7.8 2D imaging basics

Spatial localization is necessary for MRI. The simplest approach is acquiring data slice by slice instead of a full 3D object. This can be achieved by using a slice selective RF excitation.

Let's say we applied a z gradient. Then the resonance frequency of each position along z-axis becomes different.

Slice selection for 2D imaging

Figure: slice selective excitation

- Basic procedure of 2D imaging

1) Selectively excite a slice

2) Change G_x and G_y and record FID.

3) Wait for recovery (Why?)

4) Repeat the measurement

7.9 Bloch equation revisit

Until we have learned that

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma\mathbf{B}$$

$$\frac{d\mathbf{M}}{dt} = -\frac{\mathrm{M_x}\hat{\boldsymbol{\imath}} + \mathrm{M_y}\hat{\boldsymbol{\jmath}}}{\mathrm{T_2}}$$

$$\frac{d\mathbf{M}}{dt} = -\frac{\mathrm{M_z} - \mathrm{M_o}}{\mathrm{T_1}}\hat{\boldsymbol{k}}$$

What is **M**?

What does **B** consist of?

The first term is; The second term is; The third term is

Precession happens on; T_1 recovery happens on; T_2 decay happens on

So the full version of Bloch equation becomes

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma\mathbf{B} - \frac{\mathrm{M_x}\hat{\boldsymbol{\imath}} + \mathrm{M_y}\hat{\boldsymbol{\jmath}}}{\mathrm{T_2}} - \frac{\mathrm{M_z} - \mathrm{M_o}}{\mathrm{T_1}}\hat{\boldsymbol{k}}$$

Ok now you are all set to solve the equation.

Still let's begin with the simplest one.

7.9.1 Homogeneous object, uniform field

We will only consider B_o field and the object is uniform: (i.e. it has one T_1 and T_2 values)

Let's ignore the relaxations

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma B_o \hat{\mathbf{k}}$$

Based on our memory (?), the solution is

$$M_x(t) + iM_y(t) = M_0 \exp(-i\gamma B_o t)$$

$$M_z(t) = M_z(0)$$

Another way to solve this equation is looking up a differential equation list and find out a solution but we need to change the equation a bit.

Differential equations can be written as:

$$\frac{d\mathbf{M}}{dt} = \begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \mathbf{M} \times \gamma B_o \hat{\mathbf{k}} = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ \gamma B_o \end{bmatrix}$$

$$= \begin{bmatrix} \gamma B_o M_y \\ -\gamma B_o M_x \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \gamma B_o & 0 \\ -\gamma B_o & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

i.e.

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \gamma B_o & 0 \\ -\gamma B_o & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}$$

So

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \gamma B_o \\ -\gamma B_o & 0 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix}$$

and

$$\frac{dM_z}{dt} = 0$$

The solutions for these differential equations are

$$M_x(t) + iM_y(t) = \left(M_x(0) + iM_y(0)\right) \exp(-i\gamma B_o t)$$

$$M_z(t) = M_z(0)$$

These equations can be simplified in matrix form:

$$M_x(t) + iM_y(t) = \left(M_x(0) + iM_y(0)\right)\exp(-i\omega_0 t)$$

$$= \left(M_x(0) + iM_y(0)\right)(\cos\omega_0 t \ - i\sin\omega_0 t)$$

$$= \cos\omega_0 t \cdot M_x(0) + \sin\omega_0 t \cdot M_y(0)$$

$$+i\left(-\sin\omega_0 t \cdot M_y(0) + \cos\omega_0 t \cdot M_y(0)\right)$$

$$\mathbf{M}(t) = \begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{bmatrix}$$

$$= \mathbf{R_z}(\omega_0 t)\mathbf{M^o}$$

where

$\mathbf{M^o}$**=** initial magnetization

$\omega_0 = \gamma B_o$ = Larmor frequency

For T_2 decay,

$$\frac{d\mathbf{M}}{dt} = -\frac{\mathrm{M_x}\hat{\boldsymbol{\imath}} + \mathrm{M_y}\hat{\boldsymbol{\jmath}}}{\mathrm{T_2}}$$

In matrix form

$$\begin{bmatrix} \frac{d\mathrm{M}_x}{dt} \\ \frac{d\mathrm{M}_y}{dt} \\ \frac{d\mathrm{M}_z}{dt} \end{bmatrix} = \begin{bmatrix} -1/\mathrm{T_2} & 0 & 0 \\ 0 & -1/\mathrm{T_2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{M}_x \\ \mathrm{M}_y \\ \mathrm{M}_z \end{bmatrix}$$

which can be simplified as:

$$\begin{bmatrix} \dfrac{dM_x}{dt} \\ \dfrac{dM_y}{dt} \end{bmatrix} = \begin{bmatrix} -1/T_2 & 0 \\ 0 & -1/T_2 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix}$$

The solution is

$$M_x(t) + iM_y(t) = \left(M_x(0) + iM_y(0)\right) \exp(-t/T_2)$$

In matrix form

$$\mathbf{M}(t) = \begin{bmatrix} \exp(-t/T_2) & 0 & 0 \\ 0 & \exp(-t/T_2) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{bmatrix}$$

For T_1 recovery,

$$\frac{d\mathbf{M}}{dt} = -\frac{\mathrm{M_z} - \mathrm{M_o}}{\mathrm{T_1}}\widehat{\boldsymbol{k}}$$

In matrix form

$$\begin{bmatrix} \frac{d\mathrm{M}_x}{dt} \\ \frac{d\mathrm{M}_y}{dt} \\ \frac{d\mathrm{M}_z}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1/\mathrm{T_1} \end{bmatrix} \begin{bmatrix} \mathrm{M}_x \\ \mathrm{M}_y \\ \mathrm{M}_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\mathrm{M_o}}{\mathrm{T_1}} \end{bmatrix}$$

which can be simplified

$$\frac{dM_z}{dt} = -\frac{M_z}{T_1} + \frac{M_o}{T_1}$$

The solution is

$$M_z(t) = M_z(0)\exp\left(-\frac{t}{T_1}\right) + M_o(1 - \exp\left(-\frac{t}{T_1}\right))$$

In matrix form

$$\mathbf{M}(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \exp(-t/T_1) \end{bmatrix} \begin{bmatrix} M_x(0) \\ M_y(0) \\ M_z(0) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_o(1 - \exp(-t/T_1)) \end{bmatrix}$$

If we add all three matrix forms

$$\begin{bmatrix} \frac{dM_x}{dt} \\ \frac{dM_y}{dt} \\ \frac{dM_z}{dt} \end{bmatrix} = \begin{bmatrix} -1/T_2 & \gamma B_o & 0 \\ -\gamma B_o & -1/T_2 & 0 \\ 0 & 0 & -1/T_1 \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_o/T_1 \end{bmatrix}$$

The final solution is

$$M_x(t) + iM_y(t) = \left(M_x(0) + iM_y(0)\right) \exp(-t/T_2) \exp(-i\gamma B_o t)$$

$$\text{(or simply, } M_{xy}(t) = M^o \exp(-t/T_2) \exp(-i\omega_0 t))$$

$$M_z(t) = M_z(0)\exp\left(-\frac{t}{T_1}\right) + M_o(1 - \exp\left(-\frac{t}{T_1}\right))$$

In matrix form

$$\mathbf{M}(t) = \begin{bmatrix} e^{-\frac{t}{T_2}} & 0 & 0 \\ 0 & e^{-\frac{t}{T_2}} & 0 \\ 0 & 0 & e^{-\frac{t}{T_1}} \end{bmatrix} \mathbf{R}_z(\omega_0 t)\mathbf{M}^{\mathbf{o}} + \begin{bmatrix} 0 \\ 0 \\ M_o\left(1 - e^{-\frac{t}{T_1}}\right) \end{bmatrix}$$

Note that transverse magnetization and longitudinal magnetization are independent. This is only the case when only z-directional B-field is considered. The transverse magnetization depends on T_2 relaxation and Larmor frequency whereas the longitudinal magnetization depends on T_1 relaxation and proton density.

Q: Why the rotation matrix is before the relaxation matrix?

Q: What happens when the initial magnetization is (1, 0, 0), (0, 1, 0), or (0, 0, 1)?