Herding: Information Cascade

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2014

Herding Process

- Sequential process
  - Observe prior person's decision or behavior
  - Guess the truth based on other people's behavior
  - Make decisions weighing her own information & prior actions
- Rational inference
- Irrational, social(peer) pressure

Experiments

- Anderson & Holt, 1996
- There are two bags each of which contains three balls
  - MB(Majority Blue) bag: Contains two blue balls and one red ball
  - MR(Majority Red) bag: Contains two red balls and one blue ball
- Randomly select one bag
  - Pr[MB bag] = Pr[MR bag] = 1/2
- In a class, students sequentially draw a ball from the selected bag
  - A student sees the color of the drawn ball and announces her guess whether the bag is MB or MR
  - Each student makes decision based on her own private information and prior announcements
- If the first student draws a blue ball, then what is her educated guess?

Bayes' Rule - 1

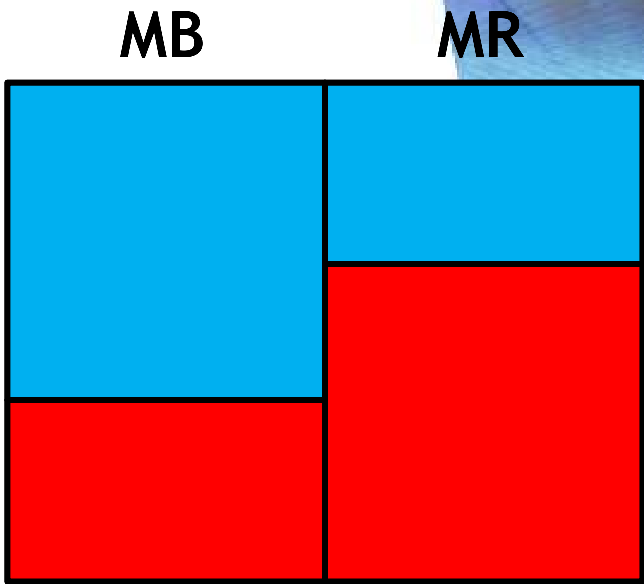
- Prob. that the bag is MB given that the first student draws a blue ball
- P[MB|b] = ?
- Bayes Rule

$$P[MB|b] = \frac{P[MB \cap b]}{P[b]}$$

$$P[b|MB] = \frac{P[MB \cap b]}{P[MB]}$$

➔ $P[MB|b] = \frac{P[b|MB] \cdot P[MB]}{P[b]}$

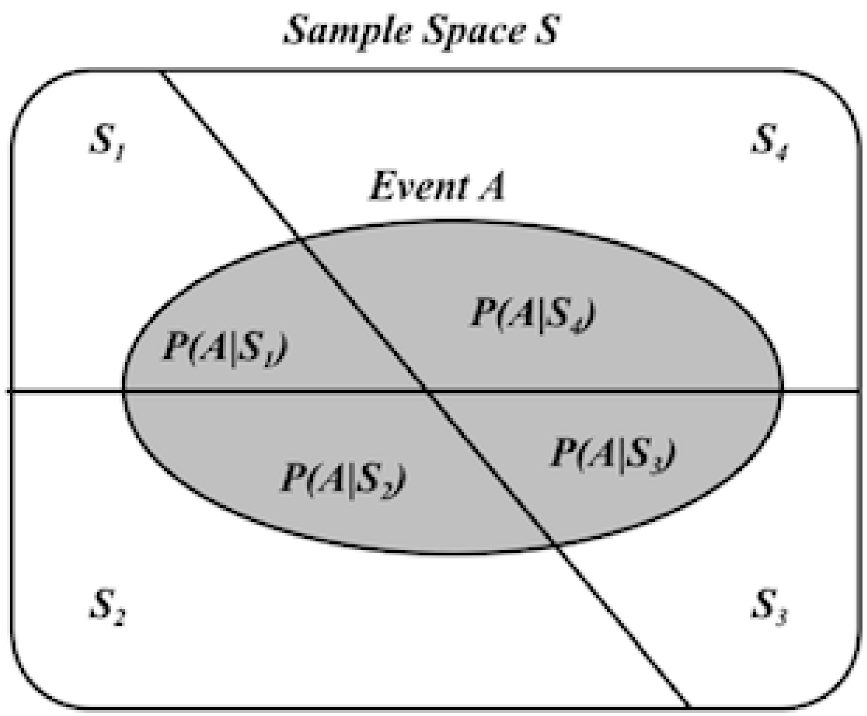
Reverend Thomas Bayes



Bayes' Rule - 2

Law of Total Probability:

$$P(A) = \sum_{i=1}^{n} P(A \mid S_i) P(S_i)$$



In this particular case:

$$P(A) = P(A|S_1)P(S_1) + P(A|S_2)P(S_2) + P(A|S_3)P(S_3) + P(A|S_4)P(S_4)$$

Bayes' Rule - 3

- P[b|MB] = 2/3
- P[b] = P[b|MB] · P[MB] + P[b|MR] · P[MR]

  = 2/3 ·1/2 + 1/3 · 1/2

  = 1/2
- $P[MB|b] = \frac{P[b|MB] \cdot P[MB]}{P[b]}$ = 2/3
- P[MR|b] = ??

Experiments

- Now, if the second student draws a blue ball, what is her educated guess?

  P[MB|bb] = ??

- If the ball sequence is (blue, red) then

  P[MB|br] = 1/2

  ➔ She announces the bag is MR

- First two students announce that the bag is MB

  ➔ Both draw blue balls

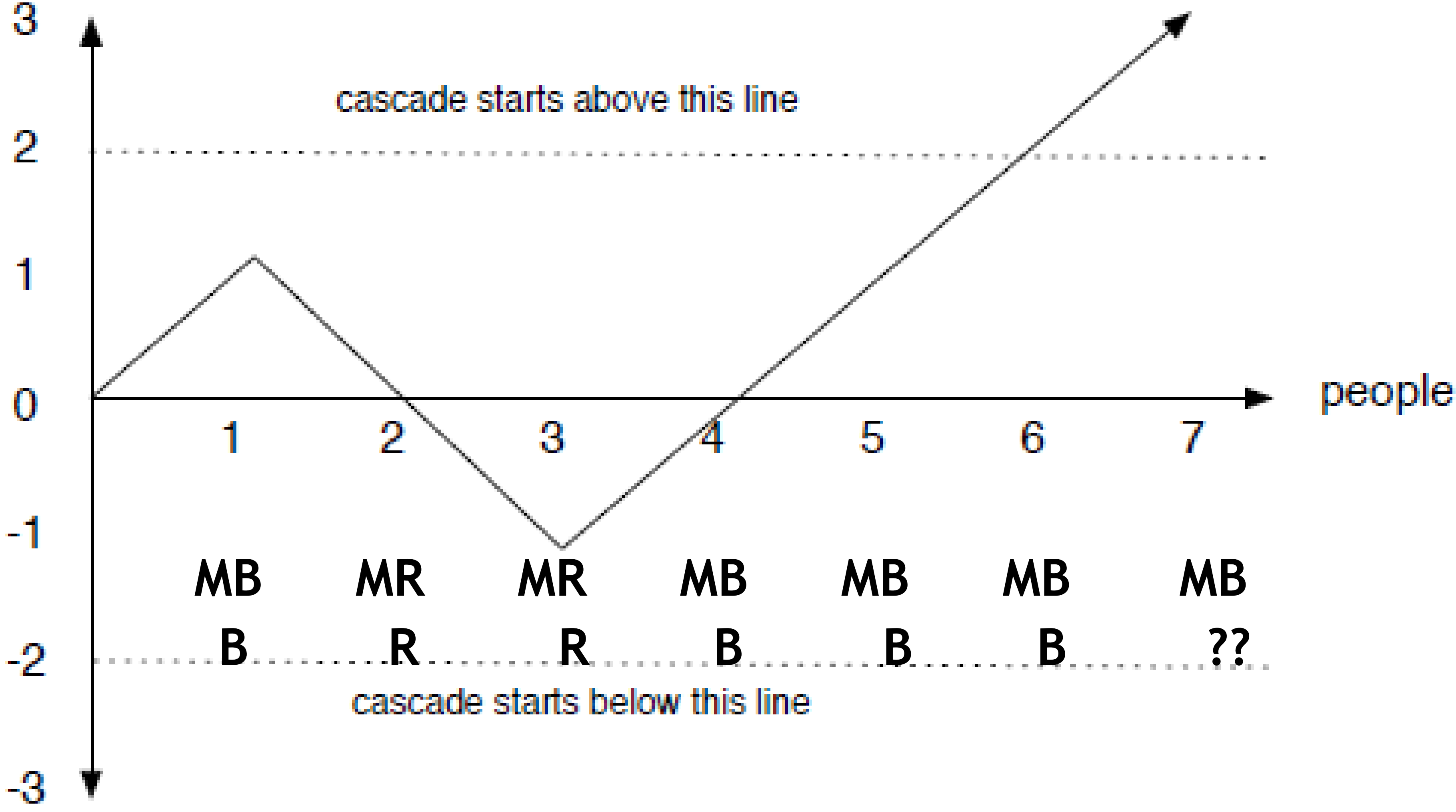
- Third student draw a red ball, what is her educated guess?

Experiments

- P[MB|bbr] = ?
- P[MR|bbr] = ?

- Information cascade
  - After the MB→ MB sequence, the best guess for the third student who draws a red ball is MB
  - For the first and second students, their decision are the same as what they saw
  - After blue-blue occurrence, the third student’s decision is not related to her private finding

- Whenever the number of MB announcements exceeds the number of MR announcements by two, information cascade occurs


3
2
1
0
-1
-2
-3
cascade starts above this line
people
1
2
3
4
5
6
7
MB
MR
MR
MB
MB
MB
MB
B
R
R
B
B
B
??
cascade starts below this line

General Cascade Model - 2

- We witness the # of accounts increased yesterday, is this an indication for bull market?
- P[Bull|Increase] = $\frac{P[Increase|Bull] \cdot P[Bull]}{P[Increase]}$

$$= \frac{P[Increase|Bull] \cdot P[Bull]}{P[Increase|Bull] \cdot P[Bull] + P[Increase|Bear] \cdot P[Bear]}$$

$$= \frac{p \cdot q}{p \cdot q + (1-p) \cdot (1-q)} >$$

- Multiple signals
  - Let E be an event consists of a increases and b decreases
  - P[E|Bull] = $q^a \cdot (1-q)^b$
  - P[E] = P[E|Bull]·P[Bull] + P[E|Bear]P[Bear]

$$= p \cdot q^a \cdot (1-q)^b + (1-p) \cdot q^b \cdot (1-q)^a$$

General Cascade Model - 3

- P[Bull|E] = $\frac{p \cdot q^a \cdot (1-q)^b}{p \cdot q^a \cdot (1-q)^b + (1-p) \cdot q^b \cdot (1-q)^a}$
- Substitute $q^b \cdot (1-q)^a$ by $q^a \cdot (1-q)^b$

  ➔ P[Bull|E] = p

- Case 1: a > b

  ➔ $q^b \cdot (1-q)^a < q^a \cdot (1-q)^b$

  ➔ P[Bull|E] > p

- Case 2: a < b

  ➔ $q^b \cdot (1-q)^a > q^a \cdot (1-q)^b$

  ➔ P[Bull|E] < p

YAE of Bayes' Rule

- In SNU the population of Yellow cats & Brown cats are 80% and 20%, respectively
- A student witnessed a cat who stole fish at the kitchen
  - She reports the cat is a Yellow cat
- Prior experience says that the probability of mistaking a Yellow cat as a Brown cat is 30%
  - Probability of mistaking a Brown cat as Yellow cat is 50%
- What is the probability that the reported cat is really a Yellow cat?
- P[real = Yellow | report = Yellow] = ?

Side Step

- Another important application area of Bayes' rule is spam filtering
- Based on the fact that "spams" contain certain words such as Viagra more frequently than "hams"
- Survey the literature and learn the important topic of "spam filtering" and several techniques such as "Naïve Bayesian" rule

Network Effects, Externalities

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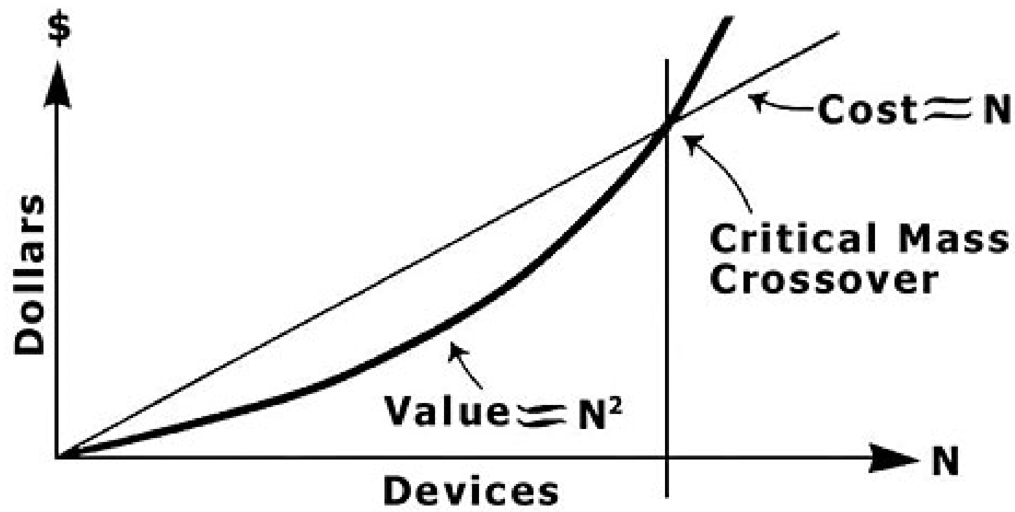
Types of Imitation

- Informational effects
  - Information cascading
- Direct benefit effects (Network effects)
  - Direct benefits from aligning behavior to others

Network Effects

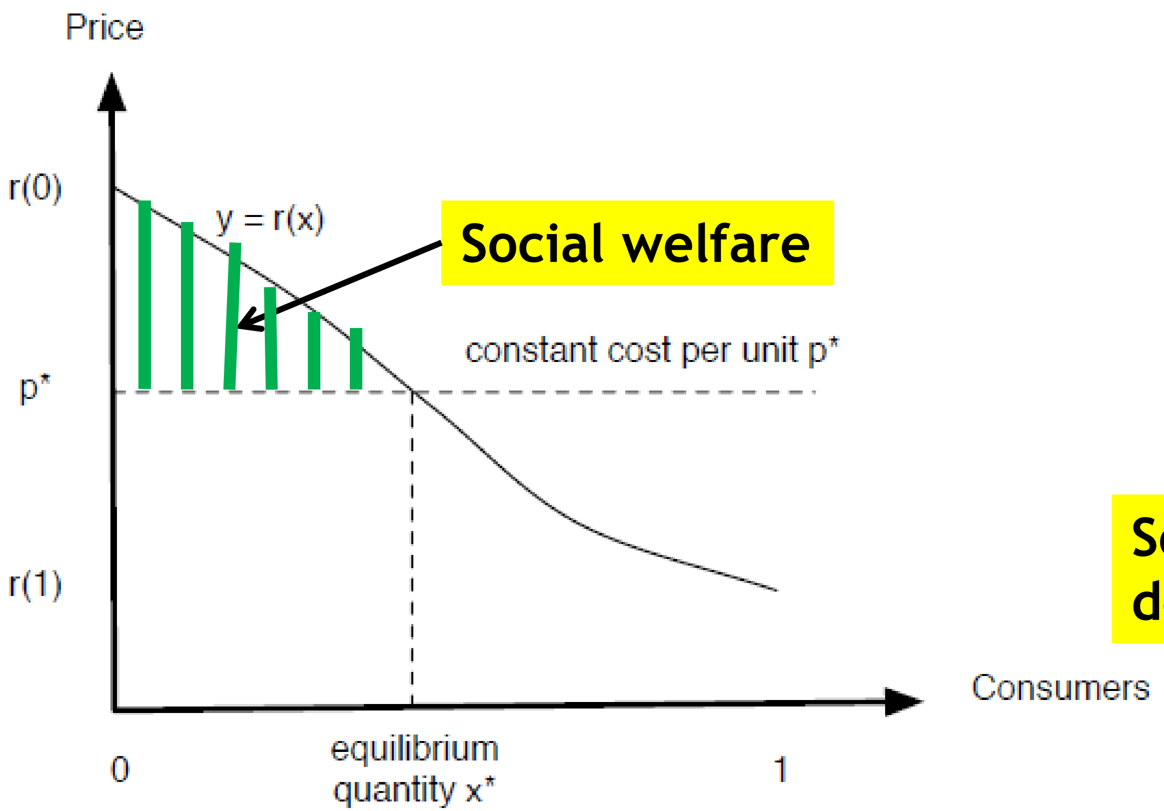
- As more people join, the benefit of participants increase
- (Network, Positive) Externality
- Examples
  - Communication devices
  - SNS
  - OS, word processor

The Systemic Value of Compatibly Communicating Devices Grows as the Square of Their Number:



No Network Effects

- Reservation price, r(·)
  - Willing to pay for a unit of product
  - Intrinsic value
  - Network effects

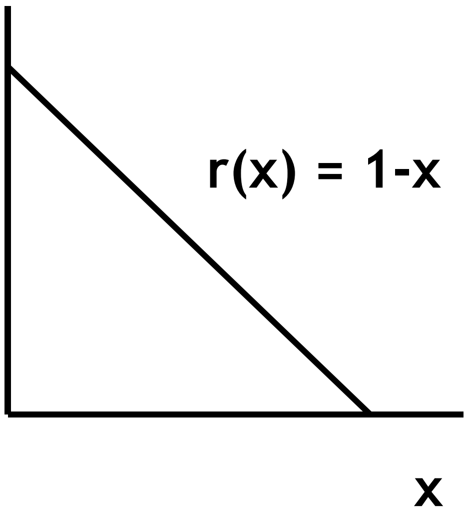


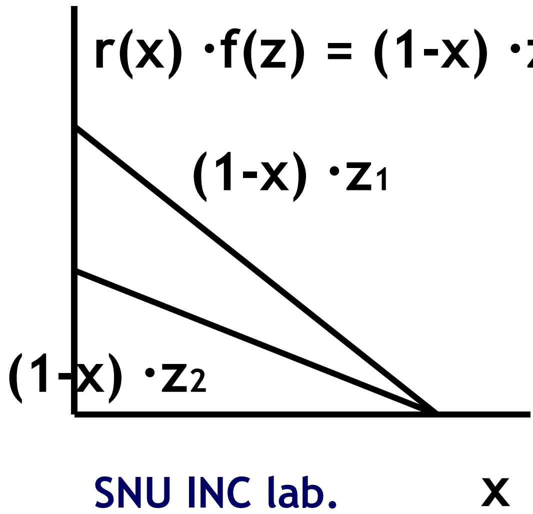
Sort customers in decreasing r(·) order

Network Effects

- Let f(z) be the network effect when z fraction of the population use a product
  - Increasing function
- Reservation price of user x when z fraction of users adopt the good
  - $r(x) \cdot f(z)$
- x will purchase the good if $r(x) \cdot f(z) > p^*$

z: Agreed expected population that will use the product



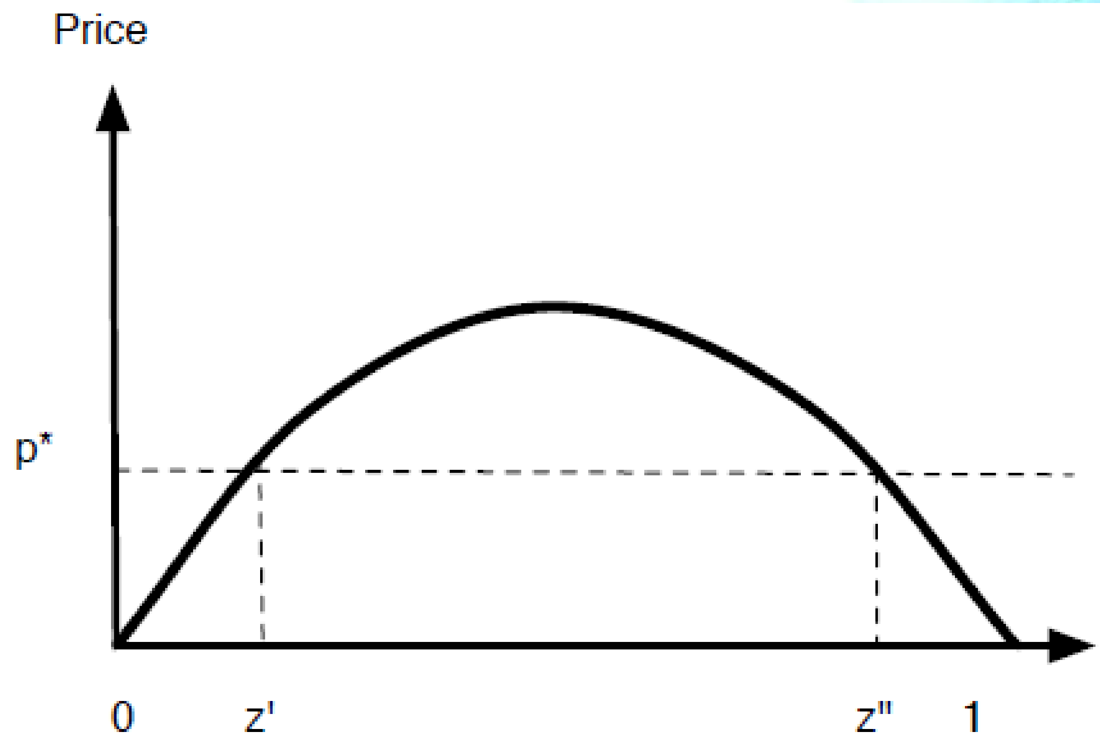


f(z) = z

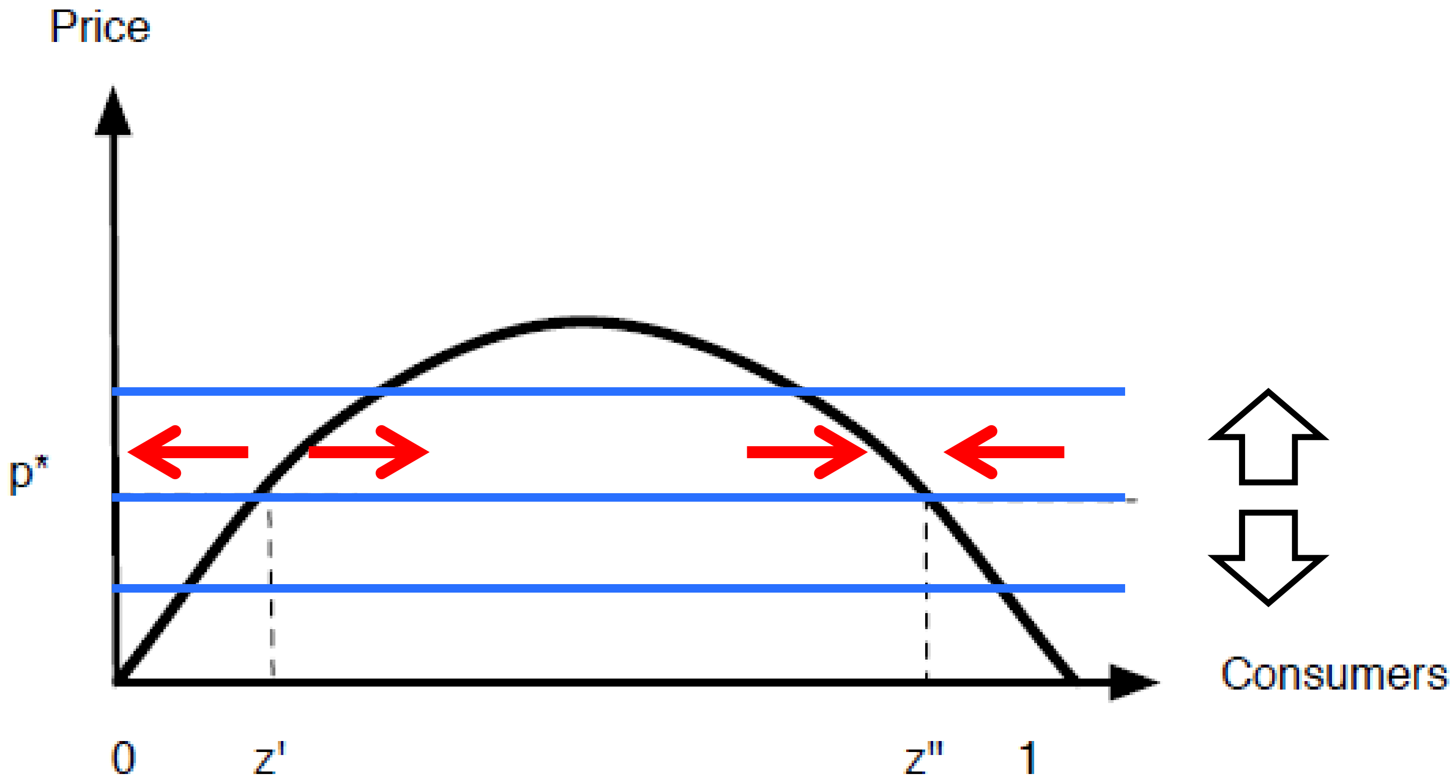
Network Effects

- Assume that each make all customers make a perfect prediction of z
- Self-fulfilling expectations equilibrium
  - Actual number of users is equal to the expected number of users

    ➔ $p^* = r(z) \cdot f(z)$



Stability & Tipping Point



Tipping Point, Critical Point

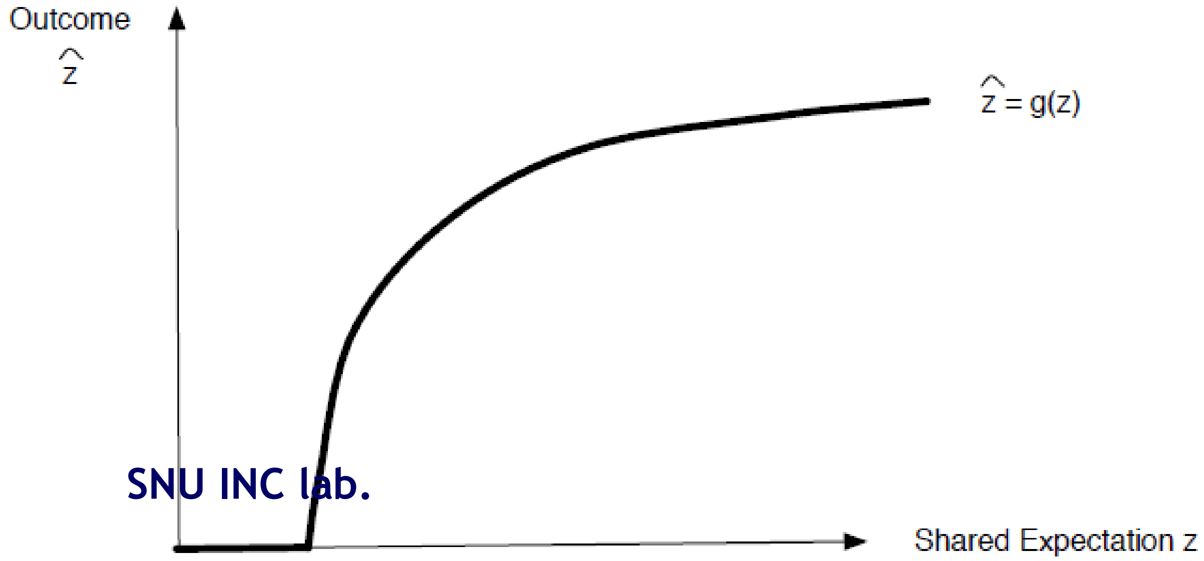
Market Dynamics

- Shared expectation z can be wrong
- x will purchase if r(x)·f(z) > p*
- Let $\hat{z}$ be a value that solves r($\hat{z}$)·f(z) = p*

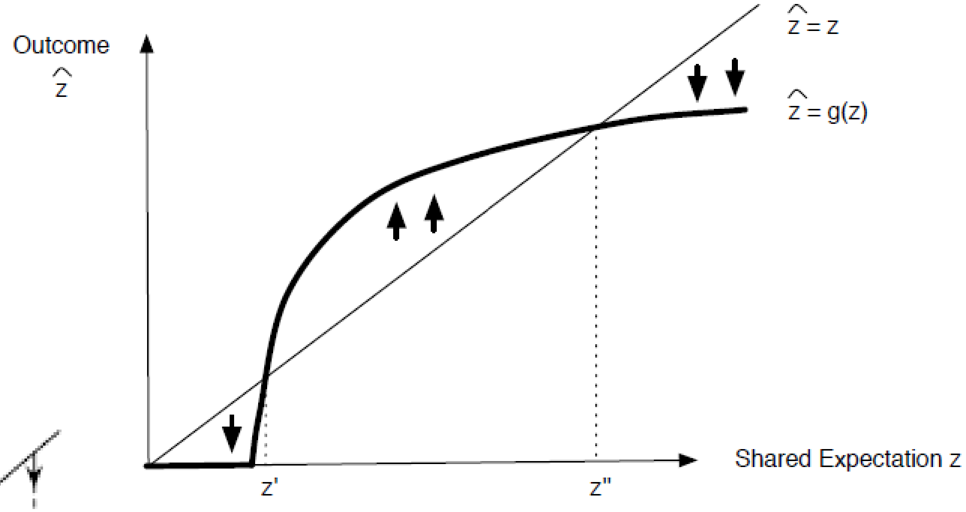
➔ r($\hat{z}$) = p*/f(z)

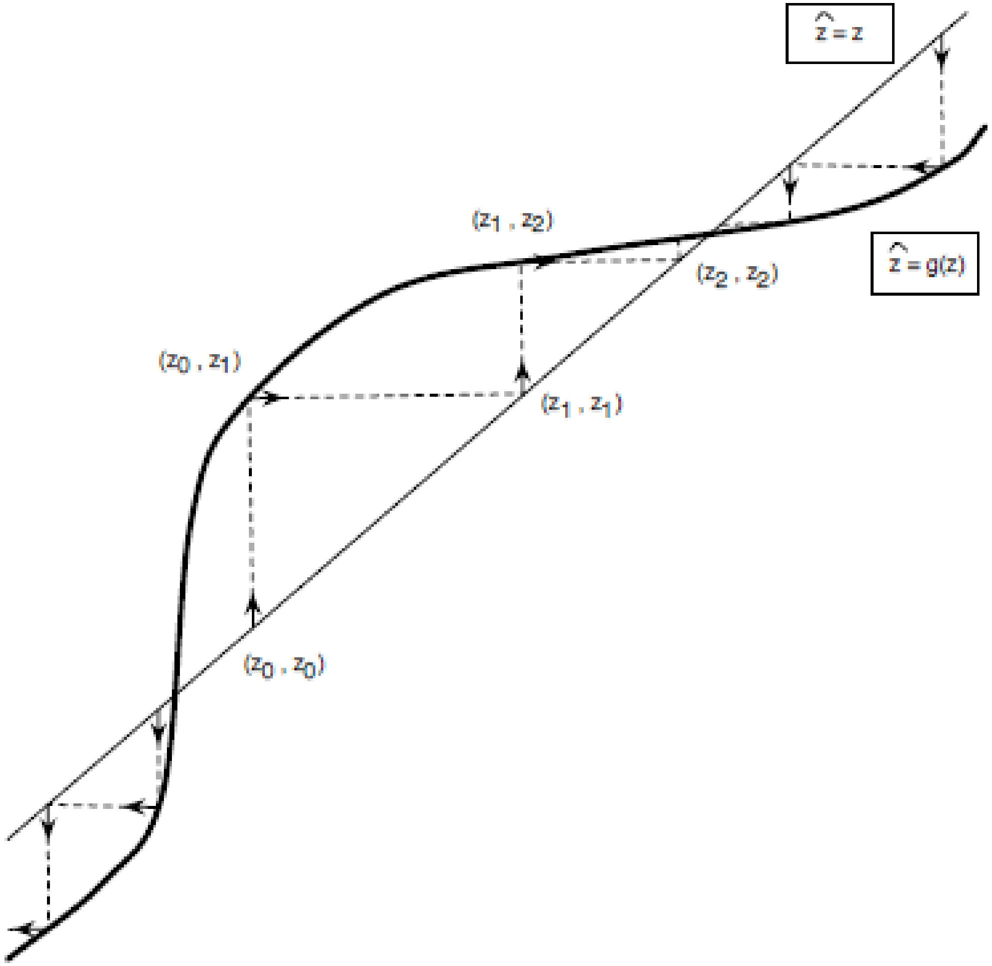
$$\hat{z} = r^{-1}\left(\frac{p^*}{f(z)}\right)$$

$$\hat{z} = 0 \; if \; \frac{p^*}{f(z)} > r(0)$$



Market Dynamics

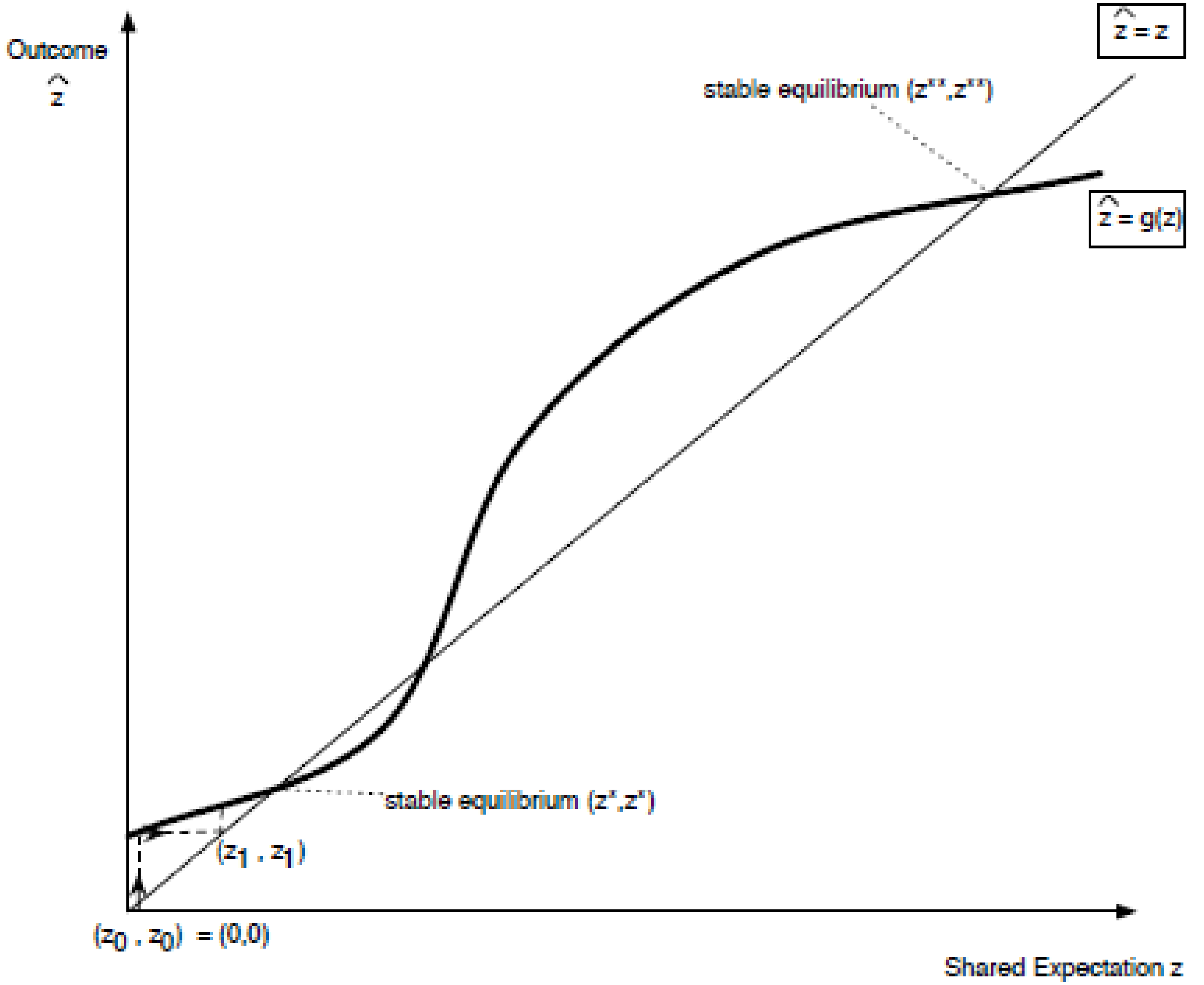




lab.

Dynamic Behavior

- Let r(x)f(z) = (1-x)·(1+a·x^2)



Dynamic Behavior - 2

- Fixed set of periods, t_0, t_1, t_2,…
- Initial audience(participant) size: z_0
- Use the previous audience size as the shared expectation