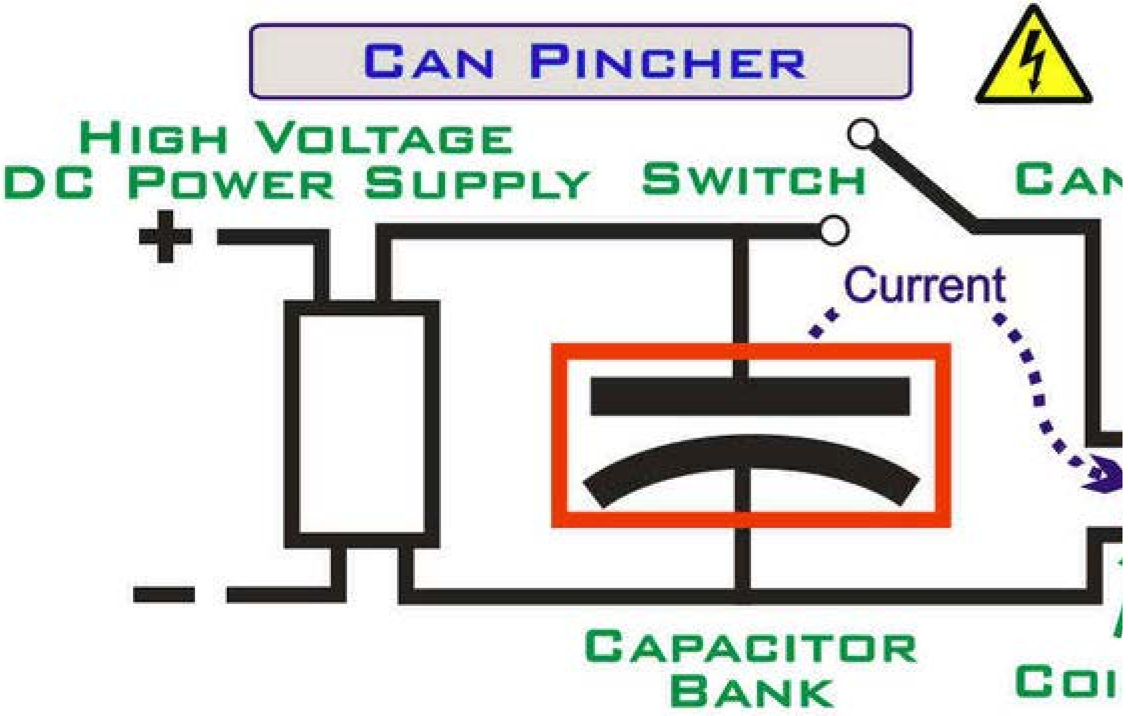
- The Lorenz force created by a lightning strike crushed this hollow lightning rod and led to the discovery of the pinch.

G. Thomson and P. Thonemann

http://joiimg.tistory.com/416
http://www.thunderbolts.info/forum/phpBB3/viewtopic.php?p=731&sid=824cd77d679b252e92355fcbf212ed7a

http://www.plasma-universe.com/Pinch_(plasma_physics)?title=Special:Booksources&isbn=0387975756
http://ask.nate.com/qna/view.html?n=6318216

Magnetic Pinch

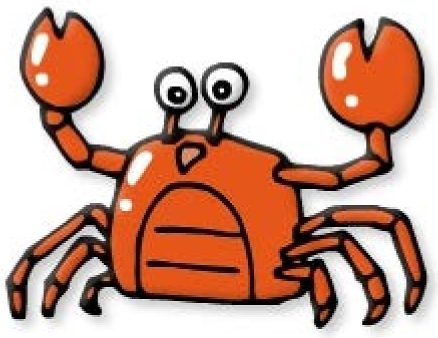


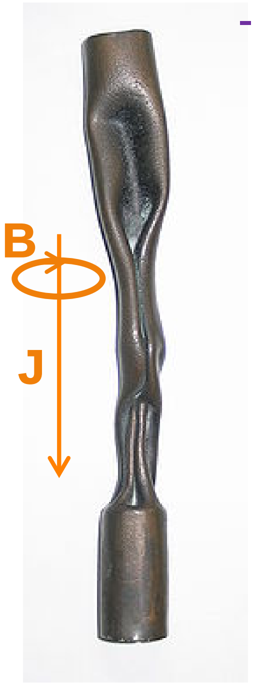
Wikipedia
The Free Encyclopedia

Magnetic Pinch

- **The *Z* Pinch**

  - The plasma carries an electric current and is confined by the magnetic field induced by this current.
  - As the current is increased, the larger magnetic field compresses the plasma and also raises its temperature by Joule-heating.
  - Confinement and heating is simultaneously provided.





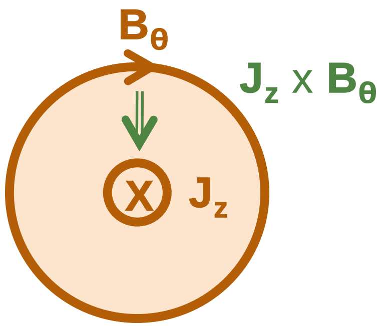
- The Lorenz force created by a lightning strike crushed this hollow lightning rod and led to the discovery of the pinch.

G. Thomson and P. Thonemann

http://joiimg.tistory.com/416
http://www.thunderbolts.info/forum/phpBB3/viewtopic.php?p=731&sid=824cd77d679b252e92355fcbf212ed7a

http://www.plasma-universe.com/Pinch_(plasma_physics)?title=Special:Booksources&isbn=0387975756
http://ask.nate.com/qna/view.html?n=6318216

Magnetic Pinch



- **The *Z* Pinch**
  - The plasma carries an electric current and is confined by the magnetic field induced by this current.
  - As the current is increased, the larger magnetic field compresses the plasma and also raises its temperature by Joule-heating.
  - Confinement and heating is simultaneously provided.

Where did the neutrons come from in ZETA?

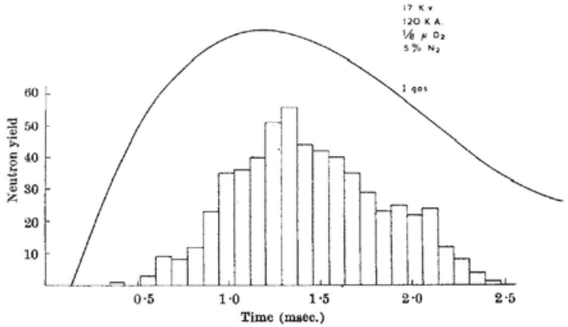


Fig. 4. Histogram showing the number of neutrons counted at various times during the current pulse

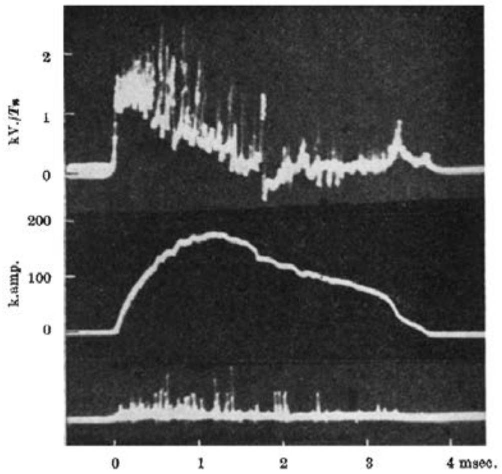
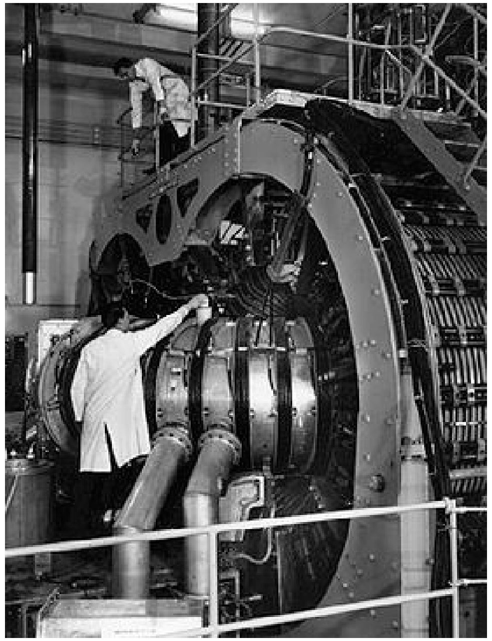


Fig. 2. Oscillograph recordings of the voltage per turn of the transformer, and the secondary current I_s. The lower trace shows the pulses produced by proton recoil in a scintillation neutron counter. Conditions: gas, deuterium + 5 per cent nitrogen + 10 per cent oxygen; pressure, $0{\cdot}13 \times 10^{-3}$ mm. mercury; axial field, 160 gauss

*P. C. Thonemann et al, Nature **181** 217 (1958)*

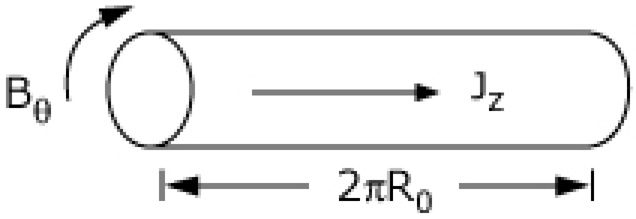


ZETA (1954-58, UK)

Magnetic Pinch

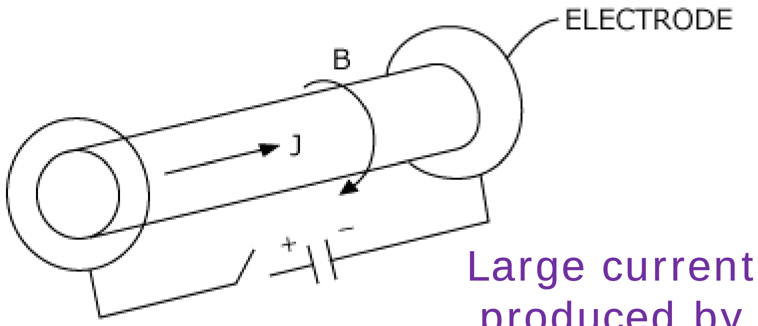
- **The *Z* Pinch**

How about the pinch effect in a electric cable/wire?



self field induced by J_z

longitudinal plasma current



Large current produced by capacitor

- Equilibrium

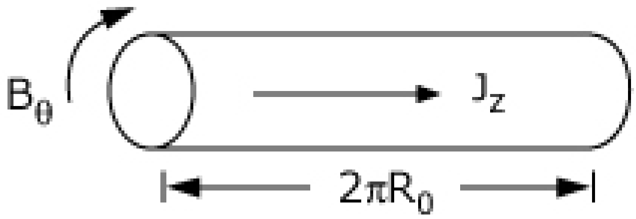
Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$
2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$
3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

$$\boxed{\begin{aligned} \nabla p &= \vec{J} \times \vec{B} \\ \nabla \times \vec{B} &= \mu_0 \vec{J} \\ \nabla \cdot \vec{B} &= 0 \end{aligned}}$$

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Magnetic Pinch



- **The *Z* Pinch**

Sequence of solution of the MHD equilibrium equations

1. The $\nabla\cdot\mathbf{B} = 0$

$$\frac{1}{r}\frac{\partial B_\theta}{\partial \theta} = 0$$

2. Ampere's law: $\mu_0\mathbf{J} = \nabla\times\mathbf{B}$

$$J_z = \frac{1}{\mu_0 r}\frac{d}{dr}(rB_\theta)$$

3. The momentum equation: $\mathbf{J}\times\mathbf{B} = \nabla p$

$$J_z B_\theta = -\frac{dp}{dr}$$

$$\frac{dp}{dr} + \frac{B_\theta}{\mu_0 r}\frac{d}{dr}(rB_\theta) = 0 \qquad \frac{d}{dr}\left(p + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

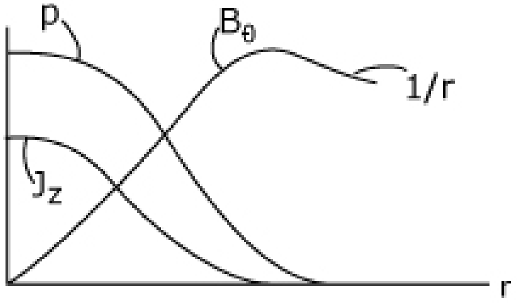
particle pressure + magnetic pressure force

tension force by the curvature of the magnetic field lines

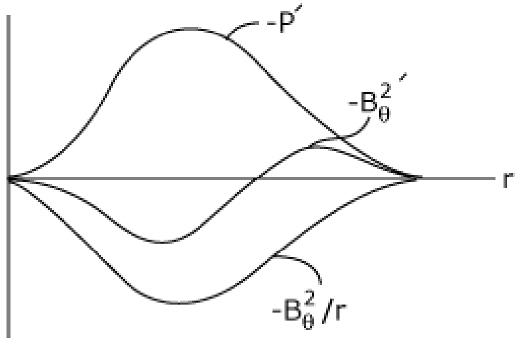
Magnetic Pinch

$$\frac{d}{dr}\left(p+\frac{B_\theta^2}{2\mu_0}\right)+\frac{B_\theta^2}{\mu_0 r}=0$$

- **The *Z* Pinch**
  - It is the tension force and not the magnetic pressure gradient that provides radial confinement of the plasma.



FIELDS

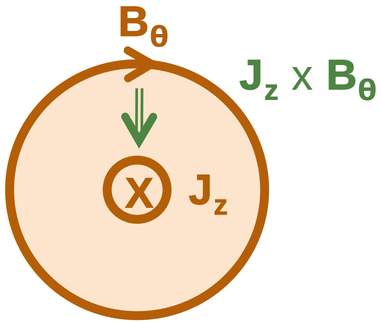


FORCES

$$B_\theta=\frac{\mu_0 I_0}{2\pi}\frac{r}{r^2+r_0^2}$$

$$J_z=\frac{I_0}{\pi}\frac{r_0^2}{(r^2+r_0^2)^2}$$

$$p=\frac{\mu_0 I_0^2}{8\pi^2}\frac{r_0^2}{(r^2+r_0^2)^2}$$



Bennett profiles
(Bennett, 1934)

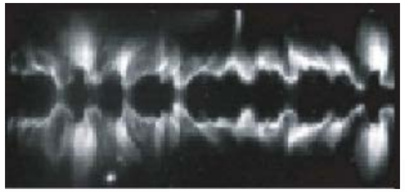
- The magnetic pressure and plasma pressure each produce positive (outward forces) near the outside of the plasma.
- The plasma is confined by magnetic tension.

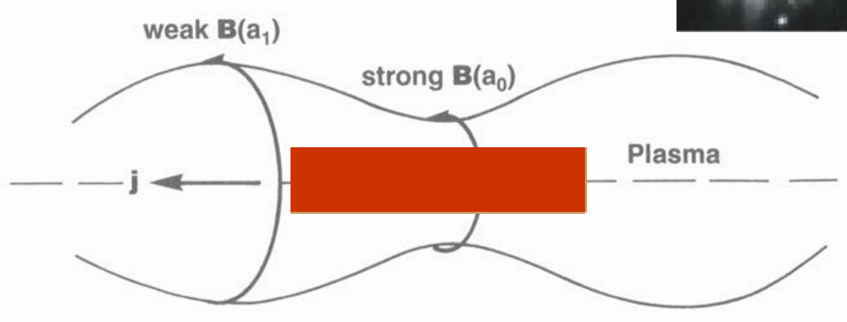
J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note
http://scienceray.com/technology/industry/where-do-rubber-bands-come-from/

Magnetic Pinch

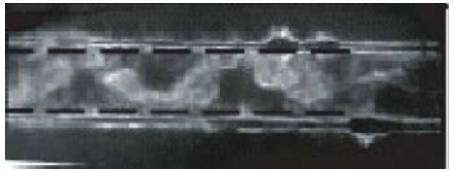
- **The *Z* Pinch**
  - A number of linear *Z*-pinch experiments were constructed during the early years of the fusion program.
  - Large currents of ~0.1 MA are needed thus rendering the *Z* pinch to operate only in short pulses.
  - Exhibiting disastrous instabilities, often leading to a complete quenching of the plasma after ~1-2 μs.

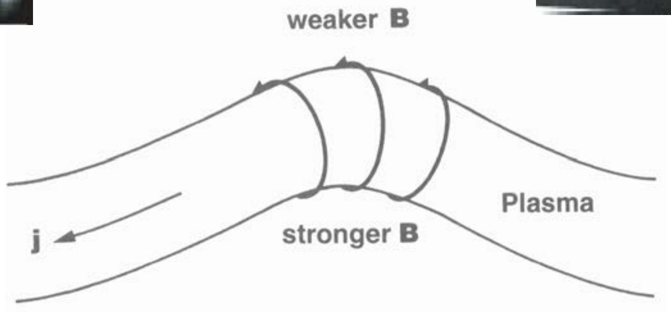
Sausage or pinch instability





Helical kink instability

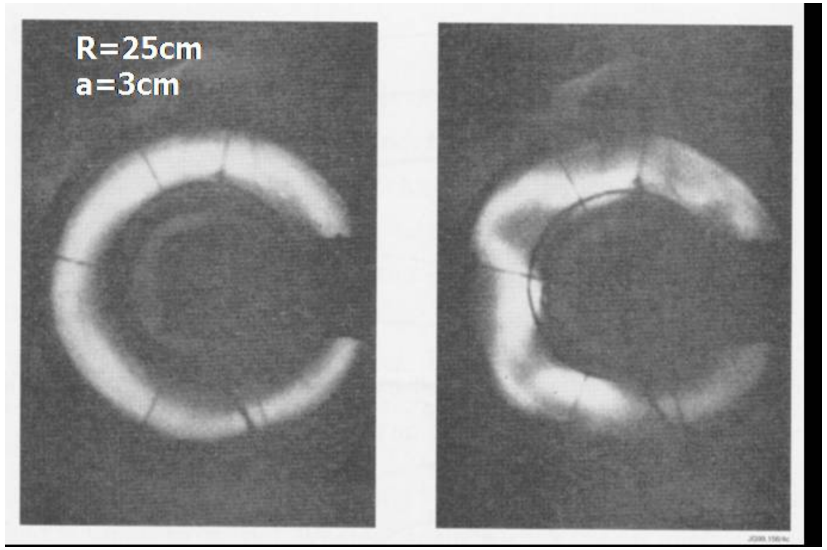




A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Magnetic Pinch

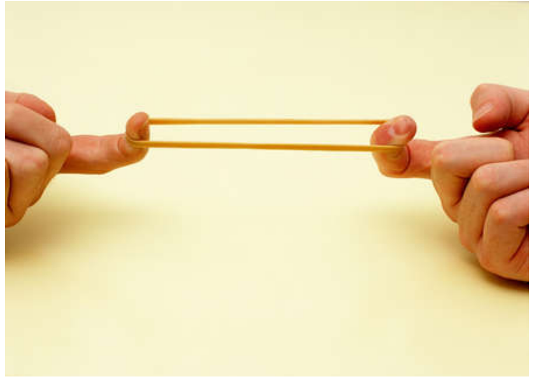
- **The *Z* Pinch**
  - A number of linear *Z*-pinch experiments were constructed during the early years of the fusion program.
  - Large currents of ~0.1 MA are needed thus rendering the *Z* pinch to operate only in short pulses.
  - Exhibiting disastrous instabilities, often leading to a complete quenching of the plasma after ~1-2 μs.

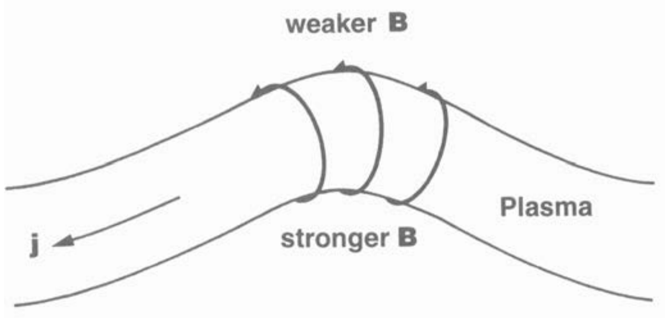


"kink instability" in one of the earliest *Z*-pinch devices, a pyrex tube used by the AEI team at Aldermaston, or earlier while still at Imperial College

Magnetic Pinch

- **The *Z* Pinch**
  - A number of linear *Z*-pinch experiments were constructed during the early years of the fusion program.
  - Large currents of ~0.1 MA are needed thus rendering the *Z* pinch to operate only in short pulses.
  - Exhibiting disastrous instabilities, often leading to a complete quenching of the plasma after ~1-2 μs.



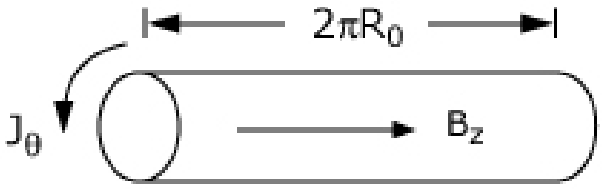


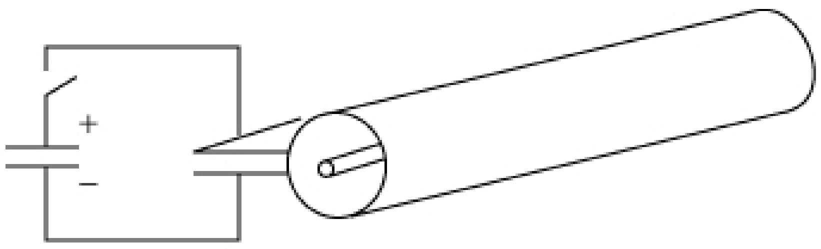
θ-direction tension for equilibrium, z-direction tension for stability

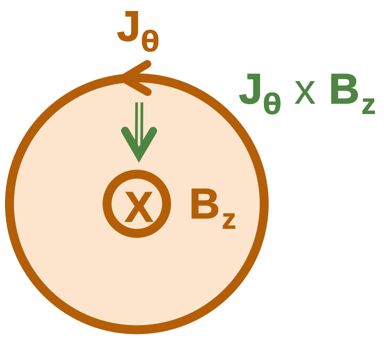
What is θ-pinch?

Magnetic Pinch

- **The θ Pinch**







Magnetic and Kinetic Pressure

- **Plasma Equilibrium**

$$\boxed{\begin{gathered}\nabla p = \vec{J} \times \vec{B} \\ \nabla \times \vec{B} = \mu_0 \vec{J} \\ \nabla \cdot \vec{B} = 0\end{gathered}}$$

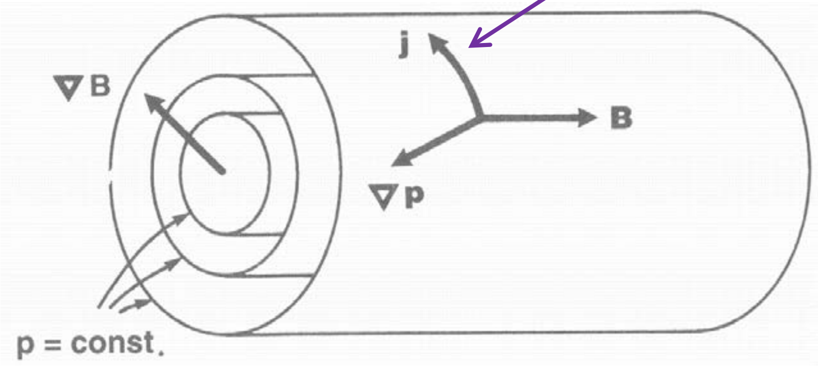
→ Force balance — kinetic pressure balanced by **J**x**B** (Lorentz) force

→ Ampere's law

→ Closed magnetic field lines

$\vec{B} \cdot \nabla p = 0 \quad \vec{J} \cdot \nabla p = 0$

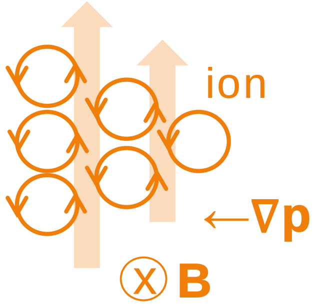
induced by the pressure gradient:
causing a decrease in **B** → diamagnetism



Diamagnetic current

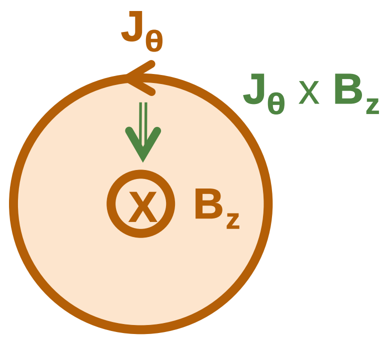
$$\vec{v}_{D,\nabla p} = -\frac{\nabla p \times \vec{B}}{nqB^2}$$

$$\vec{J} = n_i q_i \vec{v}_{D,i} + n_e q_e \vec{v}_{D,e} = \frac{\vec{B} \times \nabla p}{B^2}$$

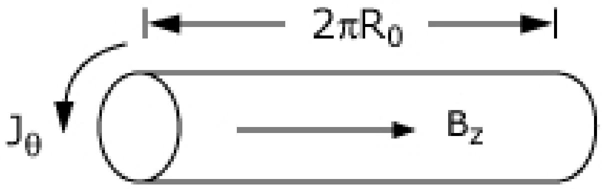


- If B is applied, plasma equilibrium can be built by itself due to induction of diamagnetic current. $\nabla p = \vec{J} \times \vec{B}$

Magnetic Pinch

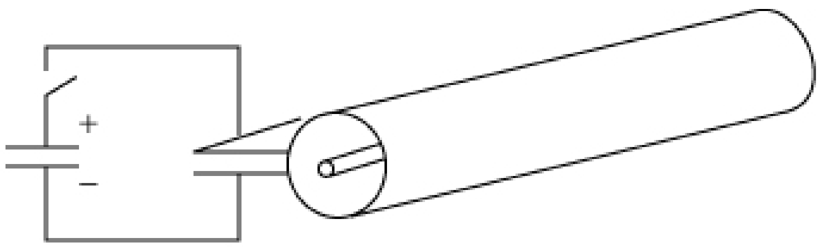


- **The θ Pinch**



induced diamagnetic current

externally driven



- Equilibrium

Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$
2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$
3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

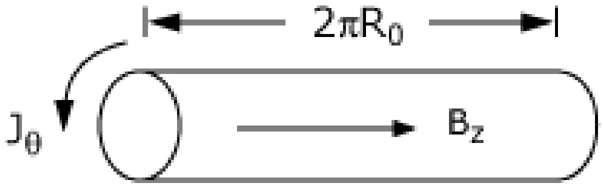
$$\nabla p = \vec{J} \times \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Magnetic Pinch



- **The *θ* Pinch**

Sequence of solution of the MHD equilibrium equations

1. The $\nabla\cdot\mathbf{B} = 0$

$$\frac{\partial B_z}{\partial z} = 0$$

2. Ampere's law: $\mu_0\mathbf{J} = \nabla\times\mathbf{B}$

$$J_\theta = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial r}$$

3. The momentum equation: $\mathbf{J}\times\mathbf{B} = \nabla p$

$$J_\theta B_z = \frac{dp}{dr}$$

$$\frac{d}{dr}\left(p + \frac{B_z^2}{2\mu_0}\right) = 0 \qquad p + \frac{B_z^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}$$

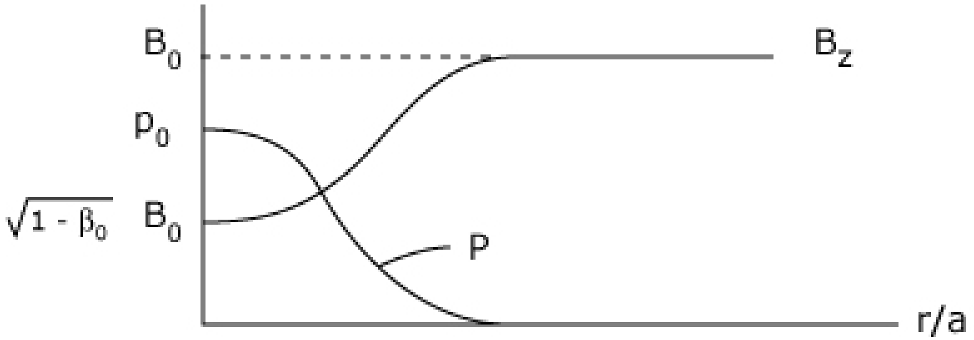
- At any local value of *r*, the sum of the particle pressure and the magnetic pressure is a constant, equal to the externally applied magnetic pressure.
- The plasma is confined radially by the pressure of the externally applied magnetic field.

Magnetic Pinch

• **The θ Pinch**

Plot the forces.

- Typical example



$$p = p_0 \exp(-r^2/a^2)$$

$$B_z = B_0[1-\beta_0 \exp(-r^2/a^2)]^{1/2}$$

$$\beta_0 \equiv 2\mu_0 p / B_0^2$$

$$p + \frac{B_z^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}$$

- The plasma is confined radially by the pressure of the externally applied magnetic field.

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Magnetic Pinch

- **The θ Pinch**

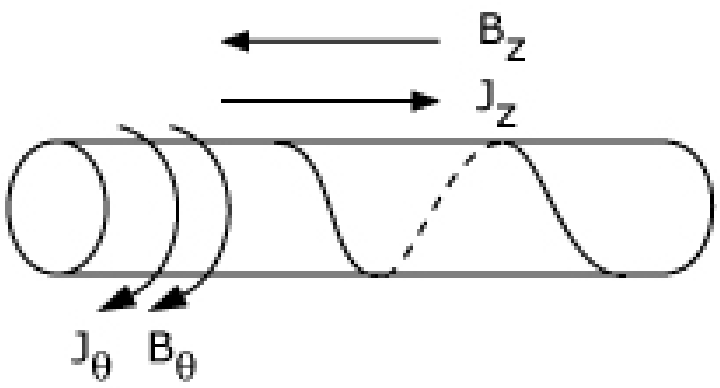
  - One of the early successes of the fusion program in terms of the performance.
  - T_i ~ 1-4 keV, n ~ 1-2x10^{22} m^{-3}, $\beta(0)$ ~ 0.7-0.9
  - High temperature using the implosion heating method: rapidly rising magnetic field acting like a piston
  - No indication of macroscopic instability (neutrally stable)
  - Severe end loss ($\tau \sim L/V_{Ti}$, e.g. 10 μs for a 5 m device)

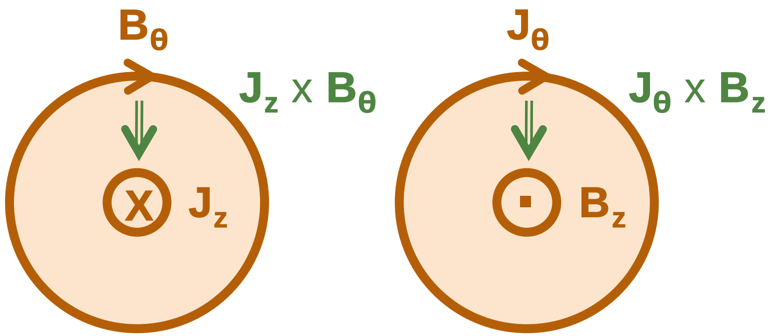
What is screw pinch?

Magnetic Pinch

- **The General Screw Pinch**

  - A hybrid combination of *Z* pinch and *θ* pinch
  - This combination of fields allows the flexibility to optimise configurations w.r.t. force balance and stability.





- Equilibrium

Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$
2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$
3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

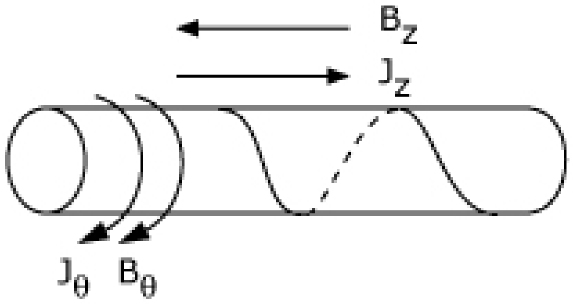
$$\nabla p = \vec{J} \times \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

Magnetic Pinch

- **The General Screw Pinch**



- Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$

$$\frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0$$

2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$

$$J_\theta = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial r} \qquad J_z = \frac{1}{\mu_0 r}\frac{d}{dr}(rB_\theta)$$

3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

$$J_\theta B_z - J_z B_\theta = \frac{dp}{dr}$$

$$\frac{d}{dr}\left(p + \frac{B_\theta^2 + B_z^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

- Even though the equations are nonlinear, the forces superpose because of symmetry.

Magnetic Pinch

- **The General Screw Pinch**

- Because of its flexibility the general screw-pinch relation describes a wide variety of configurations.