CHAPTER 10. Antennas and Antenna Arrays

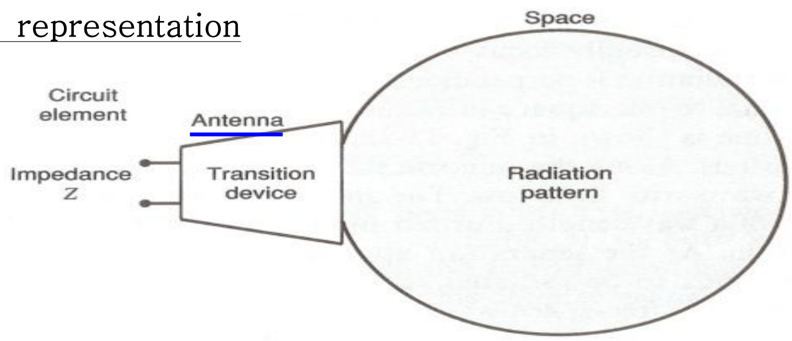
Reading assignments: Cheng Ch.10, Hayt Ch.14.8

1. Thin Linear Antennas

A. Basic Antennas

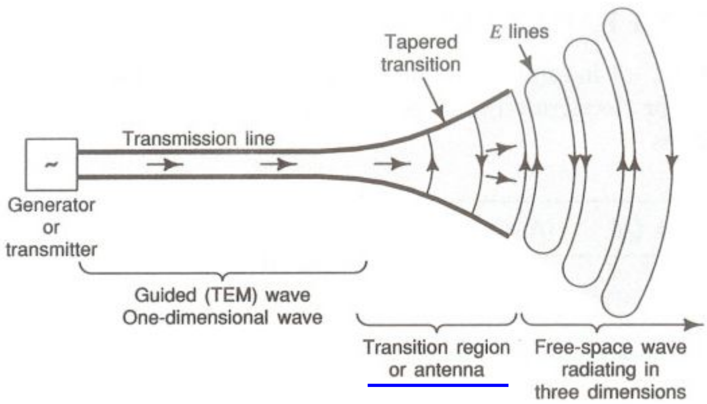
Antenna = A structures designed for radiating and receiving EM signals or energy, i.e., transition device (transducer) between a guided wave and a free-space wave.

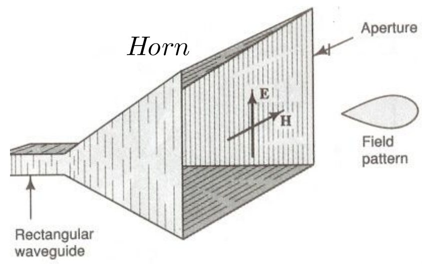
Schematic representation

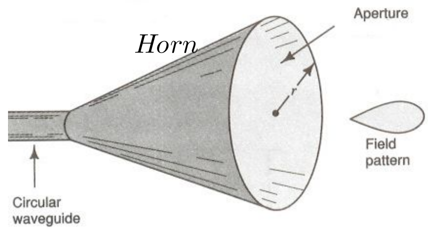


1) Types of basic antennas

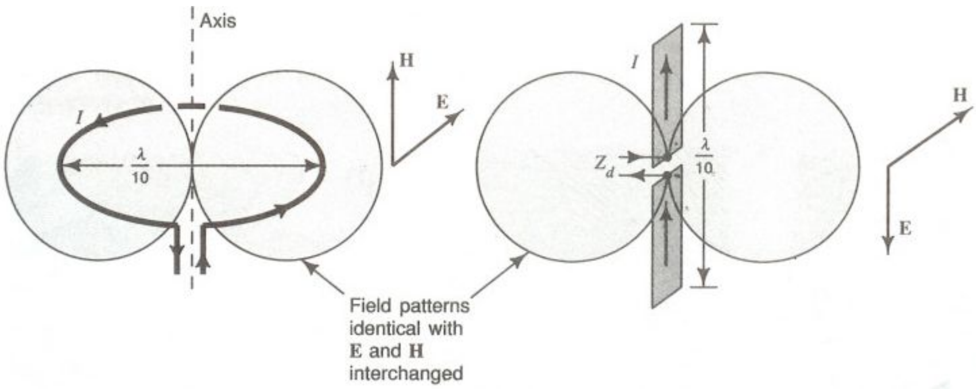
Waveguide antennas (aperture type)



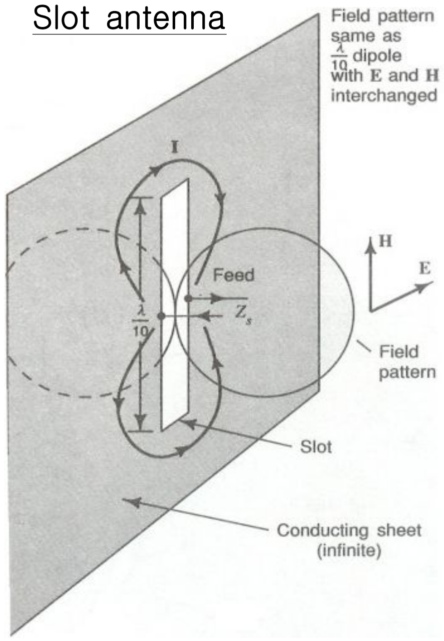




Loop antenna　　　　Dipole antenna

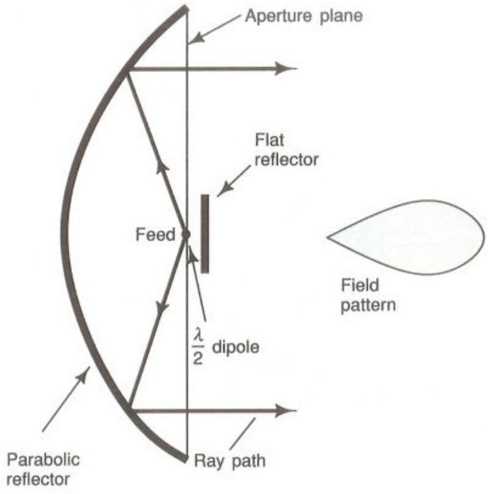


Slot antenna

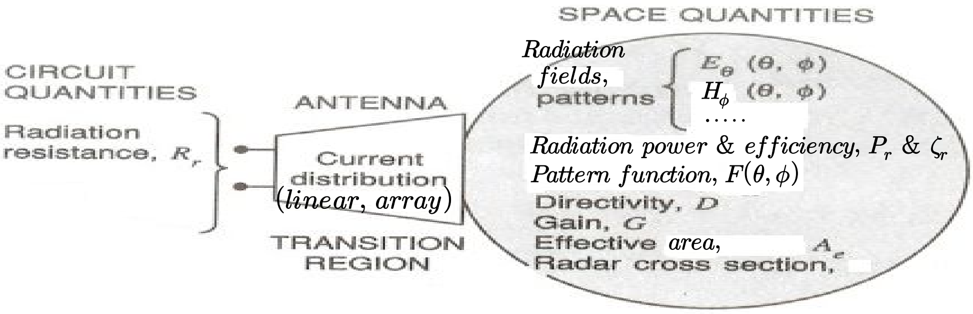


Aperture antenna

(parabolic reflector)



2) Basic antenna parameters

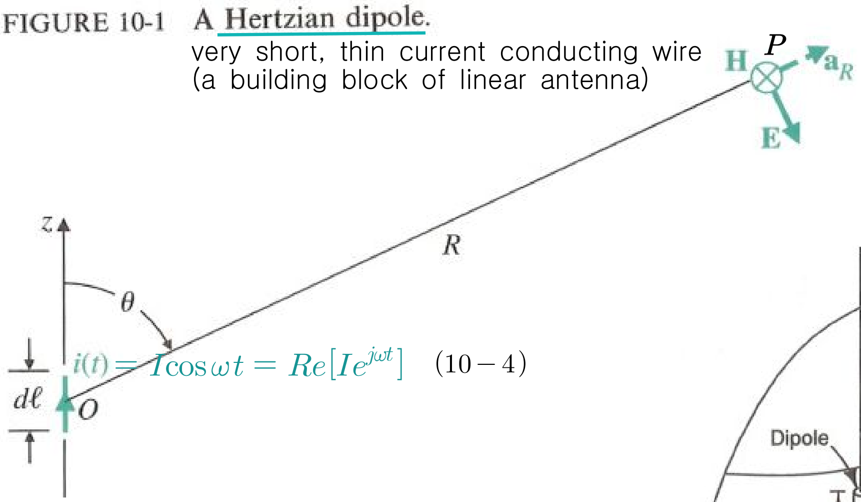


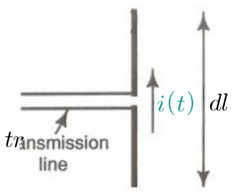
B. Elemental Electric Dipole Fields

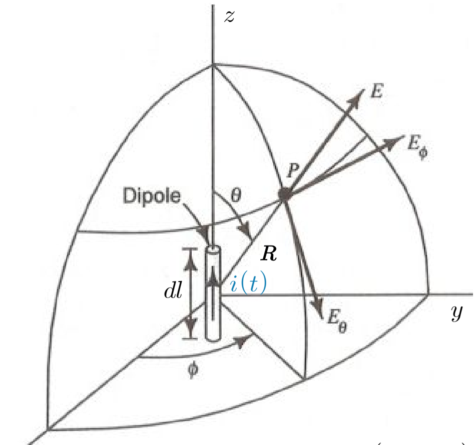
FIGURE 10-1 A Hertzian dipole.

very short, thin current conducting wire

(a building block of linear antenna)

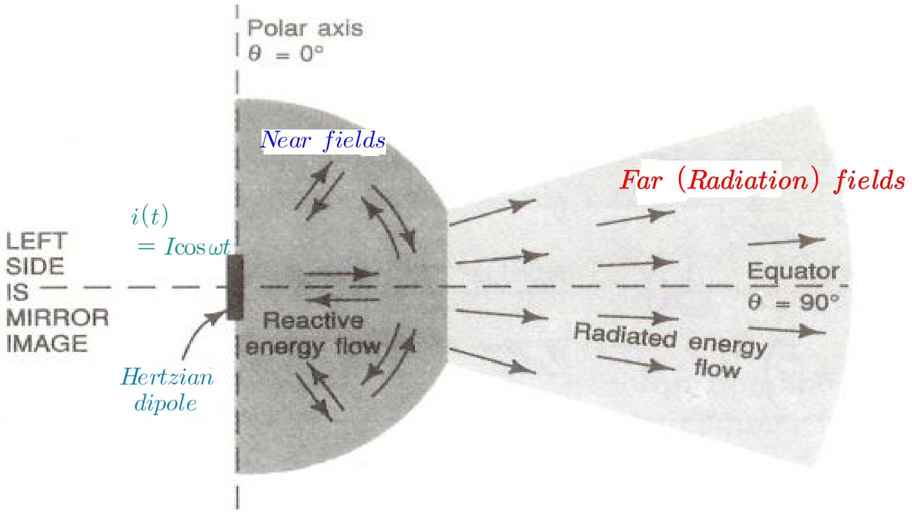






Spherical coordinates (R, θ, ϕ)

EM energy flow in near and far regions of a Hertzian dipole



Retarded vector potential phasor at P from the current source (10-1):

$$A = \frac{\mu}{4\pi}\int_V \frac{Je^{-jkR}}{R}dv' = \hat{z}\frac{\mu_o I\, dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right) \qquad (10\text{-}5)$$

where $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$ and $\beta = k_o = \omega/c = 2\pi/\lambda$ (10-6)

1) Near fields

Magnetic field from (10-5) in $\boldsymbol{B} = \nabla\times\boldsymbol{A}$:

$$\boldsymbol{H} = \hat{\phi}\frac{1}{\mu_o R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right] = -\hat{\phi}\frac{Idl}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta R} + \cancel{\frac{1}{(j\beta R)^2}}\right]e^{-j\beta R} \qquad (10\text{-}8)$$

Electric field from (10-8) in Faraday's law, $\boldsymbol{E} = \frac{1}{j\omega\epsilon_o}\nabla\times\boldsymbol{H}$:

$$E_R = -\frac{I\,d\ell}{4\pi}\eta_0\beta^2 2\cos\theta\left[\cancel{\frac{1}{(j\beta R)^2}} + \cancel{\frac{1}{(j\beta R)^3}}\right]e^{-j\beta R} \qquad (10-8a)$$

$$E_\theta = -\frac{I\,d\ell}{4\pi}\eta_0\beta^2\sin\theta\left[\frac{1}{j\beta R} + \cancel{\frac{1}{(j\beta R)^2}} + \cancel{\frac{1}{(j\beta R)^3}}\right]e^{-j\beta R} \qquad (10-8b)$$

$$E_\phi = 0 \qquad (10-8c)$$

2) Far (Radiation) fields

In the far-field zone ($R \gg \lambda/2\pi$, *i.e.*, $\beta R = 2\pi R/\lambda \gg 1$), neglecting $(\beta R)^{-2}$ and $(\beta R)^{-3}$ terms, we can get the far (radiation) fields of the elemental electric dipole:

$$H_\phi = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta \quad \text{(A/m)} \qquad (10\text{-}9)$$

$$E_\theta = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_o\beta\sin\theta = \eta_o H_\phi \quad \text{(V/m)} \qquad (10\text{-}10)$$

C. Antenna Patterns and Parameters

1) Antenna patterns of a Hertzian dipole

: Graph of the far field vs. distance at a fixed distance from an antenna

Pattern function = Normalized electric field function w.r.t. the peak value

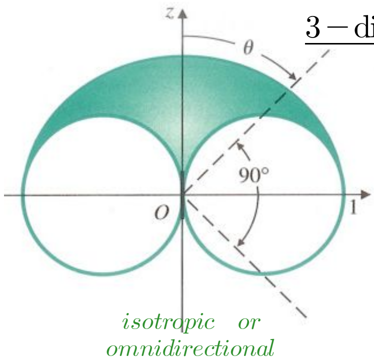
$$\Rightarrow \quad E_\theta(\theta,\phi)_n = E_\theta(\theta,\phi)/E_\theta(\theta,\phi)_{\max}$$

a) E-plane pattern

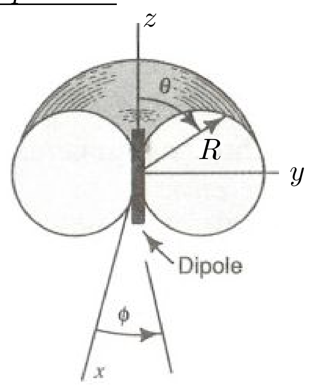
From (10-10) independent of ϕ at a given R,

$E_\theta(\theta,\phi)_n =$ Normalized $|E_\theta| = |\sin\theta|$ for $0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi$ (10-11)

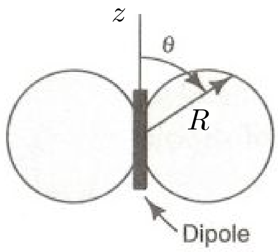
: E-plane pattern function of a Hertzian dipole



3−dim. pattern



2−dim. pattern

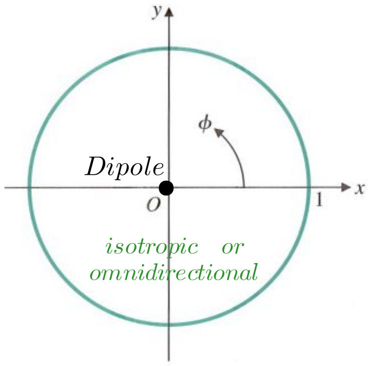


b) H-plane pattern

From (10-10) for $\theta = \pi/2$ at a given R,

$E_\theta(\theta,\phi)_n = |\sin\theta| = 1$

for $\theta = \pi/2,\ 0 \le \phi \le 2\pi$



2) Antenna parameters

Radiation intensity

= time-average power per unit solid angle

$\Rightarrow \quad U = R^2 \mathscr{P}_{av}$ (W/sr) (10-12)

Total time-ave. radiation power

$$P_r = \oint \mathscr{P}_{av} \cdot ds$$

$$= \oint \mathscr{P}_{av} R^2 d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi \quad (10\text{-}13)$$

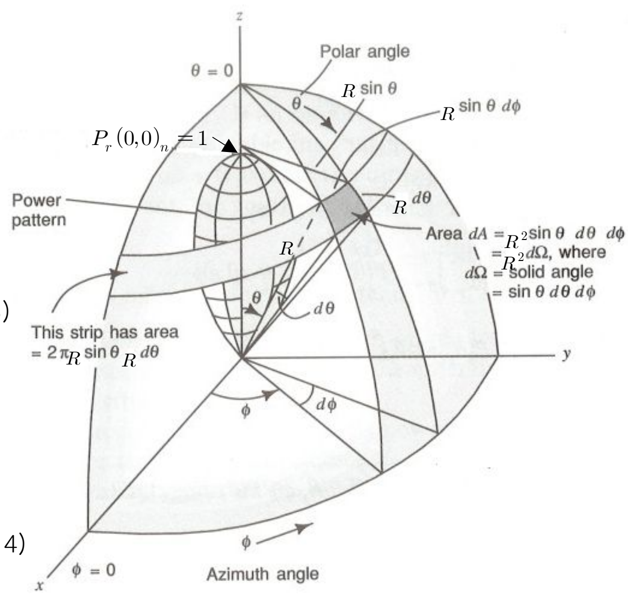
Directive gain

= rad. intens./ ave. rad. intens.

$$\Rightarrow \quad G_D(\theta,\phi) = \frac{U(\theta,\phi)}{P_r/4\pi}$$

$$= 4\pi U(\theta,\phi)/\oint U d\Omega \quad (10\text{-}14)$$

Note) $G_D = 1$ for isotropic antenna



Directivity = maximum directive gain

$$D = \frac{U_{\max(\theta,\phi)}}{P_r/4\pi} = \frac{4\pi\, U_{\max}}{P_r} \qquad (10-15)$$

$$\Rightarrow \qquad D = \frac{4\pi\, |E_{\max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta\, d\theta\, d\phi} = \frac{4\pi}{\oint P_r(\theta,\phi)_n d\Omega} \qquad (10-16)$$

(e.g. 10-2)

For a Hertzian dipole, $\mathscr{P}_{av} = (1/2)|E_\theta||H_\phi|$ in (10-12) with (10-9, 10)

$$\Rightarrow \qquad U = \frac{(Idl)^2}{32\pi^2}\eta_o \beta^2 \sin^2\theta \quad \text{in (10-14)} \qquad (10-18)$$

$$\Rightarrow \qquad G_D(\theta,\phi) = (3/2)\sin^2\theta \qquad (10-19)$$

$\Rightarrow \qquad D = G_D(\pi/2, \phi) = 1.5$ or 1.76 (dB) : omni-directional

Radiation resistance = hypothetical resistance dissipating P_r when $I_{\max}$ flows through it ($R_r I_{\max}^2/2 = P_r$)

$$R_r = 2P_r/I_{\max}^2 \qquad (10-25)^*$$

(e.g. 10-3) For a Hertzian dipole,

$$I_{\max} = I \text{ and } P_r = \frac{I^2}{2}\left[80\pi^2\left(\frac{dl}{\lambda}\right)^2\right] \quad (10-24) \text{ in } (10-25)^*$$

$$\Rightarrow \qquad R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2 \quad (\Omega) \qquad (10-25)$$

If $dl = 0.01\lambda$, $R_r \cong 0.08\,(\Omega)$: too small for practical use!

Power gain = $4\pi \times$max. rad. intens./ input power

$$G_p = \frac{4\pi\, U_{\max}}{P_i} = \frac{4\pi\, U_{\max}}{P_r + P_l} = \frac{4\pi\, U_{\max}}{(I^2/2)(R_r + R_l)} \qquad (10-21)$$

Ohmic power loss

Radiation efficiency = power gain / directivity

$$\zeta_r = \frac{G_p}{D} = \frac{P_r}{P_i} = \frac{R_r}{R_r + R_l} \qquad (10-22, 28)$$

(e.g.10-4)

For a Hertzian dipole of radius a, length dl, and conductivity σ,

$$(10-25) \text{ and } R_l = R_s\left(\frac{dl}{2\pi a}\right) = \sqrt{\frac{\pi f \mu_o}{\sigma}}\left(\frac{dl}{2\pi a}\right) \quad (10-29, 30) \text{ in } (10-28)$$

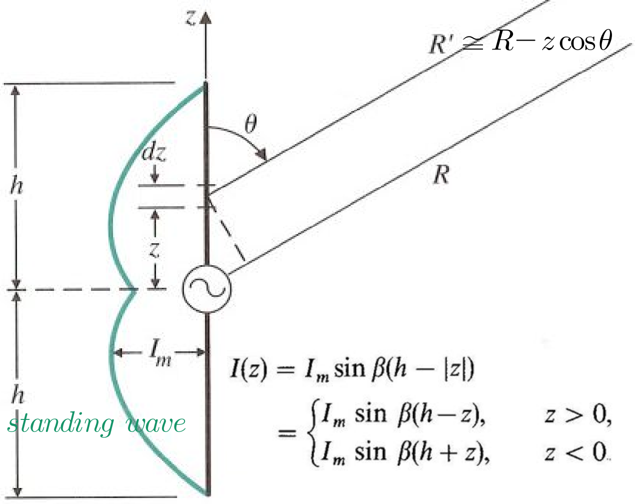
$$\Rightarrow \qquad \zeta_r = \frac{1}{1 + \frac{R_s}{160\pi^3}\left(\frac{\lambda}{a}\right)\left(\frac{\lambda}{dl}\right)} \qquad (10-31)$$

$= 58\,\%$ for $a = 1.8\,mm$, $dl = 2\,m$, $f = 1.5MHz$ and Cu

D. *Thin Linear Antenna*

1) Linear dipole antenna pattern

Consider a center-fed thin, straight dipole with sinusoidal current distribution



$$R' \cong R - z\cos\theta \qquad (10-34)$$

$$I(z) = I_m \sin\beta(h-|z|) = \begin{cases} I_m \sin\beta(h-z), & z>0, \\ I_m \sin\beta(h+z), & z<0 \end{cases} \qquad (10-32)$$

Far-field contribution from the current element Idz by (10-9, 10):

$$dE_\theta = \eta_0\, dH_\phi = j\frac{I\,dz}{4\pi}\left(\frac{e^{-j\beta R'}}{R'}\right)\eta_0\beta\sin\theta \qquad (10-33)$$

(10-32) and (10-34) in (10-33):

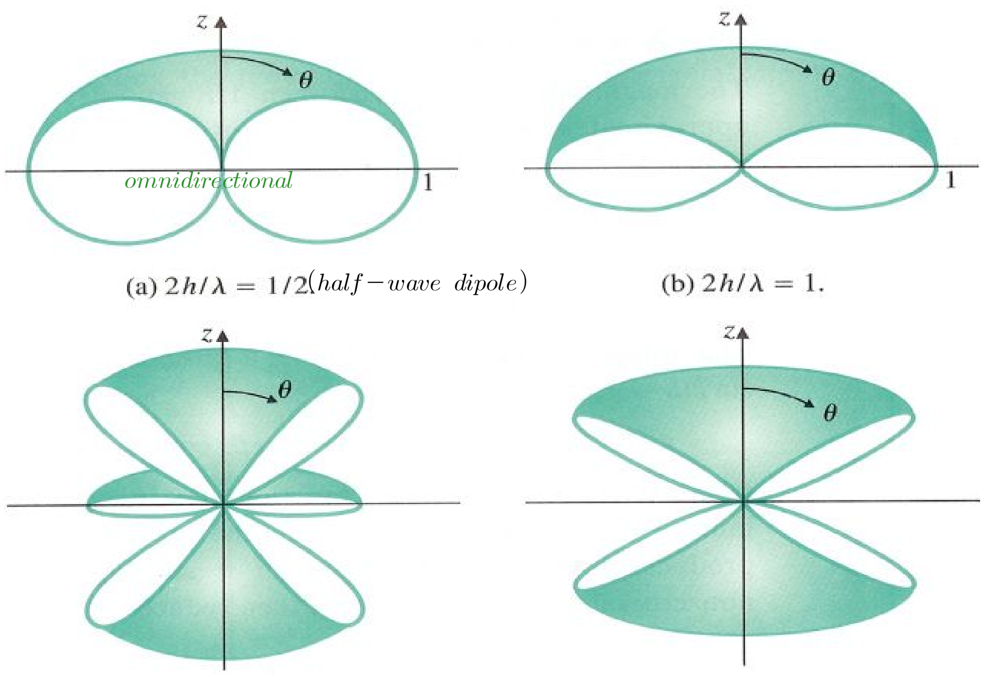
$$E_\theta = \eta_0 H_\phi = j\frac{I_m\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\sin\beta(h-|z|)e^{j\beta z\cos\theta}\,dz \qquad (10-35)$$

$$\Rightarrow\ E_\theta = \eta_0 H_\phi = j\frac{I_m\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\sin\beta(h-z)\cos(\beta z\cos\theta)\,dz$$

$$\Rightarrow\ E_\theta = \frac{j60\,I_m}{R}e^{-j\beta R}F(\theta) \qquad (10-36)$$

where $F(\theta) = \dfrac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}$: E-plane pattern function (10-37)

FIGURE 10-4 *E*-plane radiation patterns for center-fed dipole antennas.



(a) $2h/\lambda = 1/2$ (*half-wave dipole*)

(b) $2h/\lambda = 1$.

(c) $2h/\lambda = 3/2$.

(d) $2h/\lambda = 2$.

2) Half-Wave dipole

For $2h = \lambda/2$, $\beta h = 2\pi h/\lambda = \pi/2$ in (10-37):

$$F(\theta) = \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \tag{10-38}$$

Far-zone field phasor from (10-36):

$$E_\theta = \eta_0 H_\phi = \frac{j60I_m}{R} e^{-j\beta R}\left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\} \tag{10-39}$$

Time-average Poynting vector:

$$\mathscr{P}_{av}(\theta) = \frac{1}{2}E_\theta H_\phi^* = \frac{15I_m^2}{\pi R^2}\left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}^2 \tag{10-40}$$

Total power radiated by the half-wave antenna:

$$P_r = \int_0^{2\pi}\int_0^{\pi} \mathscr{P}_{av}(\theta) R^2 \sin\theta \, d\theta \, d\phi = 30I_m^2 \int_0^{\pi} \frac{\cos^2[(\pi/2)\cos\theta]}{\sin\theta} d\theta \tag{10-41}$$

$$\Rightarrow \quad P_r = 36.54\, I_m^2 \quad \text{(W)} \tag{10-42}$$

$$\Rightarrow \quad R_r = \frac{2P_r}{I_m^2} = 73.1 \quad (\Omega) \tag{10-43}$$

From (10-12) and (10-40), $U_{\max} = R^2\mathscr{P}_{av}(90^o) = 15I_m^2/\pi$ in (10-15) with (10-42):

$$D = \frac{4\pi U_{\max}}{P_r} = 1.64 \text{ or } 2.15 \text{ (dB)} : \text{ omni-directional} \tag{10-45}$$

3) Quarter-Wave monopole = Half-Wave dipole

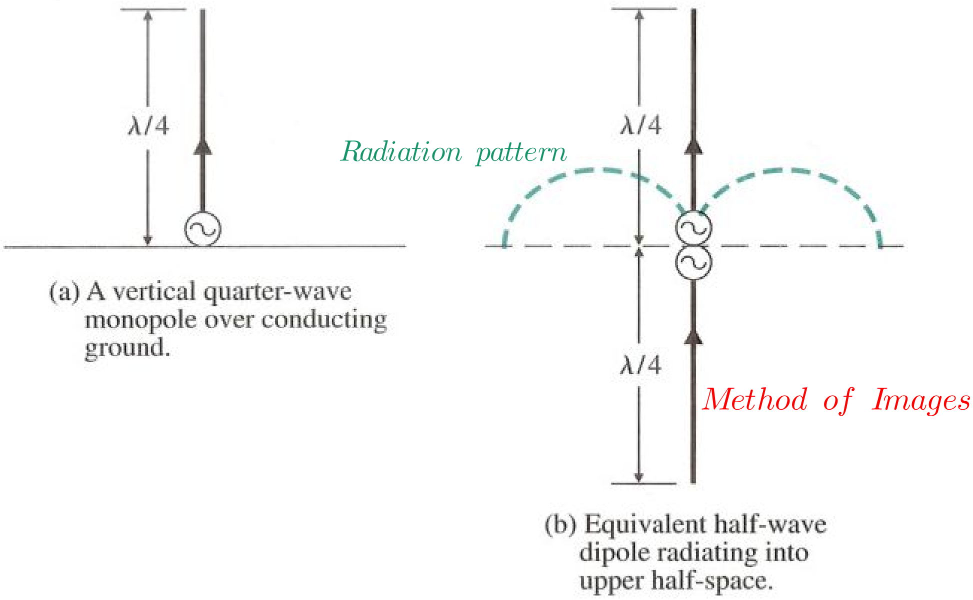


FIGURE 10-5 Quarter-wave monopole over a conducting ground and its equivalent half-wave dipole.

(10-42) for $0 \le \theta \le \pi/2 \Rightarrow P_r = 18.27\, I_m^2 \Rightarrow R_r = \dfrac{2P_r}{I_m^2} = 36.54$ (Ω) (10-46)

$$D = \frac{2\pi U_{\max}}{P_r} = 1.64 \text{ or } 2.15 \text{ (dB)} \tag{10-47}$$