Diversity

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Introduction (1)

- The idea behind diversity is to send the same data over independent fading paths
- Macro-diversity
  - Diversity to mitigate the effects of shadowing
  - is generally implemented by combining signals received by several base stations or access points
  - requires coordination among the different base stations, which is implemented as a part of networking protocols in infrastructure-based wireless networks

Introduction (2)

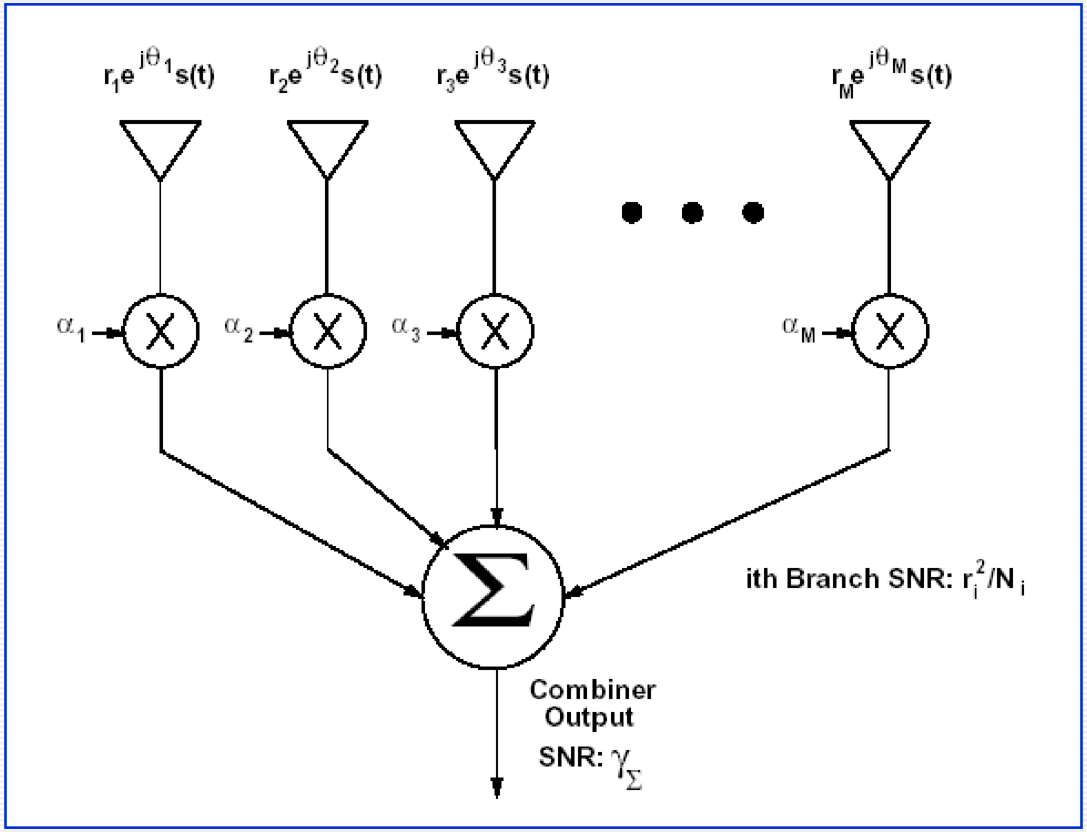
- Micro-diversity
  - Diversity techniques that mitigate the effect of multipath fading
  - Space diversity: by using multiple transmit or receive antennas
  - Angle (or directional) diversity: with smart antennas which are antenna array with adjustable phase at each antenna element
  - Frequency diversity: by transmitting the same narrowband signal at different carrier frequencies
  - Path diversity: spread spectrum with RAKE receiver
  - Time diversity: by transmitting the same date at different time (coding or interleaving)

Scope of This Chapter

- We focus on space diversity
- Receiver Diversity
  - Combining Techniques
    - Selection Combining
    - Threshold Combining
    - Maximal Ratio Combining
    - Equal Gain Combining
- Transmitter Diversity
  - Channel known at transmitter
  - Channel unknown at transmitter
    - Space Time Transmit Diversity (STTD)

Receiver Diversity

System model for Receiver Diversity (1)



- Co-phasing:

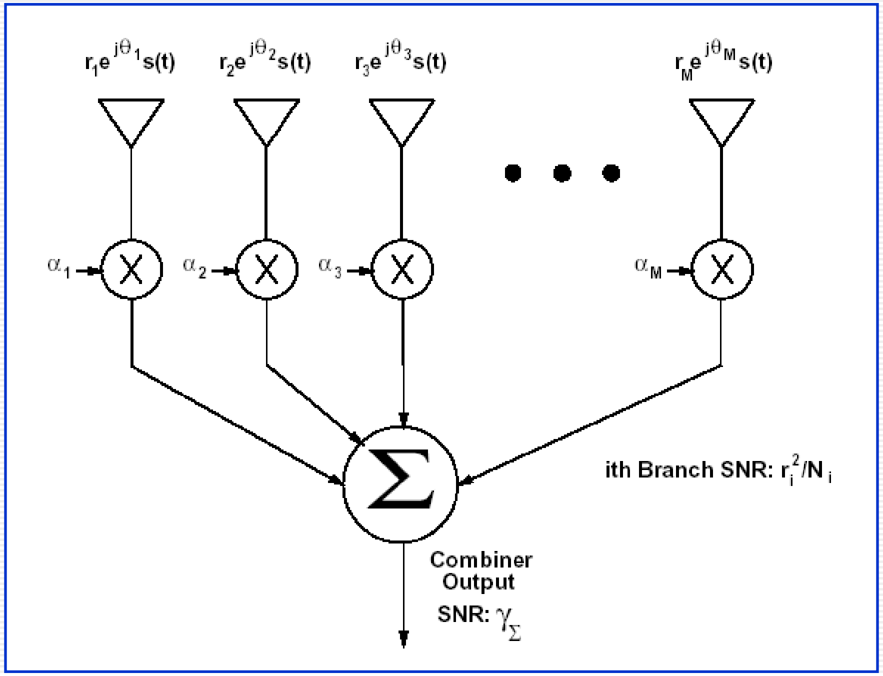
  Removal of phase through multiplication by $\alpha_i = a_i e^{-j\theta_i}$

- $$\gamma_\Sigma = \frac{\left(\sum_{i=1}^{M} a_i r_i \sqrt{E_s}\right)^2 \times B}{\sum_{i=1}^{M} a_i^2 N_0 B}$$

- Identical noise PSD $N_0/2$ on each branch and pulse shaping such that $BT_s=1$

System model for Receiver Diversity (2)

- Example (no fading)
  - $r_i = 1$
  - $a_i = r_i / \sqrt{N_0} = 1 / \sqrt{N_0}$



$$\gamma_\Sigma = \frac{\left(\sum_{i=1}^{M} a_i r_i \sqrt{E_s}\right)^2 \times B}{\sum_{i=1}^{M} a_i^2 N_0 B} = \frac{\left(\sum_{i=1}^{M} \frac{\sqrt{E_s}}{\sqrt{N_0}}\right)^2}{N_0 \sum_{i=1}^{M} \frac{1}{N_0}} = \frac{M E_s}{N_0}$$

Diversity Gain

- With fading, the combining of multiple independent fading path leads to a more favorable distribution for γ_Σ
- Performance of a diversity system
  - Average symbol error probability
    - $\overline{P}_s = \int_0^\infty P_s(\gamma) p_{\gamma_\Sigma}(\gamma) d\gamma$

    where $P_s(\gamma)$ is a symbol error probability in AWGN channel with SNR γ
  - Outage probability
    - $P_{out} = p(\gamma_\Sigma \le \gamma_0) = \int_0^{\gamma_0} p_{\gamma_\Sigma}(\gamma) d\gamma$
- Diversity Gain
  - Performance advantage in $\overline{P}_s$ and P_{out} as a result of diversity combining

Selection Combining (1)

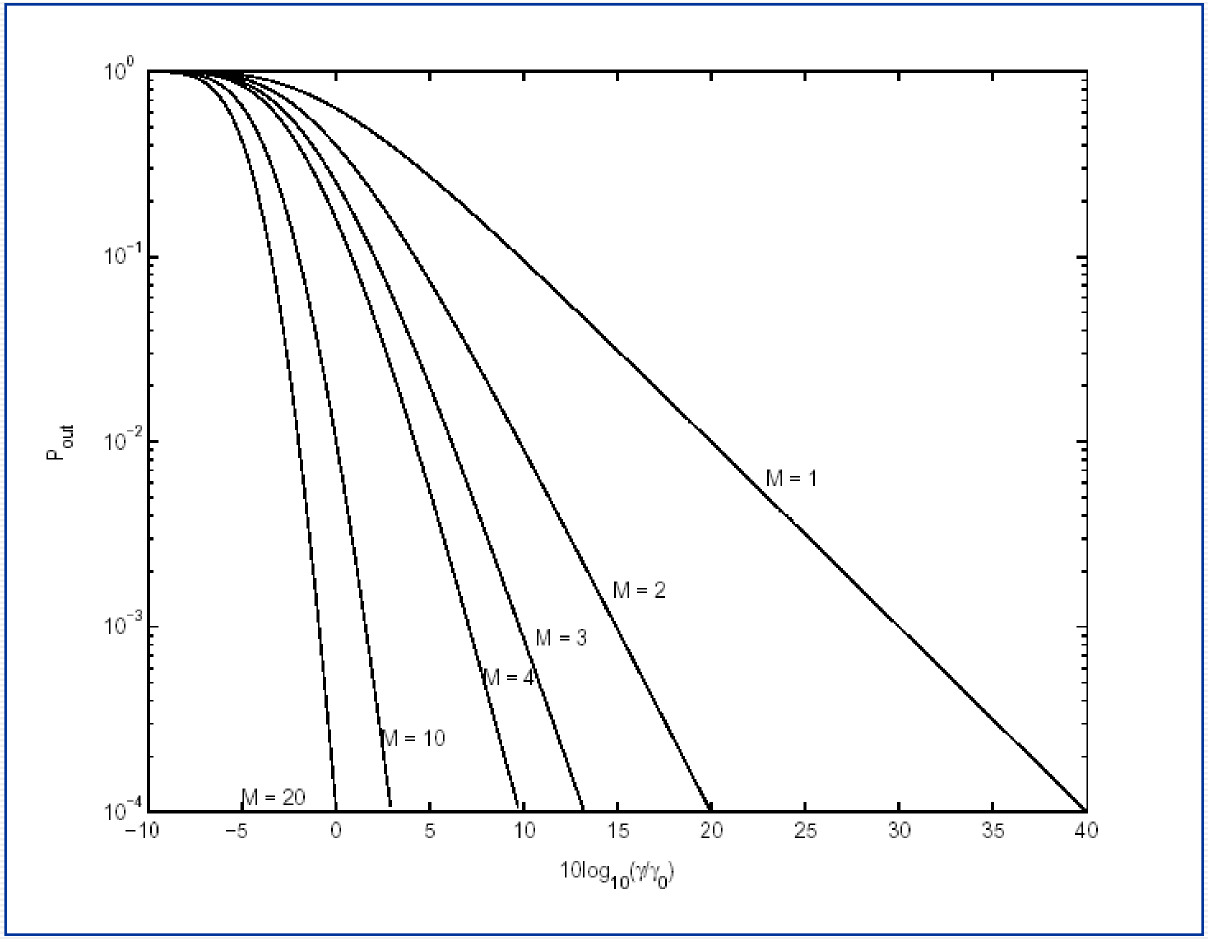
- The combiner outputs the signal on the branch with the highest SNR
- Cumulative distribution function (cdf) of γ_Σ
  - $P_{\gamma_\Sigma}(\gamma) = p(\gamma_\Sigma < \gamma) = P(\max[\gamma_1, \gamma_2, \ldots, \gamma_M] < \gamma) = \prod_{i=1}^{M} p(\gamma_i < \gamma)$
- For *M*-branch diversity with uncorrelated Rayleigh fading amplitude,
  - On *i*th branch: $p(\gamma_i) = \frac{1}{\bar{\gamma}_i} e^{-\gamma_i/\bar{\gamma}_i}$, $P_{out}(\gamma_0) = 1 - e^{-\gamma_0/\bar{\gamma}_i}$
  - Outage probability of the selection combiner for target γ_0
    - $P_{out}(\gamma_0) = p(\gamma_\Sigma < \gamma_0) = \prod_{i=1}^{M} (1 - e^{-\gamma_0/\bar{\gamma}_i}) = [1 - e^{-\gamma_0/\bar{\gamma}}]^M$
  - pdf of γ_Σ: differentiating $P_{out}(\gamma_0)$ relative to γ_0
    - $p_{\gamma_\Sigma}(\gamma) = \frac{M}{\bar{\gamma}} [1 - e^{-\gamma/\bar{\gamma}}]^{M-1} e^{-\gamma/\bar{\gamma}}$
  - Average SNR of combiner output:
    - $\bar{\gamma}_\Sigma = \int_0^\infty \gamma\, p_{\gamma_\Sigma}(\gamma) d\gamma = \int_0^\infty \frac{\gamma M}{\bar{\gamma}} [1 - e^{-\gamma/\bar{\gamma}}]^{M-1} e^{-\gamma/\bar{\gamma}} d\gamma = \bar{\gamma} \sum_{i=1}^{M} \frac{1}{i}$

The average SNR for all branches are the same

Selection Combining (2)

Outage Probability in Rayleigh fading

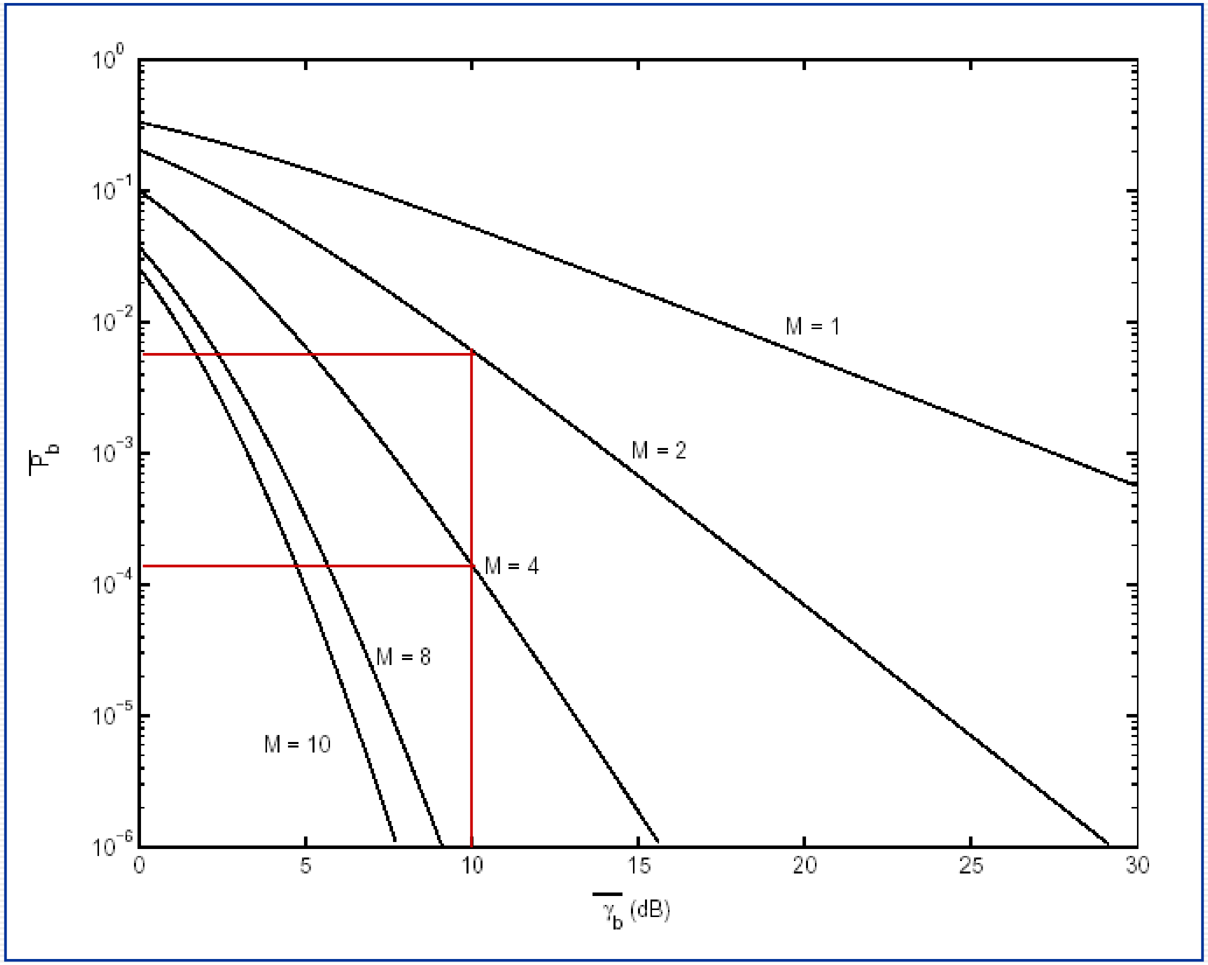
$$P_{out} = \left(1 - e^{-\gamma_0/\bar{\gamma}}\right)^M$$



Selection Combining (3)

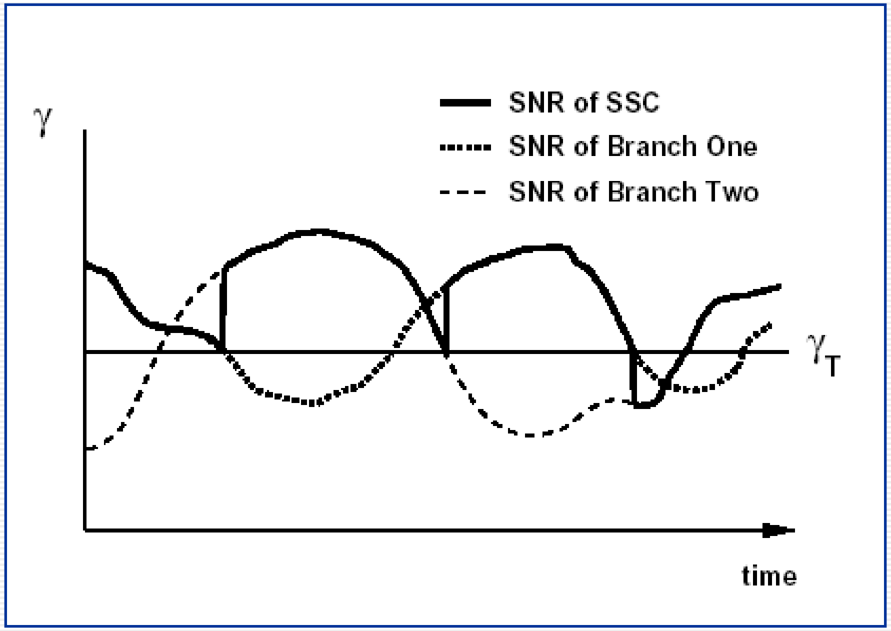
Average P_b of BPSK in Rayleigh fading

$$\int Q(\sqrt{2\gamma_b})p_{\gamma_\Sigma}(\gamma_b)d\gamma$$



Threshold Combining (1)

- The combiner scans each branch in sequential order and outputs the first signal whose SNR is above a given threshold γ_T
- Co-phasing is not required because only one branch output is used at a time
- Switch-and-stay combining (SSC)
  - Once a branch is chosen, the combiner outputs that signal as long as the SNR on that branch remains the desired threshold.



two branches

Threshold Combining (2)

- Cdf of γ_Σ, the SNR of the combiner output with two branches:

$$P_{\gamma_\Sigma}(\gamma) = \begin{cases} P_{\gamma_1}(\gamma_T)P_{\gamma_2}(\gamma) & \gamma < \gamma_\mathrm{T}, \\ p(\gamma_T \le \gamma_1 \le \gamma) + P_{\gamma_1}(\gamma_T)P_{\gamma_2}(\gamma) & \gamma \ge \gamma_\mathrm{T} \end{cases}$$

- For Rayleigh fading of each branch with $\bar{\gamma}$
  -
$$P_{\gamma_\Sigma}(\gamma) = \begin{cases} 1 - e^{-\gamma_T/\bar{\gamma}} - e^{-\gamma/\bar{\gamma}} + e^{-(\gamma_T+\gamma)/\bar{\gamma}} & \gamma < \gamma_\mathrm{T}, \\ 1 - 2e^{-\gamma/\bar{\gamma}} + e^{-(\gamma_T+\gamma)/\bar{\gamma}} & \gamma \ge \gamma_\mathrm{T}. \end{cases}$$
  - Outage probability for a given γ_0 : $P_{out}(\gamma_0) = P_{\gamma_\Sigma}(\gamma_0)$
  - Probability density function
    -
$$P_{\gamma_\Sigma}(\gamma) = \begin{cases} (1 - e^{-\gamma_T/\bar{\gamma}})(1/\bar{\gamma})e^{-\gamma/\bar{\gamma}} & \gamma < \gamma_\mathrm{T} \\ (2 - e^{-\gamma_T/\bar{\gamma}})(1/\bar{\gamma})e^{-\gamma/\bar{\gamma}} & \gamma \ge \gamma_\mathrm{T} \end{cases}$$
  - Average symbol (bit) error probability for DPSK:
    -
$$\overline{P}_b = \int_0^\infty \frac{1}{2}e^{-\gamma} p_{\gamma_\Sigma}(\gamma)d\gamma = \frac{1}{2(1+\bar{\gamma})}(1 - e^{-\gamma_T/\bar{\gamma}} + e^{-\gamma_T}e^{-\gamma_T/\bar{\gamma}})$$

Average symbol error proba. for DPSK on AWGN

Maximal Ratio Combining (1)

- Combiner Output SNR
  - $$\gamma_\Sigma = \frac{r^2}{N_{tot}} = \frac{1}{N_0}\frac{\left(\sum_{i=1}^{M} a_i r_i \sqrt{E_s}\right)^2}{\sum_{i=1}^{M} a_i^2}$$
    - Envelope of combiner output: $r = \sum_{i=1}^{M} a_i r_i \sqrt{E_s}$
    - Total noise PSD: $N_{tot}/2 = \sum_{i=1}^{M} a_i^2 \, N_0/2$
  - $$\gamma_\Sigma = \frac{1}{N_0}\frac{\left(\sum_{i=1}^{M} a_i r_i \sqrt{E_s}\right)^2}{\sum_{i=1}^{M} a_i^2} \le \sum_{i=1}^{M} \frac{r_i^2 E_s}{N_0} = \sum_{i=1}^{M} \gamma_i \quad \text{since} \left(\sum_{i=1}^{M} a_i r_i\right)^2 \le \sum_{i=1}^{M} a_i^2 \sum_{i=1}^{M} r_i^2$$
- The goal is to choose the a_i to maximize γ_Σ
  - when $a_i^2 = r_i^2 / N_0$
  - $$\gamma_\Sigma = \frac{1}{N_0}\frac{\left(\sum_{i=1}^{M} a_i r_i \sqrt{E_s}\right)^2}{\sum_{i=1}^{M} a_i^2} = \sum_{i=1}^{M} \frac{r_i^2 E_s}{N_0} = \sum_{i=1}^{M} \gamma_i$$

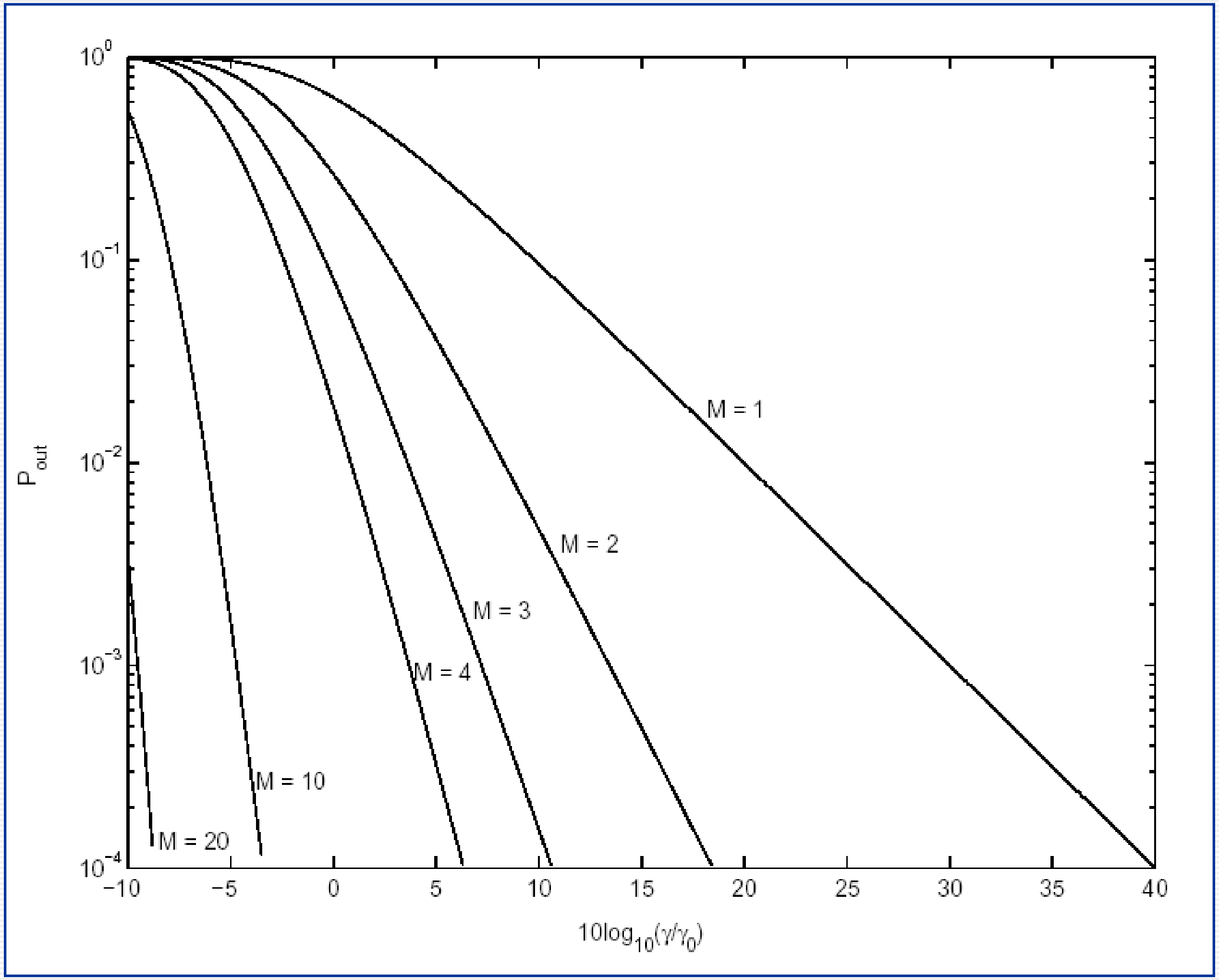
Maximal Ratio Combining (2)

- Distribution of γ_Σ
  - Assume i.i.d Rayleigh fading on each branch with the same average SNR $\bar{\gamma}$
  - pdf of γ_Σ : M-stage Erlang distribution with mean $M\bar{\gamma}$ and variance $M\bar{\gamma}^2$
    - $$p_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}^M (M-1)!} \qquad \gamma \geq 0$$
  - Outage probability for a given γ_0
    - $$P_{out} = p(\gamma_\Sigma < \gamma_0) = \int_0^{\gamma_0} p_{\gamma_\Sigma}(\gamma) d\gamma = 1 - e^{-\gamma/\bar{\gamma}} \sum_{k=1}^{M} \frac{(\gamma_0/\bar{\gamma})^{k-1}}{(k-1)!}$$
  - Average symbol (bit) error probability for BPSK modulation
    - $$\overline{P}_b = \int_0^{\infty} Q(\sqrt{2\gamma}) p_{\gamma_\Sigma}(\gamma) d\gamma = \left(\frac{1-\Gamma}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+\Gamma}{2}\right)^m$$

      where $\Gamma = \sqrt{\bar{\gamma}/(1+\bar{\gamma})}$

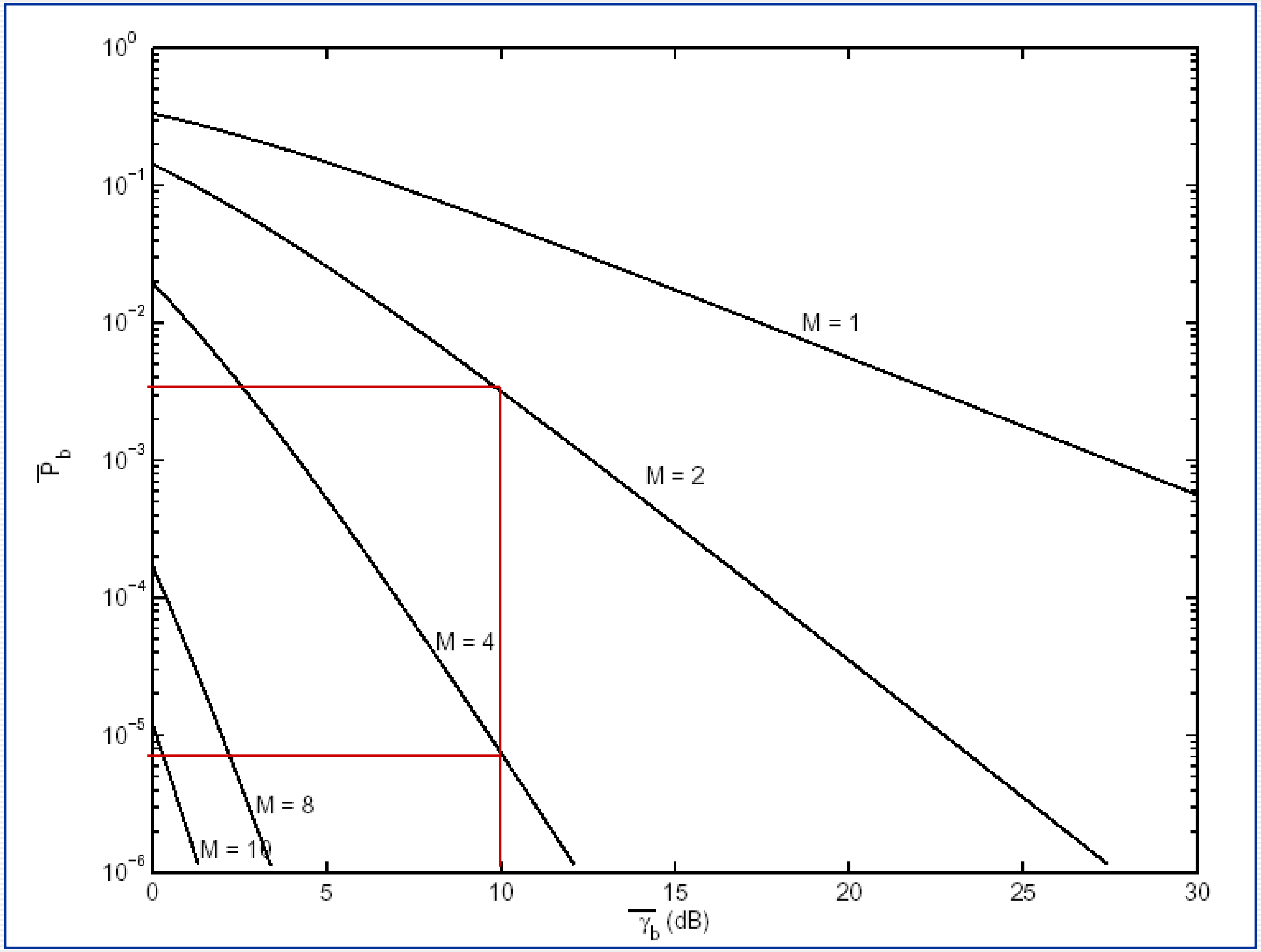
Maximal Ratio Combining (3)

Outage Probability in Rayleigh fading



Maximal Ratio Combining (4)

Average P_b
of BPSK in
Rayleigh fading



Equal Gain Combining

- Simple technique which co-phases the signal on each branch and then combines them with equal weighting, $\alpha_i = e^{-j\theta_i}$
- Combiner output SNR γ_Σ, assuming the same noise PSD $N_0/2$ in each branch
  - $\gamma_\Sigma = \frac{1}{N_0 M}\left(\sum_{i=1}^{M} r_i \sqrt{E_s}\right)^2$
- For i.i.d. Rayleigh fading with two branches having average branch SNR $\bar{\gamma}$
  - Cdf of γ_Σ: $P_{\gamma_\Sigma}(\gamma) = 1 - e^{-2\gamma/\bar{\gamma}} - \sqrt{\pi\gamma/\bar{\gamma}}\, e^{-\gamma/\bar{\gamma}}\left\{1 - 2Q\left(\sqrt{2\gamma/\bar{\gamma}}\right)\right\}$
  - Outage Probability: $P_{out} = P_{\gamma_\Sigma}(\gamma_0)$
  - Pdf of γ_Σ: $p_{\gamma_\Sigma}(\gamma) = \frac{1}{\bar{\gamma}} e^{-2\gamma/\bar{\gamma}} - \sqrt{\pi}\, e^{-\gamma/\bar{\gamma}}\left(\frac{1}{\sqrt{4\gamma\bar{\gamma}}} - \frac{1}{\bar{\gamma}}\sqrt{\frac{\gamma}{\bar{\gamma}}}\right)\left(1 - 2Q\left(\sqrt{\frac{2\gamma}{\bar{\gamma}}}\right)\right)$
  - Average bit error rate for BPSK

$$\overline{P}_b = \int_0^\infty Q(\sqrt{2\gamma})\, p_{\gamma_\Sigma}(\gamma)\, d\gamma = 0.5\left(1 - \sqrt{1 - \left(1 + \bar{\gamma}\right)^{-2}}\right)$$

Transmit Diversity

Channel Known at Transmitter

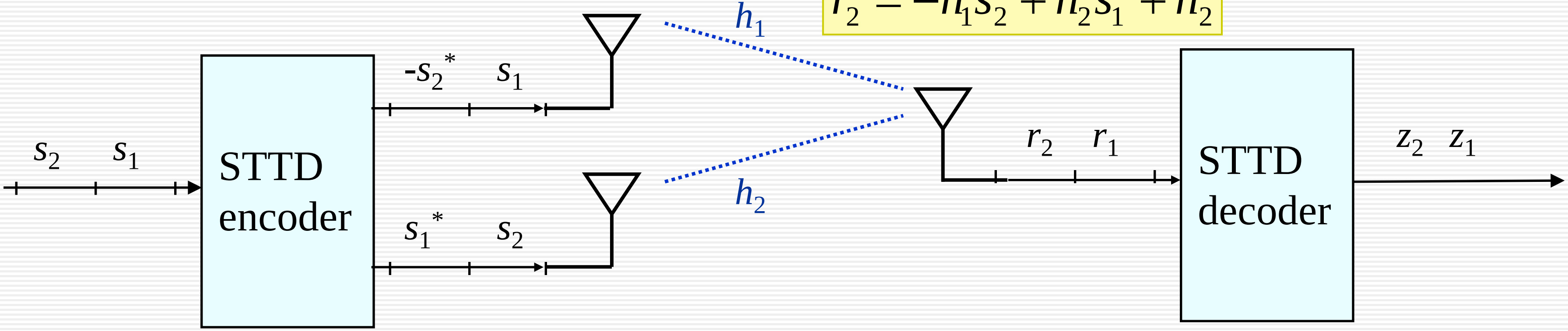
- A transmit diversity system with M transmit antennas and one receive antenna is considered
- We assume that the path gain $r_i e^{j\theta_i}$ of the ith antenna is known at transmitter.
- The signal is multiplied by $\alpha_i = a_i e^{-j\theta_i}$ and then sent through the ith antenna.
- Because the symbol energy E_s in the transmitted signal $s(t)$ is a constant, $\sum_{i=1}^{M} a_i^2 = 1$
- Received signal: $r(t) = \sum_{i=1}^{M} a_i r_i s(t)$
- The weights a_i to achieve the maximum SNR: $a_i = \dfrac{r_i}{\sqrt{\sum_{i=1}^{M} r_i^2}}$
- The resulting SNR: $\gamma_\Sigma = \dfrac{E_s}{N_0} \sum_{i=1}^{M} r_i^2 = \sum_{i=1}^{M} \gamma_i$
  - When the channel gains are known at transmitter, the transmit diversity is similar to the receiver diversity with MRC
  - If all antennas has the same gain $r_i = r$, $\gamma_\Sigma = Mr^2 E_s / N_0$
  - There is an array gain of M corresponding to an M-fold increase in SNR over a single antenna transmitting with full power

Channel Unknown at Transmitter-Alamouti Scheme

- The transmitter no longer knows the channel gain
  - If the transmit energy is divided equally among antenna, no performance advantage is obtained
- Alamouti Scheme
  - This scheme is designed for a digital communication system with two antennas
  - The scheme to combine both space TD)

$$r_1 = h_1 s_1 + h_2 s_2 + n_1$$

$$r_2 = -h_1 s_2^* + h_2 s_1^* + n_2$$



$$z_1 = h_1^* r_1 + h_2 r_2^* = (|h_1|^2 + |h_2|^2) s_1 + h_1^* n_1 + h_2 n_2^*$$

$$z_2 = h_2^* r_1 - h_1 r_2^* = (|h_1|^2 + |h_2|^2) s_2 + h_2^* n_1 - h_1 n_2^*$$

STTD-Alamouti Scheme

- Channel estimation with known data (x_1, x_2)

$$\hat{h}_1 = r_1 x_1^* - r_2 x_2 = (|x_1|^2 + |x_2|^2) h_1 + n_1 x_1^* - n_2 x_2$$

$$\hat{h}_2 = r_1 x_2^* - r_2 x_1 = (|x_1|^2 + |x_2|^2) h_2 + n_1 x_2^* - n_2 x_1$$

- Diversity gain of 2

$$z_1 = (|h_1|^2 + |h_2|^2) s_1 + \tilde{n}_1$$

$$z_2 = (|h_1|^2 + |h_2|^2) s_2 + \tilde{n}_2$$

- Array gain of 1
  - The symbols s_1 and s_2 are transmitted simultaneously with energy $E_s/2$.
  - The received SNR for z_i

$$\gamma_i = \frac{\left(|h_1|^2 + |h_2|^2\right)^2}{2} \times \frac{E_s}{N_0}$$