- Spin echo

When the gradient field is ignored, spins experience phase dispersion from field inhomogeneity

This dephasing can be restored by using an 180° RF pulse

Figure of spin echo generation (pancake)

Plot of signal decay in spin echo

Q: What should be direction of the inversion 180 pulse?

Q: At what time point does the spin echo occurs?

Figure of spin echo in k-space

Figure of spin echo sequence

In spin echo, off-resonance is removed and hence requires less strict field inhomogeneity.

Q: What are the disadvantages of SE as compared to GRE?

## 8.3 $T_2$ Relaxation

$$s_r(t) = \int_{vol} m(\boldsymbol{r}) e^{-i(\omega_0+\omega_{off}(\boldsymbol{r}))t} e^{-i2\pi k_r(t)\boldsymbol{r}} e^{-t/T_2(\boldsymbol{r})}\, dV$$

Let's ignore off-resonance effects and demodulates

$$= \int_{vol} m(\boldsymbol{r}) e^{-i2\pi k_r(t)\boldsymbol{r}} e^{-t/T_2(\boldsymbol{r})}\, dV$$

If we are acquiring data from a uniform $T_2$ object,

$$s_r(t) = e^{-t/T_2} \int_{vol} m(\boldsymbol{r}) e^{-i2\pi k_r(t)\boldsymbol{r}}\, dV = e^{-t/T_2} M(k_x(t), k_y(t))$$

Figure of $T_2$ weighting in Cartesian space

Since we are performing inverse Fourier transform to reconstruction the image, the multiplication of a decay function (in Cartesian readout) and the original object becomes a convolution.

$$\begin{aligned} m(x, y) &= \widetilde{\boldsymbol{F}}^{-\mathbf{1}}\left\{e^{-t/T_2} M(k_x(t),\ k_y(t))\right\} \\ &= \widetilde{\boldsymbol{F}}^{-\mathbf{1}}\left\{e^{-t/T_2}\right\} * \widetilde{\boldsymbol{F}}^{-\mathbf{1}}\left\{M(k_x(t),\ k_y(t))\right\} \end{aligned}$$

The FT of a decay function is a Lorentizan function so the resulting image will have a blurring of the original object.

Q: In which axis will this blurring happen in 2DFT?

For a shorter T2, the blurring will be larger.

In general, multiple T2 exists in an object and the blurring will be space-variant.

## 8.4 Other Nonidealities

1. RF ($B_1$) field inhomogeneities (flip angle and phase)

2. Gradient field considerations
    - Nonlinear components to the field
    - Eddy currents (distorts waveforms)
    - Finite rise times

## 8.5 Image Contrast

The most important power of MRI, in my opinion, is the ability to generate diverse image contrasts. This can be achieved by changing sequence timing and parameters (e.g. flip angle, TR, TE…). The contrast is determined by these as well as MRI related tissue properties such as $T_1$, $T_2$, proton density, etc.

### 8.5.1 Saturation Recovery

Excite with a constant RF for $T_2 << TR << T_1$

- Example of $90^o$ flip angle

Figure of $90^o$ flip angle and $M_z$

$$I(x,y) = K\boldsymbol{\rho}(x,y)\left[1 - e^{-\frac{TR}{T1(x,y)}}\right]$$

With T2* weighting

$$I(x,y) = K\boldsymbol{\rho}(x,y)\left[1 - e^{-\frac{TR}{T_1(x,y)}}\right]e^{-TE/T_2^*(x,y)}$$

Spin echo can be used

Figure of spin echo saturation recovery

$$I(x,y) = K\boldsymbol{\rho}(x,y)\left[1 - e^{-\frac{TR}{T_1(x,y)}}\right]e^{-TE/T_2(x,y)}$$

This is the most popular sequence in MRI.

$T_1$ contrast can be acquired using TR ~ $T_1$ and short TE.

$T_2$ contrast can be acquired using TR ~ long and TE ~ $T_2$.

- General excitation recovery

When 90 is not used, Mz is not zero after RF excitation.

Figure of general excitation recovery sequence

$$M_n^+ = M_n^- cos\theta$$

$$M_{n+1}^- = M_n^+ E_1 + M_0(1 - E_1)$$

Where $E_1 = \exp(-\frac{TR}{T_1})$

Then after a few lines of calculus

$$M_{steadystate} = M_0(1 - E_1)\sin\theta/(1 - E_1\cos\theta)$$

8.5.2 Inversion Recovery

Figure of Inversion-Recovery sequence

Remember longitudinal relaxation?

$$M_z(t) = M_0(1 - E_1) + M_z(0)E_1$$

Initially

$$M_z = M_0$$

After inversion pulse

$$M_z = -M_0$$

After T1 recovery time of TI

$$M_z = M_0(1 - 2e^{-TI/T_1})$$

After 90° excitation

$$M_z = 0$$

Recovery over TR-T1 duration

$$M_z = M_0(1 - e^{-(TR-TI)/T_1})$$

Second inversion pulse

$$M_z = -M_0(1 - e^{-(TR-TI)/T_1})$$

After TI

$$M_z = M_0(1 - e^{-\frac{TI}{T_1}}) + [-M_0(1 - e^{-\frac{TR-TI}{T_1}})]e^{-\frac{TI}{T_1}}$$

$$= M_0(1 - 2e^{-\frac{TI}{T_1}} + e^{-\frac{TR}{T_1}})$$

After excitation

$$M_{xy} = M_0(1 - 2e^{-\frac{TI}{T_1}} + e^{-\frac{TR}{T_1}})$$

Signal that we acquire at TE

$$I(x,y) = K\boldsymbol{\rho}(x,y)(1 - 2e^{-\frac{TI}{T_1(x,y)}} + e^{-\frac{TR}{T_1(x,y)}})e^{-\frac{TE}{T_2(x,y)}}$$

## 8.6 Noise consideration

SNR = signal amplitude / standard deviation of noise

CNR = signal difference / standard deviation of noise

Noise in MRI follows Rician-distribution in signal region and Rayleigh-distribution in background region

$$\text{SNR} \propto (\Delta x)\,(\Delta y)\,(\Delta z)\sqrt{total\ measurement\ time}\; f(\boldsymbol{\rho}, T_1, T_2)$$

total measurement time = # signal averaging & # phase encoding & readout time

Example: acquiring the same image twice and averaging

$$\text{SNR} \propto \frac{2A}{\sqrt{2\sigma^2}} = \sqrt{2}\,\frac{A}{\sigma}$$