2. Rectangular Waveguides

A. Boundary Value Problem (BVP) for Rectangular Waveguides

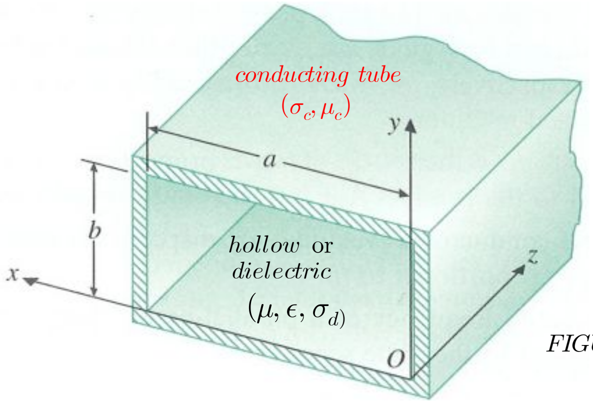


FIGURE 9−4

Wave equations in source-free hollow or dielectric region of the guide:

$$(\nabla^2 + k^2)\begin{Bmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{Bmatrix} = 0 \qquad \text{(6−98, 99)}$$

where $k^2 = \omega^2 \mu\epsilon$ (6−98)(9−5)

Assuming time-harmonic waves propagating along +z direction:

$$\boldsymbol{E}(x,y,z;t) = Re\left[\boldsymbol{E}^o(x,y)\, e^{(j\omega t - \gamma z)}\right] \qquad \text{(9−2)}$$

$$\boldsymbol{H}(x,y,z;t) = Re\left[\boldsymbol{H}^o(x,y)\, e^{(j\omega t - \gamma z)}\right] \qquad \text{(9−2)*}$$

Wave equations for longitudinal fields by using $\nabla_z^2 \rightarrow \gamma^2$:

$$(\nabla_{xy}^2 + h^2)\begin{Bmatrix} E_z^o(x,y) \\ H_z^o(x,y) \end{Bmatrix} = 0 \qquad \text{(9−22, 36)}$$

where $h^2 = \gamma^2 + k^2 \;= \gamma^2 + \omega^2 \mu\epsilon$ (9−15)

BCs : $\hat{n} \times \boldsymbol{E} = 0 \quad \Rightarrow \quad E_z^o|_{boundary\ walls} = 0$ (9−22)BC

$\hat{n} \times \boldsymbol{E} = 0$ or $\hat{n} \cdot \boldsymbol{B} = 0 \quad \Rightarrow \quad \left.\dfrac{\partial H_z^o}{\partial n}\right|_{boundary\ walls} = 0$ (9−36)BC

Wave modes :

	TM (Transverse Magnetic) (E wave)	TE (Transverse Electric) (H wave)
Logng. comp.:	$H_z^o = 0$	$E_z^o = 0$
Wave eq.:	$(\nabla_{xy}^2 + h^2) E_z^o = 0$	$(\nabla_{xy}^2 + h^2) H_z^o = 0$
BC:	$E_z^o\|_{boundary\ walls} = 0$	$\left.\dfrac{\partial H_z^o}{\partial n}\right\|_{boundary\ walls} = 0$
Transv. comps:	H_x^o, H_y^o, E_x^o, E_y^o in terms of E_z^o	H_x^o, H_y^o, E_x^o, E_y^o in terms of H_z^o

using (9−11, 12) $\quad \boldsymbol{H}_\perp^o = -h^{-2}(\gamma \nabla_\perp H_z^o + j\omega\epsilon\, \hat{z} \times \nabla_\perp E_z^o)$ (9−11, 12)*

(9−13, 14) $\quad \boldsymbol{E}_\perp^o = -h^{-2}(\gamma \nabla_\perp E_z^o - j\omega\mu\, \hat{z} \times \nabla_\perp H_z^o)$ (9−13, 14)*

B. *Properties of TM Waves* (E waves)

1) TM wave fields in the rectangular guide

Longitudinal fields: $H_z(x,y,z) = 0, \quad E_z(x,y,z) = E_z^o(x,y)\,e^{-\gamma z}$ (9-52)

BVP: $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) E_z^o(x,y) = 0 \qquad (9\text{-}53)$$

where BCs are

$$E_z^o(x,y)|_{x=0,a} = 0\,, \quad 0 \le y \le b \qquad (9\text{-}61,\ 62)$$

$$E_z^o(x,y)|_{y=0,b} = 0\,, \quad 0 \le x \le a \qquad (9\text{-}63,\ 64)$$

Separation of variables: $E_z^o(x,y) = X(x)\,Y(y)$ (9-54)

((9-54) in (9-53))/XY :

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\left[\frac{d^2Y(y)}{dy^2} + h^2\,Y(y)\right] \equiv k_x^2 = constant \qquad (9\text{-}55)$$

$$\Rightarrow \quad \frac{d^2X(x)}{dx^2} + k_x^2\,X(x) = 0 \qquad (9\text{-}56)$$

$$\frac{d^2Y(y)}{dy^2} + k_y^2\,Y(y) = 0\,, \quad k_y^2 \equiv h^2 - k_x^2 \qquad (9\text{-}57,\ 58)$$

General solutions of (9-56, 57) :

$$X(x) = A_1 \sin k_x x + A_2 \cos k_x x \qquad (9\text{-}59)$$

$$Y(y) = B_1 \sin k_y y + B_2 \cos k_y y \qquad (9\text{-}60)$$

Applying BC (9-61) to (9-59) : $A_2 = 0 \quad \Rightarrow \quad X(x) = A_1 \sin k_x x$ (9-59)TM

Applying BC (9-62) to (9-59)* : $X(a) = A_1 \sin k_x a = 0 \quad \Rightarrow \quad k_x a = m\pi$

$$\Rightarrow \quad k_x = m\pi/a \qquad ①$$

i.e., $m = \dfrac{a}{\lambda/2} = 1, 2, 3, \ldots.$ ①*

= # of half-cycle variations in the width a = x mode number

$$\Rightarrow \quad X(x) = A_1 \sin\left(\frac{m\pi}{a}x\right) \qquad (9\text{-}59)_{TM}*$$

Similarly, applying BCs (9-63, 64) to (9-60) :

$$B_2 = 0 \quad \Rightarrow \quad Y(y) = B_1 \sin k_y y \qquad (9\text{-}60)_{TM}$$

$$Y(b) = B_1 \sin k_y b = 0$$

$$\Rightarrow \quad k_y b = n\pi \qquad ②$$

i.e., $n = \dfrac{b}{\lambda/2} = 1, 2, 3, \ldots.$ ②*

= # of half-cycle variations in the height b = y mode number

$$\Rightarrow \quad Y(y) = B_1 \sin\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}60)_{TM}*$$

$(9\text{-}59)_{TM^*}$ & $(9\text{-}60)_{TM^*}$ in (9-54) by putting $E_o \equiv A_1 B_1$ to be determined by IC:

$$E_z^o(x,y) = E_o \sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \quad \text{(V/m)} : \text{ eigenmodes} \qquad (9\text{-}65)$$

①, ② in (9-58) $\Rightarrow$ $h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$: eigenvalues (9-66)

(9-15) $\Rightarrow$ $\gamma = j\beta = j\sqrt{k^2 - h^2} = j\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$ (9-67)

Transverse fields are determined by (9-11) ~ (9-14) by setting $H_z^o = 0$:

$$E_x^o(x,y) = -\frac{\gamma}{h^2}\frac{\partial E_z^o}{\partial x} = -\frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)E_o \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}65)_{Ex}$$

$$E_y^o(x,y) = -\frac{\gamma}{h^2}\frac{\partial E_z^o}{\partial y} = -\frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)E_o \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}65)_{Ey}$$

$$H_x^o(x,y) = \frac{j\omega\epsilon}{h^2}\frac{\partial E_z^o}{\partial y} = \frac{j\omega\epsilon}{h^2}\left(\frac{n\pi}{b}\right)E_o \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}65)_{Hx}$$

$$H_y^o(x,y) = -\frac{j\omega\epsilon}{h^2}\frac{\partial E_z^o}{\partial x} = -\frac{j\omega\epsilon}{h^2}\left(\frac{m\pi}{a}\right)E_o \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}65)_{Hy}$$

2) Characteristics of TM modes

Cutoff frequency of TM_{mn} modes by (9-26) or from (9-67) for $\gamma = 0$:

$$(f_c)_{mn} = \frac{hu}{2\pi} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \text{ (Hz)}, \quad m = 1, 2, ... \text{ and } n = 1, 2, ... \qquad (9\text{-}68)$$

Cutoff wavelength of TM_{mn} :

$$(\lambda_c)_{mn} = \frac{u}{(f_c)_{mn}} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}} \quad \text{(m)} \qquad (9\text{-}69)$$

If $m = 0$ or $n = 0$, all fields in (9-65)~$(9\text{-}65)_{Ey}$ vanish. Hence, $m = n = 1$, i.e., TM_{11} mode is the dominant (fundamental) mode which has the lowest cutoff frequency, $(f_c)_{TM_{11}} = \frac{u}{2}\sqrt{(1/a^2)+(1/b^2)}$ [and the longest cutoff wavelength, $(\lambda_c)_{TM_{11}} = 2/\sqrt{(1/a^2)+(1/b^2)}$].

Phase constant of TM_{mn} from (9-29):

$$(\beta)_{mn} = \omega\sqrt{\mu\epsilon}\sqrt{1-[(f_c)_{mn}/f]^2} = k\sqrt{1-[(f_c)_{mn}/f]^2} \quad \text{(rad/m)} \qquad (9\text{-}29)_{TM}$$

Wavelength of TM_{mn} from (9-30):

$$(\lambda_g)_{mn} = \frac{2\pi}{(\beta)_{mn}} = \frac{1}{f\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1-[(f_c)_{mn}/f]^2}} = \frac{\lambda}{\sqrt{1-[(f_c)_{mn}/f]^2}} \qquad (9\text{-}30)_{TM}$$

Phase velocity of TM_{mn} from (9-33)):

$$(u_p)_{mn} = \frac{\omega}{(\beta)_{mn}} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1-[(f_c)_{mn}/f]^2}} = \frac{u}{\sqrt{1-[(f_c)_{mn}/f]^2}} \qquad (9\text{-}33)_{TM}$$

Wave impedance of TM_{mn} from (9-34):

$$(Z_{TM})_{mn} = \sqrt{\mu/\epsilon}\sqrt{1-[(f_c)_{mn}/f]^2} = \eta\sqrt{1-[(f_c)_{mn}/f]^2} \qquad (9\text{-}34)_{TM}$$

C. *Properties of TE Waves* (H waves)

1) TE wave fields in the rectangular guide

Longitudinal fields: $E_z(x,y,z) = 0, \quad H_z(x,y,z) = H_z^o(x,y)\, e^{-\gamma z}$ (9-70)

BVP: $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + h^2\right) H_z^o(x,y) = 0$$ (9-71)

where BCs are

$$E_y^o(x,y)\big|_{x=0,a} = 0 \xrightarrow{(9-14)\text{ or }(9-36)_{BC}} \left.\frac{\partial H_z^o(x,y)}{\partial x}\right|_{x=0,a} = 0\ ,\quad 0 \le y \le b$$ (9-72, 73)

$$E_x^o(x,y)\big|_{y=0,b} = 0 \xrightarrow[(9-13)\text{ or }(9-36)_{BC}]{} \left.\frac{\partial H_z^o(x,y)}{\partial y}\right|_{y=0,b} = 0,\quad 0 \le x \le a$$ (9-74, 75)

Separation of variables: $H_z^o(x,y) = X(x)\, Y(y)$ (9-54)*

((9-54)* in (9-71))$/XY$:

$$-\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \frac{1}{Y(y)}\left[\frac{d^2Y(y)}{dy^2} + h^2\, Y(y)\right] \equiv k_x^2 = constant$$ (9-55)*

$$\Rightarrow \quad \frac{d^2X(x)}{dx^2} + k_x^2\, X(x) = 0$$ (9-56)*

$$\frac{d^2Y(y)}{dy^2} + k_y^2\, Y(y) = 0\ ,\quad k_y^2 \equiv h^2 - k_x^2$$ (9-57, 58)*

General solutions of (9-56, 57)* :

$$X(x) = A_1 \sin k_x x + A_2 \cos k_x x$$ (9-59)*

$$Y(y) = B_1 \sin k_y y + B_2 \cos k_y y$$ (9-60)*

Applying BC (9-72) to (9-59)* : $A_1 = 0 \quad \Rightarrow \quad X(x) = A_2 \cos k_x x$ (9-59)$_{TE}$

Applying BC (9-73) to (9-59)$_{TE}$: $\dfrac{\partial X(a)}{\partial x} = -A_2 k_x \sin k_x a = 0 \quad \Rightarrow \quad k_x a = m\pi$

$\Rightarrow \quad k_x = m\pi/a$, i.e., $m = \dfrac{a}{\lambda/2} = 0, 1, 2, 3, \ldots.$ ①

$$\Rightarrow \quad X(x) = A_2 \cos\left(\frac{m\pi}{a}x\right)$$ (9-59)$_{TE}$*

Similarly, applying BCs (9-74, 75) to (9-60)* :

$B_1 = 0 \quad \Rightarrow \quad Y(y) = B_2 \cos k_y y$ (9-60)$_{TE}$

$$\frac{\partial Y(b)}{\partial y} = -B_2 k_y \sin k_y b = 0$$

$\Rightarrow \quad k_y b = n\pi$, i.e., $n = \dfrac{b}{\lambda/2} = 0, 1, 2, 3, \ldots.$ ②

$$\Rightarrow \quad Y(y) = B_2 \cos\left(\frac{n\pi}{b}y\right)$$ (9-60)$_{TE}$*

(9-59)TE* & (9-60)TE* in (9-54)* by putting $H_o \equiv A_2 B_2$ to be determined by IC:

$$H_z^o(x,y) = H_o \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \quad \text{(A/m)} : \text{ eigenmodes} \qquad (9\text{-}76)$$

$$m,n = (either\ m \text{ or } n=0), 1, 2, ...$$

Note) If m=n=0, H_z is ind. of x and y

$\Rightarrow$ all transv. fields = 0 by (9-11)~(9-14) $\Rightarrow$ ∃ no TE modes

①, ② in (9-58) $\Rightarrow$ $h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$: eigenvalues (9-66)

(9-15) $\Rightarrow$ $\gamma = j\beta = j\sqrt{k^2 - h^2} = j\sqrt{\omega^2\mu\epsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$ (9-67)

Transverse fields are determined by (9-11) ~ (9-14) by setting $E_z^o = 0$:

$$E_x^o(x,y) = -\frac{j\omega\mu}{h^2}\frac{\partial H_z^o}{\partial y} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right)H_o\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}77)$$

$$E_y^o(x,y) = \frac{j\omega\mu}{h^2}\frac{\partial H_z^o}{\partial x} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right)H_o\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}78)$$

$$H_x^o(x,y) = -\frac{\gamma}{h^2}\frac{\partial H_z^o}{\partial x} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right)H_o\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}79)$$

$$H_y^o(x,y) = -\frac{\gamma}{h^2}\frac{\partial H_z^o}{\partial y} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right)H_o\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right) \qquad (9\text{-}80)$$

2) Characteristics of TE modes

Cutoff frequency of TE_{mn} modes by (9-26) or from (9-67) for $\gamma = 0$:

$$(f_c)_{mn} = \frac{hu}{2\pi} = \frac{u}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}, \quad m,n = (either\ m \text{ or } n=0), 1, 2, ... \qquad (9\text{-}68)$$

Note) TM_{mn} and TE_{mn} are always degenerate with the same $(f_c)_{mn}$

excluding the TE_{mn} modes for none of m and n = 0 (TE_{m0}, TE_{0n})

Cutoff wavelength of TE_{mn} :

$$(\lambda_c)_{mn} = \frac{u}{(f_c)_{mn}} = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}} \quad \text{(m)} \qquad (9\text{-}69)$$

Dominant TE mode of a waveguide with $a > b \Rightarrow TE_{10}$ (TE_{01} if $a < b$).

∵ $m = 1$ and $n = 0$ in (9-68) (or (9-69)) yields the lowest f_c (longest λ_c).

$$(f_c)_{TE_{10}} = u/2a = 1/2a\sqrt{\mu\epsilon}\ , \qquad (\lambda_c)_{TE_{10}} = 2a \qquad (9\text{-}81,\ 82)$$

Phase constant of TE_{mn} from (9-29): $(\beta)_{mn} = \omega\sqrt{\mu\epsilon}\sqrt{1-[(f_c)_{mn}/f]^2}$ (9-29)TE

Wavelength of TE_{mn} from (9-30): $(\lambda_g)_{mn} = \frac{2\pi}{(\beta)_{mn}} = \frac{1}{f\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1-[(f_c)_{mn}/f]^2}}$ (9-30)TE

Phase vel. of TE_{mn} from (9-33)): $(u_p)_{mn} = \frac{\omega}{(\beta)_{mn}} = \frac{1}{\sqrt{\mu\epsilon}}\frac{1}{\sqrt{1-[(f_c)_{mn}/f]^2}}$ (9-33)TE

Wave impedance of TE_{mn} from (9-39):

$$(Z_{TE})_{mn} = \sqrt{\mu/\epsilon}\ /\ \sqrt{1-[(f_c)_{mn}/f]^2} \qquad (9\text{-}39)_{TE}$$

D. Field Configurations for TE and TM Modes

1) TE_{10} mode : *(e.g. 9-5)*

a) Instantaneous electromagnetic fields

(9-76)~(9-80) for $m=1,\ n=0,\ h=\pi/a$

using $\boldsymbol{E}(x,y,z;t)=Re[\boldsymbol{E}^o(x,y)\,e^{j(\omega t-\beta z)}]$ and $\boldsymbol{B}(x,y,z;t)=Re[\boldsymbol{B}^o(x,y)\,e^{j(\omega t-\beta z)}]$:

$$E_x(x,y,z;t)=0 \qquad (9\text{-}83)$$

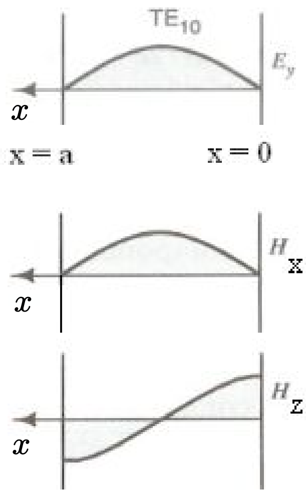
$$E_y(x,y,z;t)=\frac{\omega\mu a}{\pi}H_o\sin\left(\frac{\pi}{a}x\right)\sin(\omega t-\beta_{10}z) \qquad (9\text{-}84)$$

$$E_z(x,y,z;t)=0 \qquad (9\text{-}85)$$

$$H_x(x,y,z;t)=-\frac{\beta_{10}a}{\pi}H_o\sin\left(\frac{\pi}{a}x\right)\sin(\omega t-\beta_{10}z) \qquad (9\text{-}86)$$

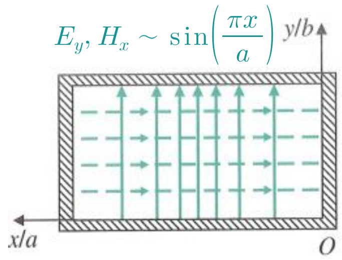
$$H_y(x,y,z;t)=0 \qquad (9\text{-}87)$$

$$H_z(x,y,z;t)=H_o\cos\left(\frac{\pi}{a}x\right)\cos(\omega t-\beta_{10}z) \qquad (9\text{-}88)$$



where $\beta_{10}=\sqrt{k^2-h_{10}^2}=\sqrt{\omega^2\mu\epsilon-(\pi/a)^2}$ (9-89)

b) Electric and magnetic field configurations



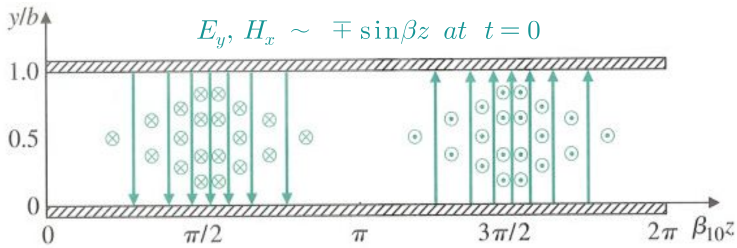
(a)

$(f_c)_{10}=u/2a$

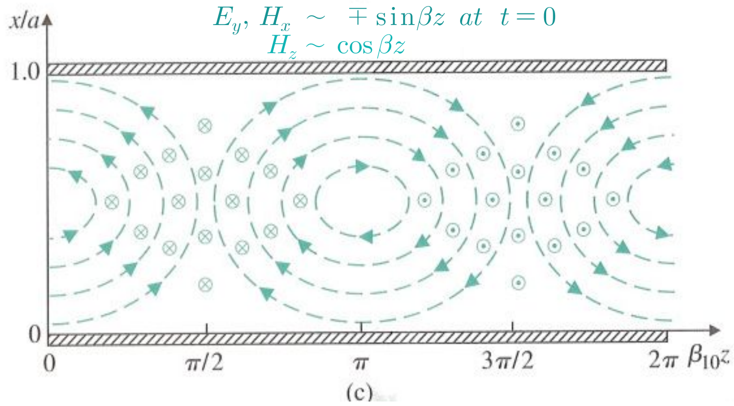
$(\lambda_c)_{10}=2a$

Electric field lines

Magnetic field lines



(b)



(c)

FIGURE 9-5

Slope of $\boldsymbol{H}$ lines at $t=0$:

$$\left(\frac{dx}{dz}\right)_H=\frac{H_x}{H_z}$$

$$=\frac{\beta}{h^2}\frac{\pi}{a}\tan\left(\frac{\pi}{a}x\right)\tan\beta z \qquad (9\text{-}90)$$

(9-86, 88) *with* $h=\pi/a$ *at* $t=0$

c) Surface current density field lines

From (6-47b),

$$\boldsymbol{J}_s = \hat{\boldsymbol{n}} \times \boldsymbol{H} \qquad \text{(9-91)}$$

(9-86)~(9-88) in (9-91) at $t=0$:

$$\boldsymbol{J}_s(x=0) = -\hat{y}H_z(0,y,z;0) = -\hat{y}H_o\cos\beta z \qquad \text{(9-92)}$$

$$\boldsymbol{J}_s(x=a) = \hat{y}H_z(a,y,z;0) = \boldsymbol{J}_s(x=0) \qquad \text{(9-93)}$$

$$\boldsymbol{J}_s(y=0) = \hat{x}H_z(x,0,z;0) - \hat{z}H_x(x,0,z;0)$$
$$= \hat{x}H_o\cos(\pi x/a)\cos\beta z - \hat{z}(\beta\pi/h^2a)H_o\sin(\pi x/a)\sin\beta z \qquad \text{(9-94)}$$

$$\boldsymbol{J}_s(y=b) = -\boldsymbol{J}_s(y=0) \qquad \text{(9-95)}$$

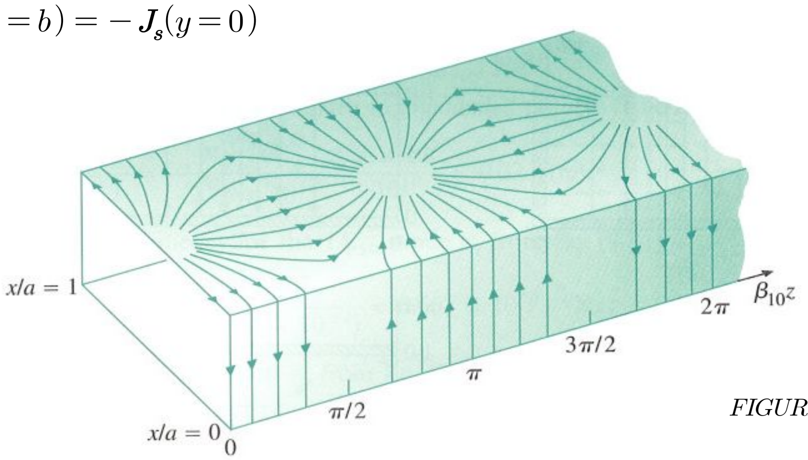
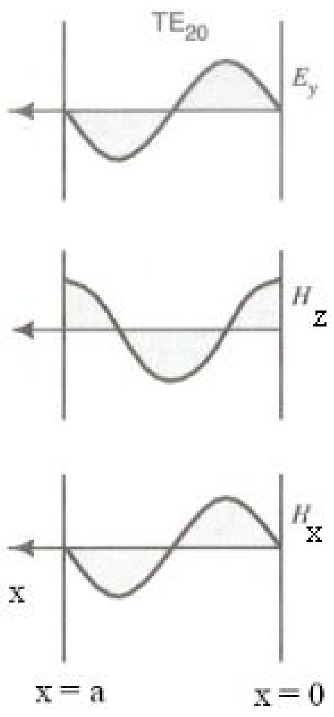
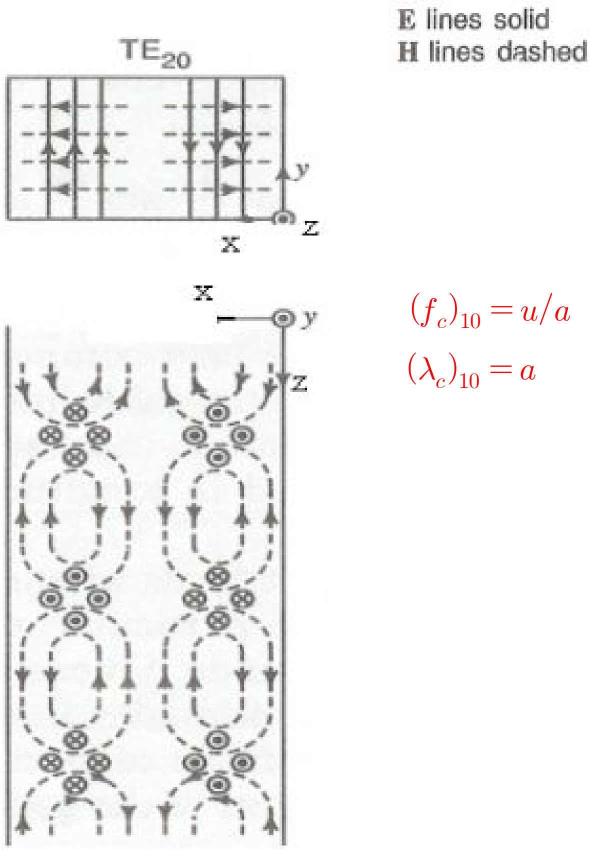


FIGURE 9−6

2) Other higher modes

a) TE_{20} mode





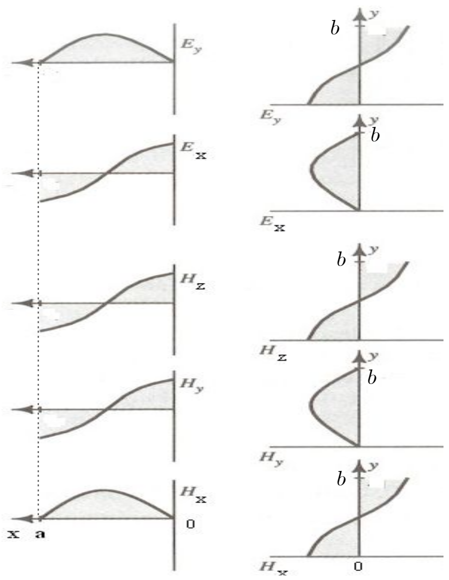
$(f_c)_{10} = u/a$

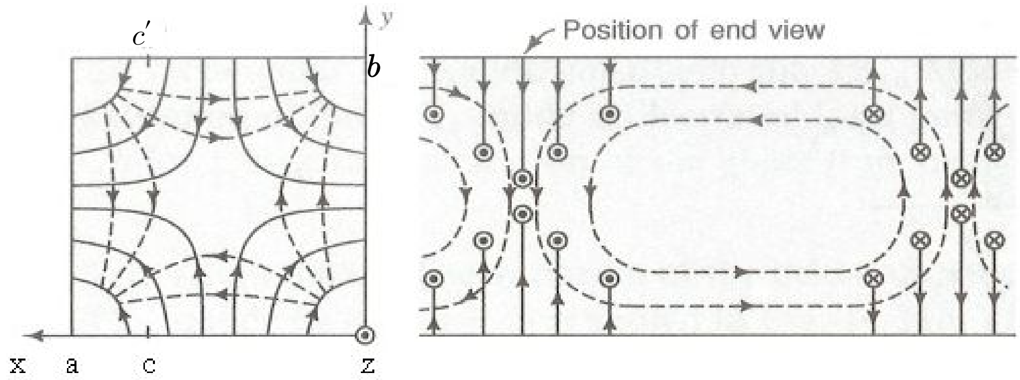
$(\lambda_c)_{10} = a$

b) TE_{11} mode

$$(f_c)_{11} = \frac{u}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

$$(\lambda_c)_{11} = 2/\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

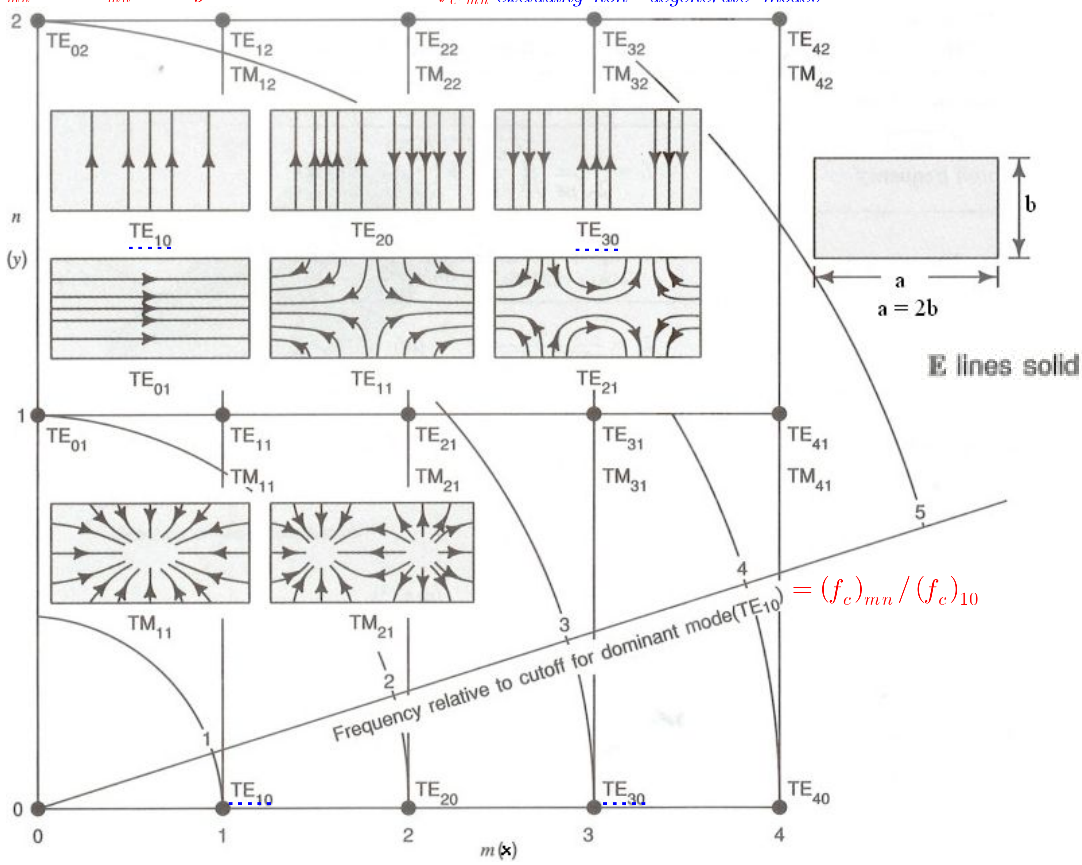




Side view through section c-c'

c) Cutoff frequencies relative the dominant mode TE_{10} (for $a = 2b$ case)

TM_{mn} and TE_{mn} are degenerate with the same $(f_c)_{mn}$ excluding non−degenerate modes ·····



E. Attenuation in Rectangular Waveguide

1) Attenuation constant α for $f > f_c$ for lossless guides ($\sigma_d = 0,\ \sigma_c \to \infty$)

$$\gamma = \sqrt{h^2 - k^2} = \sqrt{h^2 - \omega^2 \mu\epsilon} \tag{9-24}$$

$$= jk\sqrt{1-(h/k)^2} = jk\sqrt{1-(f_c/f)^2} = j\beta \tag{9-28}$$

$\Rightarrow \quad \alpha = 0$

$e^{-\gamma z} = e^{-j\beta z}$: propagating wave along z w/o power losses

2) Attenuation constant α for $f < f_c$ for lossless guides ($\sigma_d = 0,\ \sigma_c \to \infty$)

$$\gamma = h\sqrt{1-(f/f_c)^2} = \alpha \qquad (j\beta = 0) \tag{9-35}$$

$\Rightarrow e^{-\gamma z} = e^{-\alpha z}$: evanescent wave along z w/ power losses

3) Attenuation constant α for $f > f_c$ for lossy dielectrics ($\sigma_d \neq 0,\ \sigma_c \neq \infty$)

$$\alpha = \alpha_d + \alpha_c \tag{9-28}$$

where α_d = attenuation constant due to losses in the dielectric such that

$$\gamma = \alpha_d + j\beta = \sqrt{h^2 - k_c^2} = \sqrt{h^2 - \omega^2\mu\epsilon_d} \quad \text{from (7-42)}$$

$$= \sqrt{h^2 - \omega^2\mu\left(\epsilon' - j\frac{\sigma_d}{\omega}\right)} \quad \text{from (7-42) or (9-97)}$$

$\Rightarrow \quad \alpha_d \propto \sigma_d (f/f_c)$

and α_c = attenuation constant due to ohmic power loss in the nonideal conducting guide walls such that from (8-57)

$$\alpha_c = \frac{P_L(z)}{2P(z)} \propto \frac{|J_s^2| R_s}{Re[\boldsymbol{E}_\perp \times \boldsymbol{H}_\perp^*]} \tag{9-98}$$

$\Rightarrow \alpha_c \propto R_s = \sqrt{\pi f \mu / \sigma_c} \propto 1/\sqrt{\sigma_c}$ and depends on $m, n, f_c/f$

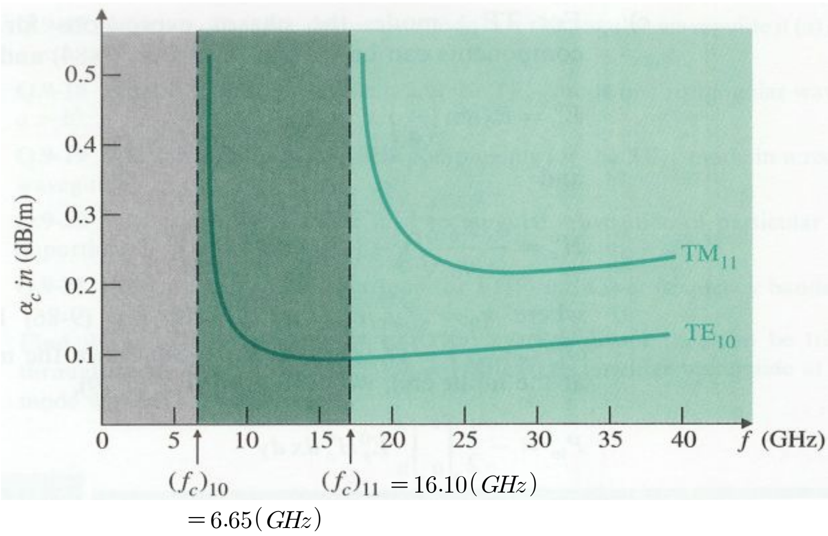


FIGURE 9−7 In a rectangular Cu waveguide for T_{10} and TM_{11} modes ($a = 2.29cm,\ b = 1.02cm$)

(e.g. 9-7)

Given: air-filled rectangular waveguide

$(\mu_o, \epsilon_o, a=5.0cm, b=2.5cm, l=0.8m, f=4.5\,GHz, P_{load}=1{,}200\,W,$

$\alpha = 0.05\ dB/m = 0.05/8.69\,Np/m = 5.75\times 10^{-3}\,Np/m\ )$

Propagating modes at 4.5 GHz $\rightarrow$ dominant mode TE_{10}

$\because\ (f_c)_{10} = 1/2a\sqrt{\mu_o\epsilon_o} = 3$ GHz for TE_{10}

$(f_c)_{20} = 1/a\sqrt{\mu_o\epsilon_o} = 6$ GHz for TE_{20}

$(f_c)_{01} = 1/2b\sqrt{\mu_o\epsilon_o} = 1/2a(2.5/5.0)\sqrt{\mu_o\epsilon_o} = 6$ GHz for TE_{01}

a) $P_{load} = P_{in}e^{-2\alpha l} \quad \Rightarrow \quad P_{in} = P_{load}e^{2\alpha l} = \underline{1{,}211 \quad \text{(W)}}$

b) $P_L = P_{in} - P_{load} = \underline{11 \quad \text{(W)}}$

c) $(E_o)_{\max} = ?$

Transverse-component phasor fields for TE_{10} from (9-84) and (9-86) :

$$E_y^o = \frac{\omega\mu_o a}{\pi}H_o\sin\left(\frac{\pi}{a}x\right) = \left(\frac{f}{f_c}\right)\eta_o H_o\sin\left(\frac{\pi}{a}x\right) = E_o\sin\left(\frac{\pi}{a}x\right) \qquad (9\text{-}99)$$

$c = 1/\sqrt{\mu_o\epsilon_o},\ f_c = c/2a,\ \eta_o = \sqrt{\mu_o/\epsilon_o}$ $\qquad E_o \equiv (f/f_c)\eta_o H_o$

$$H_x^o = -\frac{\beta_{10}a}{\pi}H_o\sin\left(\frac{\pi}{a}x\right) = -\frac{\sqrt{\omega^2\mu_o\epsilon_o - (\pi/a)^2}\ a}{\pi}H_o\sin\left(\frac{\pi}{a}x\right)$$

$$= -\sqrt{\left(\frac{f}{f_c}\right)^2 - 1}\ H_o\sin\left(\frac{\pi}{a}x\right) = -\frac{E_o}{\eta_o}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\left(\frac{\pi}{a}x\right) \qquad (9\text{-}100)$$

$c = 1/\sqrt{\mu_o\epsilon_o},\ f_c = c/2a$ $\qquad E_o \equiv (f/f_c)\eta_o H_o$

From (7-79) $\mathscr{P}_{av}(z) = \frac{1}{2}Re[\boldsymbol{E}\times\boldsymbol{H}^*]$,

$$P_{in} = -\frac{1}{2}\int_0^b\int_0^a E_y^0 H_x^o\,dx\,dy = \frac{(E_o)_{\max}^2\,ab}{4\eta_o}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 1{,}211$$

$\Rightarrow \qquad (E_o)_{\max} = \underline{44{,}283 \quad \text{(V/m)}}$

Homework Set 6

1) P.9-2 a),b) 2) P.9-3 3) P.9-7
4) P.9-9 5) P.9-11 6) P.9-15