Design Theories of Ship and Offshore Plant

Part I. Ship Design

Ch. 6 Structural Design

Fall 2014

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Ch. 6 Structural Design

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6.1 General & Materials

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- ☑ Stress Transmission
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- ☑ Criteria for the Selection of Plate Thickness, Grouping of Longitudinal Stiffener
- ☑ Material Factors

(1) Stress Transmission

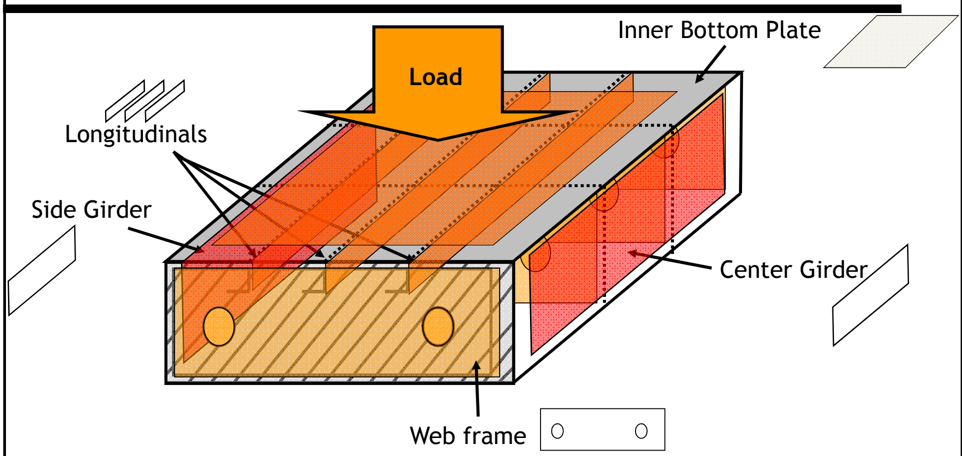


Plate ➡ Longi. ➡ Web frame ➡ Girder

➡: Stress Transmission

(2) Principal Dimensions

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.1 101

The following principal dimensions are used in accordance with DNV rule.

1) Rule length (L or L_s)

: Length of a ship used for rule scantling procedure

$$0.96 \cdot L_{WL} < L < 0.97 \cdot L_{WL}$$

- Distance on the summer load waterline (L_{WL}) from the fore side of the stem to the axis of the rudder stock
- Not to be taken less than 96%, and need not be taken greater than 97%, of the extreme length on the summer load waterline (L_{WL})
- Starting point of rule length: F.P

Ex.

L_{BP}	L_{WL}	$0.96 \cdot L_{WL}$	$0.97 \cdot L_{WL}$	L
250	261	250.56	253.17	250.56
250	258	247.68	250.26	250.00
250	255	244.80	247.35	247.35

2) Breadth

: Greatest moulded breadth in [m], measured at the summer load waterline

(DNV Pt.3 Ch.1 Sec.1 B101), 2011

B. Definitions

B 100 Symbols

101 The following symbols are used:

L = length of the ship in m defined as the distance on the summer load waterline from the fore side of the stem to the axis of the rudder stock.

L shall not be taken less than 96%, and need not to be taken greater than 97%, of the extreme length on the summer load waterline. For ships with unusual stern and bow arrangement, the length L will be especially considered.

F.P. = the forward perpendicular is the perpendicular at the intersection of the summer load waterline with the fore side of the stem. For ships with unusual bow arrangements the position of the F.P. will be especially considered.

A.P. = the after perpendicular is the perpendicular at the after end of the length L.

L_F = length of the ship as defined in the International Convention of Load Lines:

The length shall be taken as 96 per cent of the total length on a waterline at 85 per cent of the least moulded depth measured from the top of the keel, or as the length from the fore side of the stem to the axis of the rudder stock on that waterline, if that be greater. In ships designed with a rake of keel the waterline on which this length is measured shall be parallel to the designed waterline.

B = greatest moulded breadth in m, measured at the summer waterline.

D = moulded depth defined as the vertical distance in m from baseline to moulded deckline at the uppermost continuous deck measured amidships.

D_F = least moulded depth taken as the vertical distance in m from the top of the keel to the top of the freeboard deck beam at side.

In ships having rounded gunwales, the moulded depth shall be measured to the point of intersection of the moulded lines of the deck and side shell plating, the lines extending as though the gunwale was of angular design.

Where the freeboard deck is stepped and the raised part of the deck extends over the point at which the moulded depth shall be determined, the moulded depth shall be measured to a line of reference

(DNV Pt.3 Ch.1 Sec.1 B101), 2011

extending from the lower part of the deck along a line parallel with the raised part.

T = mean moulded summer draught in m.

Δ = moulded displacement in t in salt water (density 1.025 t/m^3) on draught T.

C_B = block coefficient,

$$= \frac{\Delta}{1.025\, L\, B\, T}$$

For barge rigidly connected to a push-tug C_B shall be calculated for the combination barge/ push-tug.

C_{BF} = block coefficient as defined in the International Convention of Load Lines:

$$= \frac{\nabla}{L_F\, B\, T_F}$$

∇ = volume of the moulded displacement, excluding bossings, taken at the moulded draught T_F.

T_F = 85% of the least moulded depth.

V = maximum service speed in knots, defined as the greatest speed which the ship is designed to maintain in service at her deepest seagoing draught.

g_0 = standard acceleration of gravity
= 9.81 m/s^2.

f_1 = material factor depending on material strength group. See Sec.2.

t_k = corrosion addition as given in Sec.2 D200 and D300, as relevant.

x = axis in the ship's longitudinal direction.

y = axis in the ship's athwartships direction.

z = axis in the ship's vertical direction.

E = modulus of elasticity of the material
= $2.06 \cdot 10^5$ N/mm^2 for steel
= $0.69 \cdot 10^5$ N/mm^2 for aluminium alloy.

C_W = wave load coefficient given in Sec.4 B200.

Amidships = the middle of the length L.

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.1 101

(2) Principal Dimensions

3) Depth (D)

: Moulded depth defined as the vertical distance in [m] from baseline to moulded deck line at the uppermost continuous deck measured amidships

4) Draft (T)

: Mean moulded summer draft (**scantling draft**) in [m]

5) Brock coefficient (C_B)

: To be calculated based on the rule length

$$C_B = \frac{\Delta}{1.025 \cdot L \cdot B \cdot T}$$, (Δ : moulded displacement in salt water on draft T)

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.1 101

(3) Criteria for the Selection of Plate Thickness, Grouping of Longitudinal Stiffener

1) Criteria for the selection of plate thickness

➡ When selecting plate thickness, use the provided plate thickness.

(1) 0.5 mm interval
(2) Above 0.25 mm: 0.5 mm
(3) Below 0.25 mm: 0.0 mm

Ex) 15.75 mm ➡ 16.0 mm
15.74 mm ➡ 15.5 mm

2) Grouping of longitudinal stiffener

For the efficiency of productivity, each member is arranged by grouping longitudinal stiffeners.

The grouping members should satisfy the following rule.

Average value but not to be taken less than 90% of the largest individual requirement. (DNV)

Ex. The longitudinal stiffeners have design thickness of 100, 90, 80, 70, 60 mm. The average thickness is given by 80 mm×5. However, the average value is less than 100mm×90% = 90 mm of the largest individual requirement, 100 mm.
Therefore, the average value should be taken 90 mm×5.

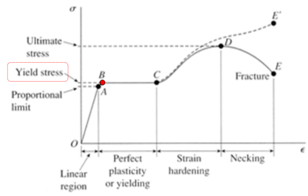
(4) Material Factors

[1] DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2
[2] James M. Gere, Mechanics of Materials 7th Edition, Thomson, Chap.1, pp.15~26

- The material factor f_1 is included in the various formulae for scantlings and in expressions giving allowable stresses.[1]

Material Designation	Yield Stress (N/mm^2)	$\dfrac{\sigma}{\sigma_{NV-NS}}$	Material Factor (f_1)
NV-NS	235	235/235 = 1.00	1.00
NV-27	265	265/235 = 1.13	1.08
NV-32	315	315/235 = 1.34	1.28
NV-36	355	355/235 = 1.51	1.39
NV-40	390	390/235 = 1.65	1.47

* NV-NS: Normal Strength Steel (Mild Steel)
* NV-XX: High Tensile Steel



* Yield Stress (σ_y) [N/mm^2] or [MPa]: The magnitude of the load required to cause yielding in the beam.[2]

* High tensile steel: A type of alloy steel that provides better mechanical properties or greater resistance to corrosion than carbon steel. They have a carbon content between 0.05-0.25% to retain formability and weldability, including up to 2.0% manganese, and other elements are added for strengthening purposes.

* A: 'A' grade 'Normal Strength Steel'
* AH: 'A' grade 'High Tensile Steel'

6.2 Global Hull Girder Strength (Longitudinal Strength)

Contents

Interest of "Ship Structural Design"

- Ship Structural Design

What is designer's major interest?

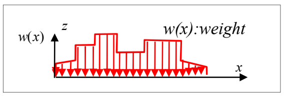
- Safety:
  Won't 'it' fail under the load?

a ship	} global
a stiffener a plate	} local

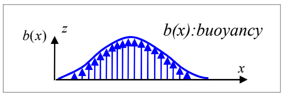
Let's consider the safety of the ship from the point of global strength first.

Dominant Forces Acting on a Ship

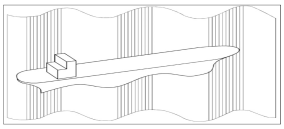
What are dominant forces acting on a ship in view of the longi. strength?



weight of light ship, weight of cargo, and consumables



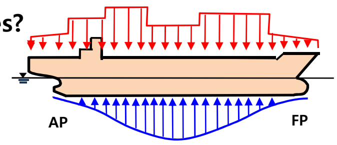
hydrostatic force (buoyancy) on the submerged hull



hydrodynamic force induced by the wave

What is the direction of the dominant forces?

The forces act in vertical (lateral) direction along the ship's length.



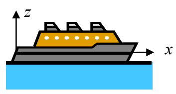
Longitudinal Strength

: Overall strength of ship's hull which resists the bending moment, shear force, and torsional moment acting on a hull girder.

Longitudinal strength loads

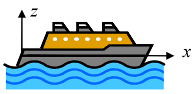
: Load concerning the overall strength of the ship's hull, such as the bending moment, shear force, and torsional moment acting on a hull girder

Static longitudinal loads



Loads are caused by differences between weight and buoyancy in longitudinal direction in the still water condition

Hydrodynamic longitudinal loads



Loads are induced by waves

[1] Okumoto,Y., Design of Ship Hull Structures, Springers, 2009, P.17

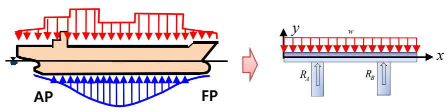
Idealization of the Ship Hull Girder Structure

How can we idealize a ship as a structural member?

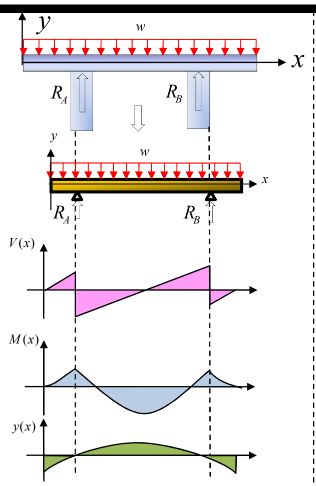
- **Structural member according to the types of loads**
  - ① Axially loaded bar: structural member which supports forces directed along the axis of the bar
  - ② Bar in torsion: structural member which supports torques (or couples) having their moment about the longitudinal axis
  - ③ Beam: structural members subjected to lateral loads, that is, forces or moments perpendicular to the axis of the bar

Since a ship has a slender shape and subject to lateral loads, it will behave like a beam from the point view of structural member.

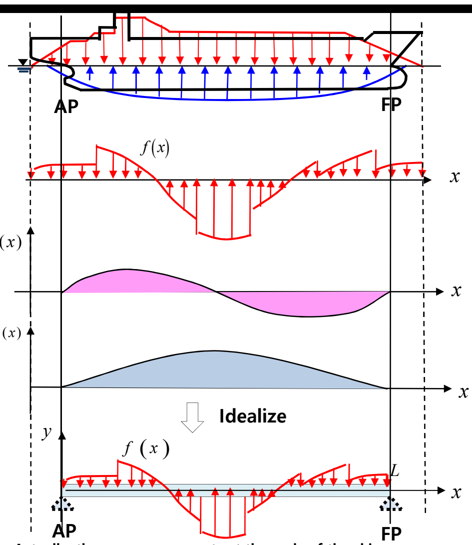
Ship is regarded as a beam.



Applying Beam Theory to a Ship



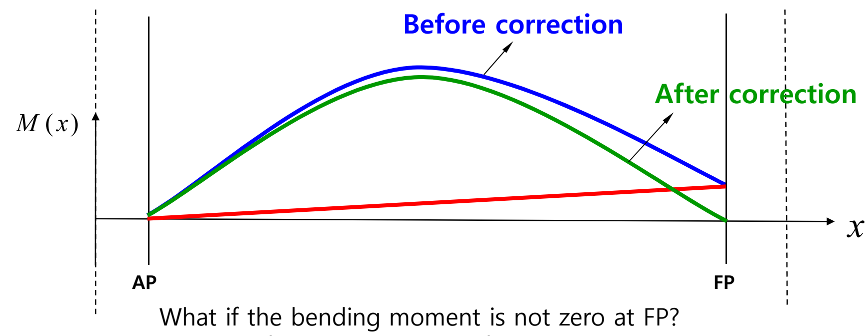
If there are supports at the ends, deflection and slope of the beam occur.



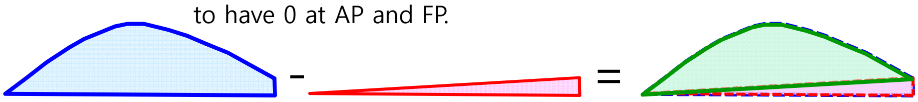
Actually, there are no supports at the ends of the ship. However, the deflection and slope could occur due to inequality of the buoyancy and the weight of a ship. For this problem, we assume that there are simple supports at the A.P and the F.P.

* James M. Gere, Mechanics of Materials, 6th Edition, Thomson, Ch. 4, p. 292

Correction of a Bending Moment Curve



What if the bending moment is not zero at FP?
➡ The deflection and slope of the beam occur at FP.
➡ Thus, we correct the bending moment curve to have 0 at AP and FP.



* James M. Gere, Mechanics of Materials, 6th Edition, Thomson, Ch. 4, p. 292

Actual Stress ≤ Allowable Stress
- Bending Stress and Allowable Bending Stress

The actual bending stress ($\sigma_{act.}$) shall not be greater than the allowable bending stress (σ_l).

M_S : Largest SWBM among all loading conditions and class rule
M_W : Calculated by class rule or direct calculation

$$\sigma_{act.} = \frac{|M_S + M_W|}{Z} 10^3 \ \ [kg/cm^2]$$

$$\sigma_{act.} \le \sigma_l$$

(DNV Pt.3 Ch.1 Sec. 5 C303)

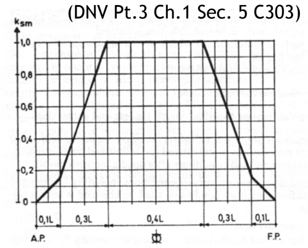


Fig. 2
Stillwater bending moment

$$\sigma_l = \sigma_{allow} = 175 f_1 \ [N/mm^2] \text{ within 0.4L amidship}$$

$$= 125 f_1 \ [N/mm^2] \text{ within 0.1L from A.P. or F.P.}$$

(f_1: Material factor. Ex. Mild steel 1.0, HT-32 1.28, HT-36 1.39)

[1)]DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.5

(DNV Pt.3 Ch.1 Sec.5 C303), 2011

303 The section modulus requirements about the transverse neutral axis based on cargo and ballast conditions are given by:

$$Z_O = \frac{|M_S + M_W|}{\sigma_l} 10^3 \quad (cm^3)$$

σ_l = 175 f_1 N/mm^2 within 0.4 L amidship
= 125 f_1 N/mm^2 within 0.1 L from A.P. or F.P.

Between specified positions σ_l shall be varied linearly.

(DNV Pt.3 Ch.1 Sec.5 C304), 2011

304 The midship section modulus about the vertical neutral axis (centre line) is normally not to be less than:

$$Z_{OH} = \frac{5}{f_1} L^{9/4} (T + 0.3B) C_B \quad (cm^3)$$

The above requirement may be disregarded provided the combined effects of vertical and horizontal bending stresses at bilge and deck corners are proved to be within 195 f_1 N/mm^2.
The combined effect may be taken as:

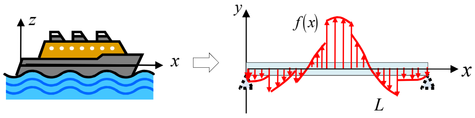
$$\sigma_s + \sqrt{\sigma_w^2 + \sigma_{wh}^2}$$

σ_s = stress due to M_S
σ_w = stress due to M_W
σ_{wh} = stress due to M_{WH}, the horizontal wave bending moment as given in B205.

Criteria of Structural Design (1/2)

● Ship Structural Design

a ship



The **actual bending stress** ($\sigma_{act.}$) shall not be greater than the **allowable bending stress** (σ_l).

$$\sigma_{act.} \le \sigma_l \quad , \sigma_{act.} = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

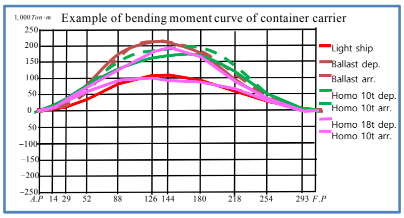
M_S : Largest SWBM among all loading conditions and class rule
M_W : calculated by class rule or direct calculation

σ_l : allowable stress

For instance, allowable bending stresses by DNV rule are given as follows:

$\sigma_l = 175 f_1 \ [N/mm^2]$ within 0.4L amidship

$= 125 f_1 \ [N/mm^2]$ within 0.1L from A.P. or F.P.



Actual bending moments at aft and forward area are smaller than that at the midship.

What is, then, the f_l?

Criteria of Structural Design (2/2)

$$\sigma_{act.} \le \sigma_l$$

$$\sigma_{act.} = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

(1) Still Water Bending Moment (Ms)
(2) Vertical Wave Bending Moment (Mw)
(3) Section Modulus ($I_{N.A}$/y)

(1) Still Water Bending Moment (Ms)

Still Water Bending Moment (Ms)

$$\boxed{\sigma_{act.} \le \sigma_l}\quad , \sigma_{act.} = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

M_S: Still water bending moment
M_W: Vertical wave bending moment

Hydrostatic loads along ship's length
caused by the weight & the buoyancy

$f_S(x)$: distributed loads in longitudinal direction in still water

↓

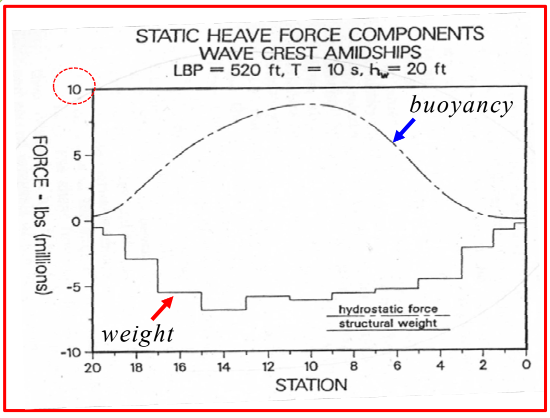
$V_S(x)$: still water shear force

$$V_S(x) = \int_0^x f_S(x)dx$$

↓

$M_S(x)$: still water bending moment

$$M_S(x) = \int_0^x V_S(x)dx$$



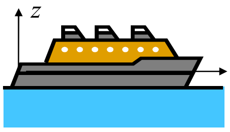
Distributed Loads in Longitudinal Direction

$$f(x) = f_S(x) + f_W(x)$$

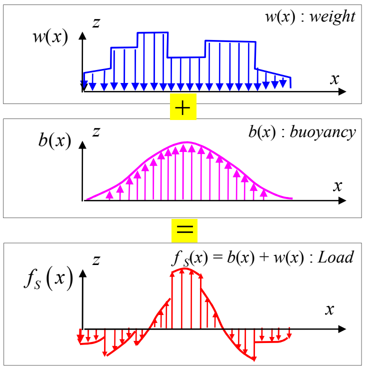
$f(x)$: Distributed loads in longitudinal direction

$f_S(x)$: **Static longitudinal loads** in longitudinal direction
$f_W(x)$: **Hydrodynamic longitudinal loads** induced by wave

In still water



$$f_S(x) = b(x) + w(x)$$



$b(x)$: Distributed buoyancy in longitudinal direction
$w(x) = LWT(x) + DWT(x)$
- $w(x)$: Weight distribution in longitudinal direction
- $LWT(x)$: Lightweight distribution
- $DWT(x)$: Deadweight distribution

Distributed Loads in Still Water

Load Curve, $f_S(x)$ → Actual Still Water Shear Force, $V_S(x)$ → Actual Still Water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$

$M_S(x) = \int_0^x V_S(x)dx$

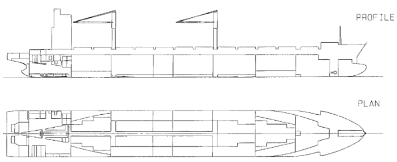
✓ Example of a 3,700 TEU Container Ship in Homogeneous 10 ton Scantling Condition

- Principal Dimensions & Plans

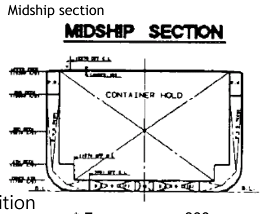
Principal dimension

LENGTH O. A.	257. 368 M
LENGTH B. P.	245. 240 M
BREADTH MOULDED	32. 20 M
DEPTH MOULDED	19. 30 M
DESIGNED DRAUGHT MOULDED	10. 10 M
SCANTLING DRAUGHT MOULDED	12. 50 M

Profile & plan



Midship section



- Loading Condition (Sailing state) in Homogeneous 10 ton Scantling Condition

* Frame space: 800mm

SAILING STATE

DRAUGHT F.P	=	12.260 M	K.M.T	=	14.889 M
DRAUGHT MIDSHIP	=	12.457 M	KG (SOLID)	=	13.586 M
DRAUGHT A.P	=	12.654 M	GM (SOLID)	=	1.303 M
TRIM BY STERN	=	.394 M	FREE SURF. CORR.(GGo)	=	.059 M
PROPELLER I/D	=	160.3 %	GoM (FLUID)	=	1.244 M
DISPLACEMENT	=	66813.6 T	KGo ACTUAL (FLUID)	=	13.645 M
DRAUGHT AT LCF	=	12.483 M	TRIM (DIS*A)/(MTC*100)	=	.394 M
LCB FROM A.P	=	115.677 M	FREE SURF. MOM.	=	3921 T-M
LCG FROM A.P	=	115.045 M	M.T.C.	=	1072.0 T-M
TRIM LEVER : A	=	.632 M	LCF FROM A.P	=	106.275 M

DEGREE =	.0	5.0	10.0	15.0	20.0	30.0	40.0	50.0	60.0	75.0
KN =	.000	1.296	2.591	3.882	5.168	7.614	9.592	10.930	11.697	11.959
KGo*SINθ =	.000	1.189	2.369	3.532	4.667	6.823	8.771	10.453	11.817	13.180
GZ =	.000	.107	.222	.350	.501	.791	.821	.477	-.120	-1.221

Distributed Loads in Still Water - Lightweight

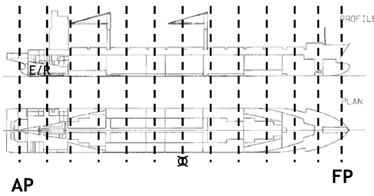
Load Curve, $f_S(x)$ → Actual Still Water Shear Force, $V_S(x)$ → Actual Still Water Bending Moment, $M_S(x)$

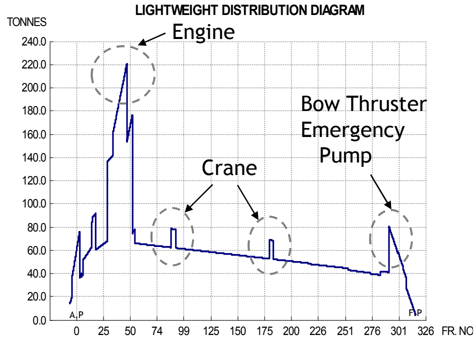
Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$

$M_S(x) = \int_0^x V_S(x)dx$

Example of a 3,700 TEU Container Ship





L I G H T W E I G H T S U M M A R Y

Hull No. : 1329. 3,700 TEU CONTAINER VESSEL

NO	AFT END	FORE END	WEIGHT	L.C.G	MOMENT
1	-5.000	14.350	616.00	7.000	4312.0
2	14.350	43.400	1387.10	31.400	43554.9
3	43.400	232.320	7591.50	128.620	976418.7
4	232.320	252.240	732.30	239.280	175224.7
5	27.200	41.600	476.40	35.800	17055.1
6	.000	245.240	30.00	122.620	3678.6
7	43.400	232.320	340.00	134.200	45628.0
8	-3.600	232.320	119.00	114.400	13613.6
9	-3.400	2.400	151.90	.000	.0
10	.000	252.240	224.00	120.000	26880.0
11	202.240	232.320	137.90	217.000	29924.3
12	43.400	202.240	1053.00	121.700	128150.1
13	143.280	146.680	55.00	144.980	7973.9
14	70.480	73.880	55.00	72.180	3969.9
15	14.350	232.320	115.90	114.360	13254.3
16	-3.600	232.320	128.00	114.360	14638.1
17	232.320	245.240	118.30	238.600	28226.4
18	36.000	170.000	3.00	81.000	243.0
19	-5.000	4.000	50.00	-.500	-25.0
20	29.000	41.600	15.50	37.100	575.0
21	-3.500	4.000	19.20	.000	.0
22	4.000	11.200	34.30	7.600	260.7
23	41.600	173.900	62.50	105.760	6610.0
24	226.160	232.320	20.40	229.240	4676.5
25	239.000	243.000	5.40	241.000	1301.4
26	11.200	232.320	39.20	121.700	4770.6
27	11.200	232.320	191.30	121.700	23281.2
28	27.200	41.600	214.50	36.000	7722.0
29	23.230	37.600	979.00	30.400	29761.6
30	11.200	41.600	289.50	22.000	6369.0
31	5.000	23.230	111.30	11.200	1246.6
32	12.000	41.600	150.70	28.000	4219.6
33	11.200	41.600	158.60	28.000	4440.8
34	11.200	41.600	95.90	28.000	2685.2
35	11.200	218.480	165.00	114.240	18849.6
36	27.200	41.600	8.50	36.000	306.0
37	11.200	41.600	43.00	30.000	1290.0
38	27.200	41.600	4.30	36.000	154.8
39	27.200	41.600	5.70	36.000	205.2
LIGHT SHIP TOTAL =			15998.10	103.228	1651446.5

31

Distributed Loads in Still Water - Deadweight

Load Curve, $f_S(x)$ → Actual Still Water Shear Force, $V_S(x)$ → Actual Still Water Bending Moment, $M_S(x)$

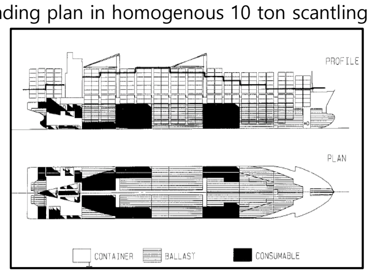
Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$

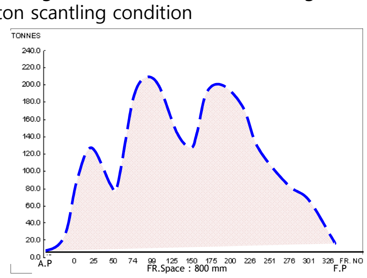
$M_S(x) = \int_0^x V_S(x)dx$

Example of a 3,700 TEU Container Ship

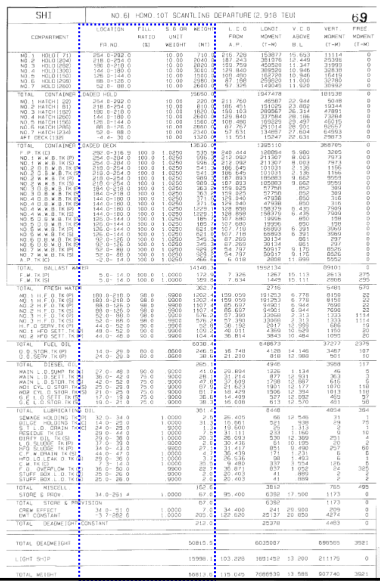
- Loading plan in homogenous 10 ton scantling condition



- Deadweight distribution curve in homogenous 10 ton scantling condition



Deadweight distribution in longitudinal direction in homogenous 10 ton scantling condition



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Distributed Loads in Still Water - Buoyancy Curve

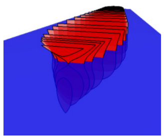
Load Curve, $f_S(x)$ → Actual Still Water Shear Force, $V_S(x)$ → Actual Still Water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

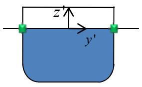
$V_S(x) = \int_0^x f_S(x)dx$ $M_S(x) = \int_0^x V_S(x)dx$

Example of a 3,700 TEU Container Ship

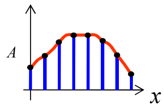
✓ Calculation of buoyancy



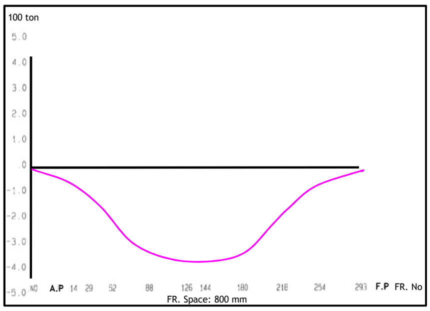
(1) Calculation of sectional area below waterline



(2) Integration of sectional area over the ship's length



✓ Buoyancy Curve in Homogeneous 10 ton Scantling Condition



DRAUGHT F.P	=	12.260 M	K.M.T	=	14.889 M
DRAUGHT MIDSHIP	=	12.457 M	KG (SOLID)	=	13.586 M
DRAUGHT A.P	=	12.654 M	GM (SOLID)	=	1.303 M
TRIM BY STERN	=	.394 M	FREE SURF. CORR.(GGo)	=	.059 M
PROPELLER I/D	=	160.3 %	GoM (FLUID)	=	1.244 M
DISPLACEMENT	=	66813.6 T	KGo ACTUAL (FLUID)	=	13.645 M
DRAUGHT AT LCF	=	12.483 M	TRIM (DIS*A)/(MTC*100)	=	.394 M
LCB FROM A.P	=	115.677 M	FREE SURF. MOM.	=	3921 T-M
LCG FROM A.P	=	115.045 M	M.T.C.	=	1072.0 T-M
TRIM LEVER : A	=	.632 M	LCF FROM A.P	=	106.275 M

Distributed Loads in Still Water - Load Curve

Load Curve, $f_S(x)$ → Actual Still Water Shear Force, $V_S(x)$ → Actual Still Water Bending Moment, $M_S(x)$

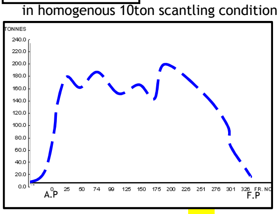
Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$ $M_S(x) = \int_0^x V_S(x)dx$

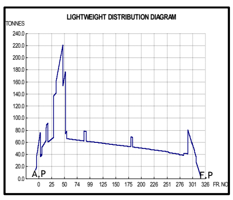
Load Curve

$f_S(x)$: Loads Curve = Weight + Buoyancy

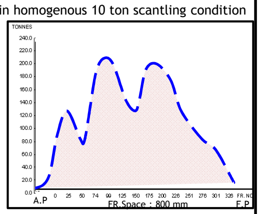
Weight Curve = Lightweight + Deadweight
in homogenous 10ton scantling condition



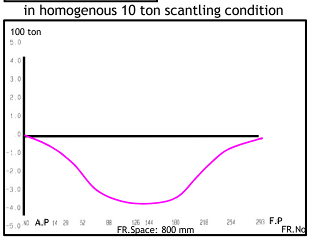
= Lightweight Distribution Curve



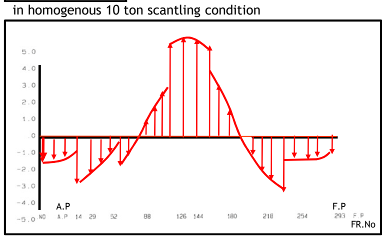
\+ Deadweight Distribution Curve
in homogenous 10 ton scantling condition



\+ Buoyancy Curve
in homogenous 10 ton scantling condition



Load Curve = Weight $w(x)$ + Buoyancy $b(x)$
in homogenous 10 ton scantling condition



Actual Still Water Shear Force

Actual Still Water Shear Force & Actual Still Water Bending Moment

Load Curve, $f_S(x)$ → Actual Still Water Shear Force, $V_S(x)$ → Actual Still Water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$ $M_S(x) = \int_0^x V_S(x)dx$

Example of a 3,700TEU Container Ship

Load Curve $f_S(x) = b(x) + w(x)$

↓

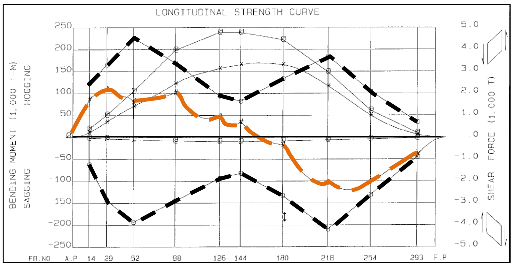
Actual Still Water Shear Force $V_S(x) = \int_0^x f_S(x)dx$

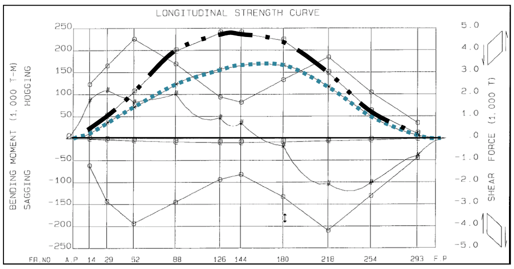
Actual still water shear force
Design still water shear force

↓

Actual Still Water Bending Moment $M_S(x) = \int_0^x V_S(x)dx$

Actual still water bending moment
Design still water bending moment





Rule Still Water Bending Moment by the Classification Rule

Recently, actual still water bending moment based on the load conditions is used for still water bending moment, because the rule still water bending moment is only for the tanker.

- The design still water bending moments amidships are not to be taken less than

(DNV Pt.3 Ch.1 Sec. 5 A105)

$$M_S = M_{SO}\ [kNm]$$

$$M_{SO} = -0.065C_{WU}L^2B(C_B + 0.7) \quad \text{[kNm] in sagging}$$

$$= C_{WU}L^2B(0.1225 - 0.015C_B) \quad \text{[kNm] in hogging}$$

C_{WU}: Wave coefficient for unrestricted service

The still water bending moment shall not be less than the large of: the largest actual still water bending moment based on the load conditions and the rule still water bending moment.

(DNV Pt.3 Ch.1 Sec. 5 A106), 2011

106 The design stillwater bending moments amidships (sagging and hogging) are normally not to be taken less than:

$$M_S = M_{SO} \quad (kNm)$$

M_{SO} = $-0.065\, C_{WU} L^2 B (C_B + 0.7)$ (kNm) in sagging
= $C_{WU} L^2 B (0.1225 - 0.015\, C_B)$ (kNm) in hogging
C_{WU} = C_W for unrestricted service.

Larger values of M_{SO} based on cargo and ballast conditions shall be applied when relevant, see 102.

For ships with arrangement giving small possibilities for variation of the distribution of cargo and ballast, M_{SO} may be dispensed with as design basis.

(DNV Pt.3 Ch.1 Sec. 5 B107), 2011

107 When required in connection with stress analysis or buckling control, the stillwater bending moments at arbitrary positions along the length of the ship are normally not to be taken less than:

$$M_S = k_{sm} M_{SO} \quad (kNm)$$

M_{SO} = as given in 106
k_{sm} = 1.0 within 0.4 L amidships
= 0.15 at 0.1 L from A.P. or F.P.
= 0.0 at A.P. and F.P.

Between specified positions k_{sm} shall be varied linearly.

Values of k_{sm} may also be obtained from Fig.3.

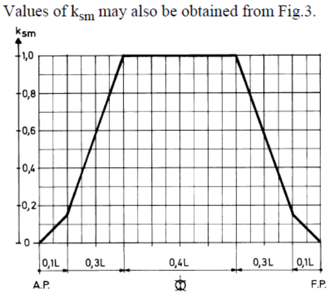


Fig. 3
Stillwater bending moment

The extent of the constant design bending moments amidships may be adjusted after special consideration.

Rule Still Water Shear Force by the Classification Rule

• The design values of still water shear forces along the length of the ship are normally not to be taken less than

(Dnv Pt.3 Ch.1 Sec. 5 B107)

$$Q_S = k_{sq} Q_{SO} (kN)$$

$$Q_{SO} = 5\frac{M_{SO}}{L}(kN)$$

k_{sq} = 0 at A.P. and F.P.
= 1.0 between 0.15L and 0.3L from A.P.
= 0.8 between 0.4L and 0.6L from A.P.
= 1.0 between 0.7L and 0.85L from A.P.

$M_{SO} = -0.065 C_{WU} L^2 B (C_B + 0.7)$ [kNm] in sagging

$= C_{WU} L^2 B (0.1225 - 0.015 C_B)$ [kNm] in hogging

C_{WU}: wave coefficient for unrestricted service

The still water shear force shall not be less than the large of: the largest actual still water shear forces based on load conditions and the rule still water shear force.

(DNV Pt.3 Ch.1 Sec. 5 B108), 2011

108 The design values of stillwater shear forces along the length of the ship are normally not to be taken less than:

$$Q_S = k_{sq} Q_{SO} \quad (kN)$$

$$Q_{SO} = 5\frac{M_{SO}}{L} \quad (kN)$$

M_{SO} = design stillwater bending moments (sagging or hogging) given in 106.
Larger values of Q_S based on load conditions ($Q_S = Q_{SL}$) shall be applied when relevant, see 102. For ships with arrangement giving small possibilities for variation in the distribution of cargo and ballast, Q_{SO} may be dispensed with as design basis

k_{sq} = 0 at A.P. and F.P.
= 1.0 between 0.15 L and 0.3 L from A.P.
= 0.8 between 0.4 L and 0.6 L from A.P.
= 1.0 between 0.7 L and 0.85 L from A.P.

Between specified positions k_{sq} shall be varied linearly.

Sign convention to be applied:

— when sagging condition positive in forebody, negative in afterbody
— when hogging condition negative in forebody, positive in afterbody.

(2) Vertical Wave Bending Moment (Mw)

Vertical Wave Bending Moment (Mw)

$$\sigma_{act.} \le \sigma_l \quad , \sigma_{act.} = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

, M_S: Still water bending moment
M_W: Vertical wave bending moment

Hydrodynamic loads induced by waves along ship's length

$f_W(x)$: distributed loads induced by waves
= Froude-Krylov force + diffraction force + added mass force + damping force

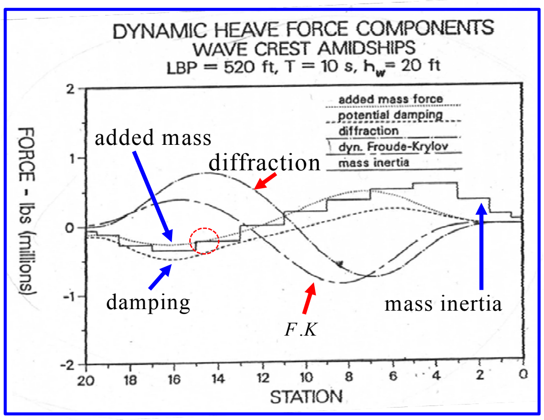
$$M\ddot{\mathbf{r}} = \sum \mathbf{F} = (Body\ Force) + (Surface\ Force)$$
$$= \mathbf{F}_{gravity}(\mathbf{r}) + \mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}})$$

$V_W(x)$: vertical wave shear force

$$V_W(x) = \int_0^x f_W(x)dx$$

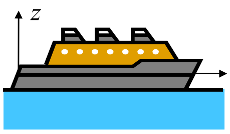
$M_W(x)$: vertical wave bending moment

$$M_W(x) = \int_0^x V_W(x)dx$$



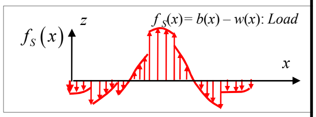
Dynamic Longitudinal Loads

In still water



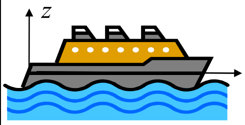
$$f_S(x) = b(x) + w(x)$$

$f(x)$: longitudinal strength loads
$f_S(x)$: static longitudinal loads
$f_W(x)$: dynamic longitudinal loads



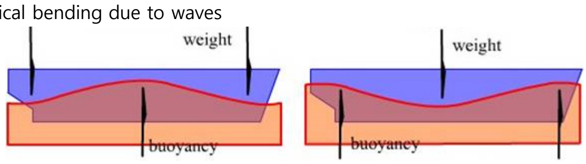
+

In wave



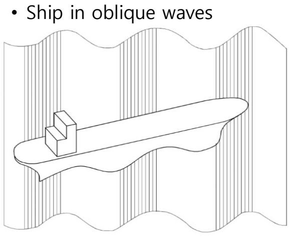
✓ Dynamic longitudinal loads
: Loads are induced by waves

Vertical bending due to waves

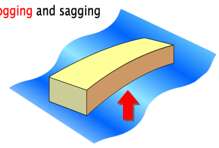


Hogging   Sagging

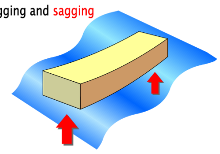
- Ship in oblique waves



Hogging and sagging



Hogging and sagging



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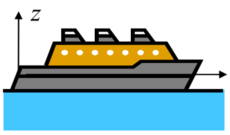
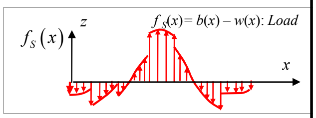
Dynamic Longitudinal Loads
- Direct Calculation of Dynamic Longitudinal Loads (1/2)

In still water



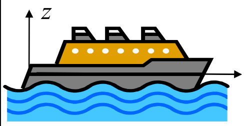
$$f_S(x) = b(x) + w(x)$$

$f(x)$: longitudinal strength loads
$f_S(x)$: static longitudinal loads
$f_W(x)$: dynamic longitudinal loads

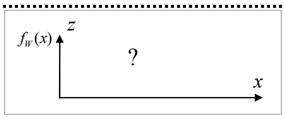


+

In wave



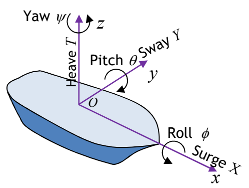
✓ Dynamic longitudinal loads
: Loads are induced by waves



✓ Direct calculation of dynamic longitudinal loads

- from 6DOF motion of ship
$\mathbf{x} = [X, Y, T, \phi, \theta, \psi]^T$

✓ Ref.> 6 DOF motion of ship



$$f(x) = f_S(x) + f_W(x)$$
$$= b(x) + w(x) + f_D(x) + f_{F.K}(x) + f_R(x)$$

additional loads in wave

where,

$$f_R(x) = -a(x)\ddot{\mathbf{x}} - b(x)\dot{\mathbf{x}}$$

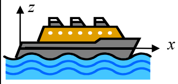
$f_D(x)$: Diffraction force at x
$f_R(x)$: Radiation force at x by damping and added mass
$f_{F.K}(x)$: Froude-Krylov force at x

In order to calculate loads in waves, first we have to determine $\ddot{\xi}_3$, $\dot{\xi}_3$.

How to determine $\ddot{\xi}_3$, $\dot{\xi}_3$?

Dynamic Longitudinal Loads
- Direct Calculation of Dynamic Longitudinal Loads (2/2)

In wave



✓ Direct calculation of dynamic longitudinal loads

Load induced by Wave

$$f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$$

where, $f_R(x) = -a(x)\,\ddot{\mathbf{x}} - b(x)\,\dot{\mathbf{x}}$

↓

Actual Vertical Wave Shear Force

$$Q_W(x) = \int_0^x f_W(x)dx$$

↓

Actual Vertical Wave Bending Moment

$$M_W(x) = \int_0^x Q_W(x)dx$$

Rule Values of Vertical Wave Bending Moments

✓ Direct calculation of dynamic longitudinal loads
- Loads are induced by waves

Actual Vertical Wave Shear Force $Q_W(x) = \int_0^x f_W(x)dx$

↓

Actual Vertical Wave Bending Moment $M_W(x) = \int_0^x Q_W(x)dx$

Recently, rule values of vertical wave moments are used, because of the uncertainty of the direct calculation values of vertical wave bending moments.

The rule vertical wave bending moments amidships are given by:

$$M_W = M_{WO} \quad [kNm]$$

(DNV Pt.3 Ch.1 Sec.5 B201)

$$M_{WO} = -0.11\alpha C_W L^2 B(C_B + 0.7) \quad \text{[kNm] in sagging}$$

$$= 0.19\alpha C_W L^2 BC_B \quad \text{[kNm] in hogging}$$

L	C_W
$L \le 100$	$0.0792 \cdot L$
$100 < L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

α = 1.0 for seagoing condition
= 0.5 for harbor and sheltered water conditions (enclosed fiords, lakes, rivers)
C_W: wave coefficient
C_B: block coefficient, not be taken less than 0.6

Direct calculation values of vertical wave bending moments can be used for vertical wave bending moment instead of the rule values of vertical wave moments, if the value of the direct calculation is smaller than that of the rule value.

Rule Values of Vertical Wave Shear Forces

✓ Direct calculation of dynamic longitudinal loads

- Loads are induced by waves

Load induced by Wave

↓

Actual Vertical Wave Shear Force

$f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$

where, $f_R(x) = -a(x)\,\ddot{\mathbf{x}} - b(x)\,\dot{\mathbf{x}}$

$Q_W(x) = \int_0^x f_W(x)dx$

The rule values of vertical wave shear forces along the length of the ship are given by: (DNV Pt.3 Ch.1 Sec.5 B203)

Positive shear force: $Q_{WP} = 0.3\beta k_{wqp} C_W LB(C_B + 0.7)$

Negative shear force: $Q_{WN} = -0.3\beta k_{wqn} C_W LB(C_B + 0.7)$

β: coefficient according to operating condition

k_{wqp}, k_{wqn}: coefficients according to location in lengthwise

C_W: wave coefficient

Direct calculation values of vertical wave shear forces can be used for vertical wave shear force instead of the rule values of vertical shear forces, if the value of the direct calculation is smaller than that of the rule value.

[Example] Rule Values of Still Water Bending Moments (Ms) and Vertical Wave Bending Moment (Mw)

Calculate L_S, $C_{B,SCANT}$, and vertical wave bending moment at amidships (0.5L) of a ship in hogging condition for sea going condition.

Dimension : $L_{OA} = 332.0\,m,\ L_{BP} = 317.2\,m,\ L_{EXT} = 322.85\,m,\ B = 43.2m,\ T_s = 14.5\,m$

$\Delta\left(Displacement\,(ton)\,at\,T_s\right) = 140,960\,ton$

(Sol.) $L_s = 0.97 \times L_{EXT} = 0.97 \times 322.85 = 313.16$

$C_{B,SCANT} = \Delta / \left(1.025 \times L_s \times B \times T_s\right) = \dfrac{140,906}{1.025 \times 313.16 \times 43.2 \times 14.5} = 0.701$

$\alpha = 1.0$, for sea going condition,

$C_W = 10.75$, if $300 \le L \le 350$ (wave coefficient)

$k_{wm} = 1.0$ between 0.4L and 0.65 L from A.P(=0.0) and F.P

$M_{WO} = 0.19 \times \alpha \times C_W \times L^2 \times B \times C_{B,SCANT}\ (kNm)$

$= 0.19 \times 1.0 \times 10.75 \times 313.16^2 \times 43.2 \times 0.701 = 6,066,303\ (kNm)$

at 0.5L, $k_{wm} = 1.0$

$M_W = 1.0 \times M_{WO}$

So, $M_W = 1.0 \times M_{WO} = 6,066,303\ (kNm)$

$M_S = M_{SO}(kNm)$

$M_{SO} = -0.065 C_{WU} L^2 B(C_B + 0.7)$, (in sagging)

$= C_{WU} L^2 B(0.1225 - 0.015 C_B)$, (in hogging)

$M_w = M_{WO}\quad (kNm)$

$M_{WO} = -0.11\alpha C_W L^2 B(C_B + 0.7)$, (in sagging)

$= 0.19\alpha C_W L^2 B C_B$, (in hogging)

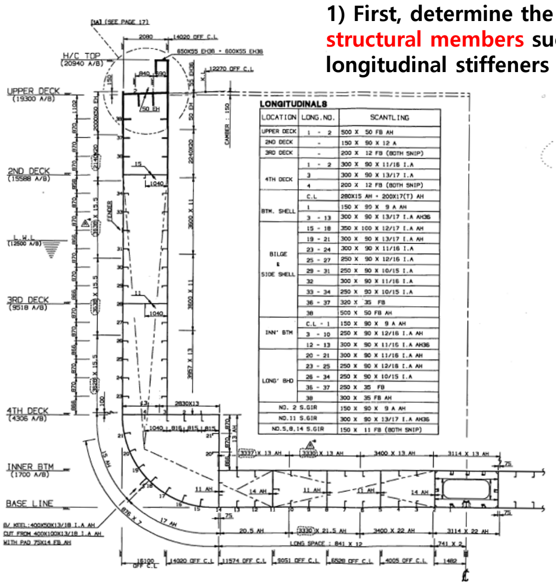
[1] 대우조선해양, 선박구조설계, 5-2 Load on Hull structure, 예제4, 2005.

1) DSME, Ship Structural Design, 5-2 Load on Hull structure, Example 4, 2005

(3) Section Modulus

Example of Midship Section of a 3,700 TEU Container Ship

1) First, determine the dimensions of the **longitudinal structural members** such as longitudinal plates and longitudinal stiffeners by rule local scantling.



Vertical Location of Neutral Axis about Baseline

2) Second, calculate the moment of sectional area about the base line.

$$\sum h_i A_i$$

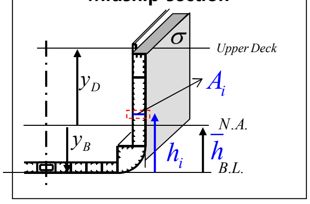
h_i : vertical center of structural member
A_i : area of structural member

3) Vertical location of neutral axis from base line ($\bar{h}$) is, then, calculated by dividing the moment of area by the total sectional area.

$$\bar{h} = \frac{\sum h_i A_i}{A}$$

$\bar{h}$: vertical location of neutral axis
A : total area

<Midship section>



By definition, neutral axis pass through the centroid of the cross section.

Midship Section Moment of Inertia about N.A

- The midship section moment of inertia about base line ($I_{B.L}$)

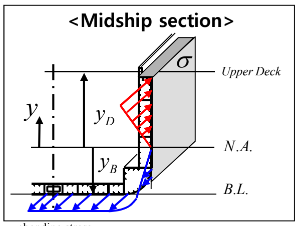
$$I_{B.L} = I_{N.A.} + A\,\bar{h}^2$$

- then calculate the midship section moment of inertia about neutral axis ($I_{N.A}$) using $I_{B.L}$.

$$I_{N.A.} = I_{B.L} - A\,\bar{h}^2$$

$$\sigma \le \sigma_l, \quad \sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$$

Calculation of Section Modulus and Actual Stress at Deck and Bottom



σ : bending stress
M_T : Total bending moment
A : Total Area
$I_{N.A}$: Inertia moment of the midship section area about neutral axis (N.A.)
B.L : Base Line

Section modulus

$$\frac{I_{N.A.}}{y_D} = Z_D, \quad \frac{I_{N.A.}}{y_B} = Z_B$$

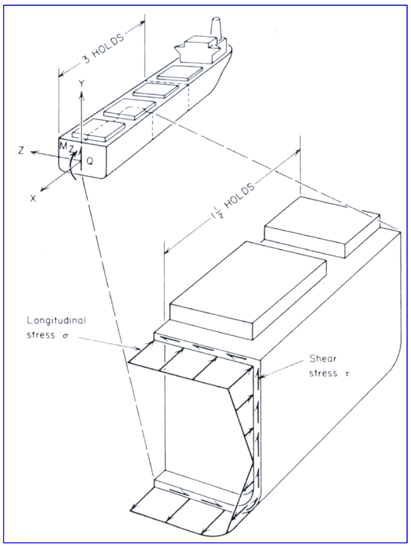
Calculation of Actual Stress at Deck and Bottom

$$\sigma_{Deck} = \frac{M}{I_{N.A}/y_D} = \frac{M}{Z_D}$$

$$\sigma_{Bottom} = \frac{M}{I_{N.A}/y_B} = \frac{M}{Z_B}$$

$$\sigma \le \sigma_l, \quad \sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$$

Global Hull Girder Strength (Longitudinal Strength) - Definition of the Longitudinal Strength Members



Application of hull girder load effects

※ Example of Requirement for Longitudinal Structural Member

DNV Rules for Classification of Ships
Part 3 Chapter 1 HULL STRUCTUREALDESIGN SHIPS WITH LENGTH 100 METERS AND ABOVE
Sec. 5 Longitudinal Strength
C 300 Section modulus
301 The requirements given in 302 and 303 will normally be satisfied when calculated for the midship section only, provided the following rules for tapering are complied with:
a) **Scantlings of all continuous longitudinal strength members shall be maintained within 0.4 L amidships**.
b) **Scantlings outside 0.4 L amidships are gradually reduced** to the local requirements at the ends, and the same material strength group is applied over the full length of the ship.
The section modulus at other positions along the length of the ship may have to be specially considered for ships with small block coefficient, high speed and large flare in the fore body or when considered necessary due to structural arrangement, see A106.

* Hughes, Ship Structural Design, John Wiley & Sons, 1983

(DNV Pt.3 Ch.1 Sec. 5 C300), 2011

C 300 Section modulus

301 The requirements given in 302 and 303 will normally be satisfied when calculated for the midship section only, provided the following rules for tapering are complied with:

a) Scantlings of all continuous longitudinal strength members shall be maintained within 0.4 L amidships. In special cases, based on consideration of type of ship, hull form and loading conditions, the scantlings may be gradually reduced towards the ends of the 0.4 L amidship part, bearing in mind the desire not to inhibit the vessel's loading flexibility.

b) Scantlings outside 0.4 L amidships are gradually reduced to the local requirements at the ends, and the same material strength group is applied over the full length of the ship.

The section modulus at other positions along the length of the ship may have to be specially considered for ships with small block coefficient, high speed and large flare in the forebody or when considered necessary due

to structural arrangement, see A106.

In particular this applies to ships of length L > 120 m and speed V > 17 knots.

The Minimum Required Midship Section Modulus and Inertia Moment by DNV Rule

DNV Rules, Jan. 2004, Pt.3 Ch.1 Sec.5

The **midship section modulus** about the transverse neutral axis shall not be less than:
(Pt.3 Ch.1 Sec.5 C302)

$$Z_O = \frac{C_{WO}}{f_1} L^2 B (C_B + 0.7) \quad [\mathrm{cm}^3]$$

C_{WO}: wave coefficient

L	C_{WO}
$L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

C_B is in this case not to be taken less than 0.60.

The midship section moment of **inertia** about the transverse neutral axis shall not be less than:
(Pt.3 Ch.1 Sec.5 C400)

$$I_{ship} = 3 C_W L^3 B (C_B + 0.7) \quad [cm^4]$$

* DNV Rules, Jan. 2004, Pt.3 Ch.1 Sec.5

(DNV Pt.3 Ch.1 Sec.5 C302), 2011

302 The midship section modulus about the transverse neutral axis shall not be less than:

$$Z_O = \frac{C_{WO}}{f_1} L^2 B (C_B + 0.7) \quad (cm^3)$$

C_{WO} = $10.75 - [(300 - L)/100]^{3/2}$ for $L < 300$
= 10.75 for $300 \leq L \leq 350$
= $10.75 - [(L - 350)/150]^{3/2}$ for $L > 350$

Values of C_{WO} are also given in Table C1.

C_B is in this case not to be taken less than 0.60.

Table C1 Values for C_{WO}

L	C_{WO}	L	C_{WO}	L	C_{WO}
		160	9.09	260	10.50
		170	9.27	280	10.66
		180	9.44	300	10.75
		190	9.60	350	10.75
100	7.92	200	9.75	370	10.70
110	8.14	210	9.90	390	10.61
120	8.34	220	10.03	410	10.50
130	8.53	230	10.16	440	10.29
140	8.73	240	10.29	470	10.03
150	8.91	250	10.40	500	9.75

For ships with restricted service, C_{WO} may be reduced as follows:

— service area notation **R0**: No reduction
— service area notation **R1**: 5%
— service area notation **R2**: 10%
— service area notation **R3**: 15%
— service area notation **R4**: 20%
— service area notation **RE**: 25%.

(DNV Pt.3 Ch.1 Sec.5 C401), 2011

C 400 Moment of inertia

401 The midship section moment of inertia about the transverse neutral axis shall not be less than:

$$I = 3\, C_W L^3 B (C_B + 0.7) \quad (cm^4)$$

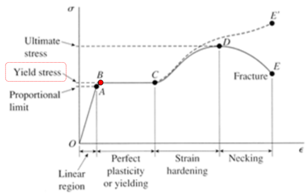
Material Factors (f_1)

[1] DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2
[2] James M. Gere, Mechanics of Materials 7th Edition, Thomson, Chap.1, pp.15~26

- The material factor f_1 is included in the various formulae for scantlings and in expressions giving allowable stresses.[1]

Material Designation	Yield Stress (N/mm²)	$\frac{\sigma}{\sigma_{NV-NS}}$	Material Factor (f_1)
NV-NS	235	235/235 = 1.00	1.00
NV-27	265	265/235 = 1.13	1.08
NV-32	315	315/235 = 1.34	1.28
NV-36	355	355/235 = 1.51	1.39
NV-40	390	390/235 = 1.65	1.47

* NV-NS: Normal Strength Steel (Mild Steel)
* NV-XX: High Tensile Steel



* Yield Stress (σ_y) [N/mm²] or [MPa]: The magnitude of the load required to cause yielding in the beam.[2]

* High tensile steel: A type of alloy steel that provides better mechanical properties or greater resistance to corrosion than carbon steel. They have a carbon content between 0.05-0.25% to retain formability and weldability, including up to 2.0% manganese, and other elements are added for strengthening purposes.

* A: 'A' grade 'Normal Strength Steel'
* AH: 'A' grade 'High Tensile Steel'

Summary of Longitudinal Strength

$\sigma \le \sigma_l$, $\sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$

Calculation of hull girder total shear force & bending moment

Still water shear forces Q_S
Still water bending moments M_S

(Q_S, M_S) based on the loading conditions

1. Weight curve $W(x)$
2. Buoyancy curve $B(x)$
3. Load curve $f_S(x) = W(x) + B(x)$
4. Shear force curve $Q_s = \int f_S\, dx$
5. Bending moment curve $M_S = \int Q_s\, dx$

(Q_S, M_S) Min. rule requirements

Larger value shall be used for the still water bending moment between the largest actual still water bending moment based on load conditions and design still water bending moment by rule.

+

Wave shear force Q_W
Wave bending moment M_W

Direct calculation (Q_W, M_W)

1. Wave Load curve $f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$
2. Vertical Wave Shear force curve $Q_W = \int f_W dx$
3. Vertical Wave Bending moment curve $M_W = \int Q_W dx$

Class rule (Q_W, M_W)

Direct calculation values can be used for wave shear force and wave bending moment.

Calculation of section modulus (Local scantling)

↓

Actual bending stress ≤ Allowable bending stress

No → **Modify longitudinal structural members** → (back to Calculation of section modulus)

Yes ↓

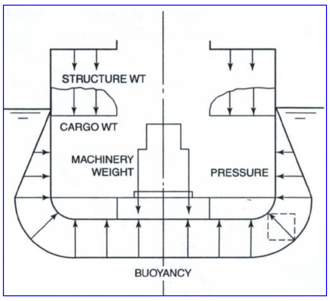
End of design of longitudinal strength

6.3 Local Strength (Local Scantling)

Contents

Local Scantling

- Ship structure members are designed to endure the loads acting on the ship structure such as hydrostatic and hydrodynamic loads[1].



2)

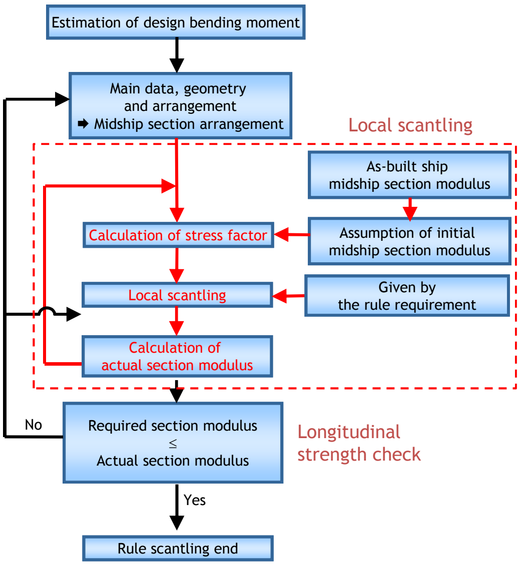
- For instance, the structural member is subjected to:

  Hydrostatic pressure due to surrounding water.
  Internal loading due to self weight and cargo weight.
  Inertia force of cargo or ballast due to ship motion.

[1] Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures - a Practical Guide for Engineers, Springer, pp. 17-32, 2009
[2] Mansour, A., Liu, D., The Principles of Naval Architecture Series – Strength of Ships and Ocean Structures, The Society of Naval Architects and Marine Engineers, 2008

(1) Procedure of Local Scantling

Procedure of Local Scantling
- Design Procedure of Structures



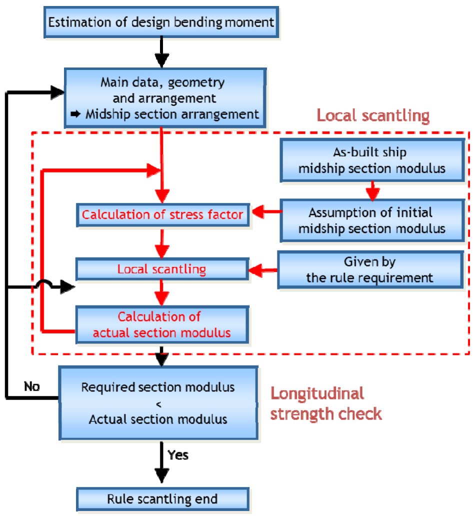
Ship structure design is carried out in accordance with the procedure shown in the figure.

Each member is adjusted to have enough local strength given by the rule of Classification Societies based on the mechanics of materials.

This is called the "local scantling".

Design Procedure of Structures
- Stress Factor

Why iteration is needed for the calculation of local scantling?



The actual midship section modulus at bottom or deck is needed.

However, the section modulus can be calculated after the scantlings of the members are determined.

➡ **Assumption!**

Therefore, actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

Design Procedure of Structures - Stress Factor

1) DNV Rules, Pt.3 Ch.1 Sec.6 C800, Jan. 2004

Why iteration is needed for the calculation of local scantling?

Example) Inner bottom longitudinals[1)]

- **Minimum longi. stiffener section modulus**

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

l: Stiffener span in m
s: Stiffener spacing in m
p: Design loads
w_k: Section modulus corrosion factor in tanks, Sec.3 C1004

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f1 for cargo holds in general cargo vessel
= 50 f1 for holds for ballast
= 85 f1 b/B for tanks for liquid cargo

Where, $\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$

f_1 : Material factor as defined in DNV Rules Pt.3 Ch.1 Sec.2

f_{2b} : stress factor

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

$5.7(M_S + M_W)$: required midship section modulus [cm³] at bottom or deck

$Z_{b,d}$: actual midship section modulus [cm³] at bottom or deck as built

M_S: Largest design SWBM[2)] [kN·m]

M_W: VWBM by class rule or direct calculation in [kN·m]

2) Largest SWBM among all loading conditions and class rule

The actual midship section modulus at bottom or deck is needed.
However, the section modulus can be calculated after the scantlings of the members are determined.
➡ Assumption!
Therefore, actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

(DNV Pt.3 Ch.1 Sec.6 C800), 2011

801 The section modulus requirement is given by:

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

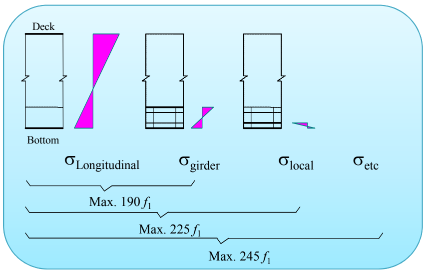
p = p_4 to p_{15} (whichever is relevant) as given in Table B1
σ = 225 f_1 – 100 f_{2B} – 0.7 σ_{db}within 0.4 L (maximum 160 f_1)
= 160 f_1 within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than:
= 20 f_1 for cargo holds in general cargo vessels
= 50 f_1 for holds for ballast
= 85 f_1 b/B for tanks for liquid cargo
f_{2b} = stress factor as given in A200
b = breadth of tank at double bottom.

(2) Local Strength & Allowable Stress

Local Strength & Allowable Stresses - Allowable Stress for Local Strength



Relationship between load and stress

1) Longitudinal load induced by waves (Hogging or Sagging)

↓

$\sigma_{Longitudinal}$

2) Cargo load

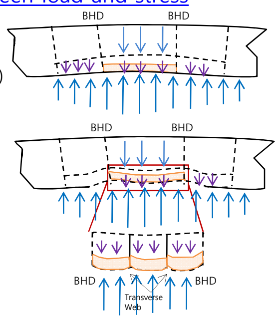
↓

σ_{girder} (σ_{db})

3) Ballasting load

↓

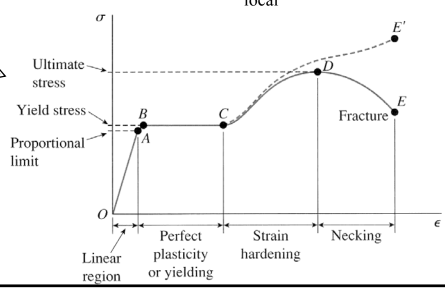
σ_{local}



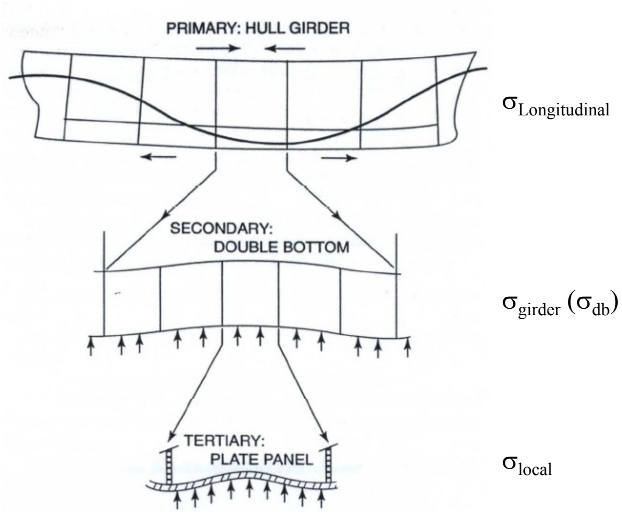
In the figure above, the meaning of the coefficients of the maximum allowable stresses is as follows:

$245 f_1$: Maximum Yield Stress
$235 f_1$: Proportional Limit

➡ $225 f_1$: The maximum allowable stress for the local strength uses the value less than the maximum yield stress. In other words, $225 f_1$ is used for the yield stress, except for the other effects.



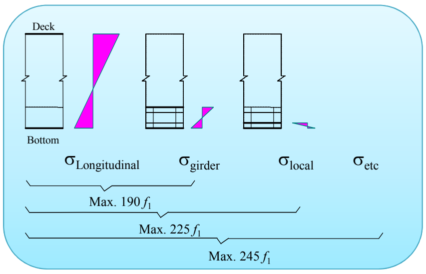
Local Strength & Allowable Stresses

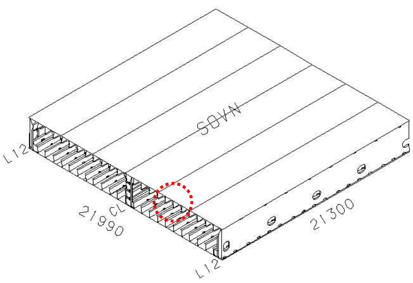


Primary, secondary, and tertiary structure

* Mansour, A., Liu, D., The Principles of Naval Architecture Series – Strength of Ships and Ocean Structures, The Society of Naval Architects and Marine Engineers, 2008

Allowable Stresses
- Allowable Stress for Local Strength





Another interpretation of the figure

Example) Inner bottom longitudinals[1)]

The section modulus requirement is given by:

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

where, p is the local pressure on bottom structure.

The nominal allowable bending stress due to lateral pressure is used except for the longitudinal stress and the double bottom stress.

$$\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

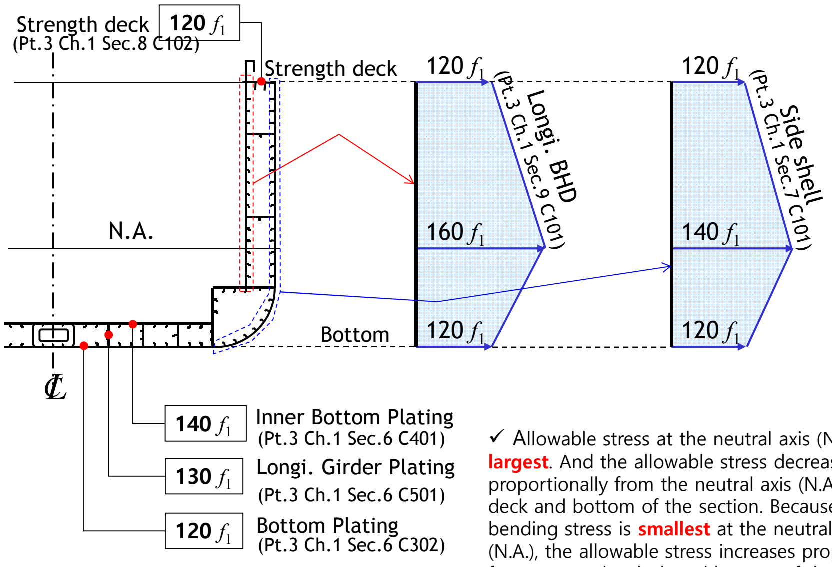
The longitudinal stress is given by the stress factor. And the double bottom stress is given by:

σ_{db}: Mean double bottom stress at plate flanges, normally not to be taken less than
- = 20 f_1 for cargo holds in general cargo vessel
- = 50 f_1 for holds for ballast
- = 85 f_1 b/B for tanks for liquid cargo

1) DNV Rules, Pt.3 Ch.1 Sec.6 C800, Jan. 2004

Allowable Stresses (σ_l) for Bottom Plating, Deck Plating, Bulkhead Plating, Side Plating

DNV Rules, Pt.3 Ch.1 Sec.6, 7, 8, 9, Jan. 2004



f_1: material factor

✓ Allowable stress at the neutral axis (N.A.) is **largest**. And the allowable stress decreases proportionally from the neutral axis (N.A) to the deck and bottom of the section. Because actual bending stress is **smallest** at the neutral axis (N.A.), the allowable stress increases proportionally from N.A to the deck and bottom of the section.

(DNV Pt.3 Ch.1 Sec.6 C302), 2011

C 300 Bottom plating

302 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_1 to p_3 (when relevant) in Table B1

σ = $175 f_1 - 120 f_{2b}$, maximum $120 f_1$ when transverse frames, within 0.4 L

= $120 f_1$ when longitudinals, within 0.4 L

= $160 f_1$ within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

f_{2b} = stress factor as given in A 200

(DNV Pt.3 Ch.1 Sec.6 C401), 2011

C 400 Inner bottom plating

401 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_4 to p_{15} (whichever is relevant) as given in Table B1

σ = 200 f_1 – 110 f_{2b}, maximum 140 f_1 when transverse frames, within 0.4 L

= 140 f_1 when longitudinals, within 0.4 L

= 160 f_1 within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

f_{2b} = stress factor as given in A200.

(DNV Pt.3 Ch.1 Sec.6 C501), 2011

501 The thickness requirement of floors and longitudinal girders forming boundaries of double bottom tanks is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_{13} to p_{15} (when relevant) as given in Table B1

p = p_1 for sea chest boundaries (including top and partial bulkheads)

σ = allowable stress, for longitudinal girders within 0.4 L given by:

Transversely stiffened	*Longitudinally stiffened*
190 f_1 – 120 f_{2b}, maximum 130 f_1	130 f_1

σ = 160 f_1 within 0.1 L from the perpendiculars and for floors in general

= 120 f_1 for sea chest boundaries (including top and partial bulkheads)

f_{2b} = stress factor as given in A200.

Between specified regions of longitudinal girders the σ-value may be varied linearly.

(DNV Pt.3 Ch.1 Sec.7 C101), 2011

101 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = $p_1 - p_8$, whichever is relevant, as given in Table B1

σ = 140 f_1 for longitudinally stiffened side plating at neutral axis, within 0.4 L amidship

= 120 f_1 for transversely stiffened side plating at neutral axis, within 0.4 L amidship.

Above and below the neutral axis the σ-values shall be reduced linearly to the values for the deck and bottom plating, assuming the same stiffening direction and material factor f_1 as for the plating considered

= 160 f_1 within 0.05 L from F.P. and 0.1 L from A.P.

Between specified regions the σ-value may be varied linearly.

(DNV Pt.3 Ch.1 Sec.8 C102), 2011

102 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = $p_1 - p_{13}$, whichever is relevant, as given in Table B1

σ = allowable stress within 0.4 L, given by:

Transversely stiffened	*Longitudinally stiffened*
175 f_1 – 120 f_{2d}, maximum 120 f_1	120 f_1

σ = 160 f_1 within 0.1 L from the perpendiculars and within line of large deck openings.

Between specified regions the σ-value may be varied linearly.

f_{2D} = stress factor as given in A 200.

(DNV Pt.3 Ch.1 Sec.9 C101), 2011

C 100 Bulkhead plating

101 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = $p_1 - p_9$, whichever is relevant, as given in Table B1

σ = 160 f_1 for longitudinally stiffened longitudinal bulkhead plating at neutral axis irrespective of ship length

= 140 f_1 for transversely stiffened longitudinal bulkhead plating at neutral axis within 0.4 L amidships, may however be taken as 160 f_1 when p_6 or p_7 are used.

Above and below the neutral axis the σ-values shall be reduced linearly to the values for the deck and bottom plating, assuming the same stiffening direction and material factor as for the plating considered

= 160 f_1 for longitudinal bulkheads outside 0.05 L from F.P. and 0.1 L from A.P. and for transverse bulkheads in general

= 220 f_1 for watertight bulkheads except the collision bulkhead, when p_1 is applied.

Between specified regions the σ-value may be varied linearly.

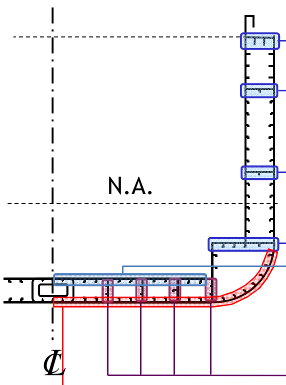
In corrugated bulkheads formed by welded plate strips, the thickness in flange and web plates may be differing. The thickness requirement then is given by the following modified formula:

$$t = \sqrt{\frac{500 s^2 p}{\sigma} - t_n^2} + t_k \quad \text{(mm)}$$

t_n = thickness in mm of neighbouring plate (flange or web), not to be taken greater than t.

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6, 7, 8, 9

Allowable Stresses (σ_l) for Longitudinal Stiffeners



On Decks (Pt.3 Ch.1 Sec.8 C301)

$225 f_1 - 130 f_{2d}, \max 160 f_1$: Strength deck ($130 f_{2d} \rightarrow \sigma_{Longitudinal}$)

$225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}, \max 160 f_1$: Continuous decks below strength deck ($\rightarrow \sigma_{Longitudinal}$)

On Inner Bottom (Pt.3 Ch.1 Sec.6 C801)

$225 f_1 - 100 f_{2b} - 0.7\sigma_{db}, \max 160 f_1$ ($100 f_{2b} \rightarrow \sigma_{Longitudinal}$)

On Double Bottom Girders (Pt.3 Ch.1 Sec.6 C901)

$225 f_1 - 110 f_{2b}, \max 160 f_1$ ($110 f_{2b} \rightarrow \sigma_{Longitudinal}$)

On Double Bottom (Pt.3 Ch.1 Sec.6 C701)

$225 f_1 - 130 f_{2b} - 0.7\sigma_{db}$ ($130 f_{2b} \rightarrow \sigma_{Longitudinal}$)

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: the largest design SWBM [kN·m]
M_W: rule VWBM in [kN·m]
$Z_{b,d}$: midship section modulus [cm³] at bottom or deck as built
(f_{2b}: Pt.3 Ch.1 Sec.6 A201)

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f1 for cargo holds in general cargo vessel
= 50 f1 for holds for ballast
= 85 f1 b/B for tanks for liquid cargo

Z_n: vertical distance in [m] from the base line or deck line to the neutral axis of the hull girder, whichever is relevant

Z_a: vertical distance in [m] from the base line or deck line to the point in question below or above the neutral axis, respectively

(DNV Pt.3 Ch.1 Sec.8 C301), 2011

301 The section modulus requirement is given by:

$$Z = \frac{83\, l^2\, s\, p\, w_k}{\sigma} \quad (cm^3), \quad \text{minimum } 15\ cm^3$$

p = $p_1 - p_{13}$, whichever is relevant, as given in Table B1.
σ = allowable stress, within 0.4 L midship given in Table C1
= 160 f_1 for continuous decks within 0.1 L from the perpendiculars and for other deck longitudinals in general.

Between specified regions the σ-value shall be varied linearly.

For longitudinals $\sigma = 160\ f_1$ may be used in any case in combination with heeled condition pressures p_9 and sloshing load pressures, p_{11} and p_{12}.

For definition of other parameters used in the formula, see A200.

(DNV Pt.3 Ch.1 Sec.8 C302), 2011

302 The section modulus requirement is given by:

$$Z = \frac{1000 l^2 s p w_k}{m \sigma} \quad (cm^3)$$

p = p_1 to p_8 whichever is relevant, as given in Pt.3 Ch.1 Sec.7 Table B1
w_k = 1.05 when calculating sectional modulus for midspan and upper end
= 1.15 when calculating sectional modulus for lower end
σ = 130 f_1 for internal loads p_3 to p_8
σ = 150 f_1 for external loads p_1, p_2 and p_{min} given above
m = 18 in general
m = 12 at upper end (including bracket) in combination with internal loads, p_3 to p_8
m = 9 at lower end (including bracket) and for upper end in combination with external loads p_1, p_2 and p_{min}.

For main frames situated next to plane transverse bulkheads, e.g. at the ends of the cargo region, the section modulus of the mid portion of the frame is generally to exceed the section modulus of the adjacent frame by a factor $3h_a/h$ where:

h_a = web height of adjacent frame
h = web height of considered frame.

The increased section modulus of the main frame adjacent to plane transverse bulkheads need not be fitted if other equivalent means are applied to limit the deflection of these frames.

(DNV Pt.3 Ch.1 Sec.6 C701), 2011

701 The section modulus requirement is given by:

$$Z = \frac{83\, l^2 s\, p\, w_k}{\sigma} \quad (cm^3)$$

p = p_1 to p_3 (when relevant) as given in Table B1
σ = allowable stress (maximum 160 f_1) given by:

— within 0.4 L:

Single bottom	*Double bottom*
$225 f_1 - 130 f_{2b}$	$225 f_1 - 130 f_{2b} - 0.7\, \sigma_{db}$

For bilge longitudinals the allowable stress σ shall be taken as $225 f_1 - 130 f_2 (z_n - z_a)/z_n$, where z_n, z_a are taken as defined in Sec.7 A201.

— within 0.1 L from perpendiculars: $\sigma = 160 f_1$

Between specified regions the σ- value may be varied linearly.

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than:
= $20 f_1$ for cargo holds in general cargo vessels
= $50 f_1$ for holds for ballast
= $85 f_1$ b/B for tanks for liquid cargo
f_{2b} = stress factor as given in A200
b = breadth of tank at double bottom.

Longitudinals connected to vertical girders on transverse bulkheads shall be checked by a direct stress analysis, see Sec.12 C.

(DNV Pt.3 Ch.1 Sec.6 C801), 2011

C 800 Inner bottom longitudinals

801 The section modulus requirement is given by:

$$Z = \frac{83\, l^2 s\, p\, w_k}{\sigma} \quad (cm^3)$$

p = p_4 to p_{15} (whichever is relevant) as given in Table B1
σ = $225 f_1 - 100 f_{2B} - 0.7\, \sigma_{db}$ within 0.4 L (maximum 160 f_1)
= $160 f_1$ within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than:
= $20 f_1$ for cargo holds in general cargo vessels
= $50 f_1$ for holds for ballast
= $85 f_1$ b/B for tanks for liquid cargo
f_{2b} = stress factor as given in A200
b = breadth of tank at double bottom.

(DNV Pt.3 Ch.1 Sec.6 C901), 2011

901 The section modulus requirement of stiffeners on floors and longitudinal girders forming boundary of double bottom tanks is given by:

$$Z = \frac{100\, l^2\, s\, p\, w_k}{\sigma} \quad (cm^3)$$

p = p_{13} to p_{15} as given in Table B1
p = p_1 for sea chest boundaries (including top and partial bulkheads)
σ = 225 f_1 – 110 f_{2b}, maximum 160 f_1 for longitudinal stiffeners within 0.4 L
= 160 f_1 for longitudinal stiffeners within 0.1 L from perpendiculars and for transverse and vertical stiffeners in general.
= 120 f_1 for sea chest boundaries (including top and partial bulkheads).

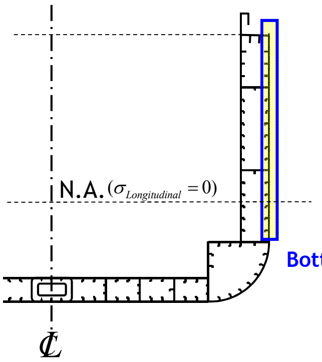
Between specified regions of longitudinal stiffeners the σ-value may be varied linearly.

f_{2b} = stress factor as given in A200.

Allowable Stresses - Longitudinal Stiffeners (1/6)

DNV Rules, Pt.3 Ch.1 Sec.6, 7, 8, 9, Jan. 2004
DSME, Dnv Rule 해설서, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



✓ $f_{2b,2d}$

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: the largest design SWBM [kN·m]
M_W: rule VWBM in [kN·m]
$Z_{b,d}$: midship section modulus [cm³] at bottom or deck as built

For example, 3,700 TEU Container Carrier: $I = 2.343e^{10}\ cm^4$

Bottom: $y_B = 9.028e^2\ cm$

$Z_B = 2.595e^7\ cm^3 \longrightarrow f_{2b} = \frac{5.7(M_S + M_W)}{Z_b} = 1.030$

Deck: $y_B = 10.272e^2\ cm$

$Z_D = 2.345e^7\ cm^3 \longrightarrow f_{2d} = \frac{5.7(M_S + M_W)}{Z_d} = 1.140$

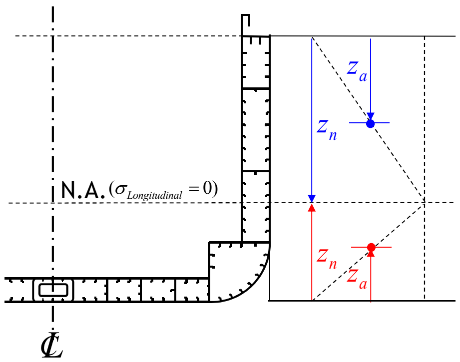
Section modulus of bottom is larger than that of deck,
So stress factor f_{2b} is smaller than f_{2d}.

Allowable Stresses - Longitudinal Stiffeners (2/6)

DNV Rules, Pt.3 Ch.1 Sec.6, 7, 8, 9, Jan. 2004
DSME, Dnv Rule 해설서, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$

✓ Z_n, Z_a



Z_n: Vertical distance in [m] from the base line or deck line to the neutral axis of the hull girder, whichever is relevant

Z_a: Vertical distance in [m] from the base line or deck line to the point in question below or above the neutral axis

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM [kN·m]
M_W : rule VWBM in [kN·m]
$Z_{b,d}$: midship section modulus [cm³] at bottom or deck as built

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Allowable Stresses - Longitudinal Stiffeners (3/6)

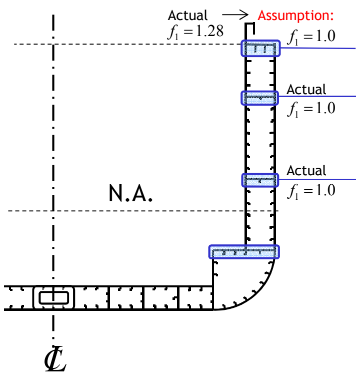
DNV Rules, Pt.3 Ch.1 Sec.6, 7, 8, 9, Jan. 2004
DSME, Dnv Rule 해설서, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$

On Decks (Pt.3 Ch.1 Sec.8 C301) $\sigma_l = 225 f_1 - 130 f_{2d} \dfrac{z_n - z_a}{z_n}$ → $\sigma_{Longitudinal}$

For example, 3,700 TEU Container Carrier:

$f_{2d} = 1.140$



$z_n = 10.272,\ z_a = 0.000 \rightarrow \dfrac{z_n - z_a}{z_n} = 1, \rightarrow \sigma_l = 70.3 [Mpa]$

$z_n = 10.272,\ z_a = 3.712 \rightarrow \dfrac{z_n - z_a}{z_n} = 0.639, \rightarrow \sigma_l = 126.147\ [Mpa]$

$z_n = 10.272,\ z_a = 9.782 \rightarrow \dfrac{z_n - z_a}{z_n} = 0.048, \rightarrow \sigma_l = 217.574\ [Mpa]$
$= 160\ [Mpa]$
(Maximum: 160)

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S: Largest design SWBM [kN·m]
M_W: Rule VWBM in [kN·m]
$Z_{b,d}$: Midship section modulus [cm³] at bottom or deck as built

Z_n: Vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a: Vertical distance in m from the baseline or deck line to the point in question below or above the neutral axis

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(DNV Pt.3 Ch.1 Sec.4 B301), 2011

301 The section modulus requirement is given by:

$$Z = \frac{83\, l^2\, s\, p\, w_k}{\sigma} \quad (cm^3), \quad \text{minimum } 15\ cm^3$$

p = $p_1 - p_{13}$, whichever is relevant, as given in Table B1.

σ = allowable stress, within 0.4 L midship given in Table C1
= 160 f_1 for continuous decks within 0.1 L from the perpendiculars and for other deck longitudinals in general.

Between specified regions the σ-value shall be varied linearly.

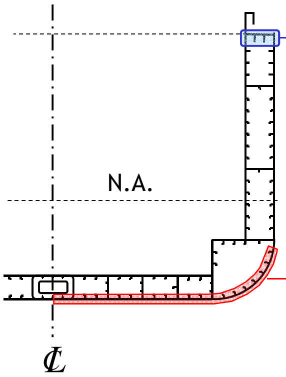
For longitudinals σ = 160 f_1 may be used in any case in combination with heeled condition pressures p_9 and sloshing load pressures, p_{11} and p_{12}.

For definition of other parameters used in the formula, see A200.

Allowable Stresses - Longitudinal Stiffeners (4/6)

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6,7,8,9
DSME, Dnv Rule 해설서, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \ (cm^3)$$



On Decks (Pt.3 Ch.1 Sec.8 C301)

$$225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}$$

For example, 3,700 TEU Container Carrier:

$$f_1 = 1.28 \quad f_{2d} = 1.140$$

$$z_n = 10.272,\ z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \rightarrow \sigma_l = 139.8 [MPa]$$

On Double Bottom (Pt.3 Ch.1 Sec.6 C701)

$$225 f_1 - 130 f_{2b} - 0.7\sigma_{db}$$

For example, 3,700 TEU Container Carrier, Assumption: σ_{db} = 0

$$f_1 = 1.0 \quad f_{2b} = 1.030$$

$$z_n = 9.208,\ z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \rightarrow \sigma_l = 154.1 [MPa]$$

Allowable stresses at deck are **smaller** than those at bottom, because the distance from N.A to bottom is shorter than deck.

If the mean double bottom stress (σ_{db}) is considered as 20,

$$225 f_1 - 130 f_{2b} - 0.7\sigma_{db}$$

$$225 \times 1.28 - 130 \times 1 - 0.7 \times 20 = \rightarrow \sigma_l = 140.1 [MPa]$$

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

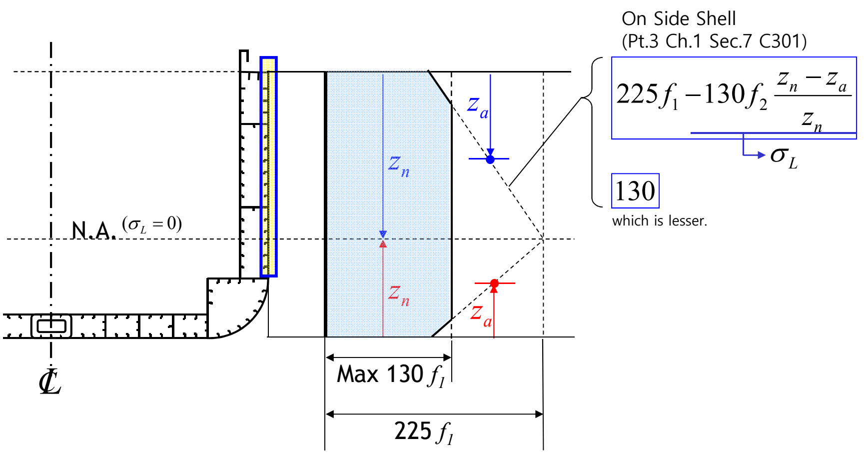
M_S : the largest design SWBM [kN·m]
M_W : rule VWBM in [kN·m]
$Z_{b,d}$: midship section modulus [cm³] at bottom or deck as built

Z_n : vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the base line or deck line to the point in question below or above the neutral axis
σ_{db} : mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f_1 for cargo holds in general cargo vessel
= 50 f_1 for holds for ballast
= 85 f_1 b/B for tanks for liquid cargo

Allowable Stresses - Longitudinal Stiffeners (5/6)

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6,7,8,9
DSME, Dnv Rule 해설서, 1991.8

$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l}$ (cm^3)



$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM [kN·m]
M_W : rule VWBM in [kN·m]
$Z_{b,d}$: midship section modulus [cm³] at bottom or deck as built

Z_n : vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the baseline or deck line to the point in question below or above the neutral axis

(DNV Pt.3 Ch.1 Sec.4 C301), 2011

301 The section modulus requirement is given by:

$$Z = \frac{83\ l^2\ s\ p\ w_k}{\sigma} \quad (cm^3), \text{ minimum } 15\ cm^3$$

p = $p_1 - p_8$, whichever is relevant, as given in Table B1
σ = allowable stress (maximum 160 f_1) given by:

Within 0.4 L amidships:

$$\sigma = 225 f_1 - 130 f_2 \frac{z_n - z_a}{z_n}$$

= maximum 130 f_1 for longitudinals supported by side verticals in single deck constructions.

Within 0.1 L from perpendiculars:

$$\sigma = 160\ f_1$$

Between specified regions the σ-value may be varied linearly.

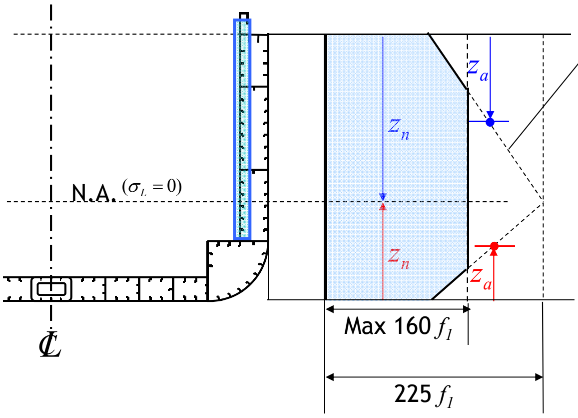
For longitudinals σ = 160 f_1 may be used in any case in combination with heeled condition pressures p_6 and p_8.

f_2 = stress factor f_{2b} as given in Sec.6 A200 below the neutral axis
= stress factor f_{2d} as given in Sec.8 A200 above the neutral axis.

Allowable Stresses - Longitudinal Stiffeners (6/6)

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6,7,8,9
DSME, Dnv Rule 해설서, 1991.8

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



On Longitudinal Bulkhead (Pt.3 Ch.1 Sec.9 C201)

$$225 f_1 - 130 f_2 \frac{z_n - z_a}{z_n} \rightarrow \sigma_L$$

160

which is lesser.

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM [kN·m]
M_W : rule VWBM in [kN·m]
$Z_{b,d}$: midship section modulus [cm³] at bottom or deck as built

z_n : vertical distance in m from the base line or deck line to the neutral axis of the hull girder, whichever is relevant
z_a : vertical distance in m from the baseline or deck line to the point in question below or above the neutral axis

[Reference] Derivation of Coefficient 5.7 of Stress Factor, f_2 (f_{2b}, f_{2d})

Rule still water bending moment

$$M_{SO} = -0.065 C_{WU} L^2 B (C_B + 0.7)$$

Rule vertical wave bending moment

$$M_{WO} = -0.11 \alpha C_W L^2 B (C_B + 0.7)$$

$$f_2 = \frac{Z_O}{Z_A}$$

Z_O: In case of mild steel (f_1 = 1.0), rule midship section modulus

Pt.3 Sh.1 Sec5 C302

$$Z_O = \frac{C_{WO}}{f_1} L^2 B (C_b + 0.7) \quad (cm^3)$$

Z_A: Actual midship section modulus [cm³]
f_2 is the ratio of the rule midship section modulus (Z_O) to the actual midship section modulus (Z_A)

$$f_2 = \frac{C_{WO} L^2 B (C_b + 0.7)}{Z_A} = \frac{1}{0.175} \frac{0.175\, C_{WO} L^2 B (C_b + 0.7)}{Z_A}$$

$$= 5.7 \times \frac{0.065\, C_{WO} L^2 B (C_b + 0.7) + 0.11\, C_{WO} L^2 B (C_b + 0.7)}{Z_A}$$

$$\therefore f_2 = 5.7 \times \frac{(M_S + M_W)}{Z_A}$$

M_S : Normally to be taken as the largest design still water bending moment in [kNm]. Ms shall not be taken less than 0.5 Mso. When actual design moment is not known, Ms may be taken equal to Mso.

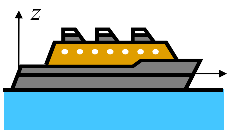
M_W : Rule wave bending moment in [kNm]. Hogging or sagging moment to be chosen in relation to the applied still water moment

(3) Design Loads

Contents

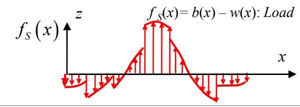
[Review] Loads in Wave

In still water



$$f_S(x) = b(x) - w(x)$$

$f(x)$: Distributed load in longitudinal direction
$f_S(x)$: Distributed load in longitudinal direction in still water
$f_W(x)$: Distributed load in longitudinal direction in wave



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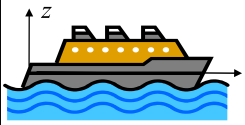
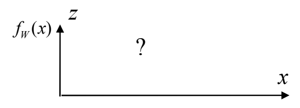
In wave



✓ Loads in wave

- From 6DOF motion of ship

$$\mathbf{x} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$$



- For example, consider heave motion.

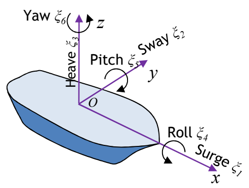
$$f(x) = f_S(x) + f_W(x)$$
$$= b(x) - w(x) + f_D(x) + f_{F.K}(x) + f_R(x)$$

additional loads in wave

Where,

$$f_R(x) = -a_{33}(x)\ddot{\xi}_3 - b_{33}(x)\dot{\xi}_3$$

✓ Ref.> 6 DOF motion of ship



$f_D(x)$: Diffraction force in a unit length
$f_R(x)$: Radiation force in a unit length
$f_{F.K}(x)$: Froude-Krylov force in a unit length

In order to calculate loads in wave, we have to know $\ddot{\xi}_3$, $\dot{\xi}_3$.

How to know $\ddot{\xi}_3$, $\dot{\xi}_3$?

[Review] 6 DOF Equation of Motion of Ship

How to know $\ddot{\mathbf{x}}, \dot{\mathbf{x}}$?

By solving equations of motion, we can get the velocities and accelerations.

✓ Pressure acting on hull

Linearized Bernoulli Eq. $\quad P_{Fluid} = -\rho g z - \rho \frac{\partial \Phi}{\partial t} = -\rho g z - \rho\left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t}\right)$

✓ Fluid force acting on hull

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = -\iint_{S_B} \rho g z\,\mathbf{n}dS - \rho\iint_{S_B}\left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t}\right)dS$$
$$= \mathbf{F}_{Static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

✓ 6 D.O.F equations of motion of a ship in waves

Newton's 2nd Law

Pressure force acting as surface force on hull

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \mathbf{F}_{Body} + \mathbf{F}_{Surface}$$
$$= \mathbf{F}_{Gravity} + \mathbf{F}_{Fluid} + \mathbf{F}_{External}$$

External force excluding wave exciting force (ex. control force)

Body force Surface force

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{Gravity} + \mathbf{F}_{Static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

$\mathbf{F}_{Restoring}$ $\quad \mathbf{F}_{Wave\ exciting}$ $\quad \mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$

Added mass, Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = (\mathbf{F}_{Gravity} + \mathbf{F}_{Static}) + \mathbf{F}_{Wave\ exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}} + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

Linearization, $(\mathbf{F}_{Restoring} = (\mathbf{F}_{Gravity} + \mathbf{F}_{Static}) \approx -\mathbf{C}\mathbf{x})$

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{Wave\ exciting} + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

By solving equations of motion, we can get the velocities and accelerations of the ship!

$F_{F.K}$: Froude-Krylov force
F_D: Diffraction force
F_R: Radiation force

Φ_I : Incident wave velocity potential
Φ_D: Diffraction wave velocity potential
Φ_R: Radiation wave velocity potential

$\mathbf{M}_A$: 6×6 added mass matrix
$\mathbf{B}$: 6×6 damping coeff. matrix
$\mathbf{C}$: 6×6 restoring coeff. matrix

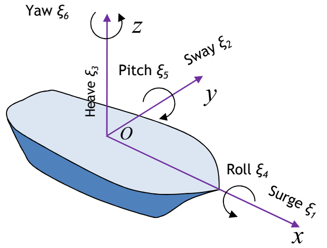
(1) Ship Motion and Acceleration - Empirical Formula of DNV Rule

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4

Common Acceleration Parameter	$a_0 = \frac{3C_w}{L} + C_v C_{v1}$
Surge Acceleration	$a_x = 0.2 g_0 a_0 \sqrt{C_b}$
Combined Sway/Yaw Acceleration	$a_y = 0.3 g_0 a_0$
Heave Acceleration	$a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$
Tangential Roll Acceleration	$a_r = \phi \left(\frac{2\pi}{T_r}\right)^2 R_r$
Tangential Pitch Acceleration	$a_p = \theta \left(\frac{2\pi}{T_p}\right)^2 R_p$

g_0 : standard acceleration of gravity =9.81m/s²

✓ Ref. 6 DOF motion of ship



Common Acceleration Parameter, a_0

$$a_0 = \frac{3C_W}{L} + C_V C_{V1}$$

$C_V = \frac{\sqrt{L}}{50}$, maximum 0.2

$C_{V1} = \frac{V}{\sqrt{L}}$, minimum 0.8

C_W = Wave coefficient

L	C_W
$L \le 100$	$0.0792 \cdot L$
$100 < L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

(DNV Pt.3 Ch.1 Sec.4 B400) Roll Motion and Acceleration

B 300 Surge, sway /yaw and heave accelerations

301 The surge acceleration is given by:

$$a_x = 0.2 g_0 a_0 \sqrt{C_B} \quad (m/s^2)$$

302 The combined sway/yaw acceleration is given by:

$$a_y = 0.3 g_0 a_0 \quad (m/s^2)$$

303 The heave acceleration is given by:

$$a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_B}} \quad (m/s^2)$$

B 400 Roll motion and acceleration

401 The roll angle (single amplitude) is given by:

$$\phi = \frac{50c}{B + 75} \quad (rad)$$

c = $(1.25 - 0.025\, T_R)\, k$
k = 1.2 for ships without bilge keel
 = 1.0 for ships with bilge keel
 = 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

402 The period of roll is generally given by:

$$T_R = \frac{2k_r}{\sqrt{GM}} \quad (s)$$

k_r = roll radius of gyration in m
GM = metacentric height in m.

(DNV Pt.3 Ch.1 Sec.4 B400) Roll Motion and Acceleration
(DNV Pt.3 Ch.1 Sec.4 B500) Pitch Motion and Acceleration

The values of k_r and GM to be used shall give the minimum realistic value of T_R for the load considered.
In case k_r and GM have not been calculated for such condition, the following approximate design values may be used:

k_r = 0.39 B for ships with even transverse distribution of mass
= 0.35 B for tankers in ballast
= 0.25 B for ships loaded with ore between longitudinal bulkheads
GM = 0.07 B in general
= 0.12 B for tankers and bulk carriers.
= 0.05 B for container ship with B < 32.2 m
= 0.08 B for container ship with B > 40.0 m
with interpolation for B in between.

403 The tangential roll acceleration (gravity component not included) is generally given by:

$$a_r = \phi \left(\frac{2\pi}{T_R}\right)^2 R_R \quad (m/s^2)$$

R_R = distance in m from the centre of mass to the axis of rotation.
The roll axis of rotation may be taken at a height z m above the baseline.

z = the smaller of $\left[\frac{D}{4} + \frac{T}{2}\right]$ and $\left[\frac{D}{2}\right]$

404 The radial roll acceleration may normally be neglected.

B 500 Pitch motion and acceleration

501 The pitch angle is given by:

$$\theta = 0.25 \frac{a_0}{C_B} \quad (rad)$$

502 The period of pitch may normally be taken as:

$$T_P = 1.8 \sqrt{\frac{L}{g_0}} \quad (s)$$

503 The tangential pitch acceleration (gravity component not included) is generally given by:

(DNV Pt.3 Ch.1 Sec.4 B600) Combined Vertical Acceleration

$$a_p = \theta \left[\frac{2\pi}{T_P}\right]^2 R_P \quad (m/s^2)$$

T_P = period of pitch
R_P = distance in m from the centre of mass to the axis of rotation.

The pitch axis of rotation may be taken at the cross- section 0.45 L from A.P. z meters above the baseline.

z = as given in 403.

With T_P as indicated in 502 the pitch acceleration is given by:

$$a_p = 120\ \theta \frac{R_P}{L} \quad (m/s^2)$$

504 The radial pitch acceleration may normally be neglected.

B 600 Combined vertical acceleration

601 Normally the combined vertical acceleration (acceleration of gravity not included) may be approximated by:

$$a_v = \frac{k_v\, g_0\, a_0}{C_B} \quad (m/s^2)$$

k_v = 1.3 aft of A.P.
= 0.7 between 0.3 L and 0.6 L from A.P.
= 1.5 forward of F.P.

Between mentioned regions k_v shall be varied linearly, see Fig.3.

(1) Ship Motions and Accelerations
- Roll Angle & Roll Period

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4

✓ Roll angle

$$\phi = \frac{50c}{B+75} \quad \text{(rad)}$$

c = $(1.25 - 0.025\,T_R)\,k$
k = 1.2 for ships without bilge keel
= 1.0 for ships with bilge keel
= 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

✓ Pitch angle

$$\theta = 0.25\,\frac{a_0}{C_B} \quad \text{(rad)}$$

$$a_0 = \frac{3C_W}{L} + C_V C_{V1}$$

✓ Roll period

$$T_R = \frac{2k_r}{\sqrt{GM}} \quad \text{(s)}$$

k_r = 0.39B for ships with even transverse distribution of mass
=0.35B for tankers in ballast
=0.25B for ships loaded with ore between longitudinal bulkheads
GM= 0.07B in general
= 0.12B for tankers and bulk carriers

✓ Pitch period

$$T_P = 1.8\sqrt{\frac{L}{g_0}} \quad \text{(s)}$$

g_0 : standard acceleration of gravity
$=9.81\text{m/s}^2$

(2) Combined Acceleration
- Combined Vertical Acceleration (a_v)

1) DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B602
2) DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B401
3) DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B402
4) DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B303

✓ The acceleration along the ship's vertical axis considering combined effect of heave, pitch & roll motion[1)]

$$a_v = \frac{k_v g_o a_o}{C_b}$$

K_v = Acceleration distribution factor along the length of vessel
= 0.7 between 0.3L and 0.6L from A.P.
a_0 = Common Acceleration Parameter

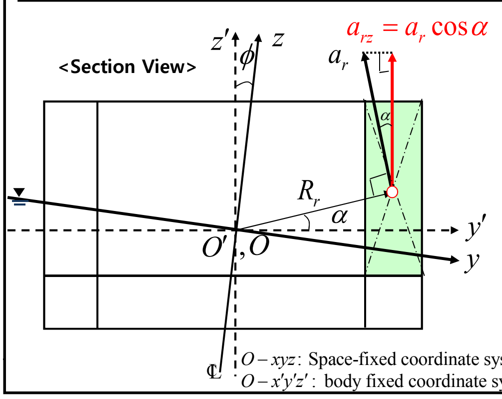
$$a_v = \max\left\{\sqrt{a_z^{\,2} + a_{rz}^{\,2}},\ \sqrt{a_z^{\,2} + a_{pz}^{\,2}}\right\}$$

Heave Acceleration[4)] $a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$

Vertical component of tangential roll acceleration

Vertical component of tangential pitch acceleration

<Section View>



$O-xyz$: Space-fixed coordinate system
$O-x'y'z'$: body fixed coordinate system

$$\phi = \phi^A \cos\left(\frac{2\pi}{T_R}t\right)$$

$$\ddot{\phi} = \phi^A \left(\frac{2\pi}{T_R}\right)^2 \cdot \cos\left(\frac{2\pi}{T_R}t\right)$$

$$a_r = \ddot{\phi} \cdot R_r = \phi^A \left(\frac{2\pi}{T_r}\right)^2 R_r$$

a_r : tangential roll acceleration
R_r : distance in m from the center of the mass to the axis of rotation
α : angle of center of mass about the body fixed coordinate system
ϕ : roll angle
ϕ^A : roll angle amplitude[2)]
T_R : period of roll[3)]
g_0 : standard acceleration of gravity
$=9.81\text{m/s}^2$

(DNV Pt.3 Ch.1 Sec.4 B303, B401, B401), 2011

B 400 Roll motion and acceleration

401 The roll angle (single amplitude) is given by:

$$\phi = \frac{50c}{B+75} \quad \text{(rad)}$$

c = $(1.25 - 0.025\ T_R)\ k$
k = 1.2 for ships without bilge keel
= 1.0 for ships with bilge keel
= 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

402 The period of roll is generally given by:

$$T_R = \frac{2k_r}{\sqrt{GM}} \quad \text{(s)}$$

k_r = roll radius of gyration in m
GM = metacentric height in m.

The values of k_r and GM to be used shall give the minimum realistic value of T_R for the load considered.
In case k_r and GM have not been calculated for such condition, the following approximate design values may be used:

k_r = 0.39 B for ships with even transverse distribution of mass
= 0.35 B for tankers in ballast
= 0.25 B for ships loaded with ore between longitudinal bulkheads
GM = 0.07 B in general
= 0.12 B for tankers and bulk carriers.
= 0.05 B for container ship with B < 32.2 m
= 0.08 B for container ship with B > 40.0 m
with interpolation for B in between.

303 The heave acceleration is given by:

$$a_z = 0.7\ g_0 \frac{a_0}{\sqrt{C_B}} \quad (\text{m/s}^2)$$

(2) Combined Acceleration
- Combined Vertical Acceleration (a_v)

1) DNV Rules, Jan. 2004, Pt.3 Ch.1 Sec.4 B602
2) DNV Rules, Jan. 2004, Pt.3 Ch.1 Sec.4 B401
3) DNV Rules, Jan. 2004, Pt.3 Ch.1 Sec.4 B402
4) DNV Rules, Jan. 2004, Pt.3 Ch.1 Sec.4 B303

✓ The acceleration along the ship's vertical axis considering combined effect of heave, pitch & roll motion[1)]

$$a_v = \frac{k_v g_o a_o}{C_b}$$

K_v = Acceleration distribution factor along the length of vessel
= 0.7 between 0.3L and 0.6L from A.P.
a_0 = Common Acceleration Parameter

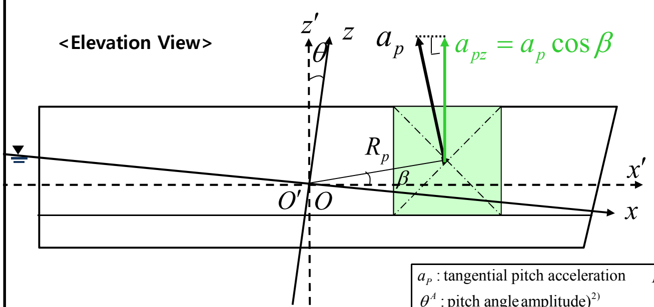
$$a_v = \max\left\{\sqrt{a_z^2 + a_{rz}^2},\ \sqrt{a_z^2 + a_{pz}^2}\right\}$$

Heave Acceleration[4)] $a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$

Vertical component of tangential roll acceleration

Vertical component of tangential pitch acceleration

<Elevation View>



$$\theta = \theta^A \cos\left(\frac{2\pi}{T_P} t\right)$$

$$\ddot{\theta} = \theta^A \left(\frac{2\pi}{T_P}\right)^2 \cdot \cos\left(\frac{2\pi}{T_P} t\right)$$

$$a_P = \ddot{\theta} \cdot R_r$$

$$= \theta^A \left(\frac{2\pi}{T_P}\right)^2 R_r$$

$O - xyz$: Space-fixed coordinate system
$O - x'y'z'$: body fixed coordinate system

a_p : tangential pitch acceleration
θ^A : pitch angle amplitude)[2)]
θ : pitch angle
T_P : period of pitch[3)]

R_p : distance in m from the center of the mass to the axis of rotation
β : angle of center of mass about the body fixed coordinate system

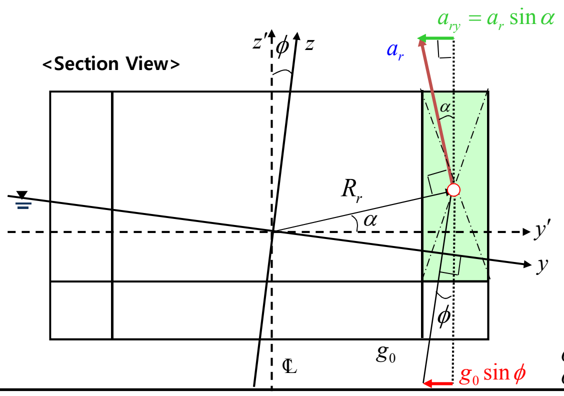
1) DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B700

(2) Combined Acceleration
- Combined Transverse Acceleration (a_t)

✓ The acceleration along the ship's transverse axis considering combined effect of sway, yaw & roll motion[1)]

$$a_t = \sqrt{a_y^2 + (g_o \sin\phi + a_{ry})^2}$$

- Combined Sway & yaw acceleration: $a_y = 0.3 g_0 a_0$
- Transverse component of acceleration of gravity by roll angle
- Transverse component of the tangential roll acceleration



$$a_r = \phi^A \left(\frac{2\pi}{T_r}\right)^2 R_r$$

a_r : tangential roll acceleration

R_r : distance in m from the center of the mass to the axis of rotation

ϕ : roll angle

ϕ^A : roll angle amplitude

T_R : period of roll[3)]

g_0 : standard acceleration of gravity =9.81m/s^2

$O-xyz$: Space-fixed coordinate system

$O-x'y'z'$: body fixed coordinate system

(DNV Pt.3 Ch.1 Sec.4 B701), 2011

701 Acceleration along the ship's transverse axis is given as the combined effect of sway/yaw and roll calculated as indicated in 100, i.e.:

$$a_t = \sqrt{a_y^2 + (g_0 \sin\phi + a_{ry})^2} \quad (\text{m/s}^2)$$

a_{ry} = transverse component of the roll acceleration given in 403.

Note that a_{ry} is equal to a_r using the vertical projection of R_R.

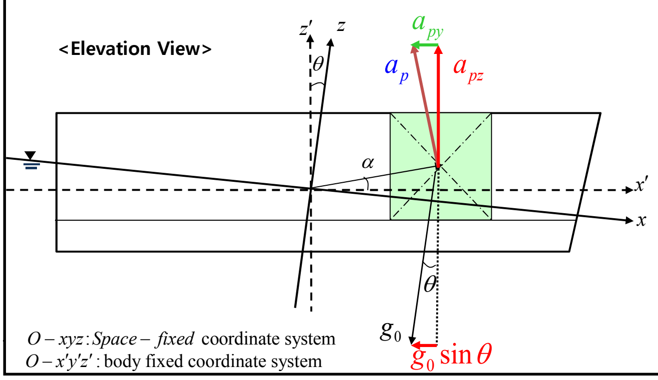
(2) Combined Acceleration
- Combined Longitudinal Acceleration (a_l)

1) DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B800

✓ The acceleration along the ship's longitudinal axis considering combined effect of surge & pitch motion[1)]

$$a_l = \sqrt{a_x^{\,2} + \left(g_o \sin\theta + a_{px}\right)^2}$$

- a_x: Combined Sway & yaw acceleration, $a_x = 0.2 g_0 a_0 \sqrt{C_b}$
- $g_o \sin\theta$: Longitudinal component of gravitational acceleration by pitch angle
- a_{px}: Longitudinal component of the pitch acceleration



$$a_P = \theta^A \left(\frac{2\pi}{T_P}\right)^2 R_r$$

a_P : tangential pitch acceleration

R_P : distance in m from the center of the mas to the axis of rotation

θ : pitch angle

θ^A : pitch angle amplitude[2)]

T_P : period of pitch[3)]

g_0 : standard acceleration of gravity =9.81m/s^2

(2) Combined Acceleration
- [Example] Vertical Acceleration

(Example) Calculate the vertical acceleration of a given ship at 0.5L (amidships) by DNV Rule.

[Dimension] L_s=315.79 m, V=15.5 knots, C_B=0.832

$$a_v = \frac{k_v g_o a_o}{C_b}$$

K_v = Acceleration distribution factor along the length of vessel = 0.7 between 0.3L and 0.6L from A.P.

a_0 = Common Acceleration Parameter

g_0 = Standard acceleration of gravity (=9.81m/sec^2)

(Sol.) $a_v = \left(k_v\, g_0\, a_0\right) / C_B = \left(0.7 \times 9.81 \times 0.277\right) / 0.832$

$= 2.286 \left(m/\sec^2\right)$

where, $k_v = 0.7$ at mid ship

$a_0 = 3C_W / L + C_v C_{v1} = 3 \times 10.75 / 315.79 + 0.2 \times 0.872 = 0.277$

$C_v = L^{0.5} / 50 = 315.79^{0.5} / 50 = 0.355$ or Max. 0.2

$= 0.2$

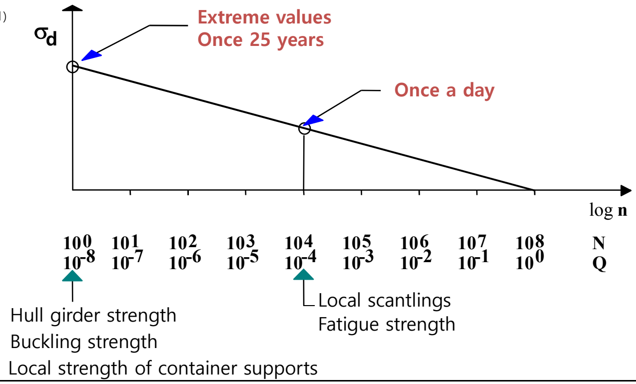
$C_{v1} = V / L^{0.5} = 15.5 / 315.79^{0.5} = 0.872$ or Min. 0.8

$= 0.872$

1) DNV, Fatigue Assessment of Ship Structures, p.18, 2003

(3) Design Probability Level

- Probability Level[1)]



- Design Probability Level[2)]
  - ✓ Number of waves that the ship experiences during the ship's life (for 25 years): about 10^8
    - ➡ The ship is designed to endure the extreme wave (10^{-8} probability) which the ship encounter once for 25 years.
      (Extreme condition: Ship motion, acceleration is given as extreme value.)
  - ✓ In case of design pressure, use the reduced value of 10^{-4} (Reduction value = 0.5×Extreme value)

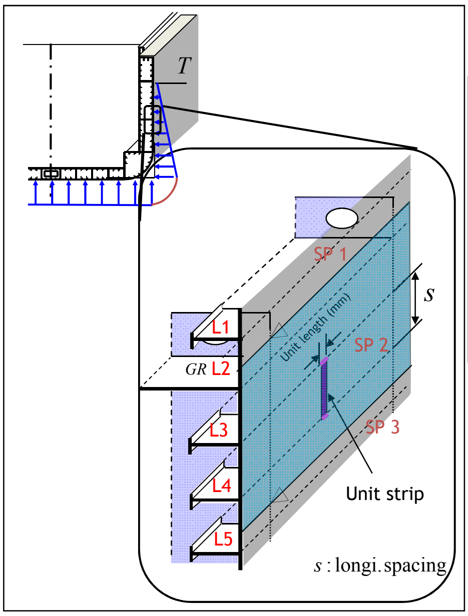
Ex) Liquid Tank Pressure: Pressure, P_1, considering vertical acceleration

$$p_1 = \rho(g_o + 0.5a_v)h_s$$

(4) Load Point - Horizontally Stiffened Plate

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 A 202

$$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi} \right]$$

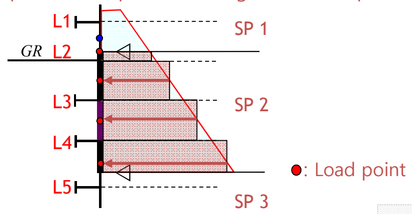


✓ **The pressure at the load point** is considered as **uniform load** of unit strip

✓ Definition of load point

1. General
   : Midpoint of stiffened plate field
2. Seam & butt (In case two plates are welded)
   1) When considered plate includes the midpoint of stiffened plate field
   : Midpoint of stiffened plate field
   2) When considered plate does not include the midpoint of stiffened plate field
   : Nearest seam or butt line from midpoint

✓ Load point of sea pressure acting on the side plate



(DNV Pt.3 Ch.1 Sec.4 A201, 202), 2011

A 200 Definitions

201 Symbols:

p = design pressure in kN/m^2
ρ = density of liquid or stowage rate of dry cargo in t/m^3.

202 The load point for which the design pressure shall be calculated is defined for various strength members as follows:

a) For plates:
midpoint of horizontally stiffened plate field.
Half of the stiffener spacing above the lower support of vertically stiffened plate field, or at lower edge of plate when the thickness is changed within the plate field.

b) For stiffeners:
midpoint of span.
When the pressure is not varied linearly over the span the design pressure shall be taken as the greater of:

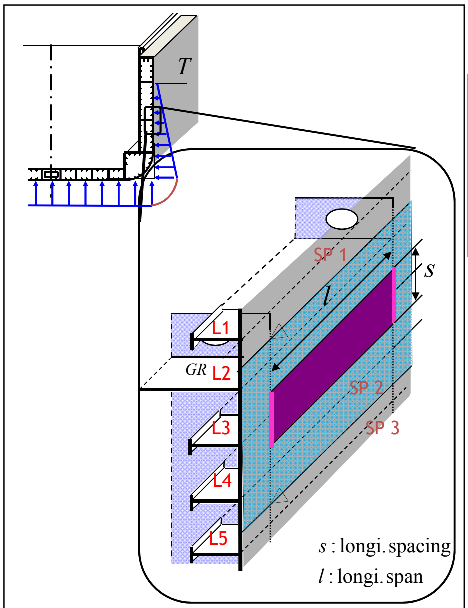
$$p_m \text{ and } \frac{p_a + p_b}{2}$$

p_m, p_a and p_b are calculated pressure at the midpoint and at each end respectively.

c) For girders:
midpoint of load area.

(4) Load Point - Longitudinal Stiffeners (1/2)

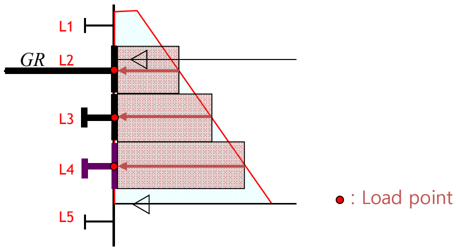
DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 A 202



✓ **The pressure at the load point** is considered as **uniform load**

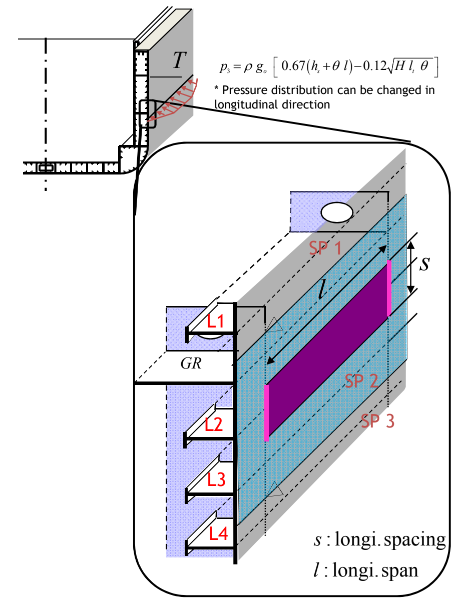
✓ Definition of load point
1. In vertical direction
: The point of intersection between a plate and a stiffener
2. In longitudinal direction
: Midpoint of span

✓ Load point of sea pressure acting on the side plate
- In vertical direction



(4) Load Point - Longitudinal Stiffeners (2/2)

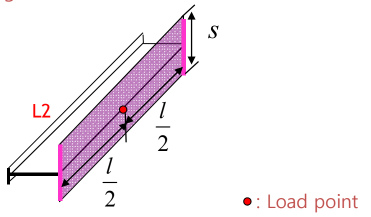
DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 A 202



✓ **The pressure at the load point** is considered as **uniform load**

✓ Definition of load point
1. In vertical direction
: The point of intersection between a plate and a stiffener
2. In longitudinal direction
: Midpoint of span

✓ Load point of sea pressure acting on the side plate
- In longitudinal direction

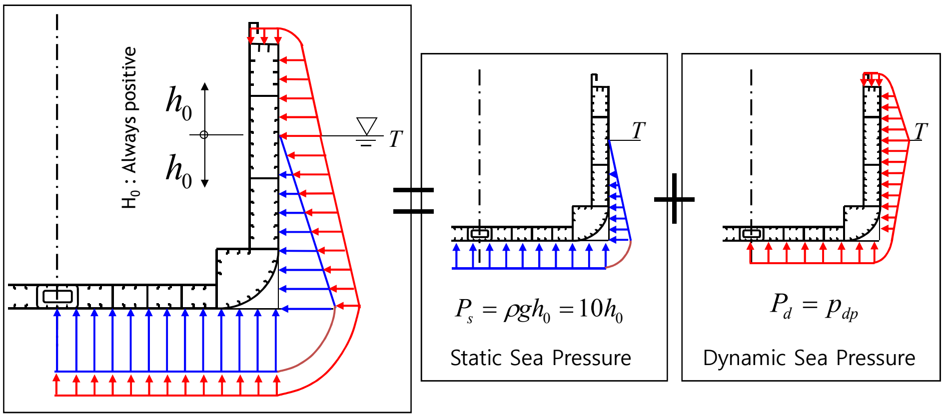


(5) Pressure and Force - Sea Pressure

DNV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 C 201

✓ Sea pressures = Static sea pressure + Dynamic sea pressure

$$P = P_s + P_d$$



(5) Pressure and Force - Liquid Tank Pressure (1/7)

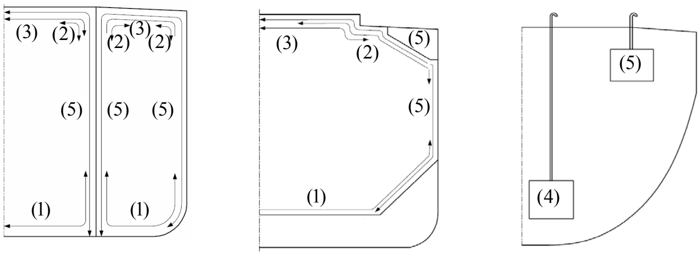
1) DNV Rules, Pt.3 Ch.1 Sec.4 C300, Jan. 2004

✓ The pressure in full tanks shall be taken as the greater of $p_1 \sim p_5$[1)]

$p_1 = \rho\,(g_o + 0.5a_v)\,h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi} \right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta} \right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

- a_v : Vertical acceleration
- ϕ : Roll angle
- b : The largest athwartship distance in [m] form the load point to the tank corner at top of tank
- b_t & l_t : Breadth and length in [m] of top of tank
- ρ : Density of liquid cargo
- h_s : Vertical distance from the load point to tank top in tank
- h_p : Vertical distance from the load point to the top of air pipe
- p_o : 25 kN/m² general
- Δp_{dyn} : Calculated pressure drop

✓ Maximum pressure is different depending on locations



(DNV Pt.3 Ch.1 Sec.4 C301, C302), 2011

301 Tanks for crude oil or bunkers are normally to be designed for liquids of density equal to that of sea water, taken as $\rho = 1.025$ t/m³ (i.e. $\rho\, g_0 \approx 10$). Tanks for heavier liquids may be approved after special consideration. Vessels designed for 100% filling of specified tanks with a heavier liquid will be given the notation **HL(ρ)**, indicating the highest cargo density applied as basis for approval. The density upon which the scantling of individual tanks are based, will be given in the appendix to the classification certificate.

302 The pressure in full tanks shall be taken as the greater of:

$$p = \rho\,(g_0 + 0.5\,a_v)\,h_s \quad (\text{kN/m}^2) \qquad [1]$$

$$p = \rho\, g_0\,[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}] \quad (\text{kN/m}^2) \qquad [2]$$

$$p = \rho\, g_0\,[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}] \quad (\text{kN/m}^2) \qquad [3]$$

$$p = 0.67\,(\rho\, g_0\, h_p + \Delta\, p_{dyn}) \quad (\text{kN/m}^2) \qquad [4]$$

$$p = \rho\, g_0\, h_s + p_0 \quad (\text{kN/m}^2) \qquad [5]$$

- a_v = vertical acceleration as given in B600, taken in centre of gravity of tank.
- ϕ = as given in B400
- θ = as given in B500
- H = height in m of the tank
- ρ = density of ballast, bunkers or liquid cargo in t/m³, normally not to be taken less than 1.025 t/m³ (i.e. $\rho\, g_0 \approx 10$)
- b = the largest athwartship distance in m from the load point to the tank corner at top of the tank which is situated most distant from the load point. For tank tops with stepped contour, the uppermost tank corner will normally be decisive

(DNV Pt.3 Ch.1 Sec.4 C302), 2011

b_t = breadth in m of top of tank

l = the largest longitudinal distance in m from the load point to the tank corner at top of tank which is situated most distant from the load point. For tank tops with stepped contour, the uppermost tank corner will normally be decisive

l_t = length in m of top of tank

h_s = vertical distance in m from the load point to the top of tank, excluding smaller hatchways.

h_p = vertical distance in m from the load point to the top of air pipe

p_0 = 25 kN/m^2 in general

= 15 kN/m^2 in ballast holds in dry cargo vessels

= tank pressure valve opening pressure when exceeding the general value.

Δp_{dyn} = calculated pressure drop according to Pt.4 Ch.6 Sec.4 K201.

For calculation of girder structures the pressure [4] shall be increased by a factor 1.15.

The formulae normally giving the greatest pressure are indicated in Figs. 4 to 6 for various types.

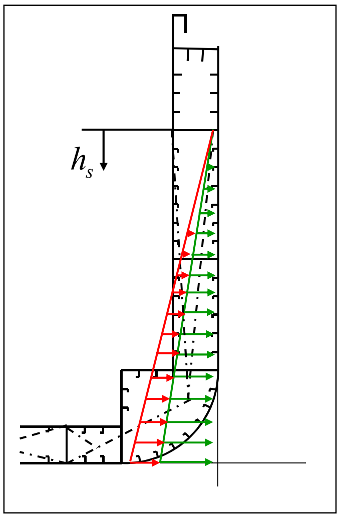
For sea pressure at minimum design draught which may be deduced from formulae above, see 202.

Formulae [2] and [3] are based on a 2% ullage in large tanks.

(5) Pressure and Force - Liquid Tank Pressure (2/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

✓ **Design pressure P_1 considering vertical acceleration (General)**



$$P_1 = \rho g_0 h_s + 0.5 \rho a_v h_s$$

Static Pressure — Dynamic Pressure

Reduced value of 10^{-4} by probability level is used.
(Reduction value=0.5×Extreme value)

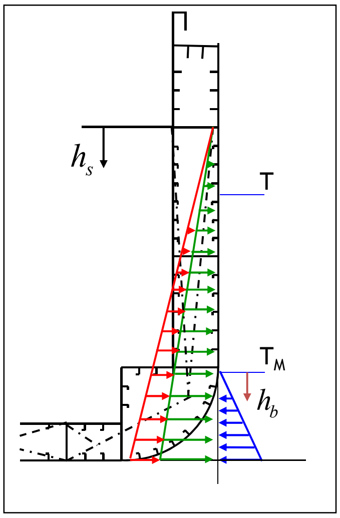
$$p = \rho (g_0 + 0.5 a_v) h_s$$

a_v : Vertical acceleration
h_s : virtical distance in m from load point to top of tank

(5) Pressure and Force - Liquid Tank Pressure (3/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

✓ **Design pressure P_1 considering vertical acceleration (In case of side shell)**



In case of side shell, the effect of sea pressure is considered.

$$P = \rho g_0 h_s + 0.5\rho a_v h_s - 10h_b$$

Static Pressure ($\rho g_0 h_s$), Dynamic Pressure ($0.5\rho a_v h_s$), Sea Pressure ($-10h_b$)

When we consider the design pressure, the largest value shall be applied. The liquid cargo pressure acting on the side shell is the highest when the sea pressure is the lowest, i.e. in case of minimum draft.

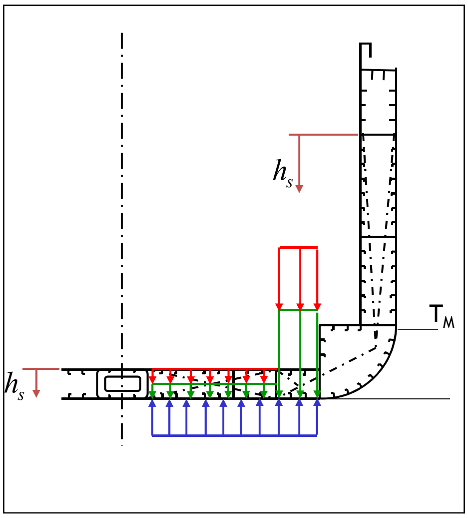
$$p = \rho(g_0 + 0.5a_v)h_s - 10h_b$$

h_b: vertical distance in m from load point to minimum design draft
= 2 + 0.02L for Tanker
= 0.35 T for Dry Cargo
(T : Rule Draft)

(5) Pressure and Force - Liquid Tank Pressure (4/7)

$p_1 = \rho (g_o + 0.5a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

✓ **Design pressure P_1 considering vertical acceleration (In case of bottom shell)**



In case of bottom shell, the effect of sea pressure is considered

$$P = \rho g_0 h_s + 0.5\rho a_v h_s - 10T_M$$

Static Pressure ($\rho g_0 h_s$), Dynamic Pressure ($0.5\rho a_v h_s$), Sea Pressure ($-10T_M$)

When we consider the design pressure, the largest value shall be applied. The liquid cargo pressure acting on the bottom shell is the highest when the sea pressure is the lowest, i.e. in case of minimum draft.

$$p = \rho(g_0 + 0.5a_v)h_s - 10T_M$$

T_M : vertical distance in m from load point to minimum design draught
= 2 + 0.02L for Tanker
= 0.35 T for Dry Cargo
(T : Rule Draft)

DNV Rules, Pt.3 Ch.1 Sec.4 B800, Jan. 2004

(5) Pressure and Force
- Example) Calculation of P_1 Pressure

(Example) When the tank is filled up, calculate the P_1 pressure of inner bottom and deck by using vertical acceleration (a_v=2.286 m/s²) and dimensions of tank which is given below.

[Dimension] Inner bottom height: 3.0 m, Deck height: 31.2m, ρ = 1.025 ton/m³

$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$

ρ = density (ton/m³)
a_v = Vertical acceleration
g_0 = Standard acceleration of gravity (=9.81m/sec²)
h_s : virtical distance in m from load point to top of tank

(Sol.) $a_v = 2.286$ m/s²

① Inner Bottom

$$h_s = 31.2 - 3.0 = 28.8\ m$$
$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$
$$= 1.025\left(9.81 + 0.5 \times 2.286\right) \times 28.2$$
$$= 316.6\,kN/m^2$$

② Deck

$$h_s = 31.2 - 31.2 = 0\ m$$
$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$
$$= 1.025\left(9.81 + 0.5 \times 2.286\right) \times 0$$
$$= 0\,kN/m^2$$

(DNV Pt.3 Ch.1 Sec.4 B801), 2011

B 800 Combined longitudinal accelerations

801 Acceleration along the ship's longitudinal axis is given as the combined effect of surge and pitch calculated as indicated in 100, i.e.:

$$a_l = \sqrt{a_x^2 + (g_0 \sin\theta + a_{px})^2} \quad (\text{m/s}^2)$$

a_{px} = longitudinal component of pitch acceleration given in 503.

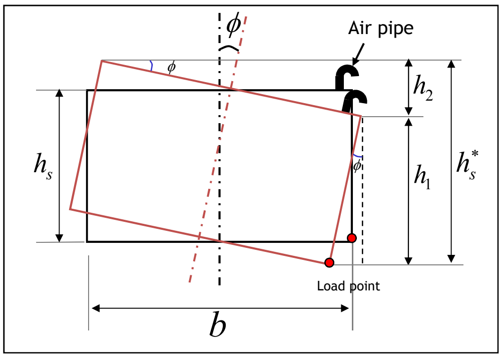
Note that a_{px} is equal to a_p using the vertical projection of R_P.

(5) Pressure and Force - Liquid Tank Pressure (5/7)

$p_1 = \rho(g_o + 0.5a_v)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3
DNV Rules, Pt.3 Ch.1 Sec.4, Jan. 2004

✓ Design pressure P_2 considering the rolling motion



When the ship is rolling, the higher static pressure is applied.

Assumption: $\phi << 1$

$$h_1 = h_s \cos\phi \approx h_s$$
$$h_2 = b\sin\phi \approx b\phi$$
$$\therefore h_s^* = h_1 + h_2 = (h_s + b\phi)$$

$$p_2 = \rho g_0[0.67(h_s + \phi b) - 0.12\sqrt{H\phi b_t}]$$

In case of rolling of a ship, two third (=0.67) of actual pressure is applied considering pressure drop by overflow.

The filling ratio of the most tank is about 98%. That (about 2%) is considered.

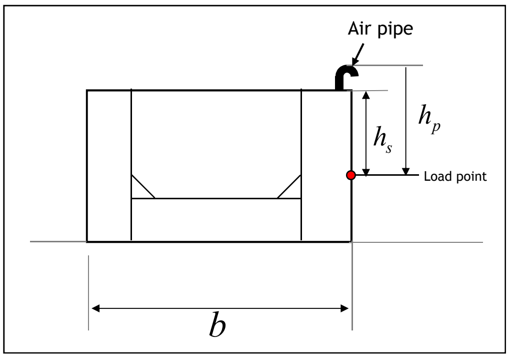
H: Height in m of the tank
b_t: Breadth in m of top of tank

(5) Pressure and Force - Liquid Tank Pressure (6/7)

$p_1 = \rho(g_o + 0.5a_v)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3
DNV Rules, Pt.3 Ch.1 Sec.4, Jan. 2004

✓ Design pressure P_4 considering the tank overflow



The liquid of tank is filled up to air pipe in case of tank overflow.
So, h_p is used for calculating static pressure.

h_p = vertical distance in m from the load point to the top of air pipe

$$p = 0.67(\rho g_0 h_p + \Delta P_{dyn})$$

Calculated pressure drop
Generally, 25kN/m²

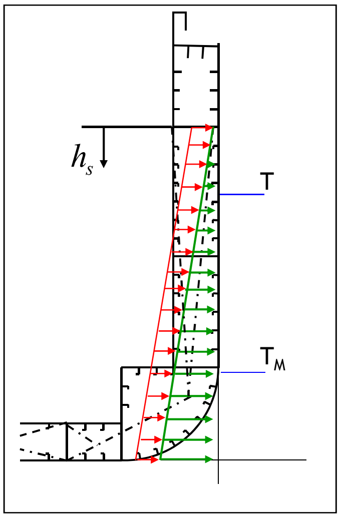
In case of rolling of a ship, two third (=0.67) of actual pressure is applied considering pressure drop by overflow.

(5) Pressure and Force - Liquid Tank Pressure (7/7)

$p_1 = \rho(g_o + 0.5a_v)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3
DNV Rules, Pt.3 Ch.1 Sec.4, Jan. 2004

✓ **Design pressure P_5 considering the tank test pressure**



Over-pressure is applied in order to have the water head of 'tank height + 2.5' [m] in case of tank test for leakage.
(Water head of over-pressure of tank test: 2.5m)

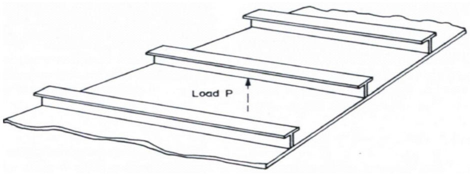
$$p = \rho g_0 h_s + p_o$$

$$p_o = \rho g_0 \times 2.5 = 10 \times 2.5 = 25kN/m^2$$

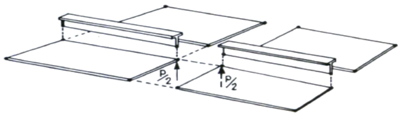
(4) Scantling of Plates

Scantling of Plates (1/3)

Use of eccentric beam element



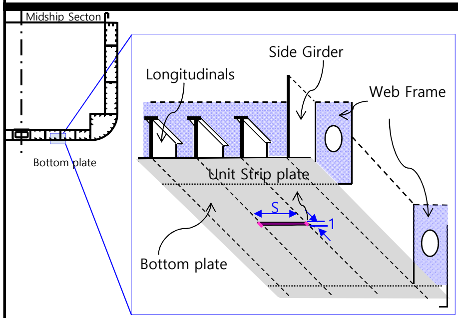
(a) Beams attached to plating

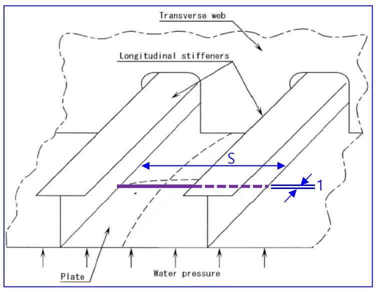


(b) Structural model using eccentric beam element

Scantling of Plates (2/3)

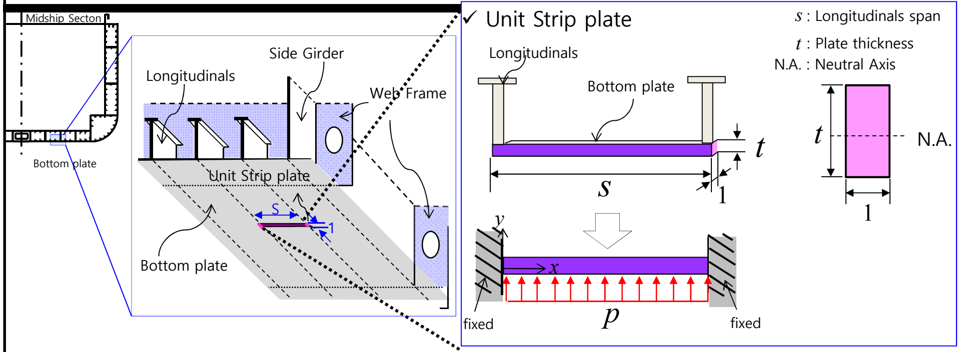
p : "pressure" on the load point for the stiffener





Scantling of Plates (3/3)

p : "pressure" on the load point for the stiffener



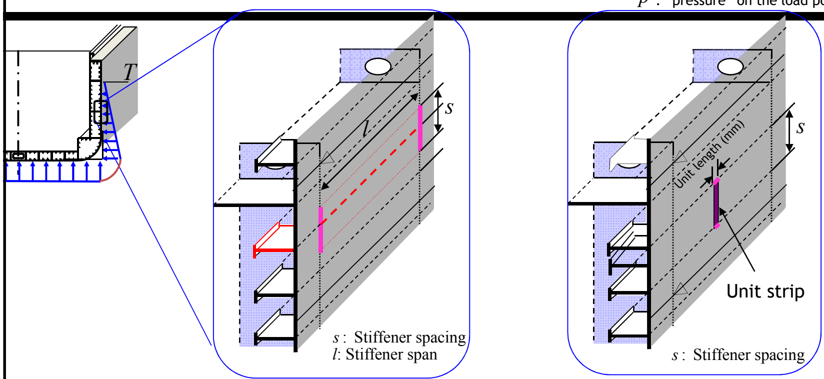
Assumption 1. Cut off the **unit strip plate** supported by the **longitudinals** or **girder.** And consider the unit strip plate as a **"fixed-end beam"** which has a span 's', thickness 't'.

Assumption 2. Consider the lateral load of the beam as a uniformly distributed load. (Assume the pressure on the load point as an intensity of uniformly distributed load.)

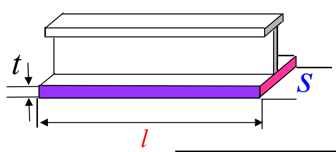
Assumption 3. The design of plates is based on the **plastic design**.

Comparison between Stiffener and Plate

p : "pressure" on the load point for the stiffener



✓ Longitudinal stiffener attached to the plate

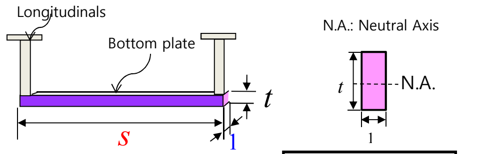


l : Stiffener span

s : Stiffener spacing

$$M = \frac{1}{12} p \cdot s \cdot l^2$$

✓ Unit strip plate



s : Stiffener spacing

1 : Unit length of strip

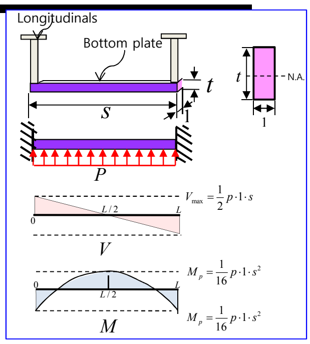
$$M_P = \frac{1}{16} p \cdot 1 \cdot s^2$$

[Reference] Derivation of the Thickness Requirement of the Plates (1/2)

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given: Bending moment "M"
Find: Thickness requirement "t"



Substituting formula: $\sigma \le \sigma_l$ → $\sigma = \sigma_l$

$$Z_{req.} = \frac{M}{\sigma_l}$$

Substituting formula:

Plastic moment (M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

$$Z_{req.} = \frac{p \cdot 1 \cdot s^2}{16 \cdot \sigma_l}$$

Substituting formula:

Plastic section modulus (Z_p)

$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

$$\frac{t_{req.}{}^2}{4} = \frac{p \cdot 1 \cdot s^2}{16 \cdot \sigma_l} \Rightarrow t_{req.} = \frac{s\sqrt{p}}{2\sqrt{\sigma}}$$

For example, the allowable stress of bottom plating is given by:

$$\sigma_l = 120 f_1$$

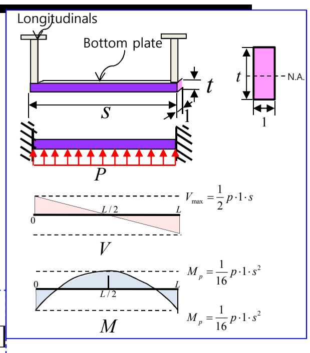
where, f_1 : Material factor

[Reference] Derivation of the Thickness Requirement of the Plates (2/2)

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given: Bending moment "M"
Find: Thickness requirement "t"



$$t_{req.} = \frac{s\sqrt{p}}{2\sqrt{\sigma_l}}$$

Considering different units: $t(mm)$, $s(m)$, $p(kN/m^2)$, $\sigma(N/mm^2)$

$$t_{req.} = \frac{s\sqrt{p}}{2\sqrt{\sigma_l}} \cdot \frac{1000[mm] \cdot \sqrt{1000/1000^2\ [N/mm^2]}}{\sqrt{1\ [N/mm^2]}}\ [mm]$$

$$= \frac{15.8 s\sqrt{p}}{\sqrt{\sigma_l}}$$

$$t_{req.} = \frac{15.8 k_a s\sqrt{p}}{\sqrt{\sigma_l}} + t_k\ [mm]$$

k_a = correction factor for aspect ratio of plate field
t_k = corrosion addition

Comparison of the Elastic and Plastic Design of the Plate - Overview

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Plastic Design

Substituting formula:

Plastic moment (M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus (Z_p)

$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

$$\sigma = \frac{M_p}{Z_p} = \frac{ps^2}{4t^2}, \quad t = \frac{s\sqrt{p}}{2\sqrt{\sigma}}$$

assumption: $\sigma = \sigma_l$

$$t_{req.} = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

Elastic Design

Substituting formula:

Elastic moment (M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$

$$\sigma = \frac{M}{Z} = \frac{ps^2}{2t^2}, \quad t = \frac{s\sqrt{p}}{\sqrt{2}\sqrt{\sigma}}$$

assumption: $\sigma = \sigma_l$

$$t_{req.} = \frac{22.4 k_a s \sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

k_a = correction factor for aspect ratio of plate field $\quad t_k$ = corrosion addition

Comparison of the Elastic and Plastic Design - [Example] Thickness Requirements

Plastic moment (M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus (Z_p)

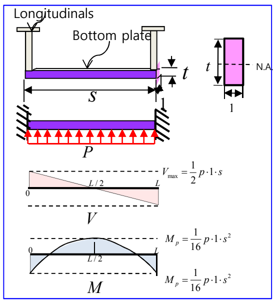
$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

Elastic moment (M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$



① Ex. A mild steel plate carries the uniform pressure of 100 kN/m² on a span length of 800 mm.
Compare the thickness requirement depending on the plastic and elastic design.

$$t_{req.\,plastic} = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma_l}} = \frac{15.8 \times 1 \times 0.8 \times \sqrt{100}}{\sqrt{235}} = 8.24 \ (mm)$$

$$t_{req.\,elastic} = \frac{22.4 k_a s \sqrt{p}}{\sqrt{\sigma_l}} = \frac{22.4 \times 1 \times 0.8 \times \sqrt{100}}{\sqrt{235}} = 11.69 \ (mm)$$

The thickness requirement of the plate of plastic design is smaller than that of the elastic design at the same pressure and on the same span.

k_a = correction factor for aspect ratio of plate field

Comparison of the Elastic and Plastic Design - [Example] Design Pressure

Plastic moment (M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus (Z_p)

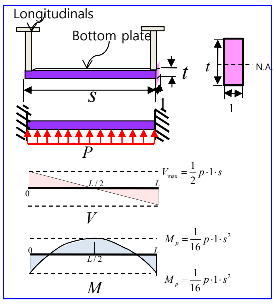
$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

Elastic moment (M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$



② Ex. A mild steel plate has a thickness of 10 mm on a span length of 800 mm.
Compare the design pressure that the maximum stresses of the plate reaches the yield stress depending on the plastic design and elastic design.

$$p_{plastic} = \frac{t^2 \sigma_l}{15.8^2 s^2} = \frac{10^2 \times 235}{15.8^2 \, 0.8^2} = 147 \; [kN/m^2]$$

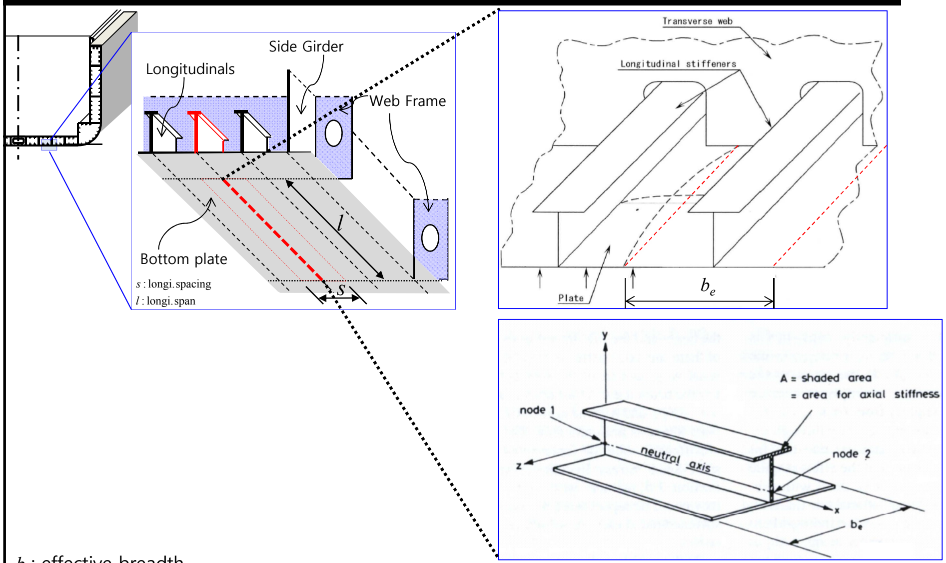
$$p_{elastic} = \frac{t^2 \sigma_l}{22.4^2 s^2} = \frac{10^2 \times 235}{22.4^2 \, 0.8^2} = 73 \; [kN/m^2]$$

The design pressure of plastic design that reaches the yield stress, is higher than that of the elastic design on the same span with the same thickness.

(5) Scantling of Stiffeners

Scantling of Stiffeners (1/3)

p : "pressure" on the load point for the stiffener

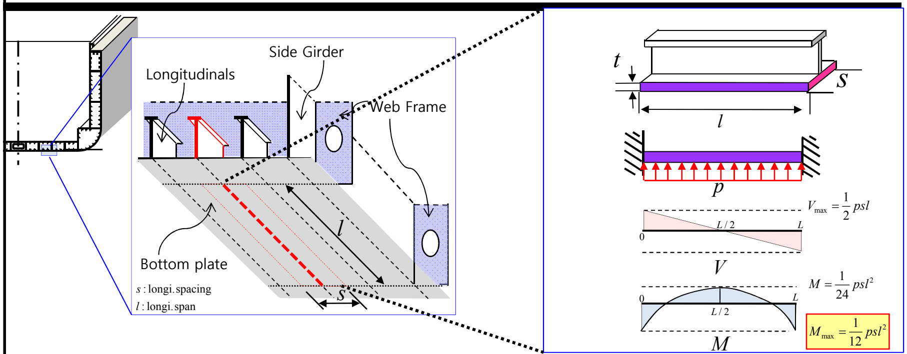


b_e: effective breadth

* Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures - a Practical Guide for Engineers, Springer, pp. 17-32, 2009

Scantling of Stiffeners (2/3)

p : "pressure" on the load point for the stiffener



Assumption 1. Cut off the **stiffener and attached plate with effective breadth**. Sectional properties of stiffener are calculated including attached plate.

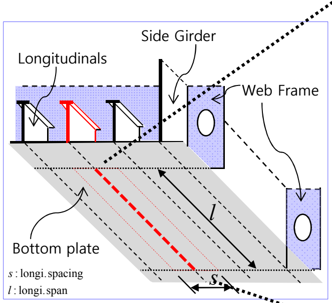
Assumption 2. Consider the stiffener and attached plate as a **"fixed-end beam"** supported by the **web frames**.

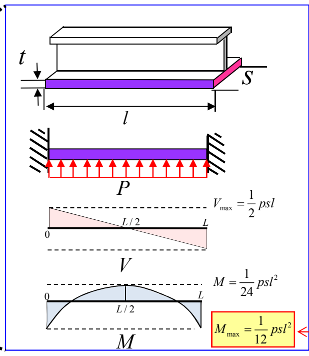
Assumption 3. Consider the **lateral load** of the beam as a **uniformly distributed load.** (Assume the **"pressure"** on the load point as an intensity of uniformly distributed load.)

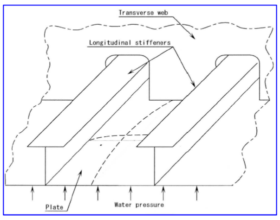
Assumption 4. The design of stiffener is based on the **elastic design** (when the load is removed, the material returns to its original dimensions)

Scantling of Stiffeners (3/3)

p : "pressure" on the load point for the stiffener

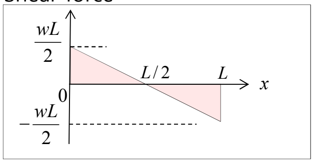




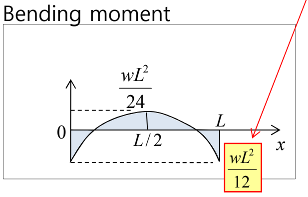


$M_{max} = \frac{1}{12} psl^2$

Shear force



Bending moment



$\frac{wL^2}{12}$

Relation between $p{\cdot}s$ and w

p: pressure (load per unit area)

$p{\cdot}s$: distributed load (load per unit length)

||

w: distributed load (load per unit length)

$\frac{wL^2}{12} \xrightarrow[L=l]{w=p\cdot s} \frac{psl^2}{12}$ *Same!*

[Reference] Derivation of the Formula for the Scantling of the Stiffener (1/2)

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given: Bending moment "M"
Find: Required section modulus "Z"

$\sigma \le \sigma_l$

$\sigma = \sigma_l$

Substituting into the formula:

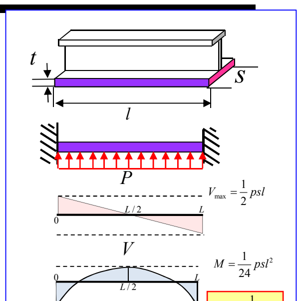
$$Z_{req.} = \frac{M}{\sigma_l}$$

Substituting into the formula:

Elastic moment (M)

$$M = \frac{p \cdot s \cdot l^2}{12}$$

$$Z_{req.} = \frac{p \cdot s \cdot l^2}{12\sigma_l}$$



$M_{max} = \frac{1}{12} psl^2$

For example, the allowable stress of bottom longitudinal stiffener is given by:

$$\sigma_l = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

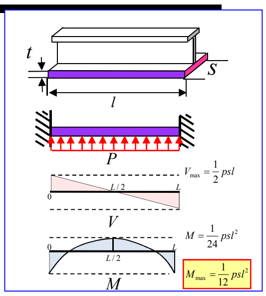
f_l : material factor f_{2b} : stress factor
σ_{db} : mean double bottom stress

[Reference] Derivation of the Formula for the Scantling of the Stiffener (2/2)

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given: Bending moment "M"
Find: Required section modulus "Z"



$$Z_{req.} = \frac{p \cdot s \cdot l^2}{12\sigma_l}$$

Considering different units: $p(kN/m^2),\ s(m),\ l(m),\ \sigma(N/mm^2)$

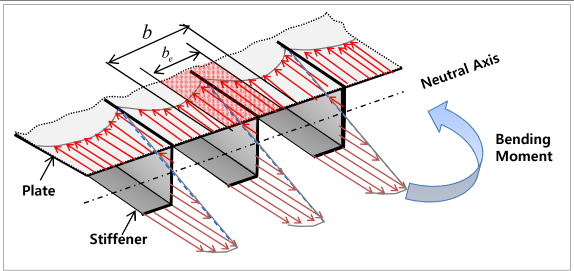
$$Z_{req.} = \frac{p \cdot s \cdot l^2}{12\sigma_l} = \frac{1}{12}\frac{p}{\sigma_l}\left(\frac{1000/1000^2[N/mm^2]}{1[N/mm^2]}\right)\frac{s \cdot l^2}{1}\left(\frac{100[cm] \cdot 100^2[cm^2]}{1}\right)$$

$$= \frac{83\, p \cdot s \cdot l^2}{\sigma_l}\ [cm^3]$$

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l}\ [cm^3]$$

w_k: Section modulus corrosion factor in tanks

[Reference] Effective Breadth of Attached Plates



When the lateral pressure is imposed, the stress distribution in plates and stiffeners is complicated as shown in the figure.

The longitudinal stress in the attached plate will be a maximum at the connection line to the stiffener and become smaller gradually beyond this line.

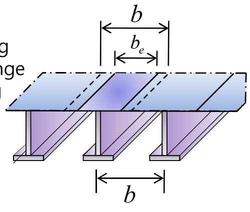
Considering the strength, **the stiffened panel will be assumed to be a collection of beams which include some parts of the attached plate. The breadth of this plate is called the "effective breadth".**

Ex. DNV Rule: Effective flange of girder[2)]

The effective plate flange area is defined as the cross sectional area of plating within the effective flange width. Continuous stiffeners within the effective range may be included. The effective flange width b_e is determined by the following formula:

$$b_e = C \bullet b\ (m)$$

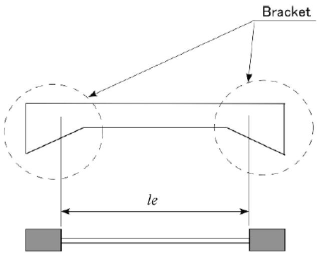
Reduction factor (C); Effective Breadth (b_e); Actual Breadth (b)



C: As given in table for various numbers of evenly spaced point loads (r) on the span
b: Sum of plate flange width on each side of girder, normally taken to half the distance from nearest girder or bulkhead

1) DSME, Ship Structural Design, 3.4 Section Properties of Hull Structure, 2005
2) DNV Rules for Ships, Pt.3 Ch.1 Sec.3 C400

[Reference] Span Point of a Beams



(a) Definition of span point

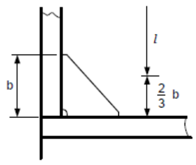
Ship structure members are usually connected with brackets or other structures.

When we consider a member as a beam, **it is convenient to assume the member to be a uniform section beam, having an equivalent length between two span points**, and to assume the outside structures of the span points to be rigid bodies as illustrated in the figure.

The span point depends on structural details and loading conditions.

Ex. DNV Rule: Definition of span for stiffeners and girders.[1)]

The effective span of a stiffener (l) or girder (S) depends on the design of the end connections in relation to adjacent structures. Unless otherwise stated the span points at each end of the member, between which the span is measured, shall be determined as shown in Fig.
It is assumed that brackets are effectively supported by the adjacent structure.



Example of span point

1) DNV Rules for Ships, Pt.3 Ch.1 Sec.3 C100

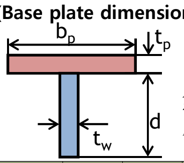
(6) Sectional Properties of Steel Sections

Sectional Properties of Steel Sections for Ship Building[1)] (1/12)

<Sectional properties of steel sections including attached plate>

[1)] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

(Base plate dimension : $b_p \times t_p$ = 420 x 8)



b_p = breadth of plate (mm)
t_p = thickness of plate (mm)

A = Area including plate(cm²)
Z = Section modulus including plate (cm³)
I = Moment of inertia of area including plate(cm⁴)

d \ t_w		6	9	11	12.7	14
50	A	3.00	4.5	5.50	6.35	7.00
	Z	6.05	8.81	10.6	12.1	13.3
	I	31.2	44.5	53.0	59.7	75.2
65	A	3.90	5.85	7.15	8.26	9.10
	Z	9.55	14.0	16.8	19.3	21.1
	I	62.3	88.8	105	119	129
75	A	4.50	6.75	8.25	9.53	10.5
	Z	12.3	18.1	21.8	25.0	27.3
	I	91.4	130	154	174	189
90	A	5.40	8.10	9.90	11.4	12.6
	Z	17.2	25.3	30.5	34.8	38.0
	I	150	214	252	284	307
100	A	6.00	9.00	11.0	12.7	14.0
	Z	20.9	30.6	37.0	42.2	46.1
	I	200	284	335	376	407
125	A	7.50	11.3	13.8	15.9	17.5
	Z	31.7	46.4	55.8	63.6	69.5
	I	370	521	612	685	738
150	A	9.00	13.5	16.5	19.1	21.0
	Z	44.7	65.2	78.3	89.1	97.2
	I	614	856	1000	1120	1200

d \ t_w		16	19	22	25.4	28	32	35	38
200	A	32.0	38.0	44.0	50.8	56.0	64.0	70.0	76.0
	Z	215	259	305	359	401	469	521	576
	I	3900	4730	5600	6640	7460	8790	9830	10900
250	A	40.0	47.5	55.0	63.5	70.0	80.0	87.5	95.0
	Z	325	390	458	536	597	694	769	845
	I	7120	8600	10100	11900	13400	15600	17400	19200
300	A	48.0	57.0	66.0	76.2	84.0	96.0	105.0	114.0
	Z	455	546	639	746	829	961	1060	1160
	I	11700	14000	16500	19300	21600	25100	27800	30700
350	A	56.0	66.5	77.0	88.9	98.0	112.0	122.5	133.0
	Z	606	726	847	988	1100	1270	1400	1530
	I	17700	21200	24800	29100	32400	37600	41600	45700
400	A	64.0	76.0	88.0	101.6	112.0	128.0	140.0	152.0
	Z	776	928	1080	1260	1400	1610	1780	1940
	I	25300	30300	35400	41400	46000	53300	58900	64600
450	A	72.0	85.5	99.0	114.3	126.0	144.02	157.5	171.0
	Z	965	1150	1340	1560	1730	2000	2200	2400
	I	34700	41500	48500	56500	62800	72600	80100	87700
500	A	80.0	95.0	110.0	127.0	140.0	160.0	175.0	190.0
	Z	1170	1400	1630	18907	2100	2420	2660	2900
	I	46000	55000	64200	4700	82900	95700	10500	11500

A = 42*0.8 + 15*1.4 = 21 [cm²]
Z_{Top} = 349.6 [cm³]
Z_{Bottom} = 97.2 [cm³]

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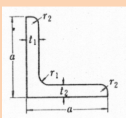
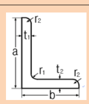
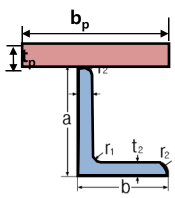
Sectional Properties of Steel Sections for Ship Building[1)] (2/12)

[1)] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

<Sectional properties of steel sections including attached plate>

- Use the standard dimension of plate depending on "a" ($b_p \times t_p$) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

	Dimension						Area	Including plate	
Symbol	a	b	t_1	t_2	r_1	r_2	A	I	Z
Unit	mm						cm²	cm⁴	cm³
Equal angle	∟							ㄷ	
	50		6		6.5	4.5	5.64		
	65		6		8.5	4	7.53		
	65		8		8.5	6	9.76		
	75		6		8.5	4	8.73		
	75		9		8.5	6	12.69	90.1	18.7
	75		12		8.5	6	16.56	191	31.9
	90		10		10	7	17.00	229	39.7
	90		13		10	7	21.71	284	42.5
	100		10		10	7	19.00	369	58.2
	100	‘’	13	‘’	10	7	24.31	433	71.6
	130		9		12	6	11.74	767	96.0
	130		12		12	8.5	19.76	905	117
	130		15		12	8.5	36.75	1030	119
	150		12		14	7	34.77	1220	147
	150		15		14	10	42.74		
	150		19		14	10	53.38		
	200		20		17	12	76.00		
	200		25		17	12	93.75		
	200		29		17	12	107.6		
Unequal angle	∟							ㄷ	
	100	75	7		10	5	11.87	674	72.5
	100	75	10		10	7	16.50	860	96.2
	125	75	7	‘’	10	5	13.62	110	97.2
	125	75	10		10	7	19.00	1420	130
	150	90	9		12	6	20.94	2490	181
	150	90	12		12	8.5	27.36	3060	230



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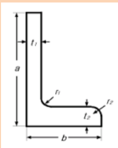
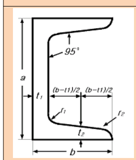
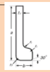
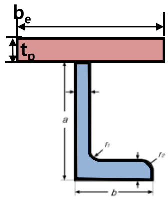
Sectional Properties of Steel Sections for Ship Building[1] (3/12)

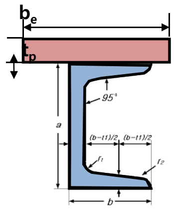
[1] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

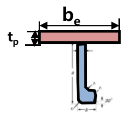
<Sectional properties of steel sections **including attached plate**>

- Use the standard dimension of plate depending on "a" ($b_p \times t_p$) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

	Dimension						Area	Including plate	
Symbol	a	b	t_1	t_2	r_1	r_2	A	I	Z
Unit	mm						cm^2	cm^4	cm^3
Unequal angle 									
	200	90	9	14	14	7.0	29.66	5870	340
	250	90	10	15	17	8.5	37.47	10300	494
	250	90	12	16	17	8.5	42.95	11000	540
	300	90	11	16	19	9.5	46.22	16400	681
	300	90	13	17	19	9.5	52.67	17600	743
	400	100	11.5	16	24	12	61.09	34200	1120
	400	100	13	18	24	12	68.59	36700	1230
	450	125	11.5	18	24	12	73.11	51200	1570
	450	150	11.5	15	24	12	73.45	51700	1590
	500	150	11.5	18	24	12	83.6	70400	2020
	550	150	12	21	24	12	95.91	93300	2520
	600	150	12.5	23	24	12	107.6	118000	3000
Channels 									
	150	75	6.5	10	10	5	23.71	2160	154
	200	90	8	13.5	14	7	38.65	5650	322
	250	90	9	13	14	7	44.07	9420	439
	250	90	11	14.5	17	8.5	51.17	10500	499
	300	90	9	13	14	7	48.57	14300	567
	300	90	10	15.5	19	9.5	55.74	16000	646
	300	90	12	16	19	9.5	61.90	16900	693
	380	100	10.5	16	18	9	69.39	29900	989
	380	100	13	20	24	12	85.71	34900	1190
Bulb flats 									
	180	32.5	9.5	-	7	2	21.06	2860	172
	200	36.5	10	-	8	2	25.23	4160	231
	230	41	11	-	9	2	31.98	6610	330
	250	45	12	-	10	2	38.13	8960	424







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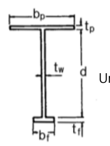
Sectional Properties of Steel Sections for Ship Building[1] (4/12)

[1] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

<Sectional properties of steel sections **including attached plate**>

(Base plate dimension: b_p x t_p = 610 x 15)

$d \times t_w$ \ $b_f \times t_f$		100×16	125×16	150×16	150×19	150×22	150×25·4	180×25·4	200×25·4	200×28	200×32	230×32	230×35	230×38
	A_f	16·0	20·0	24·0	28·5	33·0	38·1	45·6	50·8	56·0	64·0	73·6	80·5	87·4
300×11·5	A	50·5	54·5	58·5	63·0	67·5	72·6							
	Z	775	890	1000	1130	1250	1390							
	I	19400	21600	23800	26300	28700	31300							
350×11·5	A	56·3	60·3	64·3	68·8	73·3	78·4							
	Z	955	1090	1220	1360	1500	1660							
	I	27100	30100	32900	36100	39300	42700							
400×11·5	A	62·0	66·0	70·0	74·5	79·0	84·1							
	Z	1150	1300	1450	1610	1770	1950							
	I	36500	40200	43800	47900	51800	56100							
450×11·5	A	67·8	71·8	75·8	80·3	84·8	89·9							
	Z	1350	1520	1690	1870	2050	2250							
	I	47600	52200	56500	61500	66300	71500							
500×11·5	A	73·5	77·5	81·5	86·0	90·5	95·6							
	Z	1570	1760	1940	2140	2340	2560							
	I	60400	65900	71200	77100	82900	89200							
550×12	A	82·0	86·0	90·0	94·5	99·0	104·1							
	Z	1840	2040	2240	2460	2680	2920							
	I	76300	82700	88900	95800	103000	110000							
600×12·7	A	92·2	96·2	100·2	104·7	109·2	114·3	121·9	127·0	132·2	140·2	149·8	156·7	163·6
	Z	2150	2370	2590	2820	3050	3310	3720	3990	4260	4660	5170	5510	5850
	I	95300	103000	110000	118000	125000	134000	145000	152000	160000	171000	182000	190000	198000
650×12·7	A	98·6	102·6	106·6	111·1	115·6	120·7	128·3	133·4	138·6	146·6	156·2	163·1	170·0
	Z	2430	2660	2890	3140	3390	3670	4110	4410	4690	5130	5670	6050	6420
	I	115000	123000	131000	141000	149000	159000	172000	180000	189000	202000	215000	224000	234000
700×12·7	A	104·9	108·9	112·9	117·4	121·9	127·0	134·6	139·7	144·9	152·9	162·5	169·4	176·3
	Z	2720	2960	3210	3480	3750	4050	4520	4830	5140	5610	6190	6590	6990
	I	137000	146000	156000	166000	176000	187000	202000	211000	221000	236000	251000	262000	272000
700×16	A	128·0	132·0	136·0	140·5	145·0	150·1	157·7	162·8	168·0	176·0	185·6	192·5	199·4
	Z	3070	3310	3560	3820	4070	4370	4830	5130	5430	5890	6460	6850	7230
	I	150000	159000	168000	178000	187000	198000	212000	222000	231000	245000	260000	271000	282000
800×12·7	A	117·6	121·6	125·6	130·1	134·6	139·7	147·3	152·4	157·6	165·6	175·2	182·1	189·0
	Z	3330	3610	3890	4200	4500	4840	5370	5720	6080	6600	7260	7720	8170
	I	188000	200000	211000	224000	237000	251000	270000	282000	294000	313000	332000	346000	360000
800×16	A	144·0	148·0	152·0	156·5	161·0	166·1	173·7	178·8	184·0	192·0	201·6	208·5	215·4
	Z	3780	4060	4330	4630	4920	5250	5770	6110	6450	6960	7610	8050	8490
	I	207000	218000	229000	242000	354000	267000	285000	297000	309000	327000	346000	360000	374000
900×14	A	142·0	146·0	150·0	154·5	159·0	164·1	171·7	176·8	182·0	190·0	199·6	206·5	213·4
	Z	4220	4530	4840	5170	5500	5880	6460	6860	7240	7820	8550	9050	9550
	I	259000	274000	287000	303000	318000	335000	358000	373000	388000	410000	435000	452000	469000
900×18	A	178·0	182·0	186·0	190·5	195·0	200·1	207·7	212·8	218·0	226·0	235·6	242·5	249·4
	Z	4880	5190	5490	5810	6130	6490	7060	7440	7810	8370	9090	9570	10100
	I	290000	304000	317000	332000	346000	362000	384000	399000	413000	435000	459000	476000	493000
1000×16	A	176·0	180·0	184·0	188·5	193·0	198·1	205·7	210·8	216·0	224·0	233·6	240·5	247·4
	Z	5390	5730	6070	6420	6780	7180	7820	8250	8660	9290	10100	10600	11200
	I	355000	372000	388000	407000	424000	444000	471000	489000	507000	534000	563000	584000	605000
1000×19	A	206·0	210·0	214·0	218·5	223·0	228·1	235·7	240·8	246·0	254·0	263·6	270·5	277·4
	Z	5990	6320	6650	7000	7340	7740	8360	8780	9180	9800	10600	11100	11600
	I	386000	402000	418000	435000	452000	472000	498000	515000	533000	559000	588000	609000	630000



Unit: mm

A_f: Sectional area (cm^2)
A: Sectional area including plate (cm^2)
Z: Section modulus including plate (cm^3)
I: Moment of inertia including plate (cm^4)

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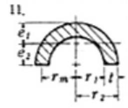
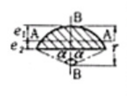
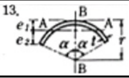
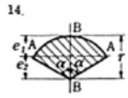
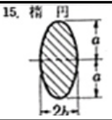
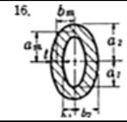
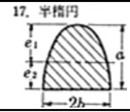
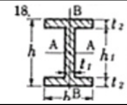
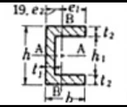
Sectional Properties of Steel Sections for Ship Building[1] (5/12)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape	A	I	Z_e	Z_P
11. 	$\frac{1}{2}\pi(r_2^2-r_1^2)$ t/r_mが小さいとき $A_{rm}=\pi r_m t$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)(r_2^4-r_1^4)$ $-\frac{8r_2^2r_1^2(r_2-r_1)}{9\pi(r_2+r_1)}$ $I_{rm}=\left(\frac{\pi}{2}-\frac{4}{\pi}\right)r_m^3t$ $\fallingdotseq 0{\cdot}2976\,r_m^3t$	$e_1=r_2-e_2$ $e_2=\frac{4(r_2^2+r_2r_1+r_1^2)}{3\pi(r_2+r_1)}$ $e_{2rm}=\frac{2}{\pi}r_m\fallingdotseq 0{\cdot}6366\,r_m$	$2[2(r_2^3\sin^3\theta_2$ $-r_1^3\sin^3\theta_1)$ $-(r_2^3-r_1^3)]/3$ ここに, $r_1\cos\theta_1=r_2\cos\theta_2$
12. 	$\frac{1}{2}r^2(2\alpha-\sin 2\alpha)$	$I_A=r^4\left[\frac{1}{16}(4\alpha-\sin 4\alpha)-\frac{8\sin^6\alpha}{9(2\alpha-\sin 2\alpha)}\right]$ $I_B=\frac{r^4}{12}\left[3\alpha-2\sin 2\alpha+\frac{1}{4}\sin 4\alpha\right]$ $e_1=r\left(1-\frac{4\sin^3\alpha}{6\alpha-3\sin 2\alpha}\right)$ $e_2=r\left(\frac{4\sin^3\alpha}{6\alpha-3\sin 2\alpha}-\cos\alpha\right)$		$\frac{2}{3}r^3(2\sin^3\alpha_0-\sin^3\alpha)$ ここに, $\frac{2\alpha-\sin 2\alpha}{2\alpha_0-\sin 2\alpha_0}=4$
13. 	$2\alpha rt$	$I_A=r^3t(\alpha+\sin\alpha\cos\alpha$ $-2\frac{\sin^2\alpha}{\alpha})$ $I_B=r^3t(\alpha-\sin\alpha\cos\alpha)$	$e_1=r\left(1-\frac{\sin\alpha}{\alpha}\right)$ $e_2=r\left(\frac{\sin\alpha}{\alpha}-\cos\alpha\right)$	$2rt(r-t/2)$ $\times(2\sin\frac{\alpha}{2}-\sin\alpha)$
14. 	αr^2	$I_A=\frac{1}{4}r^4(\alpha+\sin\alpha\cos\alpha$ $-\frac{16\sin^2\alpha}{9\alpha})$ $I_B=\frac{1}{4}r^4(\alpha-\sin\alpha\cos\alpha)$	$e_1=r\left(1-\frac{2\sin\alpha}{3\alpha}\right)$ $e_2=r\frac{2\sin\alpha}{3\alpha}$	$\alpha>0{\cdot}996$, $(2\alpha'-\sin 2\alpha'=\alpha)$ $2r^3(2\sin\alpha'-\sin\alpha)/3$ $\alpha<0{\cdot}996$ $\frac{2r^3}{3}\left[\sin\alpha-\sqrt{\frac{\alpha^3}{2\tan\alpha}}\right]$
15. 楕円 	πab	$\frac{\pi}{4}a^3b\fallingdotseq 0{\cdot}7854\,a^3b$	$\frac{\pi}{4}a^2b\fallingdotseq 0{\cdot}7854\,a^2b$	$\frac{4}{3}a^2b$

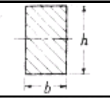
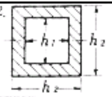
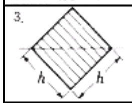
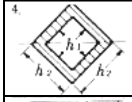
Sectional Properties of Steel Sections for Ship Building[1] (6/12)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape	A	I	Z_e	Z_P
16. 	$\pi(a_2b_2-a_1b_1)$ t/a_m, t/b_m が小さいとき $A_m=\pi(a_m+b_m)t$	$\frac{\pi}{4}(a_2^3b_2-a_1^3b_1)$ $I_m=\frac{\pi}{4}a_m^2(a_m+3b_m)t$	$\frac{\pi}{4}\ \frac{a_2^3b_2-a_1^3b_1}{a_2}$ $Z_m=\frac{\pi}{4}a_m(a_m+3b_m)t$	$\frac{4}{3}(a_2^2b_2-a_1^2b_1)$
17. 半楕円 	$\frac{1}{2}\pi ab$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)a^3b$ $\fallingdotseq 0{\cdot}1098\,a^3b$	$e_1=\left(1-\frac{4}{3\pi}\right)a\fallingdotseq 0{\cdot}5756a$ $Z_1\fallingdotseq 0{\cdot}1908\,a^2b$ $e_2=\frac{4r}{3\pi}\fallingdotseq 0{\cdot}4244\,a$ $Z_2\fallingdotseq 0{\cdot}2587\,a^2b$	$\fallingdotseq 0{\cdot}35362\,a^2b$
18. 	$2bt_2+h_1t_1$	$I_A=\frac{bh^3-(b-t_1)h_1^3}{12}$ $I_B=\frac{2b^3t_2+h_1t_1^3}{12}$	$Z_A=\frac{bh^3-(b-t_1)h_1^3}{6h}$ $Z_B=\frac{2b^3t_2+h_1t_1^3}{6b}$	$\frac{h_1^2t_1}{4}+\frac{bt_2}{2}(h+h_1)$
19. 	$2bt_2+h_1t_1$	$I_A=\frac{bh^3-(b-t_1)h_1^3}{12}$ $I_B=\frac{2b^3t_2+h_1t_1^3}{3}-Ae_2^2$	$e_1=b-e_2$ $e_2=\frac{2b^2t_2+h_1t_1^2}{4bt_2+2h_1t_1}$	18. と同じ

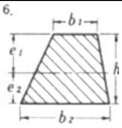
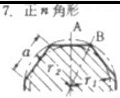
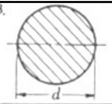
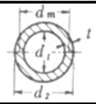
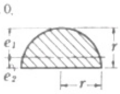
Sectional Properties of Steel Sections for Ship Building[1] (7/12)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape	A	I	Z_e	Z_P
1. 	bh	$\frac{1}{12}bh^3$	$\frac{1}{6}bh^2$	$\frac{1}{4}bh^2$
2. 	$h_2^2-h_1^2$	$\frac{1}{12}(h_2^4-h_1^4)$	$\frac{1}{6}\frac{h_2^4-h_1^4}{h_2}$	$\frac{1}{4}(h_2^3-h_1^3)$
3. 	h^2	$\frac{1}{12}h^4$	$\frac{\sqrt{2}}{12}h^3$	$\frac{\sqrt{2}}{6}h^3$
4. 	$h_2^2-h_1^2$	$\frac{1}{12}(h_2^4-h_1^4)$	$\frac{\sqrt{2}}{12}\frac{h_2^4-h_1^4}{h_2}$	$\frac{\sqrt{2}}{6}(h_2^3-h_1^3)$
5. 	$\frac{1}{2}bh$	$\frac{1}{36}bh^3$	$e_1=\frac{2}{3}h,\ Z_1=\frac{bh^2}{24}$ $e_2=\frac{1}{3}h,\ Z_2=\frac{bh^2}{12}$	$\frac{2-\sqrt{2}}{6}bh^2$

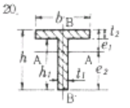
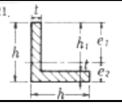
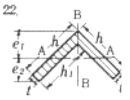
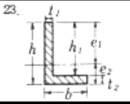
Sectional Properties of Steel Sections for Ship Building[1] (8/12)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape	A	I	Z_e	Z_P
6. 	$\frac{1}{2}(b_1+b_2)h$	$\frac{h^3(b_1^2+4b_1b_2+b_2^2)}{36(b_1+b_2)}$	$e_1=\frac{h(b_1+2b_2)}{3(b_1+b_2)}$ $Z_1=\frac{h^2(b_1^2+4b_1b_2+b_2^2)}{12(b_1+2b_2)}$ $e_2=\frac{h(2b_1+b_2)}{3(b_1+b_2)}$ $Z_2=\frac{h^2(b_1^2+4b_1b_2+b_2^2)}{12(2b_1+b_2)}$	$\frac{Ah}{3}\frac{(b_1b_2+b_2b_3+b_3b_1)}{(b_1+b_3)(b_2+b_3)}$ ここに、 $b_3^2=(b_1^2+b_2^2)/2$
7. 正n角形 	$\frac{1}{2}nar_1$	$\frac{A}{24}(6r_2^2-a^2)$ $=\frac{A}{48}(12r_1^2+a^2)$	$Z_A=\frac{A}{48r_1}(12r_1^2+a^2)$ $Z_B=\frac{A}{24r_2}(6r_2^2-a^2)$	n：偶数，$Z_{PA}=\frac{a^2r_1}{6}+\frac{2}{3}ar_1^2\sum_{k=1}^{\frac{n}{2}-1}\sin\frac{2k\pi}{n}$
8. 	$\frac{1}{4}\pi d^2$	$\frac{1}{64}\pi d^4$	$\frac{1}{32}\pi d^3$	$\frac{1}{6}d^3$
9. 	$\frac{1}{4}\pi(d_2^2-d_1^2)$ t/d_mが小さいとき $A_{dm}=\pi d_m t$	$\frac{1}{64}\pi(d_2^4-d_1^4)$ $I_{dm}=\frac{1}{8}\pi d_m^3 t$	$\frac{\pi}{32}\frac{d_2^4-d_1^4}{d_2}$ $Z_{dm}=\frac{1}{4}\pi d_m^2 t$	$\frac{1}{6}(d_2^3-d_1^3)$
10. 	$\frac{1}{2}\pi r^2$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)r^4$ $\doteqdot 0.1098\,r^4$	$e_1=\left(1-\frac{4}{3\pi}\right)r\doteqdot 0.5756r$ $Z_1\doteqdot 0.1908\,r^3$ $e_2=\frac{4r}{3\pi}\doteqdot 0.4244\,r$ $Z_2\doteqdot 0.2587\,r^3$	$\doteqdot 0.37982\,r^3$

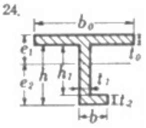
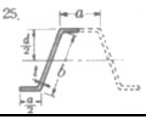
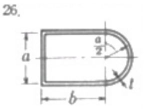
Sectional Properties of Steel Sections for Ship Building[1] (9/12)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape	A	I	Z_e	Z_P
20. 	$bt_2+h_1t_1$	$I_A=\frac{h^3t_1+(b-t_1)t_2^3}{3}-Ae_1^2$ $I_B=\frac{b^3t_2+h_1t_1^3}{12}$	$e_1=\frac{h^2t_1+(b-t_1)t_2^2}{2(bt_2+h_1t_1)}$ $e_2=h-e_1$	$t_2 \le h_1 t_1 / b$ のとき $\frac{bt_2}{2}\left(h-\frac{t_2}{t_1}b\right)+\frac{h_1t_1}{4}\left[h_1+\left(\frac{t_2}{t_1}\right)^2\times\left(\frac{b}{h_1}\right)b\right]$ $t_2 > h_1 t_1 / b$ のとき $\frac{bt_2^2}{4}\left[1-\left(\frac{h_1t_1}{bt_2}\right)^2\right]+\frac{h\,h_1t_1}{2}$
21. 	$(h+h_1)t$	$\frac{t}{3}(h^3+h_1t^2)-Ae_2^2$	$e_1=h-e_2$ $e_2=\frac{h^2+h_1t}{2(h+h_1)}$	$\frac{t}{4}[(h-t)^2+h^2]$
22. 	$(h+h_1)t$	$I_A=\frac{(h+t)^4}{24}-\frac{h_1^4+2t^4}{24}-Ae_2^2$ $I_B=\frac{1}{12}(h^4-h_1^4)$	$e_1=\frac{h^2+h_1t}{\sqrt{2}(h+h_1)}$ $e_2=\frac{h^2}{\sqrt{2}(h+h_1)}$	$\frac{t}{\sqrt{2}}[h(h-t)+t^2]$
23. 	$bt_2+h_1t_1$	$\frac{h^3t_1+(b-t_1)t_2^3}{3}-Ae_2^2$	$e_1=h-e_2$ $e_2=\frac{h^2t_1+(b-t_1)t_2^2}{2(bt_2+h_1t_1)}$	20. と同じ

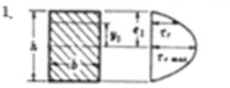
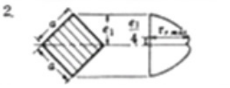
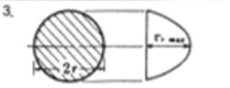
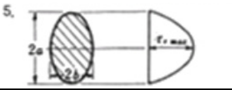
Sectional Properties of Steel Sections for Ship Building[1] (10/12)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape	A	I	Z_e	Z_P
24. 	$b_0t_0+bt_2+h_1t_1$	$I=\frac{b_0t_0^3}{3}+\frac{bh^3}{3}-\frac{(b-t_1)h_1^3}{3}-A(e_1-t_0)^2$ $e_1=t_0+\frac{bh^2-(b-t_1)h_1^2-b_0t_0^2}{2A}$ $e_2=h-\frac{bh^2-(b-t_1)h_1^2-b_0t_0^2}{2A}$		$t_0 \le (bt_2+h_1t_1)/b_0$ のとき $\frac{b_0t_0}{2}(h_1+t_0)+\frac{bt_2h}{2}+\frac{h_1^2t_1}{4}-\frac{1}{4t_1}\times(bt_2-b_0t_0)^2$ $t_0 > (bt_2+h_1t_1)/b_0$ のとき $\frac{b_0t_0^2}{4}-\frac{1}{4b_0}(bt_2+h_1t_1)^2+\frac{(h_1+t_0)(h_1t_1+bt_2)}{2}+\frac{bt_2h}{2}$
25. 	$t(a+b)$	$\frac{td^2}{12}(3a+b)$	$\frac{td}{6}(3a+b)$	$\frac{adt}{2}+\frac{bdt}{4}$
26. 	$at\left(1+\frac{\pi}{2}\right)+2bt$ $\fallingdotseq 2\cdot5708at+2bt$	$\frac{a^3t}{12}\left(1+\frac{3}{4}\pi\right)+\frac{1}{2}a^2bt$ $\fallingdotseq 0\cdot2797a^3t+0\cdot5a^2bt$	$\frac{a^2t}{6}\left(1+\frac{3}{4}\pi\right)+abt$ $\fallingdotseq 0\cdot5594a^2t+abt$	$\frac{3}{4}a^2t+abt+\frac{t^3}{6}$

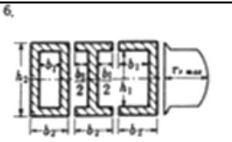
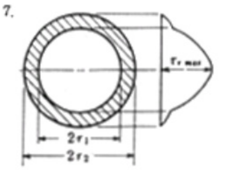
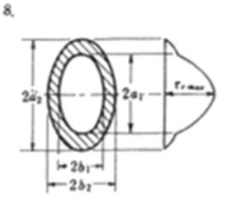
Sectional Properties of Steel Sections for Ship Building[1] (11/12)

[1] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

Section shape and distribution of shear force	$\tau_y = \frac{F}{z_1 I}\int_{y_1}^{e_1} zy\,dy$	$\tau_{ymax} = \frac{\alpha F}{A}$
1. 	$\frac{3}{2}\cdot\frac{F}{bh}\left\{1-\left(\frac{2y_1}{h}\right)^2\right\}$	$\frac{3}{2}\cdot\frac{F}{bh}=\frac{3}{2}\cdot\frac{F}{A}$
2. 	$\sqrt{2}\frac{F}{a^2}\left\{1+\sqrt{2}\frac{y_1}{a}-4\left(\frac{y_1}{a}\right)^2\right\}$	$\frac{9}{8}\sqrt{2}\frac{F}{a^2}=1{\cdot}591\frac{F}{A}$
3. 	$\frac{4}{3}\cdot\frac{F}{\pi r^2}\left\{1-\left(\frac{y_1}{r}\right)^2\right\}$	$\frac{4}{3}\cdot\frac{F}{\pi r^2}=\frac{4}{3}\cdot\frac{F}{A}$
4. 	$\frac{F}{\pi rt}\left\{1-\left(\frac{y_1}{r}\right)^2\right\}$	$\frac{F}{\pi rt}=2\frac{F}{A}$
5. 	$\frac{4}{3}\cdot\frac{F}{\pi ab}\left\{1-\left(\frac{y_1}{a}\right)^2\right\}$	$\frac{4}{3}\cdot\frac{F}{\pi ab}=\frac{4}{3}\cdot\frac{F}{A}$

Sectional Properties of Steel Sections for Ship Building[1] (12/12)

[1] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

6. 	$\frac{h_2}{2}\geq y_1\geq\frac{h_1}{2}$: $\frac{3F}{2(b_2h_2^3-b_1h_1^3)}(h_2^2-4y_1^2)$ $\frac{h_1}{2}\geq y_1\geq 0$: $\frac{3F}{2(b_2h_2^3-b_1h_1^3)}\left(\frac{b_2h_2^2-b_1h_1^2}{b_2-b_1}-4y_1^2\right)$	$\frac{3(b_2h_2^2-b_1h_1^2)F}{2(b_2h_2^3-b_1h_1^3)(b_2-b_1)}$ $=\frac{3(b_2h_2^2-b_1h_1^2)(b_2h_2-b_1h_1)}{2(b_2h_2^3-b_1h_1^3)(b_2-b_1)}\cdot\frac{F}{A}$
7. 	$r_2\geq y_1\geq r_1$: $\frac{4F}{3\pi(r_2^4-r_1^4)}(r_2^2-y_1^2)$ $r_1\geq y_1\geq 0$: $\frac{4F}{3\pi(r_2^4-r_1^4)}\{r_2^2+r_1^2-2y_1^2+\sqrt{(r_2^2-y_1^2)(r_1^2-y_1^2)}\}$	$\frac{4(r_2^2+r_2r_1+r_1^2)F}{3\pi(r_2^4-r_1^4)}$ $=\frac{4(r_2^2+r_2r_1+r_1^2)}{3(r_2^2+r_1^2)}\cdot\frac{F}{A}$
8. 	$a_2\geq y_1\geq a_1$: $\frac{4F}{3\pi(a_2^3b_2-a_1^3b_1)}(a_2^2-y_1^2)$ $a_1\geq y_1\geq 0$: $\frac{4F}{3\pi(a_2^3b_2-a_1^3b_1)}\times\frac{\frac{b_2}{a_2}(a_2^2-y_1^2)^{\frac{3}{2}}-\frac{b_1}{a_1}(a_1^2-y_1^2)^{\frac{3}{2}}}{\frac{b_2}{a_2}(a_2^2-y_1^2)^{\frac{1}{2}}-\frac{b_1}{a_1}(a_1^2-y_1^2)^{\frac{1}{2}}}$	$\frac{4(a_2^2b_2-a_1^2b_1)F}{3\pi(a_2^3b_2-a_1^3b_1)(b_2-b_1)}$ $=\frac{4(a_2^2b_2-a_1^2b_1)(a_2b_2-a_1b_1)}{3(a_2^3b_2-a_1^3b_1)(b_2-b_1)}\cdot\frac{F}{A}$

6.4 Buckling Strength

Contents

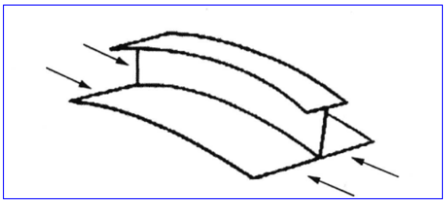
James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748-762
Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.1

Buckling

- **Definition: The phenomenon where lateral deflection may arise in the athwart direction* against the axial working load**

  *선측(船側)에서 선측으로 선체를 가로지르는

- **This section covers buckling control for plate and longitudinal stiffener.**



Flexural buckling of stiffeners plus plating

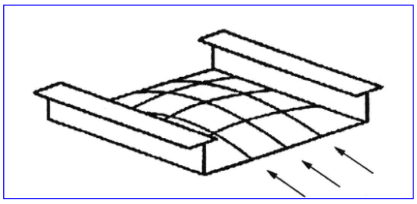


Plate alone buckles between stiffeners

* Mansour, A., Liu, D., The Principles of Naval Architecture Series - Strength of Ships and Ocean Structures, The Society of Naval Architects and Marine Engineers, 2008

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748-762

(1) Column Buckling - The Equation of the Deflection Curve

- **Differential equation for column buckling:** $EIy'' + Py = 0$

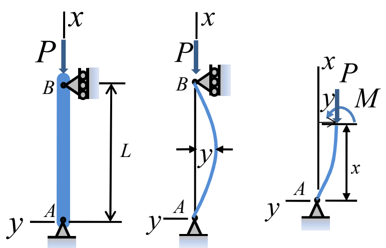
$$\frac{P}{EI} = k^2, k = \frac{n\pi}{l}$$

Using the notation $k^2 = \frac{P}{EI}$, $y'' + k^2 y = 0$

General solution of the equation: $y = C_1 \sin kx + C_2 \cos kx$

Boundary conditions:
$y(0) = 0,\ y(l) = 0$

$$y(0) = C_2 = 0$$
$$y(l) = C_1 \sin kL = 0$$



1) **If** $C_1 = 0,\ y = 0$ **(trivial solution).**

2) **If** $\sin kl = 0$, **(**$\sin kl = 0$**: buckling equation)**

① **If** $kl = 0,\ y = 0$ **(trivial solution).**

② **If** $kl = n\pi$ **(n=1, 2, 3) or** $P = \left(\frac{n\pi}{l}\right)^2 EI$, **it is nontrivial solution.**

$$\therefore y = C_1 \sin kx = C_1 \sin \frac{n\pi x}{L}, n = 1, 2, 3...$$

E = modulus of elasticity
I = 2nd moment of the section area
EI = flexural rigidity
P = axial load
v = deflection of column
L = length of column

(1) Column Buckling - Critical Stress

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748-762

- **Differential equation for column buckling :** $EIy'' + Py = 0$

$$\frac{P}{EI} = k^2, k = \frac{n\pi}{l}$$

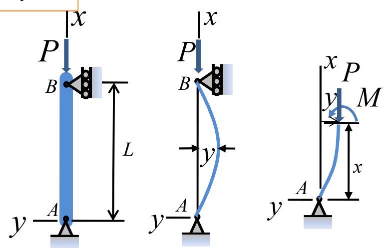
The equation of the deflection curve : $y = C_1 \sin\frac{n\pi x}{L},\ n = 1, 2, 3...$

The critical loads : $P = k^2 EI = \left(\frac{n\pi}{L}\right)^2 EI$

The lowest critical load (n=1) : $P_{cr} = \left(\frac{\pi}{L}\right)^2 EI = \frac{\pi^2 EI}{L^2}$

The corresponding critical stress : $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{AL^2}$

Euler's formula



E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
v = deflection of column
A = area of column
L = length of column

(1) Column Buckling - Critical Load

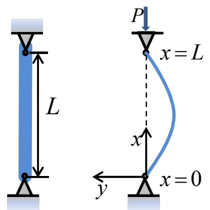
James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748-762

- **Differential equation for column buckling :** $y'' + \lambda y = 0\ , y(0) = 0\ , y(L) = 0$
$, where\ \lambda = P / EI$

The equation of the deflection curve : $y_n(x) = C_1 \sin(n\pi x / L)$

The critical loads : $P_n = n^2\pi^2 EI / L^2, n = 1,2,3...$

The lowest critical load (n=1) : $P_{cr} = P_1 = \pi^2 EI / L^2$



E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
y = deflection of column
A = area of column
L = length of column

(1) Column Buckling - Critical Buckling Stress

A critical buckling stress is often used instead of a buckling load and it can be derived by dividing P_{cr} by A, the cross sectional area of the column.

Euler's formula

The corresponding critical stress :

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{Al^2} = \pi^2 E \left(\frac{k}{l}\right)^2$$

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
y = deflection of column
A = area of column
l = length of column

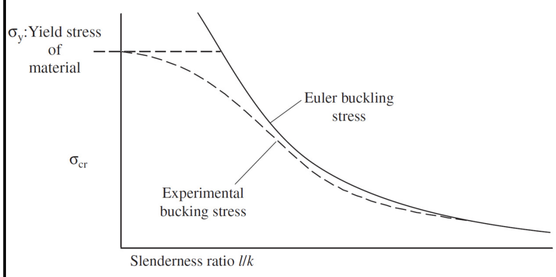
, where $k\left(k^2 = I / A\right)$ is the radius of gyration[1)] of the section of the column.

The ratio l / k, often called the slenderness ratio, is the main factor which governs the critical stress

For large value of l/k the critical stress tends toward zero, and at small values of l/k it tends to infinity. In Euler's formula, the buckling stress may become infinite for a small value of l/k, however, buckling stress never goes up above the yield stress of the material in actual conditions, because the material would fail if the stress exceeded the yield stress.

1) The radius of gyration describes a circular ring whose area is the same as the area of interest.

(1) Column Buckling - Curve of Buckling Stress



by theoretical consideration, a horizontal line of yield stress connected to Euler buckling stress is specified as an upper limit of Euler's buckling curve.

$$\sigma_{cr} = a - b\left(\frac{l}{k}\right)$$ Tetmayer's formula

$$\sigma_{cr} = a - b\left(\frac{l}{k}\right)^2$$ Johnson's formula

$$\sigma_{cr} = \frac{a}{1 + b\left(l / k\right)^2}$$ Rankine's formula

For example, one of the Classification Societies, ABS (American Bureau of Shipping) specifies the permissible load of a pillar or strut of mild steel material in the following equation :

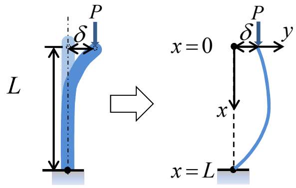
$$\sigma_{cr} = 1.232 - 0.00452\left(\frac{l}{k}\right) \quad [ton \cdot f / cm^2]$$

From the above equation, we can see that the ABS formula is theoretically based on Tetmayer's experimental result.

(1) Column Buckling
- Buckling of Thin Vertical Column Embedded at Its Base and Free at Its Top (1/2)

Suppose that a tin vertical homogeneous column is embedded at its base ($x=0$) and free at its top ($x=L$) and that a constant axial load P is applied to its free end.
The load either causes a small deflection δ, or does not cause such a deflection. In either case the differential equation for the deflection $y(x)$ is

$$EI\frac{d^2y}{dx^2} = P(\delta - y) \Rightarrow EI\frac{d^2y}{dx^2} + Py = P\delta \quad \cdots(1)$$



(1) What is the predicted deflection when $\delta = 0$?

- The general solution of the differential equation (1) is

$$y = c_1 \cos\sqrt{\frac{P}{EI}}x + c_2 \sin\sqrt{\frac{P}{EI}}x + \delta$$

- The boundary conditions of the differential equation (1) are

$$y(0) = y'(0) = 0$$

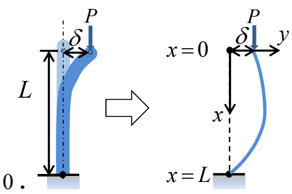
- If $\delta = 0$, this implies that $c_1 = c_2 = 0$ and $y(x) = 0$. That is, there is no deflection.

* Zill, D.G., Advanced Engineering Mathematics, 3rd edition, pp.166-174, 2006

(1) Column Buckling
- Buckling of Thin Vertical Column Embedded at Its Base and Free at Its Top (2/2)

Suppose that a tin vertical homogeneous column is embedded at its base ($x=0$) and free at its top ($x=L$) and that a constant axial load P is applied to its free end.
The load either causes a small deflection δ, or does not cause such a deflection. In either case the differential equation for the deflection $y(x)$ is

$$EI\frac{d^2y}{dx^2} = P(\delta - y) \Rightarrow EI\frac{d^2y}{dx^2} + Py = P\delta \quad \cdots(1)$$



(2) When $\delta \neq 0$, show that the Euler load for this column is one-fourth of the Euler load for the hinged column?

- If $\delta \neq 0$, the boundary conditions give, in turn, $c_1 = -\delta,\ c_2 = 0$. Then

$$y = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}x\right)$$

- In order to satisfy the boundary condition $y(L) = \delta$, we must have

$$\delta = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}L\right) \longrightarrow \cos\sqrt{\frac{P}{EI}}L = 0 \longrightarrow \sqrt{\frac{P}{EI}}L = n\pi/2$$

- The smallest value of P_n, the Euler load, is then

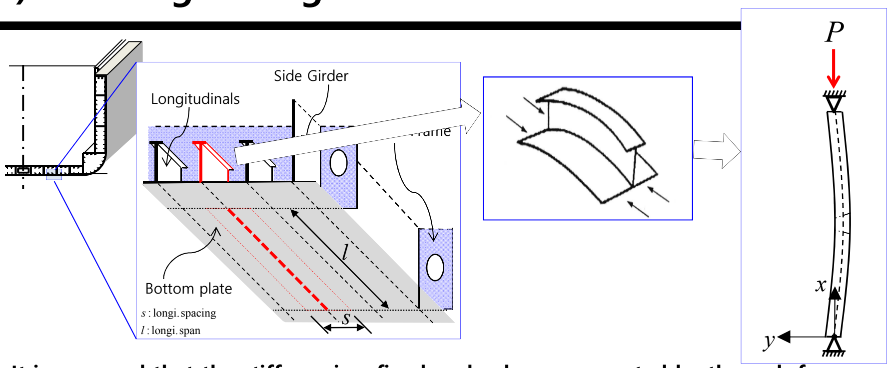
$$\sqrt{\frac{P_1}{EI}}L = \frac{\pi}{2} \quad \text{or} \quad P_1 = \frac{1}{4}\left(\frac{\pi^2 EI}{L^2}\right)$$

One-fourth of the Euler load

Euler load

* Zill, D.G., Advanced Engineering Mathematics, 3rd edition, pp.166-174, 2006

(2) Buckling Strength of Stiffener



It is assumed that the stiffener is a fixed-end column supported by the web frames.

Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation. And it is acting on the each stiffener as axial load from the point view of local deformation.

what is our interest?

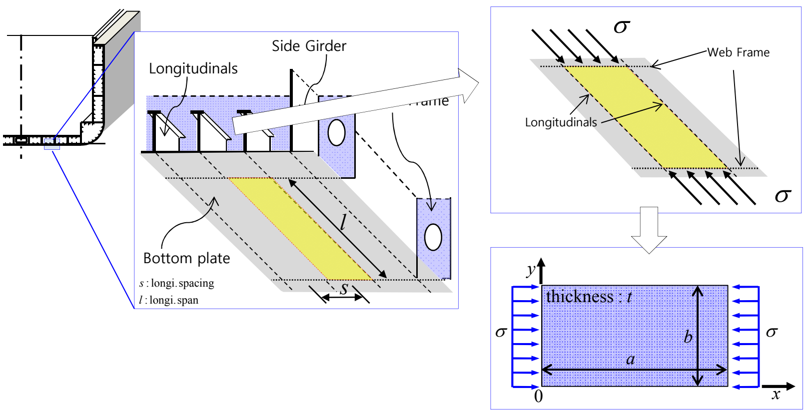
- **Safety :**

Won't it fail under the load?

The actual compressive stress (σ_a) shall not be great than the critical buckling stress (σ_{cr})

$$\sigma_a \le \sigma_{cr}$$, where $\sigma_a = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$, $Z = Z(y)$

(3) Buckling Strength of Plate (1/7)

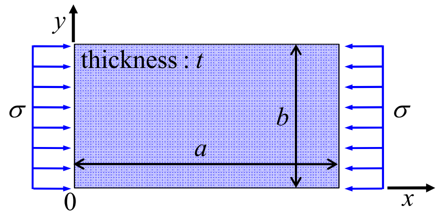


A ship hull is a stiffened-plate structure, the plating supported by a system of transverse or longitudinal stiffeners.

For practical design purpose, it is often assumed that the plate is simply supported at the all edges, since it gives the least critical stress and is on the safe side.

(3) Buckling Strength of Plate (2/7)

Let us consider the rectangular plate with only supported edges as shown in this figure.



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a : plate length
b : plate width
t : thickness of the plate

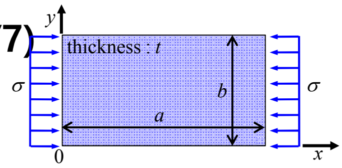
- The equation of elastic buckling stress of the plate under uni-axial compressive stress:

$$\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0 \quad \cdots(1)$$

where, $w = w(x, y)$: deflection of the plate

* Okumoto, Y., Design of Ship Hull Structures, pp.57-60, 2009

(3) Buckling Strength of Plate (3/7)



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a: plate length
b : plate width
t : thickness of the plate

- The equation of elastic buckling stress of the plate under uni-axial compressive stress:

$$\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0 \quad \cdots(1)$$

where, $w = w(x, y)$ **: deflection of the plate**

- Because all four edges are simply supported, the boundary condition can be expressed in the form:

$$w(0,y)=w(a,y)=0$$
$$w(x,0)=w(x,b)=0$$

← deformation at the edges are zero

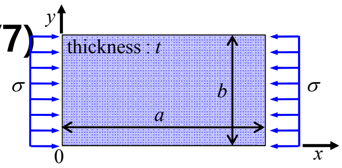
- Let us assume the following formula for the solution of the equation (1), so that the solution satisfies the boundary conditions.

$$w = f\sin\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right)\cdots(2)$$

where, m, n are integers presenting the number of half-wave of buckles.

* Okumoto, Y., Design of Ship Hull Structures, pp.57-60, 2009

(3) Buckling Strength of Plate (4/7)



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a : plate length
b : plate width
t : thickness of the plate

- **The equation of elastic buckling stress of the plate under uni-axial compressive stress:**

$$\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0 \quad \cdots(1)$$

where, $w = w(x, y)$: deflection of the plate

- **Substituting the formula (2) into the equation (1),**

$$w = f\sin\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right) \quad \cdots(2)$$

$$\sigma = \frac{Et^3}{12(1-\nu^2)}\frac{\pi^2}{b^2 t}\left(\frac{m}{\alpha}+n^2\frac{\alpha}{m}\right)^2 \quad \cdots(3)$$

where, $\alpha = \frac{a}{b}$

- **Elastic buckling stress is a minimum critical stress, therefore, we put n=1 in the equation (3),**

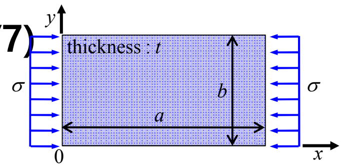
Ideal elastic (Euler) compressive buckling stress:

$$\sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 K$$

where, K = Minimum value of k, $k = \left(\frac{m}{\alpha}+\frac{\alpha}{m}\right)^2$

* Okumoto, Y., Design of Ship Hull Structures, pp.57-60, 2009

(3) Buckling Strength of Plate (5/7)



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a : plate length
b : plate width
t : thickness of the plate

Ideal elastic (Euler) compressive buckling stress:

$$\sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 K$$

where, K = Minimum value of k

$k = \left(\frac{m}{\alpha}+\frac{\alpha}{m}\right)^2,\ \alpha = \frac{a}{b}$

- **For the small b in comparison with t, the elastic buckling stress becomes more than the yield stress of the plate material.**
- **Therefore, it is usual to use Johnson's modification factor η_p and the critical buckling stress σ_c for the full range of value of t/b as follows :**

- **Bryan's formula[1]**

$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)}\cdot\left(\frac{t}{b}\right)^2\cdot K$$

σ_c : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
K : plate factor (corresponding to the boundary conditions and a/b)
η_p : plasticity reduction factor

$$\eta_p = 1 \quad \text{, when } \sigma_{el} < \frac{\sigma_y}{2}$$

$$\eta_p = \frac{\sigma_y}{\sigma_{el}}\left(1-\frac{\sigma_y}{4\sigma_{el}}\right) \quad \text{, when } \sigma_{el} \geq \frac{\sigma_y}{2}$$

≤ 1

σ_y = upper yield stress in [N/mm²]

ex) Coefficient K when all four edges are simply supported

$K = 4.0 \quad a/b \geq 1.0$

$K = \left(a/b + b/a\right)^2,\ a/b < 1.0$

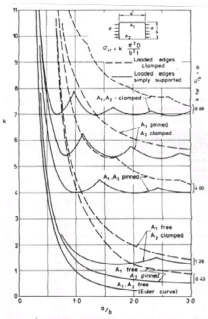


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

1) DSME, "선박구조설계" 13-18 Buckling, 2005.8

(3) Buckling Strength of Plate (6/7)
- Buckling Strength of Web Plate

1) DSME, "Ship Structural Design", 13-18 Buckling, 2005.8

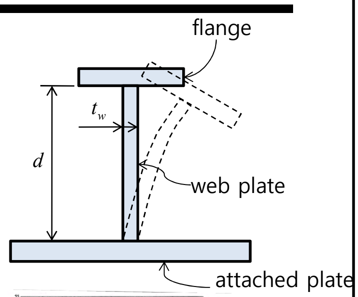
Web plate of stiffener have to be checked about buckling.

In case of T-bar, it is assumed that the <u>web plate of stiffener is the plate simply supported by flange and attached plate</u>.

$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t}{d}\right)^2 \cdot K$$, (Bryan's formula) , K = 4.0

$$\rightarrow \frac{d}{t_w} \le \sqrt{\frac{\pi^2 EK}{12\left(1-\nu^2\right)} \frac{1}{\sigma_{el}}}$$

σ_{cr} : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
ν : Poisson's ratio
K : Plate factor (corresponding to the boundary conditions and a/b)
d : depth of web plate
t : thickness of web plate
E : Modulus of elasticity



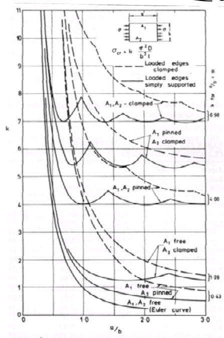


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

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(3) Buckling Strength of Plate (7/7)
- Buckling Strength of Flange Plate

1) DSME, "Ship Structural Design", 13-18 Buckling, 2005.8

Flange of stiffener have to be checked about buckling.

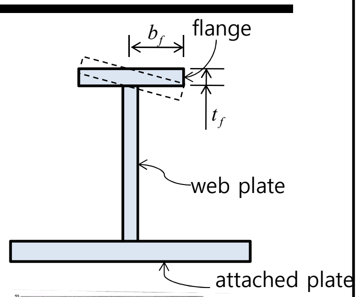
It is assumed that the <u>flange of stiffener is the rectangular plate simply supported on one end by web plate</u>.

$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t_f}{b_f}\right)^2 \cdot K$$, (Bryan's formula) , K = 0.5

$$\rightarrow \frac{b}{t_f} \le \sqrt{\frac{K\pi^2 E}{12\left(1-\nu^2\right)} \frac{1}{\sigma_{el}}}$$

In general, b/t_f does not exceed 15.

σ_c : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
ν : Poisson's ratio
K : Plate factor (corresponding to the boundary conditions and a/b)
b_f : breadth of flange plate
t_f : thickness of flange plate
E : Modulus of elasticity



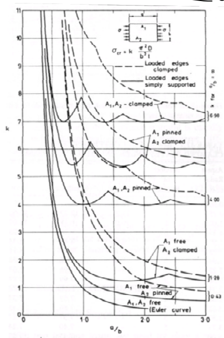


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

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(4) Buckling Strength by DNV Rule

◆ Criteria for buckling strength

$$\sigma_c > \frac{\sigma_a}{\eta}$$

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

σ_c = critical buckling stress in N/mm²
σ_a = calculated actual stress in N/mm²
η = usage factor

◆ Critical buckling stress σ_c

▪ σ_f is yield stress of material in N/mm²

$$\sigma_c = \sigma_{el} \quad \text{, when } \sigma_{el} < \frac{\sigma_f}{2}$$

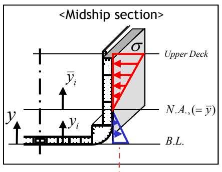
$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{,when } \sigma_{el} > \frac{\sigma_f}{2}$$

◆ Calculated actual stress σ_a

▪ σ_a is calculated actual stress in general
▪ In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,(N/mm^2)$$

$$= \text{minimum } 30 \text{ } f_1 \text{ N/mm}^2 \text{ at side}$$



consider each different stress according to location

◆ σ_{el} for Plate in uni-axial compression[1)]

Plate

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

◆ σ_{el} for stiffener in uni-axial compression[1)]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

Stiffener

◆ σ_{el} for stiffener in lateral buckling mode

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2}$$

M_S : still water bending moment as given in Sec.5
M_W : wave bending moment as given in Sec.5
I_{NA} : moment of inertia in cm⁴ of the hull girder
σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_c : critical buckling stress
σ_f : upper yield stress in [N/mm²]
t : thickness in [mm]
t_k : corrosion addition
t_w : web thickness, h_w : web height
E : modulus of elasticity
s : stiffener spacing in [m]
I_A : moment of inertia in [cm⁴] about the axis perpendicular to the expected direction of buckling
A : cross-sectional area in [cm²]
l : length of member in [m]

(DNV Pt.3 Ch.1 Sec.13, B100, B102, B103), 2011

B 100 General

101 Local plate panels between stiffeners may be subject to uni-axial or bi-axial compressive stresses, in some cases also combined with shear stresses. Methods for calculating the critical buckling stresses for the various load combinations are given below.

102 Formulae are given for calculating the ideal compressive buckling stress σ_{el}. From this stress the critical buckling stress σ_c may be determined as follows:

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

103 Formulae are given for calculating the ideal shear buckling stress τ_{el}. From this stress the critical buckling stress τ_c may be determined as follows:

$$\tau_c = \tau_{el} \quad \text{when} \quad \tau_{el} < \frac{\tau_f}{2}$$

$$= \tau_f\left(1 - \frac{\tau_f}{4\tau_{el}}\right) \quad \text{when} \quad \tau_{el} > \frac{\tau_f}{2}$$

τ_f = yield stress in shear of material in N/mm²

$$= \frac{\sigma_f}{\sqrt{3}}.$$

(5) Buckling Strength of Stiffener by DNV Rule - Stiffener in Uni-axial Compression (1/2)

◆ **Criteria for Buckling Strength**
(in the same way of plate)

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c : critical buckling stress in [N/mm²]
σ_a : calculated actual compressive stress in [N/mm²]
η : usage factor

Usage Factor (η)

η = 1.0	Deck, Single bottom & Side shell (long stiff)
η = 0.9	Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0	Extreme loads (Q = 10^{-8})
η = 0.8	Normal loads (Q = 10^{-4})

◆ **Critical buckling stress σ_c**

$$\sigma_c = \sigma_{el} \quad \text{, when } \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{,when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_f : yield stress of material in N/mm²

◆ **Calculated actual stress σ_a**
(Uni-axial compression)

- σ_a is calculated actual compressive stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,(N/mm^2)$$
$$= \text{minimum } 30\ f_1\ \text{N/mm}^2 \text{ at side}$$

(※ Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation.
And it is acting on the each stiffener as axial load from the point view of local deformation.)

M_S = still water bending moment as given in Sec.5
M_W = wave bending moment as given in Sec.5
I_N = moment of inertia in cm⁴ of the hull girder

(DNV Pt.3 Ch.1 Sec.13, B205), 2011

205 The critical buckling stress calculated in 201 shall be related to the actual compressive stresses as follows:

$$\sigma_c \geq \frac{\sigma_a}{\eta}$$

σ_a = σ_a calculated compressive stress in plate panels. With linearly varying stress across the plate panel, shall be taken as the largest stress.

In plate panels subject to longitudinal stresses, σ_a is given by:

$$\sigma_{al} = \frac{M_S + M_W}{I_N}(z_n - z_a)10^5 \quad (\text{N/mm}^2)$$
$$= \text{minimum } 30\ f_1\ \text{N/mm}^2 \text{ at side}$$

η = 1.0 for deck, single bottom and longitudinally stiffened side plating
= 0.9 for bottom, inner bottom and transversely stiffened side plating
= 1.0 for local plate panels where an extreme load level is applied (e.g. impact pressures)
= 0.8 for local plate panels where a normal load level is applied
M_S = stillwater bending moment as given in Sec.5
M_W = wave bending moment as given in Sec.5
I_N = moment of inertia in cm⁴ of the hull girder.

For reduction of plate panels subject to elastic buckling, see 207.

M_S and M_W shall be taken as sagging or hogging values for members above or below the neutral axis respectively.

For local plate panels with cut-outs, subject to local compression loads only, σ_a shall be taken as the nominal stress in panel without cut-outs.

An increase of the critical buckling strength may be necessary in plate panels subject to combined in-plane stresses, see 400 and 500.

(5) Buckling Strength of Stiffener by DNV Rule - Stiffener in Uni-axial Compression (2/2)

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

◆ Critical buckling stress σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when } \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_f : yield stress of material in [N/mm²]

'σ_{el}' is determined according to specific load.

◆ Ideal compressive buckling stress σ_{el} of stiffener in uni-axial compression[1)]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_W}\right)^2$$

▪ Derivation of the coefficient '3.8'

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K$,

$\frac{\pi^2}{12(1-\nu^2)} = 0.9038(\fallingdotseq 0.9)$

And substituting K=4(for simply supported plate), the coefficient is approximately equal to 3.8.

σ_{el} : ideal compressive buckling stress
σ_c : critical buckling stress
σ_s : minimum upper yield stress
t_w : web thickness, h_w : web height
E : modulus of elasticity
s : stiffener spacing (m)
ν : 0.3 (Poisson's ratio of steel)

◆ Ideal compressive buckling stress σ_{el} of stiffener in lateral buckling mode

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{A l^2}$$

▪ Derivation of the coefficient '0.001'

From Euler's formula $\sigma_{cr} = \frac{\pi^2 EI}{Al^2} \quad \frac{\pi^2 \; N/mm^2 \; cm^4}{cm^2 \; m^2}$,

$$\frac{\pi^2 \; N/mm^2 \; cm^4}{cm^2 \; m^2} = \frac{\pi^2 \; N/mm^2 \; (10mm)^4}{(10mm)^2 \, (1000mm)^2} \fallingdotseq 0.001 \; N/mm^2$$

◆ Thickness of flange

For flanges on angles and T-sections of longitudinals and other highly compressed stiffeners, the thickness shall not be less than

$$t_f = 0.1 b_f + t_k \quad (mm)$$

b_f = flange width in mm for angles, half the flange width for T-Section(m)
t_k = corrosion addition(DNV Rule : Pt.3 Ch.1 Sec.2 - Page15)

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(DNV Pt.3 Ch.1 Sec.13, B102, B103), 2011

102 Formulae are given for calculating the ideal compressive buckling stress σ_{el}. From this stress the critical buckling stress σ_c may be determined as follows:

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

103 Formulae are given for calculating the ideal shear buckling stress τ_{el}. From this stress the critical buckling stress τ_c may be determined as follows:

$$\tau_c = \tau_{el} \quad \text{when} \quad \tau_{el} < \frac{\tau_f}{2}$$
$$= \tau_f\left(1 - \frac{\tau_f}{4\tau_{el}}\right) \quad \text{when} \quad \tau_{el} > \frac{\tau_f}{2}$$

τ_f = yield stress in shear of material in N/mm²
$= \frac{\sigma_f}{\sqrt{3}}$.

Design Theories of Ship and Offshore Plant, Fall 2014, Myung-Il Roh

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(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (1/4)

◆ **Criteria for buckling strength**

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c : critical buckling stress in [N/mm^2]
σ_a : calculated actual compressive stress in [N/mm^2]
η : usage factor

Usage Factor (η)
η = 1.0: Deck, Single bottom & Side shell (longl. stiff)
η = 0.9: Bottom, Inner bottom & Side shell (trans. stiff)
η = 1.0: Extreme loads (Q = 10^{-8})
η = 0.8: Normal loads (Q = 10^{-4})

◆ **Critical buckling stress** σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when } \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_f : upper yield stress in [N/mm^2]

From Bryan's formula,

$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)}\cdot\left(\frac{t}{b}\right)^2\cdot K$$

when $\sigma_{el} < \frac{\sigma_f}{2}$, $\eta_p = 1$

$$\sigma_c = \eta_p \sigma_{el} \rightarrow \sigma_c = \sigma_{el}$$

when $\sigma_{el} \geq \frac{\sigma_f}{2}$, $\eta_p = \frac{\sigma_f}{\sigma_{el}}\left(1-\frac{\sigma_f}{4\sigma_{el}}\right)$

$$\sigma_c = \eta_p \sigma_{el} \rightarrow \sigma_c = \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right)$$

(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (2/4)

◆ **Criteria for buckling strength**

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c : critical buckling stress in [N/mm^2]
σ_a : calculated actual compressive stress in [N/mm^2]
η : usage factor

Usage Factor (η)
η = 1.0: Deck, Single bottom & Side shell (long stiff)
η = 0.9: Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0: Extreme loads (Q = 10^{-8})
η = 0.8: Normal loads (Q = 10^{-4})

◆ **Critical buckling stress** σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when } \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_{el} : ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_f : upper yield stress in [N/mm^2]

◆ **Calculated actual stress** σ_a

(Uni-axial compression)

- σ_a is calculated actual compressive stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,(N/mm^2)$$
$$= \text{minimum } 30\, f_1 \text{ N/mm}^2 \text{ at side}$$

(※ Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation.
And it is acting on the each plate as axial load from the point view of local deformation.)

M_S : still water bending moment as given in Sec.5
M_W : wave bending moment as given in Sec.5
I_N : moment of inertia in cm^4 of the hull girder

(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (3/4)

◆ **Critical buckling stress** σ_c

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

$$\sigma_c = \sigma_{el} \quad ,\text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

σ_f : minimum upper yield stress of material in [N/mm²]

'σ_{el}' is determined according to specific load.

◆ **Ideal compressive buckling stress** σ_{el} **in uni-axial compression**[1)]

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

▪ **Derivation of the coefficient '0.9'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K,$

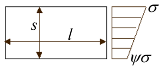
$$\frac{\pi^2}{12(1-\nu^2)} = \frac{3.141593^2}{12(1-0.3^2)} = 0.9038 \ (\fallingdotseq 0.9)$$

σ_{el} : ideal compressive buckling stress
σ_c : critical buckling stress
σ_f : upper yield stress in N/mm²
t : thickness (mm)
t_k : corrosion addition
E : modulus of elasticity
s : stiffener spacing (m)
ν : 0.3 (Poisson's ratio of steel)

◆ **factor** k

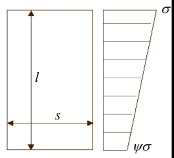
- For plating with longitudinal stiffeners (in direction of compression stress) :

$$k = k_l = \frac{8.4}{\psi + 1.1}$$



- For plating with transverse stiffeners (perpendicular to compression stress) :

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1}$$



ψ = ratio between the smaller and the larger compressive stress (positive value) $(0 \le \psi \le 1)$
c =1.21 when stiffeners are angles or T sections
=1.10 when stiffeners are bulb flats
=1.05 when stiffeners are flat bars
=1.30 when plating is supported by deep girders

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(6) Buckling Strength of Plate by DNV Rule - Plate Panel in Uni-axial Compression (4/4)

◆ **Critical buckling stress** σ_c

[1] Rules for classification of ships, Det Norske Veritas, Pt.3 Ch.1 Sec.13, pp.92-93, January 2004

$$\sigma_c = \sigma_{el} \quad ,\text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

σ_f : minimum upper yield stress of material in [N/mm²]

'σ_{el}' is determined according to specific load.

◆ **Ideal compressive buckling stress** σ_{el} **in uni-axial compression**[1)]

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

▪ **Derivation of the coefficient '0.9'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K,$

$$\frac{\pi^2}{12(1-\nu^2)} = \frac{3.141593^2}{12(1-0.3^2)} = 0.9038 \ (\fallingdotseq 0.9)$$

σ_{el} = ideal compressive buckling stress
σ_c = critical buckling stress
σ_f = upper yield stress in N/mm²
t = thickness (mm)
t_k =corrosion addition
E = modulus of elasticity
s = stiffener spacing (m)
ν = 0.3 (Poisson's ratio of steel)

◆ **factor** k

- For plating with longitudinal stiffeners (in direction of compression stress) : $k = k_l = \frac{8.4}{\psi + 1.1}$
- For plating with transverse stiffeners (perpendicular to compression stress) : $k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1}$

Example) If $\psi = 1.0, c = 1.05, s/l = 1/10$

$$k = k_l = \frac{8.4}{1.0 + 1.1} = 4$$

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} = 1.05\left[1 + \left(\frac{1}{10}\right)^2\right]^2 \frac{2.1}{1.0 + 1.1} = 1.071$$

Thus, the plate with longitudinal stiffeners can endure much stress than the plate with transverse stiffeners

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(DNV Pt.3 Ch.1 Sec.13, B201), 2011

201 The ideal elastic buckling stress may be taken as:

$$\sigma_{el} = 0.9\,k\,E\left(\frac{t - t_k}{1000 s}\right)^2 \quad (N/mm^2)$$

For plating with longitudinal stiffeners (in direction of compression stress):

$$k = k_l = \frac{8.4}{\psi + 1.1} \quad \text{for } (0 \le \psi \le 1)$$

For plating with transverse stiffeners (perpendicular to compression stress):

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} \quad \text{for } (0 \le \psi \le 1)$$

c = 1.21 when stiffeners are angles or T-sections
= 1.10 when stiffeners are bulb flats
= 1.05 when stiffeners are flat bars
c = 1.3 when the plating is supported by floors or deep girders.

For longitudinal stiffened double bottom panels and longitudinal stiffened double side panels the c-values may be multiplied by 1.1.

ψ is the ratio between the smaller and the larger compressive stress assuming linear variation, see Fig.1.

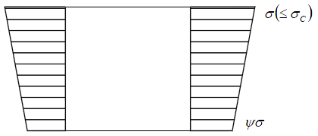

Fig. 1
Buckling stress correction factor

The above correction factors are not valid for negative ψ-values.
The critical buckling stress is found from 102.

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